ELECTRON-ION COLLIDER KINEMATICS

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MASTER OF SCIENCE

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by
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ABSTRACT

The study of nuclear matter and its emergence from the partonic structure of quarks and gluons represents a fundamental challenge in modern nuclear and particle physics. The Electron-Ion Collider (EIC), a next-generation facility expected to begin construction in 2025 with an estimated cost of nearly 3 billion dollars, is designed to address this challenge by enabling high-precision electron-proton (ep) and electron-ion (eA) collisions. Supported by the U.S. Department of Energy and endorsed by the National Academies of Sciences, the EIC will build upon the existing RHIC infrastructure and utilize advanced accelerator technologies to achieve high luminosity and broad kinematic coverage. Through these collisions, the EIC will provide tomographic imaging of quarks and gluons within nucleons and nuclei, probing phenomena such as gluon saturation and seeking answers to key questions about the role of gluons in nucleon mass, spin, and high-density systems. At the heart of this effort is the accurate reconstruction of deep inelastic scattering (DIS) kinematic variables—namely the Bjorken scaling variable x, inelasticity y, and squared four-momentum transfer Q^2 from measurable detector observables. This research project focuses on developing and evaluating several methods for reconstructing these kinematic variables, ultimately contributing to improved resolution and precision, which are essential for maximizing the scientific potential of the EIC.

This thesis is dedicated to
my family, whose love and support
made this journey possible.

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CHAPTER 1

INTRODUCTION

1.1 Electron Ion Collider

The Electron-Ion Collider (EIC) is a next-generation facility aimed at exploring the fundamental structure of matter by examining the interactions of quarks and gluons, which are the building blocks of protons and neutrons. Decades of research in nuclear and particle physics have culminated in the establishment of Quantum Chromo-dynamics, (QCD), which is the theory that describes the strong force binding quarks together through gluons. Experiments conducted at HERA, RHIC, and the LHC have highlighted the prevailing role of gluons in nuclear matter, yet several key questions remain regarding their contributions to nucleon mass, spin, and dense systems. The EIC will facilitate high-energy, polarized electron-proton and electron-ion collisions, allowing for precise tomographic imaging of quarks and gluons and probing the yet-unobserved saturation of gluon densities. It will build upon the existing infrastructure at RHIC while incorporating advanced accelerator technologies to achieve high luminosity and extensive kinematic coverage. Supported by the U.S. Department of Energy and endorsed by the National Academies of Sciences, the EIC marks a significant advancement in nuclear physics, promising to provide groundbreaking insights into the fundamental properties of matter. The following figure shows the components of the Electron-Ion Collider; 1 The new Electron-Ion Collider (EIC) will incorporate key components, including an electron accelerator (red) and an electron storage ring (blue), all within the existing tunnel that currently houses the Relativistic Heavy Ion Collider (RHIC, yellow). The EIC will repurposed

one of RHIC's two ion storage rings to facilitate electron-ion collisions, which will be analyzed using one or more advanced detectors.

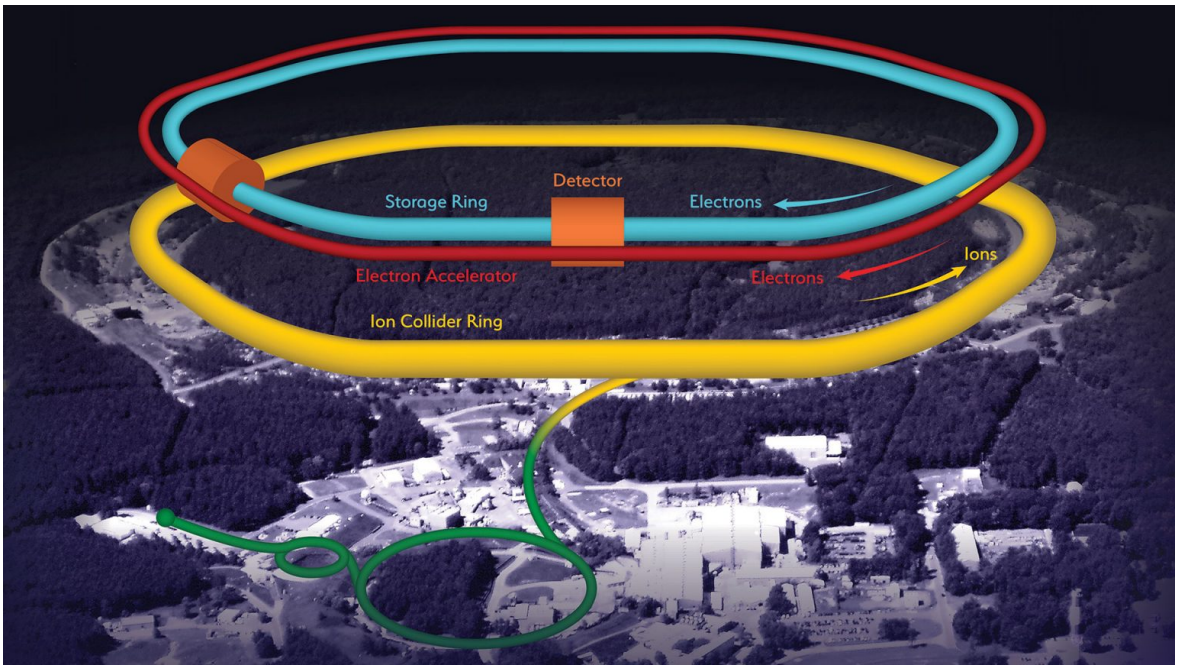

Figure 1. The new Electron-Ion Collider (EIC). [1]

1.2 Deep Inelastic Scattering

Deep inelastic scattering (DIS) is a powerful technique used to investigate the internal structure of the proton. By employing high-energy electrons to scatter off protons, this method breaks them apart and uncovers their substructure. As the energy of the probing electron increases, the resolution improves, allowing scientists to initially identify an extended charge distribution, then individual quarks, and ultimately a dynamic system comprising quarks, anti-quarks, and gluons. The SLAC-MIT experiment, which confirmed the quark structure of the proton, was awarded the Nobel Prize in 1990. Subsequent experiments, such as HERA, provided even deeper insights into parton distributions. The observed scaling behavior and the increase in low-momentum partons at high energies can be explained through gluon radiation and splitting, as formalized in the DGLAP equations [**?**]. Parton Distribution Functions (PDFs) characterize the probability of finding a parton that carries a fraction of the proton's momentum, illustrating that gluons account for a substantial portion of the

total momentum of the proton. These findings are essential for calculating scattering cross-sections and for advancing the understanding of quantum chromodynamics (QCD), the theory governing strong interactions.

1.3 Relativistic Invariant Variables

Consider a standard model electroweak gauge boson exchange process in ep collisions [2]

$$e(k) + P(p) \rightarrow l(k^{'}) + X(p^{'})$$

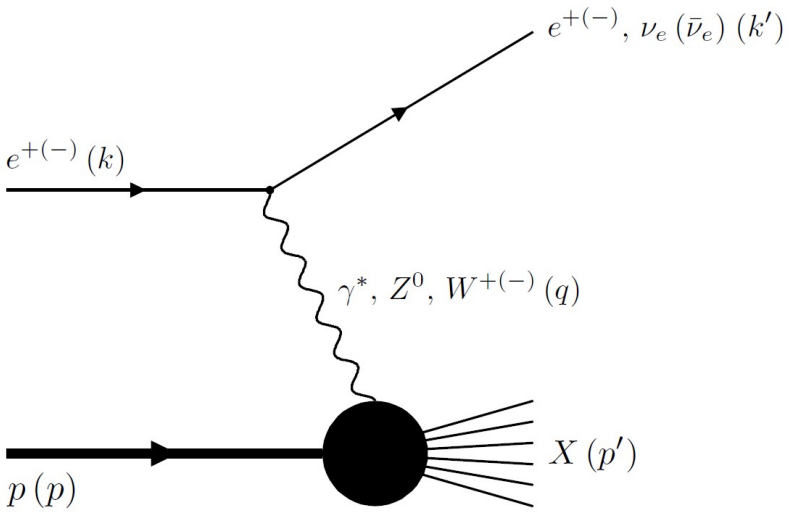

Figure 2. Feynman diagram of e-p collision.

Lorentz scalars,

$$s = (k + P)^2 = k^2 + P^2 + 2k.P = {m_e}^2 + {m_p}^2 + 4E_eE_p \approx 4E_eE_p$$

$$t = (P + P^{'})^2$$

$$u = (k^{'} - P)^2$$

The square of the momentum transfer;

$$Q^2 = -(k - k^{'})^2 = -(P - P^{'})^2 = -t = -q^2$$

Bjorken scaling variable;

$$x = \frac{Q^2}{2(P.q)} \approx -\frac{t}{u+s} \qquad 0 \leq x \leq 1$$

Inelasticity parameter;

$$y = \frac{P.q}{2(P.k)} \approx -\frac{u+s}{s} \qquad 0 \leq y \leq 1$$

Invariant mass W of the hadronic final state X;

$$W^2 = (P+q)^2 = (P^{'})^2 = {m_p}^2 + \frac{Q^2}{x}(1-x) \approx s + t + u$$

Consider a scattering of a lepton (e) with a parton resulting in a scattering lepton (e) (Energy $E^{'}$ and polar angle $\theta^{'}$) and a scattered parton (Energy F and polar angle γ) in the final state.

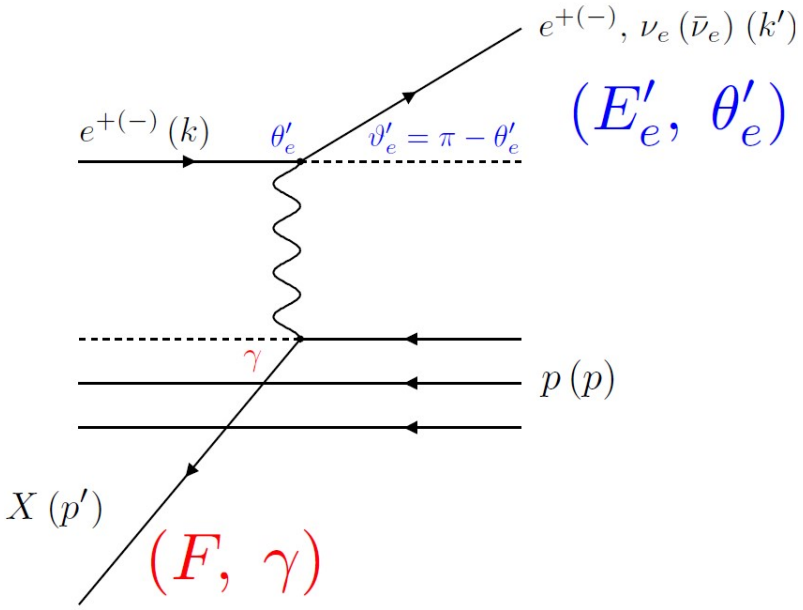

Figure 3. Feynman diagram of e-p collision with associated four-vector momenta.

Consider the four-momentum vectors,

$$k_e = \begin{pmatrix} E_e \\ 0 \\ 0 \\ -E_e \end{pmatrix} \quad k_e{}' = \begin{pmatrix} E_e{}' \\ E_e{}' \sin\theta_e' \cos\phi \\ E_e{}' \sin\theta_e' \sin\phi \\ E_e{}' \cos\theta_e' \end{pmatrix}$$

$$P = \begin{pmatrix} E_p \\ 0 \\ 0 \\ E_p \end{pmatrix} \qquad P' = \begin{pmatrix} F_e \\ F \sin\gamma \cos\beta \\ F \sin\gamma \sin\beta \\ F \cos\gamma \end{pmatrix}$$

since $E \gg m_0$ (rest mass) we can neglect the rest mass of leptons and hadrons;

$$P^2 = E^2 - m = E^2 \tag{1.1}$$

Using the conservation of 4-momentum; Energy conservation ;

$$E_p + E_e = E_e{}' + F \tag{1.2}$$

Longitudinal momentum conservation;

$$E_p - E_e = E_e{}' \cos\theta_e' + F \cos\gamma \tag{1.3}$$

Conservation of transverse momentum ;

$$E_e{}' \sin\theta_e' = F \sin\gamma \tag{1.4}$$

CHAPTER 2

REPORT

2.1 Reconstruction Methods of DIS variables Q^2, y and x

2.1.1 Introduction

Inclusive DIS processes necessitate the utilization of two independent variables, designated as y and Q^2. These variables can be established through a combination of the four measurable observable variables at our disposal. $E^{'}, \theta^{'}, F, \gamma$. In fixed target experiments, we rely solely on the combination of E' and $\theta^{'}$ to reconstruct deep inelastic scattering (DIS) kinematic variables. However, in a collider environment, the capacity to also reconstruct F and γ offers a way to optimize the reconstruction of kinematic variables using combinations that differ from E' and $\theta^{'}$.

In this chapter, we will discuss the construction of three DIS kinematic variables x, y and Q^2 using six combinations of two observable parameters mentioned above.

2.1.2 Reconstruction using observable variables $E_e^{'}$ and $\theta_e^{'}$

In this section, we are discussing the reconstruction of kinematic variables using the Energy ($E_e^{'}$) and the Polar angle ($\theta_e^{'}$) of the scattered electron.

Using the definition of momentum transfer,

$$
\begin{aligned}
Q^2 &= -q^2 \\
&= -(k_e - k_e{}')^2 \\
&= -(ke^2 + (k_e{}')^2 - 2k_e.k_e{}')
\end{aligned}
$$

Substituting the dot product of the four vector momentum of the incident and scattered electrons $k_e.k_e{}'$,

$$
\begin{aligned}
Q^2 &= -(m_e{}^2 + m_e{}^2 - 2(E_e E_e{}' + E_e E_e{}' \cos\theta e') \\
Q^2 &= 2E_e E_e{}'(1 + \cos\theta_e{}')
\end{aligned}
\tag{2.1}
$$

Using the definition of inelasticity parameter and substituting for $q = (k_e - k_e{}')$ using the four-vector momentum of incoming and scattered electrons.

$$
\begin{aligned}
y &= \frac{P.q}{P.k_e} \\
&= \frac{P.(k_e - k_e{}')}{P.k_e} \\
y &= 1 - \frac{P.ke'}{P.ke}
\end{aligned}
$$

Substituting the dot product of four vector momentum between the initial parton state and the incident lepton $(p.ke')$ and the initial parton state and the scattering lepton $(p.ke)$.

$$
\begin{aligned}
y &= 1 - \frac{E_p E_e{}'(1 - \cos\theta_e{}')}{2E_p E_e} \\
&= 1 - \frac{E_e{}'(1 - \cos\theta_e{}')}{2E_e}
\end{aligned}
\tag{2.2}
$$

Using the definition of the Bjorken scaling variable and substituting for Q^2 from equation2.1;

$$
\begin{aligned}
x &= \frac{Q^2}{2P.q} \\
&= \frac{Q^2}{sy} \\
&= \frac{2E_e E_e{}'(1+\cos\theta_e{}')}{4E_e E_p\left(1-\frac{E_e{}'(1-\cos\theta_e{}')}{2E_e}\right)} \\
x &= \frac{2E_e{}'(1+\cos\theta_e{}')}{4E_p\left(1-\frac{E_e{}'(1-\cos\theta_e{}')}{2E_e}\right)}
\end{aligned} \tag{2.3}
$$

$$
s = (k_e + P)^2 = 2P.k_e = 4E_e E_p \tag{2.4}
$$

2.1.3 Reconstruction using observable variables F and γ

In this section we are discussing the reconstruction of kinematic variables using the energy (F) and the polar angle (γ) of the scattered parton.

Taking the difference between 1.2 - 1.3 we have;

$$
\begin{aligned}
2E_e &= E_e{}'(1-\cos\theta_e{}') + F(1-\cos\gamma) \\
1 &= \frac{E_e{}'(1-\cos\theta_e{}')}{2E_e} + \frac{F(1-\cos\gamma)}{2E_e} \\
1 - \frac{E_e{}'(1-\cos\theta_e{}')}{2E_e} &= \frac{F(1-\cos\gamma)}{2E_e}
\end{aligned} \tag{2.5}
$$

To find the Inelasticity parameter y, substitute the result in 2.5 in the result for y (2.2)in the first method;

$$
\begin{aligned}
y &= \frac{E_e{}'(1-\cos\theta_e{}')}{2E_e} \\
&= \frac{F(1-\cos\gamma)}{2E_e} \\
y &= \frac{F(1-\cos\gamma)}{2E_e}
\end{aligned} \tag{2.6}
$$

From equation 2.1 substitute for E_e using equation 2.5,

$$\begin{aligned} Q^2 &= 2E_e E_e^{'}(1+\cos\theta_e^{'}) \\ &= \frac{(E_e^{'})^2(1+\cos\theta_e^{'})(1-\cos\theta_e^{'})}{1-y} \\ &= \frac{(E_e^{'})^2(1-\cos^2\theta_e^{'})}{1-y} \\ &= \frac{(E_e^{'})^2(\sin^2\theta_e^{'})}{1-y} \end{aligned}$$

From 1.4 ,

$$Q^2 = \frac{F^2\sin^2\gamma}{1-y} \tag{2.7}$$

Using the definition of the Bjorken scaling variable and substituting for Q^2 from equation2.7,

$$\begin{aligned} x &= \frac{Q^2}{2P.q} \\ &= \frac{Q^2}{sy} \\ &= \frac{F^2\sin^2\gamma}{4E_eE_p(1-y)y} \end{aligned} \tag{2.8}$$

2.1.4 Reconstruction using observable variables γ and $\theta_e^{'}$

In this section, we discuss the reconstruction of kinematic variables using the polar angle ($\theta_e^{'}$) of the scattered electron and the polar angle of the scatted Parton (γ).

Taking the difference between equation 1.2 and 1.3 we have;

$$2E_e = E_e^{'}(1-\cos\theta_e^{'}) + F(1-\cos\gamma) \tag{2.9}$$

Using the above result in equation 1.4 and simplify further to get an expression for $E_e^{'}$,

$$
\begin{aligned}
2E_e - E_e{}'(1-\cos\theta_e{}') &= \frac{E_e{}'\sin\theta_e{}'}{\sin\gamma}(1-\cos\gamma) \\
E_e{}' &= \frac{2E_e\sin\gamma}{(1-\cos\theta_e{}')\sin\gamma + (1-\cos\gamma)\sin\theta_e{}'} \\
&= \frac{2E_e\sin\gamma}{\sin\gamma + \sin\theta_e{}` - (\sin\theta_e{}'\cos\gamma + \cos\theta_e{}'\sin\gamma)} \\
&= \frac{2E_e\sin\gamma}{\sin\gamma + \sin\theta_e{}' - \sin(\theta_e{}' + \gamma)}
\end{aligned}
$$

Substitute for $E_e{}'$ in 2.1;

$$
\begin{aligned}
Q^2 &= 2E_eE_e{}'(1+\cos\theta_e{}') \\
&= 2E_e(1+\cos\theta_e{}')\left(\frac{2E_e\sin\gamma}{\sin\gamma + \sin\theta_e{}` - \sin(\theta_e{}' + \gamma)}\right) \\
&= \frac{4E_e{}^2(1+\cos\theta_e{}')\sin\gamma}{\sin\gamma + \sin\theta_e{}' - \sin(\theta_e{}' + \gamma)}
\end{aligned}
\tag{2.10}
$$

To obtain the inelasticity parameter y, substitute for $E_e{}'$ in equation 2.2;

$$
\begin{aligned}
y &= 1 - \frac{(1-\cos\theta'_e)}{2E_e}\cdot\frac{2E_e^2\sin\gamma}{\sin\gamma + \sin\theta'_e - \sin(\theta'_e+\gamma)} \\
&= \frac{\sin\theta'_e - \cos\gamma\sin\theta'_e}{\sin\gamma + \sin\theta'_e - \sin(\theta'_e+\gamma)} \\
&= \frac{\sin\theta'_e(1-\cos\gamma)}{\sin\gamma + \sin\theta'_e - \sin(\theta'_e+\gamma)}
\end{aligned}
\tag{2.11}
$$

Using the definition of Bjorken scaling variable and substituting for Q^2 from equation 2.10,

$$
\begin{aligned}
x &= \frac{Q^2}{2P \cdot q} \\
&= \frac{Q^2}{sy} \\
&= \frac{\left(\frac{4E_e^2(1+\cos\theta_e')\sin\gamma}{\sin\gamma + \sin\theta_e' - \sin(\theta_e'+\gamma)}\right)}{4E_eE_p\left(\frac{\sin\theta_e'(1-\cos\gamma)}{\sin\gamma+\sin\theta_e'-\sin(\theta_e'+\gamma)}\right)} \\
&= \frac{E_e^2(1+\cos\theta_e')\sin\gamma}{E_p\sin\theta_e'(1-\cos\gamma)} \\
&= \frac{E_e \cdot 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}(2-2\sin^2\frac{\theta_e'}{2})}{E_p\cdot(2\sin^2\frac{\gamma}{2})\cdot 2\sin\frac{\theta_e'}{2}\cos\frac{\theta_e'}{2}} \\
&= \frac{E_e\cos\frac{\gamma}{2}(1-\sin^2\frac{\theta_e'}{2})}{E_p\sin\frac{\gamma}{2}\sin\frac{\theta_e'}{2}\cos\frac{\theta_e'}{2}} \\
x &= \frac{E_e}{E_p}\cot\frac{\gamma}{2}\cot\frac{\theta_e'}{2}
\end{aligned}
\tag{2.12}
$$

2.1.5 Reconstruction using observable variables F and $\theta_e{}'$

In this section, we discuss the reconstruction of the kinematic variables using the polar angle ($\theta_e{}'$) of the scattered electron and the energy of the scattered parton (F).

From Equation 2.9,

$$
F\cos\gamma = E_e{}'(1-\cos\theta_e{}') + F - 2E_e \tag{2.13}
$$

Taking the sum of the squares of the equations of 1.4 and 2.13;

$$
\begin{aligned}
F^2\sin^2\gamma + F^2\cos^2\gamma &= E^2\sin^2\theta_e + (E_e'(1-\cos\theta_e') + F - 2E_e)^2 \\
F^2 &= E^2\sin^2\theta_e' + (E_e')^2(1-\cos\theta_e')^2 + F \\
&\quad - 4E_e^2 + 2E_e'F\cos\theta_e' - 4E_e'E_e\cos\theta_e' - 4FE_e \\
0 &= 2(E_e')^2\cos\theta_e' + E_e'(F-2E_e)(1-\cos\theta_e') + 2E_e^2 - 2E_eF
\end{aligned}
\tag{2.14}
$$

Now consider two cases for $\cos\theta_e{}'$ in Equation 2.14;
For $\cos\theta_e{}' = 1$

$$2E_e{}^2 - 2E_eF = 0$$
$$F = E_e$$

For $\cos\theta_e{}' \neq 1$

$$E'_e = \frac{-(F - 2E_e) \pm \sqrt{(2E_e - F)^2 - \frac{8E_e^2 - E_eF}{1-\cos\theta'_e}}}{2}$$

To obtain the inelasticity parameter y, substitute for $E_e{}'$ in equation 2.2,

$$y = 1 - \frac{1-\cos\theta_e{}'}{4E_e}\frac{(2E_e - F) \pm \sqrt{(2E_e - F)^2 - \frac{8E_e{}^2 - EeF}{1-\cos\theta_e{}'}}}{2} \tag{2.15}$$

To obtain Q, substitute for $E_e{}'$ in equation 2.1,

$$\begin{aligned} Q^2 &= 2E_e(1+\cos\theta_e{}')\frac{(2E_e - F) \pm \sqrt{(2E_e - F)^2 - \frac{8E_e{}^2 - EeF}{1-\cos\theta_e{}'}}}{2} \\ &= 4E_e{}^2(1-y)\frac{1+\cos\theta_e{}'}{1-\cos\theta_e{}'} \end{aligned} \tag{2.16}$$

Using the definition of the Bjorken scaling variable and substituting for Q^2 from equation 2.1;

$$\begin{aligned} x &= \frac{Q^2}{2P.q} \\ &= \frac{Q^2}{sy} \\ &= \frac{(4E_e{}^2(1-y)\frac{1+\cos\theta_e{}'}{1-\cos\theta_e{}'})^2}{4E_eE_py} \end{aligned} \tag{2.17}$$

2.1.6 Reconstruction using observable variables $E_e{}'$ and F

In this section, we discuss the reconstruction of kinematic variables using the energy of the scattered electron (E) and the energy of the scattered parton (F).

From equation 2.14 obtain an expression for $(1 - \cos\theta_e{}')$,

$$\begin{aligned} 0 &= 2E_e'^2 \cos\theta_e' + E_e'(F - 2E_e)(1 - \cos\theta_e') + 2E_e^2 - 2E_eF \\ 1 - \cos\theta_e' &= \frac{2E_eF - 2E_e^2}{E_e'^2 + E_e'(F - 2E_e)} \end{aligned} \tag{2.18}$$

To obtain the inelasticity parameter y, substitute the above result for $(1 - \cos\theta_e{}')$ in equation 2.2,

$$\begin{aligned} y &= 1 - \frac{E_e'(1 - \cos\theta_e')}{2E_e} \\ &= 1 - \frac{E_e'}{2E_e} \cdot \frac{2E_eF - 2E_e^2}{E_e'^2 + E_e'(F - 2E_e)} \\ &= \frac{F - E_e}{E_e' + F - 2E_e} \\ &= \frac{E_e' - E_e}{E_e' + F - 2E_e} \end{aligned} \tag{2.19}$$

To obtain Q^2 substitute for $(\cos\theta_e{}')$ from 2.18 to equation 2.1;

$$\begin{aligned} Q^2 &= 2E_eE_e'(1 + \cos\theta_e') \\ &= 2E_eE_e' \left(1 + 1 - \frac{2E_eF - 2E_e^2}{E_e'^2 + E_e'(F - 2E_e)}\right) \\ &= 4E_eE_e' + \frac{4E_e^2(E_e - F)}{E_e'^2 + E_e'(F - 2E_e)} \end{aligned} \tag{2.20}$$

Using the definition of the Bjorken scaling variable and substituting for Q^2 from equation 2.1;

$$
\begin{aligned}
x &= \frac{Q^2}{2P \cdot q} \\
&= \frac{Q^2}{sy} \\
&= \frac{\left(4E_eE'_e + \frac{4E_e^2(E_e-F)}{E_e'^2+E'_e(F-2E_e)}\right)}{4E_eE_p \cdot \frac{E'_e-E_e}{E'_e+F-2E_e}} \\
&= \frac{E'_e + F - E_e}{E_p}
\end{aligned}
\tag{2.21}
$$

2.1.7 Reconstruction using observable variables $E_e^{'}$ and γ

In this section, we discuss the reconstruction of kinematic variables using the energy of the scattered electron ($E_e^{'}$) and the polar angle of the scattered parton (γ).

From equation 2.18,

$$
\begin{aligned}
2E_e &= E_e^{`}(1 - \cos\theta_e^{`}) + F(1 - \cos\gamma) \\
2E_e - E_e^{`}(1 - \cos\theta_e^{`}) &= F(1 - \cos\gamma)
\end{aligned}
\tag{2.22}
$$

Now consider 1.4/2.22 and simplify,

$$
\begin{aligned}
\frac{E'_e \sin\theta'_e}{2E_e - E'_e(1 - \cos\theta'_e)} &= \frac{\sin\gamma}{1 - \cos\gamma} \\
\left(\frac{E'_e \sin\theta'_e}{2E_e - E'_e(1 - \cos\theta'_e)}\right)^2 &= \left(\frac{\sin\gamma}{1 - \cos\gamma}\right)^2 \\
\frac{E_e'^2(1 - \cos^2\theta'_e)}{(2E_e - E'_e(1 - \cos\theta'_e))^2} &= \frac{1 - \cos^2\gamma}{(1 - \cos\gamma)^2}
\end{aligned}
$$

Simplifying further and taking both sides of the expression is equal to μ,

$$
\frac{E_e'^2(1 - \cos^2\theta'_e)}{(2E_e - E'_e(1 - \cos\theta'_e))^2} = \frac{1 + \cos\gamma}{1 - \cos\gamma} = \mu
$$

Now Consider;

$$\mu = \frac{E_e'^2(1-\cos^2\theta_e')}{\left(2E_e - E_e'(1-\cos\theta_e')\right)^2}$$
$$E_e'^2(1-\cos^2\theta_e')^2 = \mu\left(2E_e - E_e'(1-\cos\theta_e')\right)^2$$
$$0 = E_e'^2(1-\cos^2\theta_e')^2(1+\mu) - 2\left(2\frac{E_e}{E_e'}\mu + 1\right)(1-\cos^2\theta_e')^2 + 4\frac{E_e^2}{E_e'^2}\mu$$

Finding the solutions for $(1-\cos^2\theta_e{}')$;

$$(1-\cos^2\theta_e') = \frac{\left(2\frac{E_e}{E_e'}\mu+1\right) \pm \sqrt{\left(2\frac{E_e}{E_e'}\mu+1\right)^2 - 4\frac{E_e^2}{E_e'^2}(1+\mu)\mu}}{1+\mu}$$
$$= 2\frac{E_e}{E_e'} + \frac{2\frac{E_e}{E_e'}}{1+\mu} \pm \frac{\sqrt{1+4\frac{E_e}{E_e'}\left(1-\frac{E_e}{E_e'}\right)\mu}}{1+\mu}$$

Now consider,

$$\frac{1+\cos\gamma}{1-\cos\gamma} = \mu$$
$$1+\mu = \frac{2}{1-\cos\gamma}$$

Substituting for μ and $\mu+1$;

$$1-\cos^2\theta_e{}' = 2\frac{E_e}{E_e{}'} + \frac{2\frac{E_e}{E_e{}'}}{\frac{2}{1-\cos\gamma}} \pm \frac{\sqrt{1+4\frac{E_e}{E_e{}'}(1-\frac{E_e}{E_e{}'})(\frac{\sin\gamma}{1-\cos\gamma})^2}}{\frac{2}{1-\cos\gamma}}$$
$$= 2\frac{E_e}{E_e{}'} + (1-2\frac{E_e}{E_e{}'})\frac{1-\cos\gamma}{2} \pm \sqrt{\frac{(1-\cos\gamma)^2}{4} + \frac{E_e}{E_e{}'}(1-\frac{E_e}{E_e{}'})\sin^2\gamma}$$

To obtain the inelasticity parameter y, substitute for $1-\cos^2\theta_e{}'$ in equation 2.2;

$$
\begin{aligned}
y &= 1 - \frac{E_e{}'}{2E_e}(2\frac{E_e}{E_e{}'} + (1 - 2\frac{E_e}{E_e{}'})\frac{1-\cos\gamma}{2} \pm \sqrt{\frac{(1-\cos\gamma)^2}{4} + \frac{E_e}{E_e{}'}(1 - \frac{E_e}{E_e{}'})\sin^2\gamma)} \\
&= \left(1 - \frac{E_e{}'}{2E_e}\right)\frac{1-\cos\gamma}{2} \pm \frac{E_e{}'}{2E_e}\sqrt{\frac{(1-\cos\gamma)^2}{4} + \frac{E_e}{E_e{}'}(1 - \frac{E_e}{E_e{}'})\sin^2\gamma)}
\end{aligned}
\tag{2.23}
$$

To obtain Q, substitute for $\cos\theta_e{}'$ in equation 2.1 from above,

$$
\begin{aligned}
Q^2 &= 2E_eE_e{}'(1 + \cos\theta_e{}') \\
&= 2E_eE_e{}'(1 + 1 - (2\frac{E_e}{E_e{}'} + (1 - 2\frac{E_e}{E_e{}'})\frac{1-\cos\gamma}{2} \pm \\
&\sqrt{\frac{(1-\cos\gamma)^2}{4} + \frac{E_e}{E_e{}'}(1 - \frac{E_e}{E_e{}'})\sin^2\gamma)}) \\
&= 4E_eE_e{}' + 4E_e{}'(y-1)
\end{aligned}
\tag{2.24}
$$

Using the definition of Bjorken scaling variable, and substituting for Q^2 from equation2.1;

$$
\begin{aligned}
x &= \frac{Q^2}{2P.q} \\
&= \frac{Q^2}{sy} \\
&= \frac{(4E_eE_e{}' + 4E_e{}'(y-1))^2}{4E_eE_py}
\end{aligned}
\tag{2.25}
$$

2.2 Error Analysis of Relativistic Kinematic Variables

2.2.1 Introduction

In the realms of scientific and engineering measurements, uncertainties are an inherent aspect that must be meticulously analyzed to ensure the accuracy and reliability of results. Analytical error propagation is a systematic approach employed to evaluate how uncertainties in measured variables influence the final computed outcomes in mathematical expressions. Utilizing calculus, particularly through the application of partial derivatives, this technique quantifies the contribution of each variable's uncertainty to the overall error. Consider the following calculation of error propagation to the function z,

Define a function"z" where $z = f(x, y)$, in which z depends on multiple variables "x" and "y".

The first step in finding the propagation of the error to z of each involved variable is to take the partial derivatives of z with respect to each variable.

$$\left(\frac{\partial z}{\partial x}\right) \qquad\qquad \left(\frac{\partial z}{\partial y}\right)$$

To find the uncertainty involved in z we use the following expression,

$$(\delta z)^2 = \left(\frac{\partial z}{\partial x}\right)^2 \delta x + \left(\frac{\partial z}{\partial y}\right)^2 \delta y$$

The above principle is known as the total differential formula for an error equation, which represents the propagated error in a calculated value due to uncertainties in input variables. Analytical error propagation is crucial in fields such as experimental physics, engineering, and data analysis, as it empowers researchers to gauge

the precision of derived quantities, refine measurement techniques, and report results with greater confidence.

In this chapter, we will discuss the error propagation to the DIS kinematic variables derived in Chapter 2.1. It is important note that we are restricting ourselves to only four methods which are $[E_e{}', \theta_e{}']$, $[F, \gamma]$, $[\theta_e{}', \gamma]$ and $[E_e{}', F]$.

2.2.2 Error Analysis with respect to variables of $E_e{}'$ and $\theta_e{}'$

In this section, we will examine the propagation of errors related to kinematic variables within the context of the reconstruction methods associated with energy ($E_e{}'$) and the Polar angle ($\theta_e{}'$) of the scattered electron.

From equation 2.1,

$$Q^2 = 2E_e E_e{}'(1 + \cos\theta_e{}')$$

Taking the partial derivative of Q^2 with respect to $E_e{}'$;

$$\frac{\partial Q^2}{\partial E_e{}'} = 2E_e(1 + \cos\theta_e{}') \tag{2.26}$$

Taking the partial derivative of Q^2 with respect to $\theta_e{}'$;

$$\frac{\partial Q^2}{\partial \theta_e{}'} = 2E_e E_e{}' \sin\theta_e{}' \tag{2.27}$$

From equation 2.2,

$$y = 1 - \frac{E_e{}'(1 - \cos\theta_e{}')}{2E_e}$$

Taking the partial derivative of y with respect to $E_e{}'$;

$$\frac{\partial y}{\partial E_e{}'} = \frac{\cos\theta_e{}' - 1}{2E_e} \tag{2.28}$$

Taking the partial derivative of y with respect to $\theta_e{}'$

$$\frac{\partial y}{\partial \theta_e{}'} = \frac{E_e{}' \sin\theta_e{}'}{2E_e} \tag{2.29}$$

From equation 2.3,

$$x = \frac{2E_e{}'(1+\cos\theta_e{}')}{4E_p\left(1-\frac{E_e{}'(1-\cos\theta_e{}')}{2E_e}\right)}$$

Taking the partial derivative of x with respect to $E_e{}'$;

$$\frac{\partial x}{\partial E_e{}'} = \frac{(2E_e{}')^2(1+\cos\theta_e{}')}{E_p(2E_e - E_e{}' + E_e{}'\cos\theta_e{}')^2} \tag{2.30}$$

Taking the partial derivative of x with respect to $\theta_e{}'$

$$\frac{\partial x}{\partial \theta_e{}'} = \frac{2E_eE_e{}'(E_e{}' - E_e)\sin\theta_e{}'}{E_p(2E_e - E_e{}' + E_e{}'\cos\theta_e{}')^2} \tag{2.31}$$

2.2.3 Error Analysis with respect to variables of F and γ

In this section, we will examine the propagation of errors related to kinematic variables within the context of the reconstruction methods associated with Energy (F) and the Polar angle (γ) of the scattered parton.

From equation 2.6,

$$Q^2 = \frac{F^2\sin^2\gamma}{1-y}$$

Taking the partial derivative of Q^2 with respect to F;

$$\frac{\partial Q^2}{\partial F} = \frac{F^2(1-\cos\gamma)\sin^2\gamma}{2E_e\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2} + \frac{2F\sin^2\gamma}{1-\frac{F(1-\cos\gamma)}{2E_e}} \tag{2.32}$$

Taking the partial derivative of Q^2 with respect to γ;

$$\frac{\partial Q^2}{\partial \gamma} = \frac{2F^2\sin\gamma\cos\gamma}{1-\frac{F(1-\cos\gamma)}{2E_e}} + \frac{F^3\sin^3\gamma}{2E_e\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2} \tag{2.33}$$

From equation 2.7,

$$y = \frac{F(1-\cos\gamma)}{2E_e}$$

Taking the partial derivative of y with respect to F;

$$\frac{\partial y}{\partial F} = \frac{1-\cos\gamma}{2E_e} \tag{2.34}$$

Taking the partial derivative of y with respect to γ;

$$\frac{\partial y}{\partial \gamma} = \frac{F\sin\gamma}{2E_e} \tag{2.35}$$

From equation 2.8,

$$x = \frac{F^2\sin^2\gamma}{4E_eE_p(1-y)y}$$

Taking the partial derivative of x with respect to F;

$$\frac{\partial x}{\partial F} = \frac{\sin^2\gamma}{2E_p(1-\cos\gamma)\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)} + \frac{F\sin^2\gamma}{4E_eE_p\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2} \tag{2.36}$$

Taking the partial derivative of x with respect to F;

$$\frac{\partial x}{\partial \gamma} = \frac{F\cos\gamma\sin\gamma}{E_p(1-\cos\gamma)\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)} + \frac{F^2\sin\gamma\sin^2\gamma}{4E_eE_p(1-\cos\gamma)\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2} - \frac{F\sin\gamma\sin^2\gamma}{2E_p(1-\cos\gamma)^2\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)} \tag{2.37}$$

2.2.4 Error Analysis with respect to variables of γ and $\theta_e{}'$

In this section, we will examine the propagation of errors related to kinematic variables within the context of the reconstruction methods associated with the Polar angle ($\theta_e{}'$) of the scattered electron and the polar angle of the scattered Parton (γ).

From equation 2.10

$$Q^2 = \frac{4E_e^2(1+\cos\theta_e')\sin\gamma}{\sin\gamma+\sin\theta_e'-\sin(\theta_e'+\gamma)}$$

Taking the partial derivative of Q^2 with respect to γ

$$\frac{\partial Q^2}{\partial\gamma} = -2E_e^2\cos^2\left(\frac{\theta_e'}{2}\right)\cot\left(\frac{\theta_e'}{2}\right)\csc^2\left(\frac{\gamma+\theta_e'}{2}\right) \tag{2.38}$$

Taking the partial derivative of Q^2 with respect to $\theta_e{}'$;

$$\frac{\partial Q^2}{\partial\theta_e{}'} = -4{E_e}^2\sin\gamma\left(\frac{\sin\theta_e{}'}{\sin\theta_e{}'+\sin\gamma-\sin(\theta_e{}'+\gamma)}+\frac{(1+\cos\theta_e{}')(\cos\theta_e{}'+\cos\left(\theta_e{}'+\gamma\right))}{(\sin\theta_e{}'+\sin\gamma-\sin(\theta_e{}'+\gamma))^2}\right) \tag{2.39}$$

From equation 2.11

$$y = \frac{\sin\theta_e'(1-\cos\gamma)}{\sin\gamma+\sin\theta_e'-\sin\left(\theta_e'+\gamma\right)}$$

Taking the partial derivative of y with respect to γ

$$\frac{\partial y}{\partial\gamma} = \frac{\sin\theta_e{}'(-1+\cos\gamma)(-1+\cos\theta_e{}')}{(\sin\theta_e{}'+\sin\gamma-\sin(\theta_e{}'+\gamma))^2} \tag{2.40}$$

Taking the partial derivative of y with respect to $\theta_e{}'$

$$\frac{\partial y}{\partial\theta_e{}'} = \frac{\sin\gamma(1-\cos\theta_e{}')(-1+\cos\theta_e{}')}{(\sin\theta_e{}'+\sin\gamma-\sin(\theta_e{}'+\gamma))^2} \tag{2.41}$$

From equation 2.12

$$x = \frac{E_e}{E_p}\cot\frac{\gamma}{2}\cot\frac{\theta_e{}'}{2}$$

Taking the partial derivative of x with respect to γ

$$\frac{\partial x}{\partial\theta_e{}'} = -\frac{E_e\cot\left(\frac{\gamma}{2}\right)\csc^2(\frac{\theta_e{}'}{2})}{2E_p} \tag{2.42}$$

Taking the partial derivative of x with respect to $\theta_e{}'$

$$\frac{\partial x}{\partial\gamma} = -\frac{E_e\cot\left(\frac{\theta_e'}{2}\right)\csc^2\left(\frac{\gamma}{2}\right)}{2E_p} \tag{2.43}$$

2.2.5 Error Analysis with respect to variables of $E_e{}'$ and F

In this section, we will examine the propagation of errors related to kinematic variables within the context of the reconstruction methods associated with the energy of the scattered electron (E) and the energy of the scattered parton (F).

From equation 2.20

$$Q^2 = 4E_eE'_e + \frac{4E_e^2(E_e - F)}{E_e'^2 + E'_e(F - 2E_e)}$$

Taking the partial derivative of Q^2 with respect to $E_e{}'$

$$\frac{\partial Q^2}{\partial E_e{}'} = 4E_e - \frac{4{E_e}^2(E_e - F)}{(-2E_e + E_e{}' + F)^2} \tag{2.44}$$

Taking the partial derivative of Q^2 with respect to F

$$\frac{\partial Q^2}{\partial F} = \frac{4{E_e}^2(-E_e + E_e{}')}{-2E_e + E_e{}' + F} \tag{2.45}$$

From equation 2.19

$$y = \frac{E_e{}' - E_e}{E_e{}' + F - 2E_e}$$

Taking the partial derivative of y with respect to $E_e{}'$

$$\frac{\partial y}{\partial E_e{}'} = -\frac{-E_e + F}{(-2E_e + E_e{}' + F)^2} \tag{2.46}$$

Taking the partial derivative of y with respect to F

$$\frac{\partial y}{\partial F} = -\frac{-E_e + E_e{}'}{(-2E_e + E_e{}' + F)^2} \tag{2.47}$$

From equation 2.21

$$x = \frac{\left(4E_eE'_e + \frac{4E_e^2(E_e - F)}{E_e'^2 + E'_e(F - 2E_e)}\right)^2}{4E_eE_p\frac{E'_e - E_e}{E'_e + F - 2E_e}}$$

Taking the partial derivative of x with respect to $E_e{}'$

$$\frac{\partial x}{\partial E_e{}'} = \frac{1}{E_p} \tag{2.48}$$

Taking the partial derivative of x with respect to F

$$\frac{\partial x}{\partial F} = \frac{1}{E_p} \tag{2.49}$$

2.3 Resolutions for Q^2, y and x

2.3.1 Introduction

In this chapter, we examine kinematic variable resolution through six different reconstruction methods. Our goal is to determine the set of observables that yields the optimal resolution of Q^2, y, and x. It is important note that we are restricting ourselves to only four methods which are $[E_e{}', \theta_e{}'], [F, \gamma], [\theta_e{}', \gamma]$ and $[E_e{}', F]$.

Relative resolution assesses a detector's capability to differentiate between two closely spaced signals or peaks and is typically represented as a ratio or percentage. As an example for quantity X, the relative resolution R is given by

$$R = \left(\frac{\partial X}{X}\right)$$

Here, X is the measured value, and ∂X is the smallest distinguishable difference between two values.

2.3.2 Resolutions with respect to variables of $E_e{}'$ and $\theta_e{}'$

In this section, we will examine the relative resolution of kinematic variables within the context of the reconstruction methods associated with energy ($E_e{}'$) and the Polar angle ($\theta_e{}'$) of the scattered electron.

The relative resolution of Q^2 can be written as,

$$\frac{\delta Q^2}{Q^2} = \frac{\frac{\partial Q^2}{\partial E_e{}'}\delta E_e{}'}{Q^2} \oplus \frac{\frac{\partial Q^2}{\partial \theta_e{}'}\delta \theta_e{}'}{Q^2} \tag{2.50}$$

Similarly for y,

$$\frac{\delta y}{y} = \frac{\frac{\partial y}{\partial E_e{}'}\delta E_e{}'}{y} \oplus \frac{\frac{\partial Q^2}{\partial \theta_e{}'}\delta \theta_e{}'}{y} \tag{2.51}$$

Next, for x the relative resolution will be,

$$\frac{\delta x}{x} = \frac{\frac{\partial x}{\partial E_e{}'}\delta E_e{}'}{x} \oplus \frac{\frac{\partial x}{\partial \theta_e{}'}\delta \theta_e{}'}{x} \tag{2.52}$$

From chapter 2.1 we have equations 2.1, 2.2 and 2.3

$$\begin{aligned} Q^2 &= 2E_e E_e{}'(1+\cos\theta_e{}') \\ y &= 1 - \frac{E_e{}'(1-\cos\theta_e{}')}{2E_e} \\ x &= \frac{2E_e{}'(1+\cos\theta_e{}')}{4E_p(1-\frac{E_e{}'(1-\cos\theta_e{}')}{2E_e})} \end{aligned}$$

From chapter 2.2 we have the partial derivatives of Q^2 with respect to the two observables $E_e{}'$ eq 2.26 and $\theta_e{}'$ eq 2.27,

$$\frac{\partial Q^2}{\partial E_e{}'} = 2E_e(1+\cos\theta_e{}')$$
$$\frac{\partial Q^2}{\partial \theta_e{}'} = 2E_e E_e{}'\sin\theta_e{}'$$

The partial derivatives of y with respect to the two observables $E_e{}'$ eq 2.28 and $\theta_e{}'$ eq 2.29,

$$\frac{\partial y}{\partial E_e{}'} = \frac{\cos\theta_e{}' - 1}{2E_e}$$
$$\frac{\partial y}{\partial \theta_e{}'} = \frac{E_e{}'\sin\theta_e{}'}{2E_e}$$

The partial derivatives of x with respect to the two observables $E_e{}'$ eq 2.30and $\theta_e{}'$ eq 2.31

$$\frac{\partial x}{\partial E_e{}'} = \frac{(2E_e{}')^2(1+\cos\theta_e{}')}{E_p(2E_e - E_e{}' + E_e{}'\cos\theta_e{}')^2}$$
$$\frac{\partial x}{\partial \theta_e{}'} = \frac{2E_e E_e{}'(E_e{}' - E_e)\sin\theta_e{}'}{E_p(2E_e - E_e{}' + E_e{}'\cos\theta_e{}')^2}$$

Substituting equations 2.1, 2.26 and 2.27 into the equation 2.50;

$$\begin{aligned}
\frac{\delta Q^2}{Q^2} &= \frac{2E_e(1+\cos\theta'_e)\delta E'_e}{2E_e E'_e(1+\cos\theta'_e)} \oplus \frac{2E_e E'_e\sin\theta'_e\delta\theta'_e}{2E_e E'_e(1+\cos\theta'_e)} \\
\frac{\delta Q^2}{Q^2} &= \frac{\delta E'_e}{E'_e} \oplus \frac{\sin(-\theta'_e)}{1+\cos\theta'_e}\delta\theta'_e \\
\frac{\delta Q^2}{Q^2} &= \frac{\delta E'_e}{E'_e} \oplus \tan\left(\frac{\theta'_e}{2}\right)\delta\theta'_e
\end{aligned} \tag{2.53}$$

Next, we are going to plot the resolution of Q^2 ($\frac{\delta Q^2}{Q^2}$ vs Q) for two cases for different uncertainties measured in angle and energy. Note that here we assume that the incident electron energy is 10 GeV and the energy of the initial parton is 275 GeV

with a fixed scattering angle of 30 degrees.

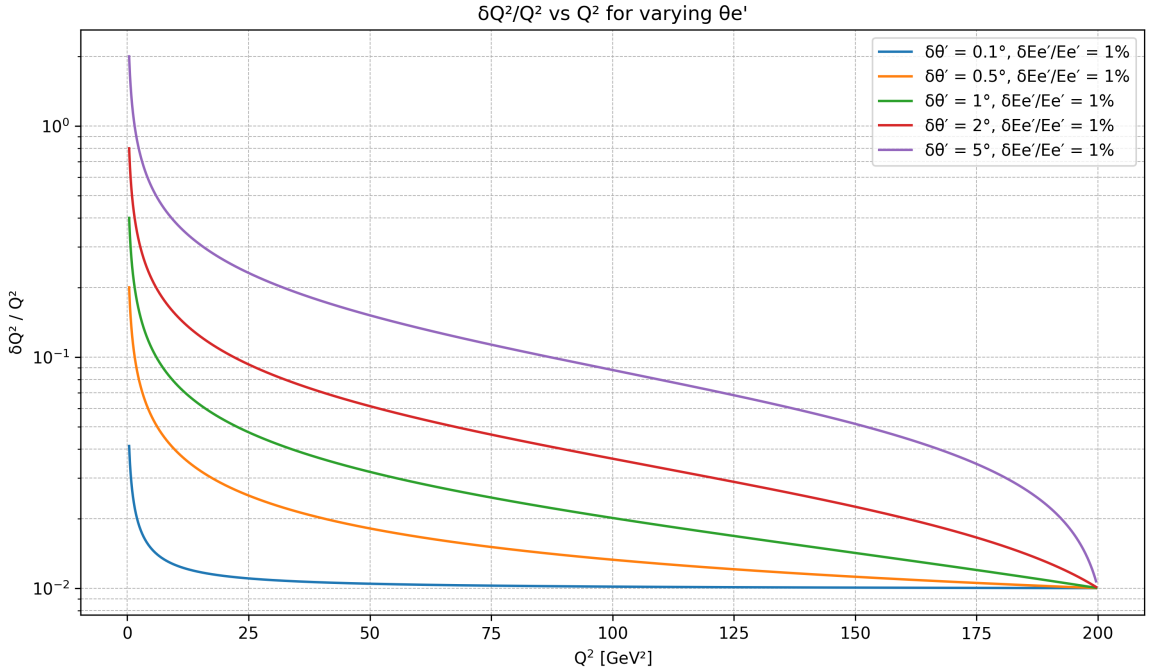

Figure 4. $\delta Q^2/Q^2$ vs. Q^2 for different $\theta_e^{'}$ uncertainties.

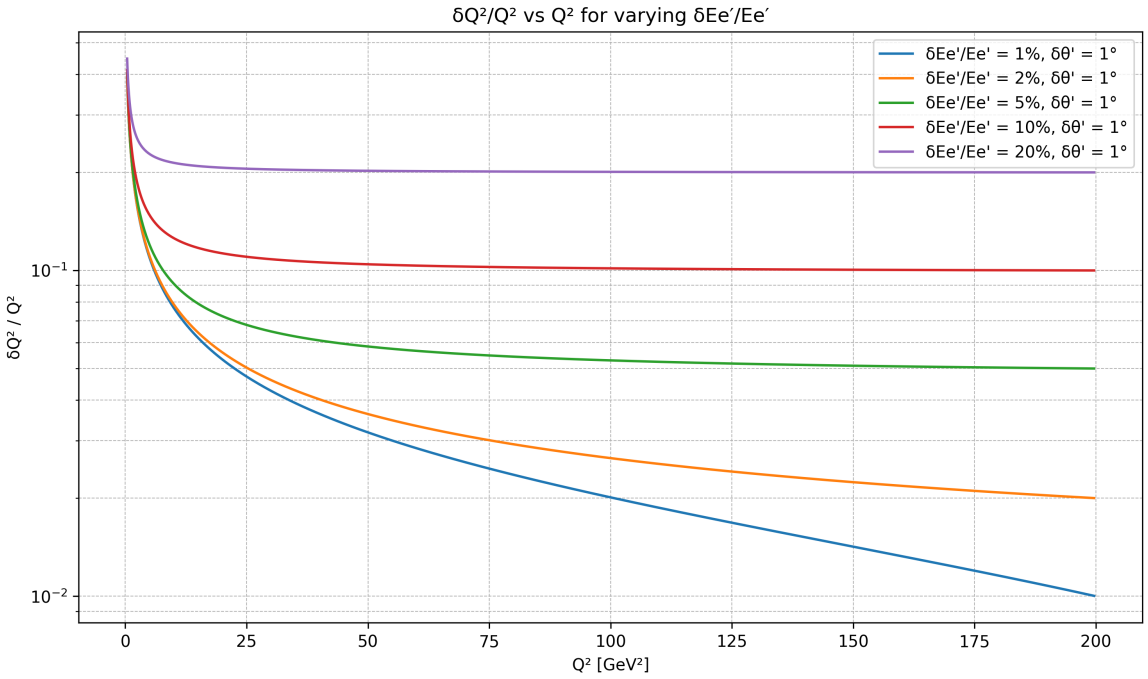

Figure 4. $\delta Q^2/Q^2$ vs. Q^2 for different E_e uncertainties.

To obtain the relative resolution of y, substitute equations 2.2, 2.28 and 2.29 into the equation 2.51

$$\frac{\delta y}{y} = \frac{\frac{\cos\theta_e{}' - 1}{2E_e}\delta E_e{}'}{1 - \frac{E_e{}'(1-\cos\theta_e{}')}{2E_e}} \oplus \frac{\frac{E_e{}'\sin\theta_e{}'}{2E_e}\delta\theta_e{}'}{1 - \frac{E_e{}'(1-\cos\theta_e{}')}{2E_e}} \tag{2.54}$$

Taking the contributing term for $\delta E_e{}'$ along gives;

$$\begin{aligned} &= \frac{\frac{\cos\theta_e{}' - 1}{2E_e}\delta E_e{}'}{1 - \frac{E_e{}'(1-\cos\theta_e{}')}{2E_e}} \\ &= \frac{\frac{1-\cos\theta_e{}'}{2E_e}\delta E_e{}'}{\frac{E_e{}'(1-\cos\theta_e{}')}{2E_e} - 1} \end{aligned} \tag{2.55}$$

Considering equation 2.2 we can replace, $(1 - \cos\theta_e{}')$ with $\frac{E_e(1-y)}{2E_e{}'}$ gives;

$$
\begin{aligned}
&= \frac{\frac{\frac{2E_e(1-y)}{E_e{}'}}{2E_e}\delta E_e{}'}{\frac{E_e{}'(1-\cos\theta_e{}')}{2E_e} - 1} \\
&= \frac{(1-y)\delta E_e{}'}{E_e{}'y} \\
&= (1-\frac{1}{y})\frac{\delta E_e{}'}{E_e{}'}
\end{aligned}
\tag{2.56}
$$

Taking the contributing term from $\delta\theta_e{}'$ alone gives;

$$
-\frac{\frac{E_e{}'\sin\theta_e{}'}{2E_e}\delta\theta_e{}'}{1-\frac{E_e{}'(1-\cos\theta_e{}')}{2E_e}} \tag{2.57}
$$

Considering equation 2.2 we can replace, $E_e{}'$ with $\frac{E_e{}'(1-y)}{1-\cos\theta_e{}'}$ gives;

$$
\begin{aligned}
&- \left(\frac{\frac{E_e{}'(1-y)}{1-\cos\theta_e{}'}\sin\theta_e{}'}{2E_e}\delta\theta_e{}'\right) / \left(1-\frac{\frac{E_e{}'(1-y)}{1-\cos\theta_e{}'}(1-\cos\theta_e{}')}{2E_e}\right) \\
&= -\frac{(1-y)\sin\theta_e{}'\delta\theta_e{}'}{y(1-\cos\theta_e{}')} \\
&= \left(\frac{1}{y}-1\right)\cot\left(\frac{\theta_e{}'}{2}\right)\delta\theta_e{}'
\end{aligned}
\tag{2.58}
$$

From equations 2.56 and 2.58 substitute to equation 2.54

$$
\frac{\delta y}{y} = (1-\frac{1}{y})\frac{\delta E_e{}'}{E_e{}'} \oplus \left(\frac{1}{y}-1\right)\cot\left(\frac{\theta_e{}'}{2}\right)\delta\theta_e{}' \tag{2.59}
$$

Next, we are going to plot the resolution of y ($\frac{\delta y}{y}$ vs y) for two cases for different uncertainties measured in angle and energy. Note that we assume that scattered electron with a fixed scattering angle of 30 degrees.

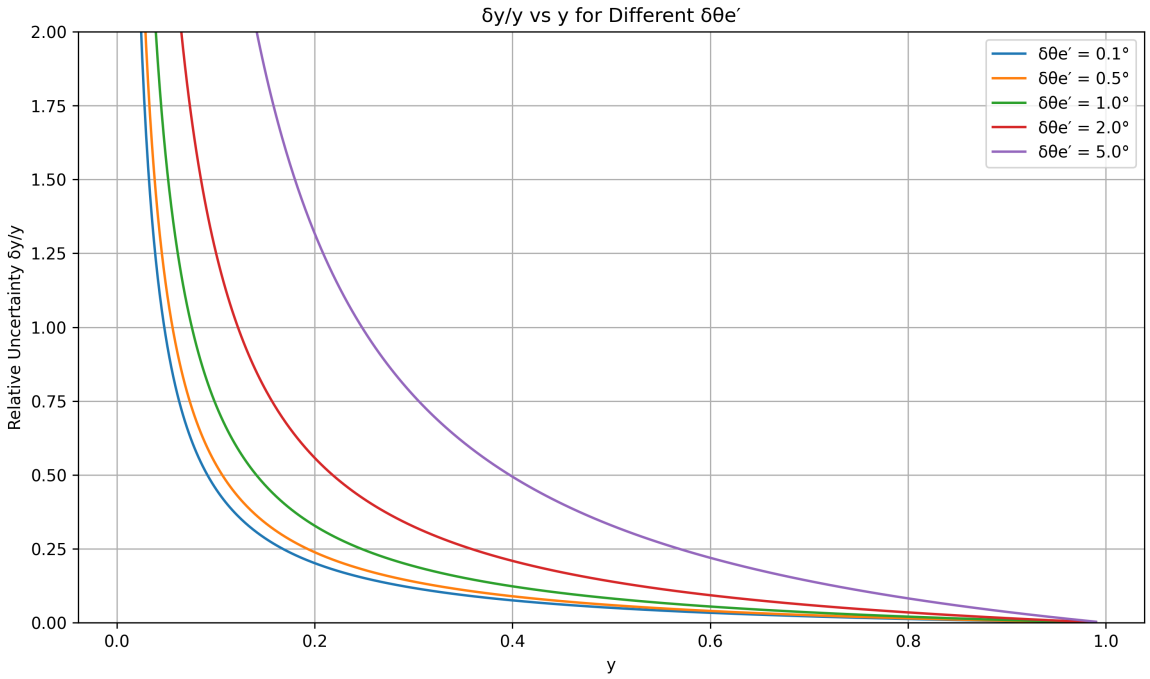

Figure 5. $\delta y/y$ vs. y for different $\theta_e^{'}$ uncertainties.

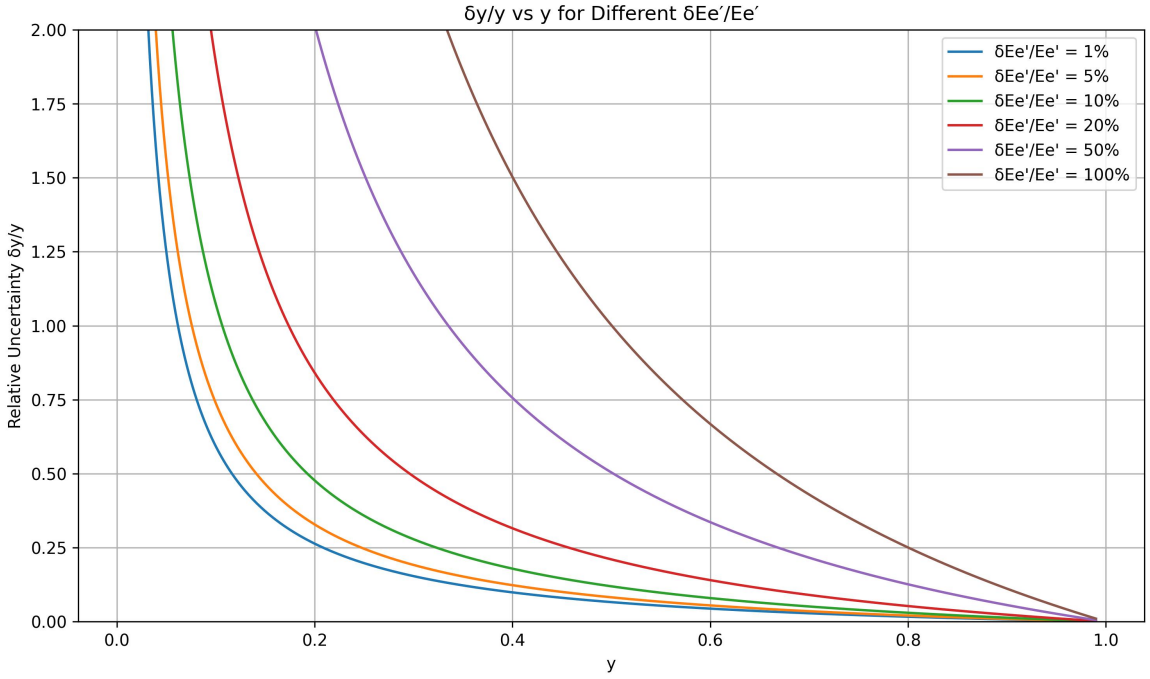

Figure 6. $\delta y/y$ vs. y for different E_e uncertainties.

To obtain the relative resolution of x, substituting equations 2.3, 2.30, 2.31, into equation 2.52 gives;

$$\frac{\left(\frac{2{E_e}^2(1+\cos{\theta_e}')}{E_p(2E_e-{E_e}'+{E_e}'\cos{\theta_e}')^2}\right)}{\left(\frac{{E_e}'(1+\cos{\theta_e}')}{2E_p\left(1-\frac{{E_e}'(1-\cos{\theta_e}')}{2E_e}\right)}\right)}\delta{E_e}' \oplus \frac{\left(\frac{2E_e({E_e}'-E_e){E_e}'\sin{\theta_e}'}{E_p(2E_e-{E_e}'+{E_e}'\cos{\theta_e}')^2}\right)}{\left(\frac{{E_e}'(1+\cos{\theta_e}')}{2E_p\left(1-\frac{{E_e}'(1-\cos{\theta_e}')}{2E_e}\right)}\right)}\delta{\theta_e}' \tag{2.60}$$

The contributing term from $\delta{E_e}'$ simplifies to;

$$\frac{2E_e}{E_p(2E_e-{E_e}'+{E_e}'\cos{\theta_e}')^2}\delta{E_e}' \tag{2.61}$$

Considering equation 2.2 we can replace, $\cos{\theta_e}'$ with $1-\frac{E_e(1-y)}{2{E_e}'}$;

$$\frac{2E_e}{{E_e}'(2E_e-{E_e}'+{E_e}'\left(1-\frac{E_e(1-y)}{2{E_e}'}\right)}\delta{E_e}' \tag{2.62}$$

$$\frac{2E_e}{{E_e}'(2E_e-{E_e}'+{E_e}'\left(1-\frac{E_e(1-y)}{2{E_e}'}\right)}\delta{E_e}' \tag{2.63}$$

$$\frac{2E_e}{{E_e}'(2E_e-2E_e(1-y))}\delta{E_e}' \tag{2.64}$$

$$\frac{1}{{E_e}'(1-(1-y))}\delta{E_e}' \tag{2.65}$$

$$\frac{1}{y}\frac{\delta{E_e}'}{{E_e}'} \tag{2.66}$$

Contribution from ${\theta_e}'$

$$\begin{aligned}&\left(\frac{2E_e({E_e}'-E_e){E_e}'\sin{\theta_e}'}{E_p(2E_e-{E_e}'+{E_e}'\cos{\theta_e}')^2}\right)\delta{\theta_e}'/\left(\frac{{E_e}'(1+\cos{\theta_e}')}{2E_p\left(1-\frac{{E_e}'(1-\cos{\theta_e}')}{2E_e}\right)}\right)\\&=\frac{2({E_e}'-E_e)\tan\left(\frac{{\theta_e}'}{2}\right)}{2E_e-{E_e}'+{E_e}'\cos{\theta_e}'}\delta{\theta_e}'\end{aligned} \tag{2.67}$$

Using the equation 2.3 replace $E_e{}'$ with $(2xE_p)/\left(1+\cos\theta_e{}'+\frac{xE_p(1-\cos\theta_e{}')}{E_e}\right)$ and simplify to,

$$\left(\frac{x}{E_e/E_p}-1\right)\tan\left(\frac{\theta_e{}'}{2}\right)\delta\theta_e{}' \tag{2.68}$$

Using equations 2.66 and 2.68 in the equation 2.60 ;

$$\frac{\delta x}{x}=\frac{1}{y}\frac{\delta E_e{}'}{E_e{}'}\oplus\left(\frac{x}{E_e/E_p}-1\right)\tan\left(\frac{\theta_e{}'}{2}\right)\delta\theta_e{}' \tag{2.69}$$

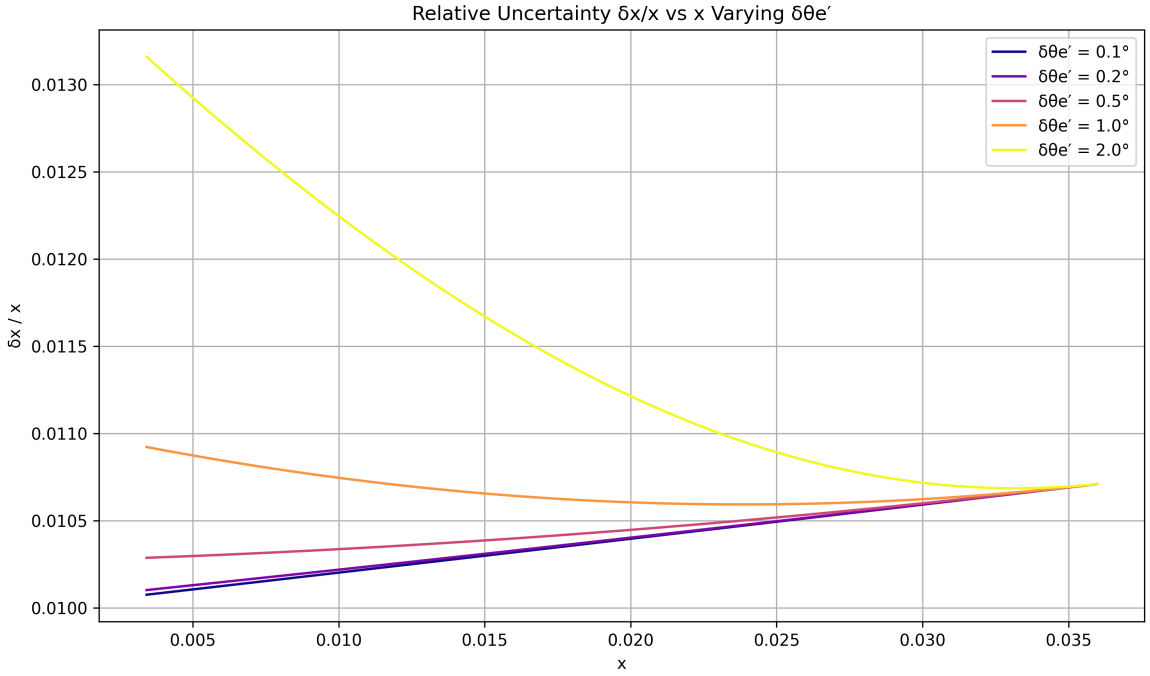

Figure 7. $\delta x/x$ vs. x for different θ_e' uncertainties.

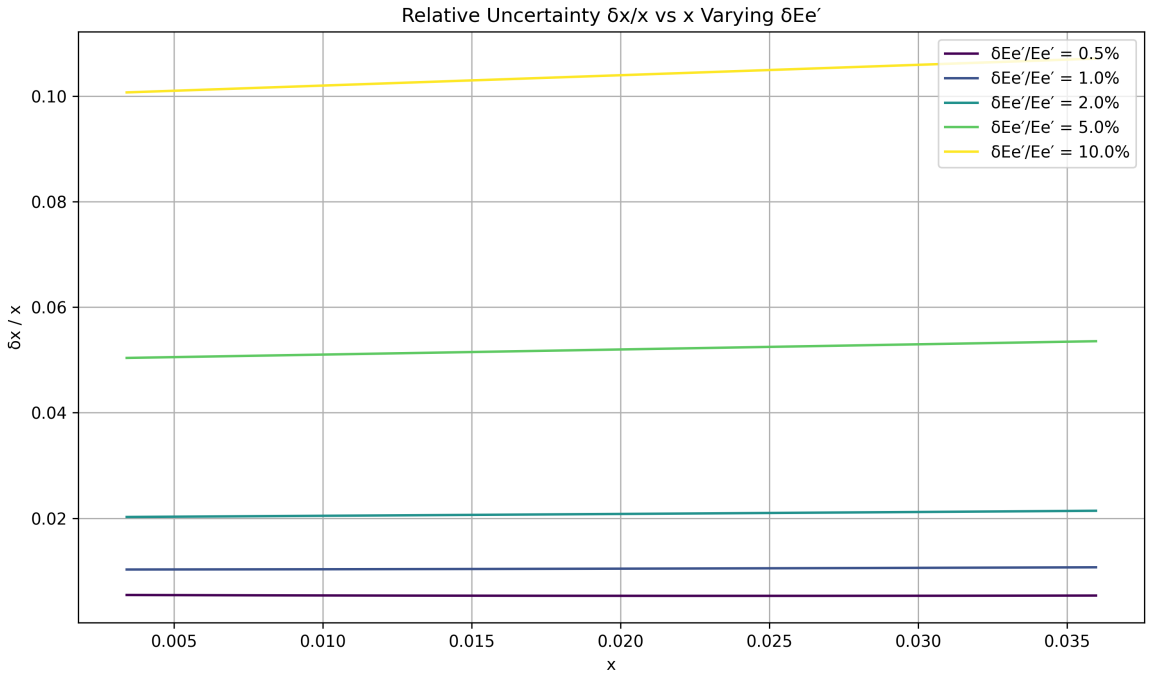

Figure 8. $\delta x/x$ vs. x for different E_e uncertainties.

2.3.3 Resolutions with respect to variables of F and γ

In this section, we will examine the relative resolution of kinematic variables within the context of the reconstruction methods associated with energy (F) and the Polar angle (γ) of the scattered parton.

The relative resolution of Q^2 can be written as,

$$\frac{\delta Q^2}{Q^2} = \frac{\frac{\partial Q^2}{\partial F}\delta F}{Q^2} \oplus \frac{\frac{\partial Q^2}{\partial \gamma}\delta \gamma}{Q^2} \tag{2.70}$$

Similarly for y,

$$\frac{\delta y}{y} = \frac{\frac{\partial y}{\partial F}\delta F}{y} \oplus \frac{\frac{\partial Q^2}{\partial \gamma}\delta \gamma}{y} \tag{2.71}$$

Next, for x the relative resolution will be,

$$\frac{\delta x}{x} = \frac{\frac{\partial x}{\partial F}\delta F}{x} \oplus \frac{\frac{\partial x}{\partial \gamma}\delta \gamma}{x} \tag{2.72}$$

From chapter 2.1 we have equations 2.7, 2.6 and 2.8;

$$\begin{aligned} Q^2 &= \frac{F^2 \sin^2 \gamma}{1-y} \\ y &= \frac{F(1-\cos\gamma)}{2E_e} \\ x &= \frac{(E_e^{'})^2 \sin^2 \theta_e^{'}}{4E_e E_p (1-y) y} \end{aligned}$$

From chapter 2.2 we have the partial derivatives of Q^2 with respect to the two observables F eq 2.32 and γ eq 2.33.

$$\frac{\partial Q^2}{\partial F} = \frac{F^2(1-\cos\gamma)\sin^2\gamma}{2E_e\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2} + \frac{2F\sin^2\gamma}{1-\frac{F(1-\cos\gamma)}{2E_e}}$$

$$\frac{\partial Q^2}{\partial \gamma} = \frac{2F^2\sin\gamma\cos\gamma}{1-\frac{F(1-\cos\gamma)}{2E_e}} + \frac{F^3\sin^3\gamma}{2E_e\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2}$$

From chapter 2.2 we have the partial derivatives of y with respect to the two observables F eq 2.34 and γ eq 2.35.

$$\frac{\partial y}{\partial F} = \frac{1-\cos\gamma}{2E_e}$$

$$\frac{\partial y}{\partial \gamma} = \frac{F\sin\gamma}{2E_e}$$

From chapter 2.2 we have the partial derivatives of x with respect to the two observables F eq 2.36 and γ eq 2.37.

$$\frac{\partial x}{\partial F} = \frac{\sin^2\gamma}{2E_p(1-\cos\gamma)\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)} + \frac{F\sin^2\gamma}{4E_eE_p\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2}$$

$$\frac{\partial x}{\partial \gamma} = \frac{F\cos\gamma\sin\gamma}{E_p(1-\cos\gamma)\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)} + \frac{F^2\sin\gamma\sin^2\gamma}{4E_eE_p(1-\cos\gamma)\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2} - \frac{F\sin\gamma\sin^2\gamma}{2E_p(1-\cos\gamma)^2\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)}$$

From equation 2.70 consider the contributing term for δF and then substitute from equation 2.32 and equation 2.8 and then for y from equation 2.6

$$\begin{aligned}
\frac{\frac{\partial Q^2}{\partial F}}{Q^2} &= \left(\frac{F^2(1-\cos\gamma)\sin^2\gamma}{2E_e\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2} + \frac{2F\sin^2\gamma}{1-\frac{F(1-\cos\gamma)}{2E_e}}\right) / \left(\frac{F^2\sin^2\gamma}{1-y}\right) \\
&= \left(\frac{F^2(1-\cos\gamma)\sin^2\gamma}{2E_e(1-y)^2} + \frac{2F\sin^2\gamma}{1-y}\right) / \left(\frac{F^2\sin^2\gamma}{1-y}\right) \\
&= \left(\frac{F(1-\cos\gamma)}{2E_e(1-y)} + \frac{2}{1-y}\right) / \left(\frac{F}{1-y}\right) \\
&= \frac{(2-y)}{(1-y)F}
\end{aligned} \tag{2.73}$$

From equation 2.70 consider the contributing term for $\delta\gamma$ and then substitute from equation 2.33 and equation 2.8 and then for y from equation 2.6

$$\begin{aligned}
\frac{\frac{\partial Q^2}{\partial \gamma}}{Q^2} &= \left(\frac{2F^2\sin\gamma\cos\gamma}{1-\frac{F(1-\cos\gamma)}{2E_e}} + \frac{F^3\sin^3\gamma}{2E_e\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2}\right) / \left(\frac{F^2\sin^2\gamma}{1-y}\right) \\
&= \left(\frac{2F^2\sin\gamma\cos\gamma}{1-y} + \frac{F^3\sin^3\gamma}{2E_e(1-y)^2}\right) / \left(\frac{F^2\sin^2\gamma}{1-y}\right) \\
&= \frac{y}{1-y}\cot\left(\frac{\gamma}{2}\right) + 2\cot\gamma
\end{aligned} \tag{2.74}$$

Substitute 2.73 and 2.74 into 2.70;

$$\frac{\delta Q^2}{Q^2} = \frac{(2-y)}{(1-y)}\frac{\delta F}{F} \oplus \left(\frac{y}{1-y}\cot\left(\frac{\gamma}{2}\right) + 2\cot\gamma\right)\delta\gamma \tag{2.75}$$

To obtain the relative resolution of y, substitute equations 2.6, 2.34 and 2.35 into the equation 2.71

$$\begin{aligned}\frac{\delta y}{y} &= \frac{\frac{1-\cos\gamma}{2E_e}\delta F}{\frac{F(1-\cos\gamma)}{2E_e}} \oplus \frac{\frac{F\sin\gamma}{2E_e}\delta\gamma}{\frac{F(1-\cos\gamma)}{2E_e}} \\ &= \frac{\delta F}{F} \oplus \frac{2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}\delta\gamma}{1-1+\sin^2\frac{\gamma}{2}} \\ &= \frac{\delta F}{F} \oplus \cot\left(\frac{\gamma}{2}\right)\delta\gamma\end{aligned} \tag{2.76}$$

To obtain the relative resolution of x, substitute equations 2.7, 2.36 and 2.37 into the equation 2.72 and then consider the contribution from the δF term;

$$\begin{aligned}\frac{\frac{\partial x}{\partial F}}{x} &= \left(\frac{\sin^2\gamma}{2E_p(1-\cos\gamma)\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)} + \frac{F\sin^2\gamma}{4E_eE_p\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2}\right) \Big/ \left(\frac{F^2\sin^2\gamma}{4E_eE_p(1-y)y}\right) \\ &= \left(\frac{\sin^2\gamma}{2E_p(1-\cos\gamma)(1-y)} + \frac{F\sin^2\gamma}{4E_eE_p(1-y)^2}\right) \Big/ \left(\frac{F^2\sin^2\gamma}{4E_eE_p(1-y)y}\right) \\ &= \frac{1}{F} + \frac{1-\cos\gamma}{4E_e(1-y)}\end{aligned} \tag{2.77}$$

Now consider the contribution from the $\delta\gamma$ term;

$$\begin{aligned}\frac{\frac{\partial x}{\partial F}}{x} &= \frac{\frac{F\cos\gamma\sin\gamma}{E_p(1-\cos\gamma)\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)} + \frac{F^2\sin\gamma\sin^2\gamma}{4E_eE_p(1-\cos\gamma)\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)^2} - \frac{F\sin\gamma\sin^2\gamma}{2E_p(1-\cos\gamma)^2\left(1-\frac{F(1-\cos\gamma)}{2E_e}\right)}}{\frac{F^2\sin^2\gamma}{4E_eE_p(1-y)y}} \\ &= 2\cot\gamma - \frac{2\sin\gamma}{1-\cos\gamma} + \frac{F\sin\gamma}{2E_e(1-y)}\end{aligned} \tag{2.78}$$

Substituting equation 2.77 and 2.78 in to 2.72 we have;

$$\frac{\delta x}{x} = \left(\frac{1}{F} + \frac{1-\cos\gamma}{4E_e(1-y)}\right)\delta F + \left(2\cot\gamma - \frac{2\sin\gamma}{1-\cos\gamma} + \frac{F\sin\gamma}{2E_e(1-y)}\right)\delta\gamma \tag{2.79}$$

2.3.4 Resolutions with respect to variables of $\theta_e{}'$ and γ

In this section, we will examine the relative resolution of kinematic variables within the context of the reconstruction methods associated with Polar angle ($\theta_e{}'$) of the scattered electron and the polar angle of the scattered Parton (γ).

The relative resolution of Q^2 can be written as,

$$\frac{\delta Q^2}{Q^2} = \frac{\frac{\partial Q^2}{\partial\gamma}\delta\gamma}{Q^2} \oplus \frac{\frac{\partial Q^2}{\partial\theta_e{}'}\delta\theta_e{}'}{Q^2} \tag{2.80}$$

Similarly for y,

$$\frac{\delta y}{y} = \frac{\frac{\partial y}{\partial\gamma}\delta\gamma}{y} \oplus \frac{\frac{\partial Q^2}{\partial\theta_e{}'}\delta\theta_e{}'}{y} \tag{2.81}$$

Next, for x the relative resolution will be,

$$\frac{\delta x}{x} = \frac{\frac{\partial x}{\partial\gamma}\delta\gamma}{x} \oplus \frac{\frac{\partial x}{\partial\theta_e{}'}\delta\theta_e{}'}{x} \tag{2.82}$$

From chapter 2.1 we have equations 2.10, 2.11 and 2.12

$$Q^2 = \frac{4E_e^2(1+\cos\theta_e')\sin\gamma}{\sin\gamma+\sin\theta_e'-\sin(\theta_e'+\gamma)}$$
$$y = \frac{\sin\theta_e'(1-\cos\gamma)}{\sin\gamma+\sin\theta_e'-\sin(\theta_e'+\gamma)}$$
$$x = \frac{E_e}{E_p}\cot\frac{\gamma}{2}\cot\frac{\theta_e'}{2}$$

From chapter 2.2 we have the partial derivatives of Q^2 with respect to the two observables $\theta_e{}'$ eq 2.39 and γ eq 2.38

$$\frac{\partial Q^2}{\partial\gamma} = -2E_e^2\cos^2\left(\frac{\theta_e'}{2}\right)\cot\left(\frac{\theta_e'}{2}\right)\csc^2\left(\frac{1}{2}(\gamma+\theta_e')\right)$$
$$\frac{\partial Q^2}{\partial\theta_e'} = -4E_e^2\sin\gamma\left(\frac{\sin\theta_e'}{\sin\theta_e'+\sin\gamma-\sin(\theta_e'+\gamma)}+\frac{(1+\cos\theta_e')(\cos\theta_e'+\cos(\theta_e'+\gamma))}{(\sin\theta_e'+\sin\gamma-\sin(\theta_e'+\gamma))^2}\right)$$

From chapter 2.2 we have the partial derivatives of y with respect to the two observables $\theta_e{}'$ eq 2.41 and γ eq 2.40

$$\frac{\partial y}{\partial\gamma} = \frac{\sin\theta_e{}'(-1+\cos\gamma)(-1+\cos\theta_e{}')}{(\sin\theta_e{}'+\sin\gamma-\sin(\theta_e{}'+\gamma))^2}$$
$$\frac{\partial y}{\partial\theta_e{}'} = \frac{\sin\gamma(1-\cos\theta_e{}')(-1+\cos\theta_e{}')}{(\sin\theta_e{}'+\sin\gamma-\sin(\theta_e{}'+\gamma))^2}$$

From chapter 2.2 we have the partial derivatives of x with respect to the two observables $\theta_e{}'$ eq 2.36 and γ eq 2.37.

$$\frac{\partial x}{\partial\theta_e'} = -\frac{E_e\cot\left(\frac{\gamma}{2}\right)\csc^2\left(\frac{\theta_e'}{2}\right)}{2E_p}$$
$$\frac{\partial x}{\partial\gamma} = -\frac{E_e\cot\left(\frac{\theta_e'}{2}\right)\csc^2\left(\frac{\gamma}{2}\right)}{2E_p}$$

To obtain the relative resolution of Q^2, substitute equations 2.10, 2.38 and 2.39 into the equation 2.80 and then consider the contribution from $\delta\theta_e{}'$

$$\begin{aligned}\frac{\frac{\partial Q^2}{\partial \theta_e^{'}}}{Q^2} &= \frac{-4E_e^2 \sin\gamma \left(\frac{\sin\theta_e^{'}}{\sin\theta_e^{'}+\sin\gamma-\sin\left(\theta_e^{'}+\gamma\right)} + \frac{(1+\cos\theta_e^{'})(\cos\theta_e^{'}+\cos\left(\theta_e^{'}+\gamma\right))}{(\sin\theta_e^{'}+\sin\gamma-\sin\left(\theta_e^{'}+\gamma\right))^2} \right)}{\frac{4E_e^2(1+\cos\theta_e^{'})\sin\gamma}{\sin\gamma+\sin\theta_e^{'}-\sin\left(\theta_e^{'}+\gamma\right)}} \\ &= -\frac{1+\cos\theta_e^{'}-\cos\gamma(1+\cos\theta_e^{'})+2\sin\gamma\sin\theta_e^{'}}{(1+\cos\theta_e^{'})\left(\sin\gamma+\sin\theta_e^{'}-\sin(\gamma+\theta_e^{'})\right)}\end{aligned} \tag{2.83}$$

Suppose in a similar way the contribution from $\delta\gamma$;

$$\begin{aligned}\frac{\frac{\partial Q^2}{\partial \gamma}}{Q^2} &= \frac{2E_e^2\cos^2\left(\frac{\theta_e^{'}}{2}\right)\cot\left(\frac{\theta_e^{'}}{2}\right)\csc^2\left(\frac{1}{2}\left(\gamma+\theta_e^{'}\right)\right)}{\frac{4E_e^2(1+\cos\theta_e^{'})\sin\gamma}{\sin\gamma+\sin\theta_e^{'}-\sin\left(\theta_e^{'}+\gamma\right)}} \\ &= -\frac{1}{2}\cos\left(\frac{\theta_e^{'}}{2}\right)\csc\left(\frac{1}{2}\left(\gamma+\theta_e^{'}\right)\right)\sec\left(\frac{\gamma}{2}\right)\end{aligned} \tag{2.84}$$

By substituting equations 2.84,2.83 to equation 2.82;

$$\begin{aligned}\frac{\delta Q^2}{Q^2} = &\left(-\frac{1+\cos\theta_e^{'}-\cos\gamma(1+\cos\theta_e^{'})+2\sin\gamma\sin\theta_e^{'}}{(1+\cos\theta_e^{'})\left(\sin\gamma+\sin\theta_e^{'}-\sin(\gamma+\theta_e^{'})\right)}\right)\delta\theta_e^{'} \\ &\oplus\left(-\frac{1}{2}\cos\left(\frac{\theta_e^{'}}{2}\right)\csc\left(\frac{1}{2}\left(\gamma+\theta_e^{'}\right)\right)\sec\left(\frac{\gamma}{2}\right)\right)\delta\gamma\end{aligned} \tag{2.85}$$

To obtain the relative resolution of y, substitute equations 2.11, 2.40 and 2.41 into the equation 2.81 and then consider the contribution from $\delta{\theta_e}'$

$$\begin{aligned}\frac{\frac{\partial y}{\partial \theta_e^{'}}}{y} &= \frac{\sin\theta_e^{'}(-1+\cos\gamma)(-1+\cos\theta_e^{'})}{\left(\sin\theta_e^{'}+\sin\gamma-\sin(\theta_e^{'}+\gamma)\right)^2} \Big/ \frac{\sin\theta_e^{'}(1-\cos\gamma)}{\sin\gamma+\sin\theta_e^{'}-\sin(\theta_e^{'}+\gamma)} \\ &= \frac{\left(-1+\cos\theta_e^{'}\right)\csc\theta_e^{'}\sin\gamma}{\sin\gamma+\sin\theta_e^{'}-\sin(\gamma+\theta_e^{'})}\end{aligned} \tag{2.86}$$

Suppose in a similar way the contribution from $\delta\gamma$

$$\begin{aligned}\frac{\frac{\partial y}{\partial \gamma}}{y} &= \left(\frac{\sin\theta_e^{'}(-1+\cos\gamma)(-1+\cos\theta_e^{'})}{\left(\sin\theta_e^{'}+\sin\gamma-\sin(\theta_e^{'}+\gamma)\right)^2}\right) / \left(\frac{\sin\theta_e^{'}(1-\cos\gamma)}{\sin\gamma+\sin\theta_e^{'}-\sin(\theta_e^{'}+\gamma)}\right) \\ &= \frac{-1+\cos(\gamma)\left(-1+\cos\theta_e^{'}\right)+\cos\theta_e^{'}-2\sin\gamma\sin\theta_e^{'}}{(\cos\gamma+1)\left(\sin\gamma+\sin\theta_e^{'}-\sin(\gamma+\theta_e^{'})\right)}\end{aligned} \tag{2.87}$$

By substituting equations 2.87, 2.86 to equation 2.81;

$$\begin{aligned}\frac{\delta y}{y} = &\left(\frac{-1+\cos(\gamma)\left(-1+\cos\theta_e^{'}\right)+\cos\theta_e^{'}-2\sin\gamma\sin\theta_e^{'}}{(\cos\gamma+1)\left(\sin\gamma+\sin\theta_e^{'}-\sin(\gamma+\theta_e^{'})\right)}\right)\delta\gamma\oplus \\ &\left(\frac{\left(-1+\cos\theta_e^{'}\right)\csc\theta_e^{'}\sin\gamma}{\sin\gamma+\sin\theta_e^{'}-\sin(\gamma+\theta_e^{'})}\right)\delta\theta_e^{'}.\end{aligned} \tag{2.88}$$

To obtain the relative resolution of x, substitute equations 2.12, 2.42 and 2.43 into the equation 2.82

$$\begin{aligned}\frac{\delta x}{x} &= \frac{\frac{\partial x}{\partial \gamma}\delta\gamma}{x} \oplus \frac{\frac{\partial x}{\partial \theta_e^{'}}\delta\theta_e^{'}}{x} \\ &= \frac{-\frac{E_e\cot\left(\frac{\theta_e^{'}}{2}\right)\csc^2\left(\frac{\gamma}{2}\right)}{2E_p}\delta\gamma}{\frac{E_e}{E_p}\cot\left(\frac{\gamma}{2}\right)\cot\left(\frac{\theta_e^{'}}{2}\right)} \oplus \frac{-\frac{E_e\cot\left(\frac{\gamma}{2}\right)\csc^2\left(\frac{\theta_e^{'}}{2}\right)}{2E_p}\delta\theta_e^{'}}{\frac{E_e}{E_p}\cot\left(\frac{\gamma}{2}\right)\cot\left(\frac{\theta_e^{'}}{2}\right)} \\ &= -\csc\theta_e^{'}\delta\theta_e^{'} \oplus \csc\gamma\delta\gamma\end{aligned} \tag{2.89}$$

2.3.5 Resolutions with respect to variables of $E_e^{'}$ and F

In this section, we will examine the relative resolution of kinematic variables within the context of the reconstruction methods associated with energy of the scattered electron (E) and the energy of the scattered parton (F).

The relative resolution of Q^2 can be written as,

$$\frac{\delta Q^2}{Q^2} = \frac{\frac{\partial Q^2}{\partial E_e^{'}}\delta E_e^{'}}{Q^2} \oplus \frac{\frac{\partial Q^2}{\partial F}\delta F}{Q^2} \tag{2.90}$$

Similarly for y,

$$\frac{\delta y}{y} = \frac{\frac{\partial y}{\partial E_e{}'}\delta E_e{}'}{y} \oplus \frac{\frac{\partial Q^2}{\partial F}\delta F}{y} \tag{2.91}$$

Next, for x the relative resolution will be,

$$\frac{\delta x}{x} = \frac{\frac{\partial x}{\partial E_e{}'}\delta E_e{}'}{x} \oplus \frac{\frac{\partial x}{\partial F}\delta F}{x} \tag{2.92}$$

From chapter 2.1 we have equations 2.20, 2.19 and 2.21;

$$\begin{aligned} Q^2 &= 4E_e E_e{}' + \frac{4E_e{}^2(E_e - F)}{E_e{}'^2 + E_e{}'(F - 2E_e)} \\ y &= \frac{E_e{}' - E_e}{E_e{}' + F - 2E_e} \\ x &= \frac{\left(4E_e E_e{}' + \frac{4E_e{}^2(E_e - F)}{E_e{}'^2 + E_e{}'(F - 2E_e)}\right)^2}{4E_e E_p \frac{E_e{}' - E_e}{E_e{}' + F - 2E_e}} \end{aligned}$$

From chapter 2.2 we have the partial derivatives of Q^2 with respect to the two observables $E_e{}'$ eq 2.44 and F eq 2.45

$$\begin{aligned} \frac{\partial Q^2}{\partial E_e{}'} &= 4E_e - \frac{4E_e{}^2(E_e - F)}{(-2E_e + E_e{}' + F)^2} \\ \frac{\partial Q^2}{\partial F} &= \frac{4E_e{}^2(-E_e + E_e{}')}{-2E_e + E_e{}' + F} \end{aligned}$$

From chapter 2.2 we have the partial derivatives of y with respect to the two observables $E_e{}'$ eq 2.46 and F eq 2.47;

$$\frac{\partial y}{\partial E_e{}'} = -\frac{-E_e + F}{(-2E_e + E_e{}' + F)^2}$$
$$\frac{\partial y}{\partial F} = -\frac{-E_e + E_e{}'}{(-2E_e + E_e{}' + F)^2}$$

From chapter 2.2 we have the partial derivatives of y with respect to the two observables $E_e{}'$ eq 2.48 and F eq 2.49;

$$\frac{\partial x}{\partial E_e{}'} = \frac{1}{E_p}$$
$$\frac{\partial x}{\partial F} = \frac{1}{E_p}$$

To obtain the relative resolution of Q^2, substitute equations 2.20, 2.44 and 2.45 into the equation 2.90 and then consider the contribution from $\partial E_e{}'$;

$$\begin{aligned}\frac{\frac{\partial Q^2}{\partial E_e{}'}}{Q^2} &= \left(4E_e - \frac{4E_e{}^2(E_e - F)}{(-2E_e + E_e{}' + F)^2}\right) / \left(4E_e E_e{}' + \frac{4E_e{}^2(E_e - F)}{E_e{}'^2 + E_e{}'(F - 2E_e)}\right) \\ &= \frac{1}{-E_e + E_e'} - \frac{1}{-2E_e + E_e' + F} + \frac{1}{-E_e + E_e' + F}\end{aligned} \tag{2.93}$$

Then consider the contribution from ∂F;

$$\begin{aligned}\frac{\frac{\partial Q^2}{\partial F}}{Q^2} &= \left(\frac{4E_e{}^2(-E_e + E_e{}')}{-2E_e + E_e{}' + F}\right) / \left(4E_e E_e{}' + \frac{4E_e{}^2(E_e - F)}{E_e{}'^2 + E_e{}'(F - 2E_e)}\right) \\ &= \frac{E_e}{(E_e - E_e' - F)(-2E_e + E_e' + F)}\end{aligned} \tag{2.94}$$

Substituting equation 2.93 and 2.94 into equation 2.90;

$$\begin{aligned}\frac{\delta Q^2}{Q^2} = &\left(\frac{1}{-E_e + E_e'} - \frac{1}{-2E_e + E_e' + F} + \frac{1}{-E_e + E_e' + F}\right) \delta E_e{}' \\ &\oplus \left(\frac{E_e}{(E_e - E_e' - F)(-2E_e + E_e' + F)}\right) \delta F\end{aligned} \tag{2.95}$$

To obtain the relative resolution of y, substitute equations 2.19, 2.46 and 2.47 into the equation 2.91 and then consider the contribution from $\partial {E_e}'$;

$$\begin{aligned}\frac{\frac{\partial y}{\partial {E_e}'}}{y} &= \left(-\frac{-E_e+F}{(-2E_e+{E_e}'+F)^2}\right) / \left(\frac{{E_e}'-E_e}{{E_e}'+F-2E_e}\right) \\ &= \frac{F-E_e}{(-E_e+E_e')\,(-2E_e+E_e'+F)}\end{aligned} \tag{2.96}$$

Similarly, consider the contribution from ∂F;

$$\begin{aligned}\frac{\frac{\partial y}{\partial F}}{y} &= \left(\frac{-E_e+{E_e}'}{(-2E_e+{E_e}'+F)^2}\right) / \left(\frac{{E_e}'-E_e}{{E_e}'+F-2E_e}\right) \\ &= -\frac{1}{-2E_e+E_e'+F}\end{aligned} \tag{2.97}$$

Substituting equation 2.96 and 2.97 into equation 2.91

$$\begin{aligned}\frac{\delta y}{y} = &\left(\frac{F-E_e}{(-E_e+E_e')\,(-2E_e+E_e'+F)}\right)\delta {E_e}' \\ &\oplus \left(-\frac{1}{-2E_e+E_e'+F}\right)\delta F\end{aligned} \tag{2.98}$$

To obtain the relative resolution of x, substitute equations 2.21, 2.48 and 2.49 into the equation 2.92 ;

$$\begin{aligned}\frac{\delta x}{x} &= \frac{\frac{1}{E_p}\delta {E_e}'}{(E_e'+F-E_e)/E_p} \oplus \frac{\frac{1}{E_p}\delta F}{(E_e'+F-E_e)/E_p} \\ &= \left(\frac{1}{-E_e+E_e'+F}\right)\delta {E_e}' \oplus \left(\frac{1}{-E_e+E_e'+F}\right)\delta F\end{aligned} \tag{2.99}$$

CHAPTER 3

SUMMARY

This research project investigates six methods for reconstructing deep inelastic scattering (DIS) kinematics at the Electron-Ion Collider (EIC), utilizing combinations of four key observables: the scattered electron energy ($E_e^{'}$), polar angle ($\theta_e^{'}$), and two collider-specific variables (F and γ). While traditional fixed-target experiments primarily rely on $E_e^{'}$ and $\theta_e^{'}$, the collider environment at the EIC facilitates more versatile and potentially more precise reconstruction techniques.

The study also employs analytical error propagation to assess how uncertainties in the measured observables influence the precision of the derived kinematic variables, such as x, y, and Q^2. By applying the total differential formula through partial derivatives, the analysis quantifies the individual contributions of measurement uncertainties to the overall error.

Finally, the resolution of each reconstruction method is evaluated, with a particular focus on relative resolution, which reflects the detector's capability to differentiate between closely spaced values. The objective of this work is to identify the optimal combinations of observables for high-accuracy kinematic reconstruction, thereby enhancing the scientific capabilities of the EIC.

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APPENDIX A

Python codes for plots of relative uncertainties

Python code for plot of $\delta Q^2/Q^2$ vs. Q^2 for different E_e uncertainties

```
import numpy as np
import matplotlib.pyplot as plt

# Constants
Ee = 10  # GeV
Ee_prime = 5  # GeV
theta_deg = np.linspace(5, 175, 500)
theta_rad = np.deg2rad(theta_deg)

# Calculate Q^2
Q2 = 2 * Ee * Ee_prime * (1 + np.cos(theta_rad))

# Function to calculate deltaQ2/Q2
def deltaQ2_over_Q2(delta_Eeprime_frac=0.01, delta_theta_deg=1):
    delta_theta_rad = np.deg2rad(delta_theta_deg)
    tan_term = np.tan(theta_rad / 2)
    return np.sqrt(delta_Eeprime_frac**2
    + (tan_term * delta_theta_rad)**2)

# Plot for varying delta_Eeprime_frac
plt.figure(figsize=(10, 6))
for delta_frac in [0.01, 0.02, 0.05, 0.1, 0.2]:  # 1% to 20%
    res = deltaQ2_over_Q2(delta_Eeprime_frac=delta_frac,
    delta_theta_deg=1)
    plt.plot(Q2, res, label=f"\u03B4Ee'/Ee'=
```

```
----{int(delta_frac*100)}%,-\u03B4 '=1  ")

plt.xlabel('Q$^2$-[GeV ]')
plt.ylabel('\u03B4Q$^2$-/-Q$^2$')
plt.title('\u03B4Q$^2$/Q$^2$-vs-Q$^2$-for-varying
\u03B4Ee / E e  ')
plt.yscale("log")  # Log scale for y-axis
plt.grid(True, which="both", linestyle="--", linewidth=0.5)
plt.legend()
plt.tight_layout()
plt.savefig("delta_Q2_vs_Q2_different_E_uncertainties_log.png",
dpi=300)
plt.show()
```

Python code for plot of $\delta Q^2/Q^2$ vs. Q^2 for different $\theta_e^{'}$ uncertainties

```
import numpy as np
import matplotlib.pyplot as plt

# Constants
Ee = 10  # GeV
Ee_prime = 5  # GeV
theta_deg = np.linspace(5, 175, 500)
theta_rad = np.deg2rad(theta_deg)

# Calculate Q^2
Q2 = 2 * Ee * Ee_prime * (1 + np.cos(theta_rad))

# Function to calculate deltaQ2/Q2
def deltaQ2_over_Q2(delta_Eeprime_frac=0.01, delta_theta_deg=1):
    delta_theta_rad = np.deg2rad(delta_theta_deg)
    tan_term = np.tan(theta_rad / 2)
    return np.sqrt(delta_Eeprime_frac**2 +
    (tan_term * delta_theta_rad)**2)

# Plot for varying delta_theta_deg
plt.figure(figsize=(10, 6))
for delta_theta in [0.1, 0.5, 1, 2, 5]:  # in degrees
    res = deltaQ2_over_Q2(delta_Eeprime_frac=0.01,
    delta_theta_deg=delta_theta)
    plt.plot(Q2, res, label=f"\u03B4  -={delta_theta}  ,
----\u03B4Ee / E e  -=1%")
```

```
plt.xlabel('Q$^2$ [GeV]')
plt.ylabel('\u03B4Q$^2$ / Q$^2$')
plt.title("\u03B4Q$^2$/Q$^2$ vs Q$^2$ for varying e '")
plt.yscale("log")  # Set y-axis to logarithmic scale
plt.legend()
plt.grid(True, which="both", linestyle='--', linewidth=0.5)
plt.tight_layout()
plt.savefig("delta_Q2_vs_Q_logscale.png", dpi=300)
plt.show()
```

Python code for plot of $\delta y/y$ vs. y different E_e uncertainties

```
import numpy as np
import matplotlib.pyplot as plt
np.cot = lambda x: 1 / np.tan(x)
# Define theta' in degrees and convert to radians
theta_deg = 30  # for example, 30 degrees
theta_rad = np.radians(theta_deg)
delta_theta = np.radians(1)  # 1 degree uncertainty

# Define range of y values
y_values = np.linspace(0.01, 0.99, 500)

# Define fractional energy uncertainties from 1% to 100%
delta_Ee_prime_fractions = [0.01, 0.05, 0.1, 0.2, 0.5, 1.0]

# Plotting
plt.figure(figsize=(10, 6))

for delta_Ee_frac in delta_Ee_prime_fractions:
    # Calculate each term in quadrature
    term1 = (1 - 1 / y_values) * delta_Ee_frac
    term2 = (1 / y_values - 1) * np.cot(theta_rad / 2) *
    delta_theta
    total_uncertainty = np.sqrt(term1**2 + term2**2)

    label = f"\u03B4Ee'/Ee' = {int(delta_Ee_frac * 100)}%"
    plt.plot(y_values, total_uncertainty, label=label)

plt.xlabel("y")
plt.ylabel("Relative Uncertainty \u03B4y/y")
plt.title("\u03B4y/y vs y for Different \u03B4Ee / E e ")
plt.grid(True)
plt.legend()
```

```
plt.ylim(0, 2)
plt.tight_layout()
plt.savefig("delta_y_vs_y_E.jpg", dpi=300)
plt.show()
```

Python code for plot of $\delta y/y$ vs. y different $\theta_e^{'}$ uncertainties

```
import numpy as np
import matplotlib.pyplot as plt

# Fixed theta' in degrees and convert to radians
theta_deg = 30
theta_rad = np.radians(theta_deg)

# Fixed energy uncertainty
delta_Ee_frac = 0.05  # 5%

# Define range of y values
y_values = np.linspace(0.01, 0.99, 500)

# Define different angle uncertainties in degrees,
then convert to radians
delta_theta_degrees = [0.1, 0.5, 1, 2, 5]
# test different angle uncertainties
delta_theta_radians = [np.radians(dt) for dt in
delta_theta_degrees]

# Plotting
plt.figure(figsize=(10, 6))

for delta_theta in delta_theta_radians:
    # Calculate each term in quadrature
    term1 = (1 - 1 / y_values) * delta_Ee_frac
    term2 = (1 / y_values - 1) * (1 / np.tan(theta_rad / 2)) *
    delta_theta
    total_uncertainty = np.sqrt(term1**2 + term2**2)

    label = f"\u03B4 e =-{np.degrees(delta_theta):.1f} "
    plt.plot(y_values, total_uncertainty, label=label)

plt.xlabel("y")
plt.ylabel("Relative Uncertainty \u03B4y/y")
```

```
plt.title("\u03B4y/y vs y for Different \u03B4 e ")
plt.grid(True)
plt.legend()
plt.ylim(0, 2)
plt.tight_layout()

# Save the plot
plt.savefig("delta_y_vs_y_different_theta_uncertainties.png",
dpi=100)

plt.show()
```

Python code for plot of $\delta x/x$ vs. x different $\theta_e^{'}$ and E_e uncertainties

```
import numpy as np
import matplotlib.pyplot as plt

# Assumed values for Constants
Ee = 10  # GeV
Ep = 275  # GeV
theta_deg = 30
theta_rad = np.radians(theta_deg)

cos_theta = np.cos(theta_rad)
sin_theta_half = np.sin(theta_rad / 2)
tan_theta_half = np.tan(theta_rad / 2)

# Calculation
def compute_Q2(Ee, Ee_prime, theta_rad):
    return 2 * Ee * Ee_prime * (1 + np.cos(theta_rad))

def compute_y(Ee, Ee_prime, theta_rad):
    return 1 - (Ee_prime * (1 - np.cos(theta_rad))) / (2 * Ee)

def compute_x(Ee, Ep, Ee_prime, theta_rad):
    numerator = 2 * Ee_prime * (1 + np.cos(theta_rad))
    denominator = 4 * Ep * (1 - (Ee_prime *
    (1 - np.cos(theta_rad))) / (2 * Ee))
    return numerator / denominator

# Relative uncertainty in x
def delta_x_over_x(x, y, delta_Ee_frac, delta_theta_rad):
    term1 = (1 / y) * delta_Ee_frac
```

```
    term2 = ((x / (Ee / Ep)) - 1) * tan_theta_half *
    delta_theta_rad
    return np.sqrt(term1**2 + term2**2)

# Generate Ee' range
Ee_prime_range = np.linspace(1, Ee - 0.1, 1000)

# Compute x and y values for given theta
x_vals = compute_x(Ee, Ep, Ee_prime_range, theta_rad)
y_vals = compute_y(Ee, Ee_prime_range, theta_rad)

# Plot 1 (Fig2.6): Vary deltaEe/Ee
delta_Ee_list = [0.005, 0.01, 0.02, 0.05, 0.1]
fixed_delta_theta = np.radians(0.5)
# Fixed angularuncertainty
colors = plt.cm.viridis(np.linspace(0, 1, len(delta_Ee_list)))

plt.figure(figsize=(10, 6))
for i, delta_Ee_frac in enumerate(delta_Ee_list):
    deltas = delta_x_over_x(x_vals, y_vals, delta_Ee_frac,
    fixed_delta_theta)
    plt.plot(x_vals, deltas, label=f"\u03B4Ee / Ee =
----{delta_Ee_frac*100:.1f}%", color=colors[i])

plt.xlabel("x")
plt.ylabel("\u03B4x / x")
plt.title("Relative Uncertainty \u03B4x/x vs x Varying
\u03B4Ee ")
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.savefig("rel_uncertainty_x_vs_x_varying_dEe_2.png",
dpi=300)
plt.show()

# Plot 2(Fig2.5): Vary delta e
delta_theta_list_deg = [0.1, 0.2, 0.5, 1.0, 2.0]
delta_theta_list_rad = np.radians(delta_theta_list_deg)
fixed_delta_Ee = 0.01  # Fixed energy uncertainty
colors = plt.cm.plasma(np.linspace
(0, 1, len(delta_theta_list_rad)))

plt.figure(figsize=(10, 6))
```

```
for i, delta_theta_rad in enumerate(delta_theta_list_rad):
    deltas = delta_x_over_x(x_vals, y_vals, fixed_delta_Ee,
    delta_theta_rad)
    plt.plot(x_vals, deltas, label=f"\u03B4  e =
----{delta_theta_list_deg[i]:.1f}  ", color=colors[i])

plt.xlabel("x")
plt.ylabel("\u03B4x / x")
plt.title("Relative Uncertainty \u03B4x/x vs x
Varying \u03B4  e ")
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.savefig("rel_uncertainty_x_vs_x_varying_dtheta_2.png",
dpi=300)
plt.show()
```