THE POWER OF CROWD IN THE BUSINESS WORLD

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ABSTRACT

In this work, we focus on two attractive crowd-based business models, specifically, user-generated content creation and freight-matching long-haul trucking. First, as elaborated in CHAPTER 2, we consider a game theoretical modeling approach for understanding the operation of non-profit UGC platforms that rely on users to create content and maintain financial sustainability. In particular, we examine several interesting research questions with practical importance and unique contributions to the literature. These research questions mainly investigate how changes in critical business factors influence the platforms' strategic effort allocation, user participation, and overall performance. Second, in CHAPTER 3, we focus on the flourishing freight-matching businesses that rely on crowdsourced drivers for long-haul trucking. In particular, although the practice suggests that shippers' ordering behaviors of freight-matching services may remarkably impact crowdsourced drivers' bidding behaviors, the literature has yet to examine this issue formally. Therefore, we collect industrial data and construct a strict empirical schema for understanding the association between shippers' order timing and freight-matching performance. Besides, by deliberately building a theoretical modeling framework and using a data-driven estimation of model parameters, we are able to simulate the freight-matching performance of adopting our empirical findings and evaluate the practical value of our study. By investigating these two prominent business models, we aim to understand the advantages of crowdsourcing businesses and the role of crowds in nowadays' business innovations. Besides, we also provide valuable managerial insights for business runners who are interested in this "young" market of crowdsourcing businesses.

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CHAPTER 1

INTRODUCTION

Business innovations and rapid growth in information technologies have marked business in the 21st century. The advancement of information technologies incubates a series of innovative business concepts, such as blockchains, crowdsourcing, and user-generated content (UGC), and brings vast opportunities to business runners and managers. Interestingly, one common feature of many emerging business innovations is the involvement of crowds. In particular, rather than solely relying on the in-house workforce (i.e., hiring and using its own employees) to complete every business task, the innovative businesses in the 21st century manage to utilize the power of crowds to speed up the business progress and reduce some aspects of operational costs, and many of them have achieved considerable successes. The success of crowd-based business models also results in a fruitful area of research and expanded literature. Despite the existing intensive investigations of these crowd-based business models, as they are still relatively "young" in the entire business world, there are still plenty of puzzles awaiting further examinations. Therefore, in this dissertation, we focus on two of the most interesting crowd-based business models, i.e., the non-profit UGC platforms and the freight-matching long-haul trucking, and examine several research questions that the practice and literature deem important but have yet to investigate. We now elaborate on each of them in a few paragraphs below.

1.1. Sustainability of Non-Profit User-Generated Content Platforms

Following the huge success made by Wikipedia in 2006, the concept of "non-profit UGC" has been flourishing and widely adopted by crowd-based peer collaboration online programs. In particular, rather than creating website content solely relying on their in-house labor force and maintaining financial sustainability by themselves, these non-profit UGC platforms utilize their users' content contributions and donations to grow their content and raise the budget. Besides, unlike traditional businesses that consider profit and sales the primary objectives, non-profit UGC platforms usually focus on improving their content quality and user experiences.

Due to the prominence of the non-profit UGC business model, earlier studies have intensively investigated its nature and user behaviors, and many of them have built profound findings and valuable insights. However, after a deliberate review of the literature, we notice that these studies have overlooked the interactive effects of the UGC creation and the platform's financial sustainability. Besides, although earlier studies have examined the effectiveness of community management strategies for UGC creation and donations, they have yet to formally investigate how the non-profit UGC platforms can optimally allocate their community management efforts to achieve better business performance. Therefore, we consider a game-theoretical model that captures interactions between users and non-profit UGC platforms from a managerial perspective.

This study investigates several interesting business phenomena that the practice and/or literature deem critical. In particular, as the recent rapid advancement of generative AI technologies allows a more efficient enrichment of non-profit UGC platforms, bringing tremendous opportunities and challenges to their business operation, we examine its influences on the community management strategy and overall performance of non-profit UGC platforms.

Second, the generousness of users is fundamental for the business operation of non-profit UGC platforms. Besides, the earlier studies have emphasized the impact of user heterogeneity on their participation in online platform businesses. However, the user heterogeneity of their generousness has yet to be formally investigated in the research context of non-profit UGC. Therefore, we analytically examine this issue and provide valuable insights for business stakeholders of non-profit UGC platforms.

Thirdly, as success only accounts for a tiny part of the business world of non-profit UGC, it is essential to understand the failures and how to overcome the potential obstacles faced by non-profit UGC platforms. Therefore, based on the analyses of some critical factors, we establish several vital findings and provide constructive suggestions on this issue. We relegate more details of this study in CHAPTER 2.

1.2. Crowd-Sourcing Workers in On-Demand Freight-Matching Platforms

With the rapid expansion of the manufacturing and retail businesses in past decades, freight transportation demand has kept a stable growth profile. Among all types of freight transportation, long-haul transportation plays a critical role in ensuring efficient movement and steady availability of production materials, products, and physical resources. Due to its competitive cost, flexibility, and simplicity, long-haul trucking owns unique business advantages over other types of transportation (e.g., railway, water, and aviation freight) and contributes considerably to national and global economics. However, traditional long-haul trucking companies (i.e., carrier companies) usually suffer from substantial expenditures in purchasing vehicles, maintaining equipment, and employing full-time drivers. Therefore, they cannot expand their business very rapidly and, thus, usually hold a limited capacity (i.e., an insufficient amount of vehicles or drivers when there is an overcrowding of shippers' transportation requests).

Fortunately, 'crowdsourced transportation' (or 'crowd-shipping') emerges as a prospective alternative for business runners of long-haul trucking who intend to improve the cost-effectiveness of their business further. In particular, crowdsourced transportation companies (or freight-matching platforms, as explained in CHAPTER 3) serve shippers' shipping requests using the crowdsourced workforce, such as drivers from carrier companies or individual drivers who work for themselves. Following the business achievement of several successful freight-matching platforms, such as C.H.Robinson, UShip, and Uber Freight, this innovative business model has attracted exceptional attention from both the practice and literature.

Although the literature has been expanded on examining the routing and dispatching issues faced by freight-matching platforms, the earlier studies of this business context barely discussed the role of shippers in the freight-matching process. Although a handful of studies, such as Park et al. (2023), noticed the critical role of shippers in impacting freight-matching performance, they have yet to examine the influence of shippers' order timing formally. Therefore, we investigate this issue by adopting a strict empirical modeling framework. In particular, we define a helpful concept named "request lead time," i.e., the time difference between shippers' request submission and pickup date of freight, to measure the shippers' order timing in a standardized manner. Besides, instead of simply adopting the profit as the performance metric of freight matching, we consider the freight-matching probability (i.e., the probability of matching a shipping request) and the freight-matching sourcing cost (i.e., the payment made by the platform to drivers) to evaluate the freight-matching performance and provide more insightful discussions.

In this study, we also propose several effective modeling manners to solve the econometric challenges in modeling the association between shippers' order timing and freight-matching performance. Specifically, we organically integrate the Heckman model and two-stage residual inclusion manner to solve the selection bias and endogeneity issues in our model estimation. To better evaluate the practical value of our empirical findings in actual business operations, we also consider a counterfactual analysis and simulation process to mimic the freight-matching performance if shippers are inspired or not inspired to provide their shipping request at the optimal request lead time. Based on the above manners, we offer valuable insights to platform stakeholders and managers, as well as unique contributions to the literature. We relegate more details of this study into CHAPTER 3.

In summary, by deliberately investigating two prominent crowd-based business models and several relevant issues that the practical/literature deems important, we aim to understand the role and power of crowds in this new era of business. We hope the insightful findings and discussions offered by our studies can generate practical values and provide inspirational contributions to future research on crowd-based businesses.

CHAPTER 2

SUSTAINABILITY OF NON-PROFIT USER-GENERATED CONTENT PLATFORMS: THE ROLE OF CONTENT CREATION AND DONATIONS

2.1. Abstract

Content on non-profit user-generated content (UGC) platforms, such as Wikipedia, is generally created and maintained as open collaboration projects among users and the platform. Furthermore, the financial sustainability of these platforms depends on funding from external parties and donations of users, which is similar to pay-what-you-want (PWYW) contexts. Although the sustainability of this business model is being questioned, it has not been investigated in the literature. We fill this critical gap by studying PWYW and UGC concurrently. We formulate a game-theoretical model that considers users' donations and content contributions, as well as the platform's fundraising, community support, and in-house quality improvement efforts. First, regarding some platforms that failed to thrive, we find that a high barrier to contributing content always leads to poor overall content quality on UGC platforms. However, a small user community does not necessarily result in low overall content quality. Second, although "making stingy users generous enough" and "making generous users more generous" both increase the aggregate generousness level of users, they reflect opposite changes in the user heterogeneity and improve the total donation at different degrees in different scenarios. Third, with higher efficiency in exerting in-house effort, a UGC platform may suffer from a lower level of aggregate content contribution from users and/or loss of content contributors. However, a loss in the number of content contributors does not necessarily mean a reduction in overall content generation. We provide several other results and managerial insights that could be useful for policy-makers and UGC platforms to formulate user-management strategies.

2.2. Introduction

User-generated content (UGC) is a form of content created and posted by users on online platforms such as social media and wikis (e.g., Wikipedia, Facebook, Twitter, and Quora). It is used for a wide range of applications, including product evaluation (e.g., Yelp and Amazon collect customer ratings of products) (Chen et al. 2021, Gutt et al. 2019, Ho et al. 2017); problem processing (e.g., people post their questions and help others on Quora); advertising (e.g., TINT helps brands collect UGC and display them in marketing campaigns); use/customization of software (e.g., see Cavusoglu et al. 2020); and information spread and research (e.g., Wikipedia and other online UGC platforms) (ReleaseWire 2021). It has become a powerful tool that virtually every manager tries to utilize. A recent report claimed that more than 86% of companies today use UGC as part of their business strategy (Bennett 2024). The spread of UGC implies that content creation is fundamentally different from that in conventional businesses. This shift has resulted in the emergence of an Internet-enabled commons-based peer production model – a socio-economic production model in which a large number of people work together (Liu and Feng 2021). A prominent example of UGCs is Wikipedia. Since its inception in 2001, Wikipedia has grown from a simple English online UGC platform to a vast platform with more than 300 languages and six million articles (Wikipedia 2024a).

Wikipedia relies on two financial streams to maintain its operations. First, it collects donations from ordinary users. To raise more money, Wikipedia exerts a fundraising effort that enhances users' generousness by operating fundraising activities and building relationships with users. Second, Wikipedia also receives funding from external parties that use its content to facilitate their businesses (e.g., Amazon and Google, see Lisa 2019). Wikipedia's parent organization, i.e., Wikimedia Foundation, received over $169 million from all sources in 2022-2023. While a significant portion of these donations were small and made from individual donors through various channels (e.g., mobile apps, web portals, and sidebars), around 19 million U.S. dollars were major gifts (i.e., donations of 1,000 U.S.

dollars and above) from big donors and third parties (as shown in Figure 7). The reliability or the quality of UGC content is a critical factor that influences the willingness of third parties to support non-profit UGC platforms (e.g., see LinkedIn 2023). Besides, Wikimedia encourages companies who use Wikipedia's content to give back in the spirit of sustainability (see Heater 2018). Considering that the quality of the content is a critical factor that attracts companies to adopt UGC content, a higher level of overall content quality can attract more companies to utilize content from the non-profit UGC platform, and hence, increase third-party funding. The way of simultaneously managing content creation and maintaining its financial sustainability makes Wikipedia's business setting unique, and we call this business model "*Wikipedia's business model*."

Following Wikipedia's success, this business model is widely adopted to create non-profit commons-based peer collaboration projects. Unlike traditional online platforms, these non-profit UGC platforms do not seek financial returns but set the platform content's quality, reliability, and enrichment as their primary objectives (Wikipedia 2024d). The measurement of quality usually consists of many different perspectives. For example, Wikipedia has developed a quality assessment system to curate its articles by accounting for the features of articles and readers' experience (Wikipedia 2024d).[1] Non-profit UGC platforms continuously improve and build up their content to keep up with their users' needs, mainly in two ways. First, members of the UGC community modify and add new content (Bhargava 2021). To encourage more non-contributor users to contribute content and inspire existing contributors to make greater content contributions, the non-profit UGC platform implements a community support effort to reduce the barrier to contributing content. Second, the UGC platform makes an in-house effort to modify existing content, add new content, and improve the content structure, design, and user experience (Wikimedia 2022a).

An increasing number of online collaboration programs treat *Wikipedia's business model*

[1] As of November 2022, over seven million articles have been assessed, and this curation helps Wikipedia measure its overall quality.

as a prominent model and adopt it in their businesses (MediaWiki 2024, Wikipedia 2024g, Zhang et al. 2012). However, because the characteristics of the technology landscape, the platform, and the user community (such as their heterogeneity) influence the decisions and performance levels of both users and platforms, it is complicated and challenging for non-profit UGC platforms to thrive in today's rapidly evolving tech environment and with varied user demographics. Indeed, only a few of them have succeeded, while others like LocalWiki have been struggling to be viable. To understand why these platforms fail to thrive and because an active UGC without a professionally managed platform is not bound to be successful and vice versa (AdAlone 2022), we study how these platforms should manage their user communities to achieve better results and how the characteristics of the platform and users influence this interaction. This is also one of our key contributions to the literature.

Furthermore, the UGC creation and financial sustainability perspectives of non-profit UGC platforms interactively impact platforms' performance and each other. Specifically, as shown in Figure 1, the content creation on a platform impacts its overall content quality, causing higher or lower funding from third parties and individuals. In turn, given financial and operational constraints, this change in funding can either constrain or enable the platform to increase its efforts in UGC. Hence, understanding these interactive impacts on the platform's performance is crucial for understanding the decision mechanisms of the users and the UGC platform. However, earlier studies mostly focus on either UGC creation or fundraising and overlook their interactions. In effect, the literature mainly examines the effectiveness of user management strategies (e.g., Burtch et al. 2022, Gallus 2017, Huang et al. 2019), users' strategic behaviors (e.g., Ahn et al. 2015, Zhang et al. 2012), or UGCs' business value (e.g., Timoshenko and Hauser 2019, Yi et al. 2019), without considering the platform's financial sustainability. On the other hand, studies of charitable giving usually stand on research contexts very different from ours, e.g., vulnerable groups (e.g., Khadjavi 2017, Sudhir et al. 2016) or disaster relief (e.g., Aflaki and Pedraza-Martinez 2023, Toyasaki and Wakolbinger 2014). Hence, their findings do not directly apply to non-profit UGC platforms. Therefore, we attempt to capture the decision mechanisms of both users and the platform by simultaneously accounting for UGC creation and donations. This is another unique contribution of our work to the literature.

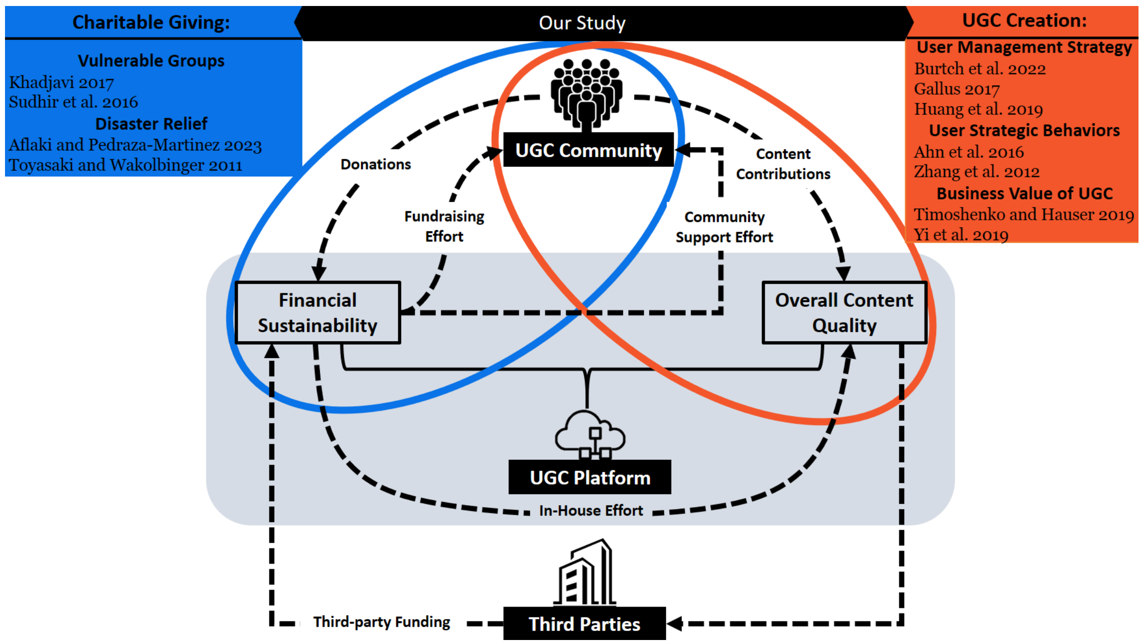

Figure 1. *Business Model of Non-Profit UGC Platforms and Literature Review of Relevant Research Contexts.*

2.3. Research Questions and Contributions

In recent years, as non-profit UGC platforms are proliferating, the industry and academia have paid increasing attention to Wikipedia's business model. Although their high annual expense on community support and fundraising implies the management of user communities is a vital component for these platforms to achieve business success, the prior studies mainly focus on the nature of UGC business (e.g., Ahn et al. 2015, Zhang et al. 2012) and the motivations of content contributors (e.g., Burtch et al. 2022, Gallus 2017, Huang et al. 2019, Liu and Feng 2021) but overlook the importance of strategic management of user communities. Besides, although financial sustainability and UGC creation interactively influence the improvement of overall content quality for non-profit UGC platforms, there is a dearth of studies examining and accounting for both donation and content contribution simultaneously. We fill these gaps by, first, building a model to investigate business settings similar to Wikipedia. Second, by utilizing a unified utility function for users that captures their heterogeneity, we endogenize the process of determining which users contribute content and/or make donations (and if they do, the extent of content contributions and donations). Based on the equilibrium solution of our model, we formally address the questions outlined in the rest of this section.

First, as discussed before, rather than solely relying on users' content contributions to enrich or modify the content, non-profit UGC platforms exert in-house quality improvement effort too (Wikimedia 2022a). Recent developments in generative AI are promising factors in this landscape that may improve the efficiency of the platforms' in-house quality improvement efforts (Maruccia 2023, Masolo 2022). However, the literature has yet to explore the impact of this issue on the platform's community management strategy, i.e., fundraising and community support efforts, but similar questions are raised in practice (Ackermann 2024). Therefore, we ask: *Do the platform's fundraising and community support efforts increase if it becomes more efficient in its in-house quality improvement effort?* Related to this issue, we also investigate the influence of better in-house efficiency (e.g., due to the improvements in generative AI[2]) on the users' content creation efforts and donations. As the literature has yet to examine this issue formally and it is debated in practice (Ackermann 2024), we further ask: *Does users' participation, at an aggregate level, increase if the platform becomes more efficient in its in-house quality improvement effort?* We address these questions and show that the answers are not trivial. In particular, our results indicate that although the increased efficiency of the in-house quality improvement effort helps the platform achieve higher overall content quality, it may reduce users' content contributions. We also find that the increased efficiency of the in-house quality improvement effort does not necessarily result in a reduction in other efforts. Our findings (i) reveal the importance and impact of the cost-effectiveness of in-house effort on the platform performance

[2]Generative AI for improving editorial efficiency but not for generating content (Deckelmann 2023).

and (ii) elucidate ways in which non-profit UGC platforms could strategically shift their effort allocation depending on changes in their cost-effectiveness of in-house effort.

Second, the practice and literature imply that people are heterogeneous in their generousness (Levina and Arriaga 2014, Marsh 2016). Furthermore, the literature has argued user heterogeneity as a critical factor for user participation (Demirezen et al. 2016, Srinivasan 2006) in online businesses. Since non-profit UGC platforms make their business model different from traditional UGC platforms by intensively relying on users' donations, understanding how user heterogeneity affects non-profit UGC platforms' financial sustainability is important for platform managers. However, to the best of our knowledge, although the literature examined the altruistic motivation of UGC contributors (e.g., Alam and Campbell 2017, Gallus 2017) and charitable giving (e.g., Andreoni and Serra-Garcia 2021, Khadjavi 2017), only a handful of them account for the user heterogeneity on businesses' financial performances. For example, Zhang et al. (2012) accounts for the heterogeneity in the value of buyers and sellers across product categories, while Yi et al. (2019) considers the heterogeneous preferences of individual reviewers in reviewing the same product. However, these studies focus on how businesses use UGC as a tool to promote sales, which is different from our research context. Since prior studies overlooked the influence of user heterogeneity in the financial sustainability of non-profit UGC businesses, but this is a highly debated issue in practice (Philipose 2020), we ask: *Whether the platform should "make the stingy generous enough" so that they make donations, or "make the generous more generous" so that they make higher levels of donations?* Note that the heterogeneity is inversely impacted if the already generous users (who are more generous than the general population) become more generous (i.e., heterogeneity increases) compared to stingy users (who do not make donations) become more generous (i.e., heterogeneity decreases). Our findings imply that although both options improve the platform's financial sustainability, the answer to the proposed question is rather nuanced. By addressing this question, we provide practical insights for non-profit UGC businesses and fill the gap in the literature on the link between user heterogeneity and the platform's financial sustainability.

Third, although some non-profit UGC platforms have been relatively successful, many others that generally cater to the interests of specific groups have failed to thrive. The literature and practice implicitly or explicitly suggest several reasons for the poor performance of these special-interest platforms. In particular, the literature conventionally associates an increased number of users with better performance metrics (e.g., see Zhang et al. 2012). As users are one of the main drivers of UGC creation, the relatively small user community may be a potential cause of these platforms' poor overall content quality. Furthermore, as these platforms usually cater to special interest groups, contributing to content generally requires a certain level of expertise on these platforms. Therefore, in addition to having a smaller user base, content contribution hurdles on the platform may hinder the improvement of the overall content quality and these issues are debated in practice (Everipedia 2019, Torres 2016). However, the literature has not addressed these issues yet. Therefore, we investigate: *do a higher barrier to contributing content and a lower number of users always lead to worse platform performance?* Our analysis helped us glean the following insights: First, while a higher contribution hurdle always results in lower overall content quality; expanding the user community improves the overall content quality, but interestingly, not always. Second, the effectiveness of improving the overall content quality by expanding the user base depends on the platform's contribution barrier.

In summary, utilizing a game-theory approach, we formally analyze the issues the practice deems important and produce several insights valuable for managers and other stakeholders. First, regarding the potential enhancements in UGC platforms' efficiency of in-house quality improvement due to recent developments in generative AI, we find that becoming more efficient in in-house efforts does not necessarily mean a reduction in other efforts. We also find that enhancements in UGC platforms' efficiency of in-house quality improvement may interestingly reduce users' content contributions. Second, regarding users' donation behaviors, we investigate the influence of generousness level on user heterogeneity and on the platform's financial sustainability. We find that the relationships between these factors are rather nuanced. Third, although some non-profit UGC platforms have been relatively successful, others (that generally cater to the interests of specific groups) have failed to thrive. Therefore, we investigate the impacts of the community size of users and the content contribution hurdle on platform performance as they are implied in the literature or practice as possible drivers of platform failure. Hence, our work contributes to several literature streams as we discuss in more detail in the next section.

2.4. Literature Review

Our study contributes to the following literature streams: (i) UGC, (ii) PWYW and advertisement, and (iii) warm glow for public goods. Next, we compare and contrast our work in these streams.

2.4.1. UGC

We utilize the literature on UGC platforms to motivate different components of our model, as discussed in detail in CHAPTER 2.5. Here, we discuss how our study is different than and how we contribute to this stream of literature. The literature focuses on examining the motivations of UGC contributors (e.g., Burtch et al. 2022, Gallus 2017, Huang et al. 2019) and the business value of commercial UGC applications (e.g., Goh et al. 2013, Song et al. 2019, Zhang et al. 2012). However, prior studies in the context of non-profit UGC platforms are scarce, and they do not focus on the strategic governance of user communities from a managerial perspective. Our work contributes to this literature stream by formally investigating this business setting by accounting for the strategic management of the user community.

Furthermore, the recent developments in generative AI (e.g., ChatGPT) are argued to influence UGC platforms either positively or negatively - by perhaps boosting the efficiency of their in-house quality improvement effort (Maruccia 2023, Masolo 2022). However, these issues have not been addressed in the literature yet. Therefore, we investigate how the efficiency of the platform's in-house quality improvement effort influences the platform's performance and users' participation. This is the second way our study is different from the literature.

Moreover, although the practice argues that user generousness is an essential factor in the financial sustainability or success of online UGC platforms (e.g., see Alam and Campbell 2017, Andreoni 1990), this issue has yet to be examined in the non-profit UGC context. Although a handful of prior studies discuss the economic value of retaining or acquiring users for UGC platforms, they are mostly in for-profit settings, and they do not account for the role of user generousness (e.g., Song et al. 2019, Zhang et al. 2012). Therefore, in this work, we formally investigate the impact of user generousness and its heterogeneity on platforms' financial sustainability, and this is the third way our study differs from and contributes to this literature stream.

2.4.2. PWYW and Advertisement

Second, our paper is related to a recent pricing practice dubbed "pay what you want" (PWYW), where consumers can pay any amount, sometimes including zero (i.e., making no payment) for a service or product they use. A body of work in this area examines the problem empirically. We refer readers to Gerpott (2017) for an overview of these studies. The focus of this stream is generally on understanding (i) the role of consumers' fairness (e.g., see Jang and Chu 2012), (ii) the tension between self-interest and the notion of helping others (e.g., see Fehr and Klaus M. 1999), or (iii) the comparison of PWYW with a fixed-price mechanism (e.g., see Mak et al. 2015). Whereas on the analytical modeling side, Chen et al. (2017) and Kim et al. (2022) are two representative examples that investigate the advantages and adaptability of PWYW pricing in different for-profit business and market settings. However, unlike our work, the literature has not formally investigated this innovative business strategy in non-profit UGC platforms where payments are actually donations - that are inherently different than pure payments.

Furthermore, by accounting for the influence of fundraising activities on users' donation behavior in Wikipedia's business model, our paper also closely relates to the literature stream on advertisements to inspire charitable giving. For an overview of the research stream, we refer readers to Wymer and Gross (2021). Earlier studies in this literature suggest that people's donation behaviors are influenced by fundraising activities and advertising strategies (Ryzhov et al. 2016). For example, Schlosser and Levy (2016) empirically finds that a downward-comparison (e.g., "I am in need and you can help") advertising strategy both inspires more people to donate and increases individuals' donation amounts. Besides, Sudhir et al. (2016) validates that the advertising content may significantly influence people's donation behaviors and individual donation amount. Our study incorporates these findings by accounting for the influence of fundraising activities on users' donation behaviors. To the best of our knowledge, our work is unique in its contribution to PWYW contexts as it incorporates the impact of advertising on user generosity.

2.4.3. Warm Glow for Public Goods

Third, our study is related to a relatively newer economic theory, "warm glow," that describes the emotional reward of charitable giving, where people experience a sense of self-pleasure and satisfaction for "doing their part" to help others (Richefort 2018). In contrast to classical pure altruism theory (Andreoni 1990), by combining both altruistic and egoistic motivations (or practical utility) of "giving" (Krishnamurthy and Tripathi 2009), the warm glow explains giving to public goods, which is our focus in this study. Kocielnik et al. (2018) examine Wikipedia pages to investigate the relationship between the characteristics of a page and the amount of donations and suggest a positive correlation between utility and users' giving, which we also capture in our model.

In particular, the literature on warm glow focuses on the mechanisms of mixed altruism and motivation. By constructing a general model, Andreoni (1990) utilizes altruism, egoism, and impure altruism to explain the warm glow that agents feel upon their contribution to charities. In a platform context, Chen et al. (2018) examines the effect of mixed motivation, which includes altruism and private benefits, on the contribution of domain experts to public information goods. However, most of these studies only consider the motivation of users to contribute (donate) or how the demographic features of users affect their contribution/ charitable giving. Our study extends the literature by incorporating a platform's reactions to environmental characteristics and user behavior.

Beyond the above literature streams that are most relevant to our research context, our study also relates to the literature streams on crowdfunding, open-source projects, business innovation, and R&D. However, these streams either do not directly relate to our research focus or consider business settings different from ours. For example, the R&D literature addresses a technology that could potentially create a breakthrough in one or more ways the business is done (or a product is manufactured). On the other hand, in our context, the in-house quality improvement effort is an improvement in the content quality on the platform; it is not about developing a technology or a new method. Therefore, we skip a detailed discussion for brevity.

In sum, we consider several important characteristics of the non-profit UGC platforms, technology landscape, and user communities in our work, such as the more efficient in-house quality improvement effort (due to improvements in generative AI), warm glow, and user heterogeneity in generousness. By doing so, we are able to comment on some important issues that are debated in practice, such as why it is hard for some non-profit UGC platforms to thrive. Furthermore, unlike the literature, we utilize a comprehensive approach and study donations and content contributions simultaneously. This is because, given financial and operational constraints, the platform has to deliberately decide on its effort allocations on fundraising efforts (so that it can raise more funding to improve the content) and on content creation (so that it can enhance the content quality and raise more funding from external parties). Furthermore, the level of content quality impacts funding, and funding impacts quality improvement efforts. Therefore, the financial aspect and content creation impact each other in a rather complicated way, so it is crucial to account for both to provide meaningful managerial insights.

2.5. Model

Consider a platform such as Wikipedia that relies mostly on UGCs to generate content, as well as external parties and ordinary users, to maintain financial sustainability. As in the case of Wikipedia, there are three different efforts of the platform: (i) the community support effort - to improve the editing effort of the UGC community, (ii) the fundraising effort - to increase donations, and (iii) the in-house content quality improvement effort. On the other hand, any interested individual can donate and/or contribute content to improve the content in the platform (Wikimedia 2022a).

Therefore, we construct a sequential game model to capture the interactions between the platform and users, where the platform implements community support effort (denoted by h) to reduce the barrier for users to contribute content (e.g., make content contribution easier), fundraising effort (denoted by f) to induce users to make donations, and in-house quality improvement effort (denoted by v) to directly improve the overall content quality (i.e., Π). Out of the population of users of size $R = R_0 + c\Pi_0$ (where R_0 indicates the base level of the user population, and $c\Pi_0$ represents the sensitivity of the user population to initial quality), each user is a potential *contributor* that adds new content or edits existing content, and a possible *donor* that donates to the platform.[3] Specifically, observing the effort levels of the platform, user i makes her donation (denoted by d_i) and contribution (denoted by e_i) decisions based on her altruistic motivations, external utilities for her efforts (e.g., reputation for making content contributions and tax deductions for donations), and expected improvement in the platform's overall content quality (i.e., reciprocal mechanism). We analyze the equilibrium behavior of the platform and its users in this setting. In particular, we are interested in how changes in the characteristics of users or the platform affect their equilibrium behaviors and the eventual impact of such changes on important measures such as overall content quality and financial sustainability. Table 1 summarizes

[3]We consider this formulation based on the fact that anyone can observe the state of the platform before they decide whether to be a user of the platform, and the quality at that point is positively correlated with the number of users. So, we capture in our stylized model that a better platform has more users.

the key variables and parameters. Next, we discuss each component of our model in detail.

Symbol	Definition
Outcome Variable	
Π	Platform's overall content quality
C (resp., D)	Total content contribution (resp., Total donation) from all users
$E[K_e]$ (resp., $E[K_d]$)	Expected number of users who make content contributions (resp., donations)
Decision Variables	
e_i	Content contribution effort exerted by user i
d_i	Donation amount by user i
f (resp., h)	Level of platform's fundraising effort (resp., community support effort)
v	Level of platform's in-house quality improvement effort
Parameters	
Π_0	Platform's initial content quality level
a_d	Platform's efficiency of in-house quality improvement effort
a_e	Platform's efficiency in utilizing users' contribution efforts
w_0	Constant for external party funding
w	Sensitivity of external party funding rate to the platform's overall quality
g (resp., x)	Fixed operational cost of the platform (resp., variable cost of serving each user)
R_0	Base number of users on the UGC platform
c	Sensitivity of the number of users to the base overall content quality
r	Platform's financial reserves
$b_{e,i}$	Generousness level of user i in contributing content, $b_{e,i} \sim U(\underline{b_e}, \overline{b_e})$
$b_{d,i}$	Generousness level of user i in making donations to the platform, $b_{d,i} \sim U(\underline{b_d}, \overline{b_d})$
q_e (resp., q_d)	Barrier to making content contributions (resp., donations)
c_e (resp., c_d)	Users' sensitivity to the disutility of contributing content (resp., making donations)

Table 1. *Key Variables and Parameters*

2.5.1. Platform's Efforts

Although we motivate our model with the business practice of Wikipedia as an example, our study can be applied to other non-profit UGC platforms that rely on user-side donations and contributions. Our model applies to settings where platforms rely on users regarding only donations or contributions or on platforms themselves. In practice, non-profit UGC platforms like Wikipedia utilize two revenue streams for financial sustainability. In 2019 alone, Google Inc. made a donation to Wikipedia worth two million dollars (Lisa 2019), while in 2018, Amazon made a one million dollar donation (Bariso 2018). Wikimedia states that companies who utilize Wikimedia's content give back in the spirit of sustainability (Bariso 2018). The reliability or quality of UGC content is then a critical factor that influences external funding from third parties (LinkedIn 2023). In other words, funding from third parties is positively related to the platform's overall content quality. Hence, we model the external funding by $w_0 + w\Pi$, where w_0 is a constant and w is the sensitivity of the funding rate to the overall content quality.

Furthermore, Wikipedia also relies on users' donations. Over the fiscal year 2020-2021, Wikipedia raised over 155 million dollars from donations (Wikimedia 2022b). Wikipedia's annual report states that fundraising activities are important annual expenses (Wikimedia 2022a). Specifically, Wikipedia exerts effort to increase its public visibility and people's awareness of the importance of Wikipedia as a non-profit platform in building a better world and allowing everyone to access free knowledge. In this way, it enhances users' feelings of belonging, invites them to a mission, and increases their awareness of the impact of their donation, eventually making users more generous while making donations. As fundraising effort plays an essential role in inducing users' generousness to donate in practice, we incorporate the fundraising effort of the platform in our model and denote it by f that increases users' generousness levels in a decreasing returns-to-scale manner. The details of this process are discussed in CHAPTER 2.5.2.

Furthermore, a significant portion of Wikimedia's annual expenditure is allocated to supporting the user community (Wikimedia 2022a). In particular, Wikipedia's community support includes training contributors, developing and maintaining several easy-to-use editing tools, improving its technologies to locate issues among articles that fall into users' knowledge field (e.g., reporting and fixing misleading content or missing links in articles), and promoting projects that the general public can easily edit (Wikimedia 2022a). Therefore, in our study, we consider community support (i.e., h) as an effort to reduce the barrier of contributing content and discuss the details in CHAPTER 2.5.3. We further consider that the cost of serving users on the platform has variable and fixed components (i.e., $xR + g$).

Furthermore, Wikipedia states that they maintain strategic cash reserves (Dewey 2015), which we capture by r below which no further expenses are incurred.

Besides, a platform needs to maintain and improve its platform structure, and upkeep and develop its user interface to improve the usability and feel of the platform. Furthermore, it needs to protect some prime or high-traffic pages by preventing random users from modifying them by managing them with its own effort. It also may develop and utilize several types of bots to help create or edit content (e.g., see Dormehl 2020). In our model, we capture these efforts by the "platform's in-house content quality improvement effort" and denote it by v. As a non-profit platform, the in-house quality improvement effort level depends on the remaining budget after the other expenses that we discussed so far and the cash reserves of the platform.

2.5.2. Donations

In practice, non-profit UGC platforms maintain their financial sustainability by collecting donations from users (Wikipedia 2024a). Literature states that as non-profit UGC platforms serve public goods, the motivation behind making donations can be explained by impure altruism or warm glow that external factors can influence rather than the innate pure altruism (Andreoni 1990). As explained in CHAPTER 2.4.3, "warm glow" implies that people who make donations to charity or others feel good and receive an emotional reward. We further consider that there might be external rewards, benefits, or incentives in donating, such as the tax deductions donors "might" receive (Mengle 2023). Therefore, we account for (i) warm glow (impure altruistic reward) and (ii) external rewards in the donation behavior. Furthermore, in line with practice, we model the utility gain of users (owing to altruistic and external rewards) by donating, i.e., d_i, to be heterogeneous.

For non-profit UGCs, some users do not donate. Even among users who donate, the donation amount may differ (Wikimedia 2014). We incorporate these facts into our model by considering an individual-level parameter $b_{d,i}$ that captures the generousness level (which consists of the warm glow and the external reward) of user i and write the benefit of making a donation as $(b_{d,i} - q_d)d_i$. We consider that $b_{d,i}$ follows a uniform distribution, although considering other finite and bounded distributions does not change our results qualitatively - as we demonstrate in CHAPTER 2.7.5. In this formulation, q_d serves as a barrier parameter that ensures that not everyone donates to the platform.

In practice, non-profit UGC platforms exert fundraising effort (denoted by f) such as sending users emails or text messages, publishing donation banners, building a tighter connection to the user community, and running advertisement campaigns (Wikimedia 2022a,b). As discussed in CHAPTER 2.5.1, these efforts affect the donation behavior of users by exposing them to marketing materials (Schlosser and Levy 2016, Sudhir et al. 2016). By implementing the fundraising effort, the UGC platform can increase users' generousness in a decreasing returns-to-scale manner that we take as $f^{1/2}$ without loss of generality.[4]

Furthermore, to represent practical scenarios where a specific user finds it harder to justify making higher levels of donations, we consider that the disutility of making donations increases quadratically, implying a decreasing return-to-scale environment. This quadratic disutility approach is prevalent in the literature streams on finance and donations (e.g., see Lim and Lee 2018, Koo et al. 2016, Roh et. al 2017, Shin et al. 2018, Toyasaki and Wakolbinger 2014). However, we also show in CHAPTER 2.7.2 that our key results are robust to other disutility settings In sum, we model the "individual utility gain" of user i due to donations (not the entire utility function) as follows.

$$\Delta_{d,i}U = \left((b_{d,i} + f^{1/2}) - q_d\right) d_i - c_d d_i^2. \tag{1}$$

2.5.3. Content Contributions

The literature on UGC highlights that the content creation effort of users exhibits heterogeneity, and this effort is due to intrinsic rewards (including pure altruism) and extrinsic rewards (see Alam and Campbell 2017, Khern-am-nuai et al. 2018, Jiang et al. 2022). First,

[4]Considering a power term more than 1 is not practical as it implies an infinite donation as f increases. Our results and insights are robust to power terms between 0 and 1 (in addition to $1/2$) as we demonstrate in CHAPTER 2.7.3.

by adding new content or editing existing content, users intrinsically satisfy their desire to "do their part" to help others or make the world better (Alam and Campbell 2017, Rafaeli and Ariel 2008, UNU 2010). Therefore, intrinsic motivation (including pure altruism) is one of the most critical factors that motivate users to contribute content. Second, the literature and surveys also note that people make contributions to fulfill extrinsic desires, such as appearing credible, confident, and reputable (Rafaeli and Ariel 2008). Therefore, external utilities acquired through voluntary contributions (or extrinsic motivation) are also important factors.

We model this individual-level incentive for content contribution with b_e (with realization $b_{e,i}$ for user i) to capture users' generousness for content creation. Hence, the benefit of making a content contribution for user i in the amount e_i can be written as $(b_{e,i} - q_e)e_i$. In this formulation, a higher value of q_e implies that making content contributions on the platform is inherently harder, so there are fewer content contributors. Therefore, q_e can be considered as a barrier to making content contributions. In line with the classical Hoteling line model, we consider that b_e follows a uniform distribution with lower and upper bounds $\underline{b_e}$ and $\overline{b_e}$. However, our results qualitatively hold for other bounded distributions. On the other hand, to ensure diseconomies of scale (so that user contribution is bounded), we consider that users' cost elasticity term for contributing content is quadratic without loss of generality. This is in line with the literature on relevant contexts (e.g., Lacroix and Fortin 1992, Ransom 1987).[5]

After discussing the different characteristics of users affecting their contribution behavior, we now explain the platform-side effort to entice the users to contribute more content. According to Torres (2016) and Everipedia (2019), the complicated editing process and users' low confidence in their knowledge level both indicate high barriers to contributing content. Hence, they are two critical factors that cause low willingness to contribute content (i.e., higher levels would indicate higher q_e). To inspire users to make more content

[5]Note that utilizing other cost elasticity terms greater than 1 does not change our key results qualitatively as we demonstrate in CHAPTER 2.7.2.

contributions, Wikipedia exerts a community support effort by developing and maintaining several easy-to-use editing tools, developing bots to locate issues in articles that fall into users' expertise (e.g., reporting and fixing misleading content or missing links), and promoting projects that the general public can easily edit (Wikimedia 2022a). This implies that the community support effort of the platform (denoted by h) decreases the "barrier" to contributing content (i.e., q_e). To rule out the possibility that infinite contributions are feasible, we consider that the reduction in the barrier parameter is by $h^{1/2}$ without loss of generality. Considering different power terms to reflect the decreasing returns-to-scale nature of this relationship yields qualitatively similar results and insights as we show in CHAPTER 2.7.3. Taking these discussions into account, the utility gain of user i that makes a content contribution is given as the following:

$$\Delta_{e,i}U = \left(b_{e,i} - (q_e - h^{1/2})\right) e_i - c_e e_i^2. \tag{2}$$

Furthermore, the reciprocal mechanism implies that users would expect higher levels of content quality from the platform if they contribute content (Marsh 2016, Qi et al. 2018) or make donations (Kocielnik et al. 2018, Marsh 2016). Hence, we also consider the improved platform's overall content quality (i.e., Π, as explained in more detail in CHAPTER 2.5.4) as another factor that affects the contribution and donation behaviors of users and refer to it as the self-driven benefit. Together with user i's utility gains from making donations (regarding the warm glow and external factors, see Expression 1) and from making content contributions (regarding the intrinsic and extrinsic motivations, see Expression 2), the utility function for user i is given as:

$$\begin{aligned} U_i =& U_{0,i} + \Delta_{d,i}U + \Delta_{e,i}U + \Pi \\ =& U_{0,i} + ((b_{e,i} - (q_e - h^{1/2}))e_i - c_e e_i^2) + ((b_{d,i} + f^{1/2} - q_d)d_i - c_d d_i^2) + \Pi, \ \ \forall i, \end{aligned}$$

where $U_{0,i}$ represents user i's base utility level. Although we consider that the generousness parameters for donations and contributions (i.e., $b_{d,i}$ and $b_{e,i}$) are independent in our main model, we relax this assumption in CHAPTER 2.7.1 for a platform where donation and content contribution decisions are correlated, we show that all our key insights are qualitatively similar in such a setting.

2.5.4. Objective of The Platform

Taking all the discussions so far into account, we now elaborate on the platform's objective to maximize content quality Π, by also remaining financially sustainable through donations (i.e., $\int_i d_i$) and external funding (i.e., $w_0 + w\Pi$). Non-profit UGC platforms aim to improve overall content quality by gathering users contributions (i.e., $\int_i e_i$) and exerting in-house quality improvement effort (i.e., v) using donations from users and third-party funding (Bariso 2018, Wikimedia 2022a). We denote the efficiency of the platform's in-house quality improvement by a_d and the platform's efficiency in utilizing users' content contribution effort by a_e.[6] Our modeling approach and findings can be generalized to other types of online platforms by adopting various parameter settings, e.g., pure UGC platforms without any in-house efforts (by setting $a_d = 0$), non-UGC platforms (by setting $a_e = 0$), non-profit UGC platforms solely relying on donations (by setting $w = w_0 = 0$), and UGC platforms whose financial stream is related to its content quality (by setting q_d to a large number).

As discussed in CHAPTER 2.5.1, the in-house quality improvement effort (i.e., v) is the difference between total revenue (i.e., $\int_i d_i + w_0 + \Pi w$) and the cost items (i) cash reserves (i.e., r), (ii) expenses on fundraising (i.e., f), (iii) expenses on community support (i.e., h), and (iv) the total hosting cost (i.e., $g + xR$). Hence, the improvement of the overall content quality due to the platform's in-house effort can be written as $a_d v = a_d \left(w_0 + \Pi w + \int_i d_i - f - h - g - xR - r\right)$. Therefore, considering both the platform's in-house quality improvement effort and the content contributions made by users (i.e.,

[6]Since many UGC platforms have mechanisms (like website rollback) to ensure that content contributions do not deteriorate the content quality (Wikipedia 2024e), we consider users' content contributions can only enhance Π in our main model. We relax this assumption in CHAPTER 2.7.4 and show that the results remain qualitatively similar.

$\int_i e_i$), the overall content quality is:

$$\Pi = \Pi_0 + a_d \left(w_0 + \Pi w + \int_i d_i - f - h - g - xR - r \right) + a_e \int_i e_i,$$

where Π_0 denotes the initial overall quality of the platform.

2.5.5. Main Model

Based on the discussions so far, we now present our model as:

$$\max_{d_i, e_i} \quad U_i = U_{0,i} + ((b_{e,i} - (q_e - h^{1/2}))e_i - c_e e_i^2) + ((b_{d,i} + f^{1/2} - q_d)d_i - c_d d_i^2) + \Pi), \quad \forall i, \tag{3}$$

$$\max_{f,h} \quad \Pi = \Pi_0 + a_d \left(w_0 + \Pi w + \int_i d_i - f - h - g - x(R_0 + c\Pi_0) - r \right) + a_e \int_i e_i, \tag{4}$$

$$subject\ to: \quad w_0 + \Pi w + \int_i d_i - f - h - g - x(R_0 + c\Pi_0) - r \geq 0, \tag{5}$$

$$d_i \geq 0, \;\; e_i \geq 0, \;\; \forall i, \;\; q_e > h^{1/2}. \tag{6}$$

In this model, Expression (3) depicts users' utilities. Expression (4) formulates the objective function of the platform. Expression (5) presents the budget constraint or the fact that donations from users and external funding should cover all the costs of the platform. Due to the non-profit nature of the platform, all the operations of the platform (such as the in-house quality improvement effort, fundraising effort, and community support effort) are organically tied together because of the limited budget. Expression (6) states that donation amounts and content contribution efforts are nonnegative and ensures that the community support effort cannot change the sign of the net barrier effect for contributing content. These and other feasibility constraints impose further bounds on parameters such as a_d and a_e that we consider in our solution. Please refer to APPENDIX A.2 for further details.

2.5.6. Rational Expectation Framework

The theory of rational expectations was first proposed in Muth (1961), and it asserts that outcomes that market participants are forecasting do not differ systematically from the equilibrium results. This approach has also been adopted in different literature streams (e.g., Demirezen et al. 2016, Veronesi 1999) and is commonly utilized, implicitly or explicitly, in the context of UGCs and other platforms (Ahn et al. 2015, Chen et al. 2022) as users' beliefs about others' participation or the future state of the platform are central to the problem of one's own participation.

In our rational expectation framework, the UGC platform and users privately form beliefs over the future state of the overall content quality, deterministically resulting from the platform's effort and the users' aggregate-level participation. Based on practice, we consider that, first, the platform develops and discloses its effort allocation plan (e.g., see Wikimedia 2022a) based on its knowledge of users' generousness levels and expectation of user participation. Second, users make their decisions on contributing content and donating to the platform, which is also contingent on other users' participation and the platform's efforts.

The platform and users may also access available public data, such as user surveys on the experience of participation (e.g., Khanna 2012, UNU 2010), the platform's historical expenditure (e.g., Wikimedia 2014), and quality improvement (e.g., Wikipedia 2024d), to come up with their expectations of variables of interest. Therefore, we solve this problem via backward induction, first solving for the user participation decisions (i.e., donation amount and content contribution effort) contingent on the platform's effort allocation and then solving for the platform's optimal decisions.

2.5.7. Solution

By utilizing the first-order conditions based on the utility function of user i, it is trivial to show that the optimal contribution effort of user i is $e_i^* = \frac{\frac{a_e}{1-a_d w}+b_{e,i}+\sqrt{h}-q_e}{2c_e}$. Without loss of generality, we order $b_{e,i}$ such that $b_{e,i} \geq b_{e,i-1}$. Observe that if user i decides to contribute, so does user j, $\forall j > i$ because $\left(b_{e,i} - \left(q_e - h^{1/2}\right)\right) e_i - c_e e_i^2$ is non-decreasing in $b_{e,i}$. We denote b_e^* as the threshold of the user generousness that is indifferent between contributing content or not. Hence, Expression (3) reveals that $b_e^* = -\frac{a_e}{1-a_d w} - \sqrt{h} + q_e$.

Therefore, the number of content contributors, i.e., $E[K_e] = (R_0 + c\Pi_0)(1 - F_e(b_e^*))$, and the total content contribution made by all users, i.e., $C = (R_0 + c\Pi_0)\int_{\underline{b_e}}^{\overline{b_e}} e^*(b_e)f_e(b_e)db_e$, where F_e and f_e are the cumulative distribution function (CDF) and the density function of users' generousness levels to contribute content to the platform, respectively.

Next, as discussed earlier, users determine if and how much they want to donate to maximize their respective utilities. Without loss of generality, we reorder $b_{d,i}$ such that $b_{d,i} \geq b_{d,i-1}$. Evidently, if user i decides to make a donation, so does user j, $\forall j > i$ because $(b_{d,i} + f^{1/2} - q)d_i - c_d d_i^2$ is non-decreasing in $b_{d,i}$. The first-order condition with respect to d_i reveals that if there is a donation, it is given as $d_i^* = \frac{\frac{a_d}{1-a_d w} + b_{d,i} + \sqrt{f} - q_d}{2c_d}$. The optimal donation further reveals that the generousness level of the user that is indifferent between making a donation or not is $b_d^* = -\frac{a_d}{1-a_d w} - \sqrt{f} + q_d$. Furthermore, the number of donors, i.e., $E[K_d] = (R_0 + c\Pi_0)(1 - F_d(b_d^*))$, and the total donation from all users, i.e., $D = (R_0 + c\Pi_0)\int_{\underline{b_d}}^{\overline{b_d}} d^*(b_d)f_d(b_d)db_d$, where F_d is the CDF of the level of users' generousness for making donations, and f_d is the corresponding density function.

Based on these arguments and the intermediate results, we solve for the equilibrium that we present in Lemma 1. For brevity, we relegate the details of the solution process to the Appendix along with the equilibrium levels of other variables of interest.

<u>Lemma 1</u>. *The fundraising effort f^* and community support effort h^* of the platform; and the donation amount d_i^* and the extent of content contribution e_i^* of user i at equilibrium are:*

$$f^* = \frac{(c\Pi_0 + R_0)^2(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)^2}{(1 - a_d w)^2(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^2};$$

$$h^* = \frac{a_e^2(c\Pi_0 + R_0)^2(a_e + (1 - a_d w)(\overline{b_e} - q_e))^2}{(1 - a_d w)^2(-4a_d c_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))^2};$$

$$e_i^* = \max\left\{0, \frac{-\frac{a_e(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))}{-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0)} + (1 - a_d w)(b_{e,i} - q_e) + a_e}{2c_e(1 - a_d w)}\right\};$$

$$d_i^* = \max\left\{0, \frac{-\frac{(c\Pi_0+R_0)(-a_d\overline{b_d}w+a_d q_d w+a_d+\overline{b_d}-q_d)}{(1-a_d w)(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)} + \frac{a_d}{1-a_d w} + b_{d,i} - q_d}{2c_d}\right\}.$$

2.6. Key Findings and Managerial Insights

In this section, we present our results and managerial insights based on the solutions obtained in the previous section.

2.6.1. The Impact of Growth in Generative AI Technologies on Non-Profit UGC Platforms

In this section, we investigate how being more efficient in the in-house quality improvement effort impacts the UGC platforms' strategic user community management and overall performance.

Whether increased efficiency of in-house effort reduces or increases platforms' community management efforts?

Non-profit online UGC platforms consider maximizing overall content quality as their primary goal (Wikipedia 2024a). To achieve a higher level of content quality, UGC platforms may exert in-house effort (e.g., directly enrich the content on the platform with the available budget, design and improve the platform, upkeep the user interface/experiences/ readability) (Dormehl 2020, Wikimedia 2022a). In recent years, the rapid growth of generative AI has made it possible for UGC platforms to efficiently exert their in-house effort to enrich and improve their content (Maruccia 2023, Masolo 2022).[7] Although this fact may have a tremendous impact on the platforms' operational management, especially user community management (i.e., the fundraising effort f^* and community support effort h^*), the literature has not formally examined this issue, but it is a debated issue in practice (Ackermann 2024). Therefore, we ask: *Do the platform's fundraising and community support efforts increase if it becomes more efficient in its in-house quality improvement effort?* We address this question and present our results in Proposition 1. All proofs and thresholds are provided in the Appendix.

Proposition 1. *If the platform's efficiency of in-house quality improvement effort (i.e., a_d) increases, the platform exerts more effort in fundraising (i.e., $\frac{df^*}{da_d} > 0$). However, interest-*

[7] Generative AI is mainly used for editorial purposes but not for generating new content in Wikipedia. Deckelmann (2023) elaborates on why generative AI cannot replace human input in the context of Wikipedia.

ingly, it also exerts more community support effort if the platform's efficiency of in-house quality improvement effort is already at a high level, i.e., $\frac{dh^*}{da_d} > 0$ *if* $a_d > \theta_{h,a_d}$.

In the economics literature, when facing multiple options to achieve a higher objective function value, business entities tend to choose the most efficient one to implement (Staiger 2018). In our context, the increased efficiency of the in-house quality improvement effort represents a more efficient utilization of financial resources to improve the overall content quality. Therefore, one may intuitively consider that the platform should reduce the effort on community support and increase the fundraising effort (to make more resources available for the in-house effort). However, Proposition 1 reveals that although the fundraising effort increases in such a case, the community support effort is more nuanced. This result can be explained as follows.

Proposition 1 suggests that the fundraising effort increases in the efficiency of in-house quality improvement effort in a monotone manner, i.e., $\frac{df^*}{da_d} > 0$. It implies that as this efficiency increases, the marginal benefit of the fundraising effort in collecting donations becomes marginally smaller. This is because of the decreasing returns-to-scale behavior of the fundraising effort discussed in CHAPTER 2.5.2. In effect, the proof of Proposition 1 implies that when the platform is already efficient in its in-house quality improvement effort (i.e., $a_d > \theta_{h,a_d}$) and it becomes more efficient, the platform finds it better to increase the community support effort as well. In particular, by exerting a higher level of community support effort to inspire users to contribute content, the platform can enhance the overall content quality and, hence, increase its external funding (because this revenue stream depends on the overall quality). The increased funding from external parties and users (as $\frac{df^*}{da_d} > 0$) is then utilized by the platform to exert a higher level of in-house effort. In other words, the impact of the community support effort on the overall quality becomes more significant because of third-party funding, increased level of donations, and increased efficiency of in-house effort. On the other hand, if the platform is not substantially efficient in its in-house quality improvement effort (i.e., $a_d < \theta_{h,a_d}$), the platform finds it better to focus only on the efficiency gain in the fundraising effort as it is at a low level and has a high marginal benefit (based on the argument provided above). This also entices the platform to reduce its spending on community support and instead utilize those resources in fundraising and in-house quality improvement efforts.

From a practical perspective, Proposition 1 suggests that the relationship between a platform's efficiency of in-house efforts and its strategy of allocating efforts to fundraising and community support is subtle. In particular, although the adoption of advanced generative AI technologies may bring non-profit UGC platforms higher efficiency in exerting their in-house effort (Maruccia 2023, Masolo 2022), the platform does not necessarily become more self-reliant by reducing its efforts in managing user community to make more content contributions. Instead, it must consider each potential source for quality improvement. There is also evidence from the industry supporting our result. In effect, by advancing its research and AI techniques, although Wikipedia has become more efficient in exerting various in-house efforts over the years (Kanowitz 2020), the annual reports from 2016 to 2019 show that Wikipedia continues to invest more in supporting the contributor community (Wikimedia 2024).

As discussed earlier, the economics literature states that a business entity chooses the most efficient way to achieve a better objective (Staiger 2018). However, this statement does not necessarily hold in the context of non-profit UGC platforms. The charitable nature of non-profit UGC platforms requires researchers and other stakeholders to consider the relationship between total donations, external funding, and the platform's effort allocation. As discussed in CHAPTER 2.4, the literature on crowdsourcing or UGC platforms has neither (i) formally examined the influence of UGC platforms' increased efficiency in exerting in-house effort because of the recent advanced generative AI technologies nor (ii) formally discussed UGC platforms' strategic management of user community from a managerial perspective. In sum, our unique modeling approach enables us to contribute to the literature in a unique way.

Does increased efficiency of in-house effort reduce or increase user participation?

Besides the influence of increased efficiency of in-house effort on platforms' strategic management of user communities, we are also interested in its impact on the platforms' performance. Although there is no doubt that the overall content quality would increase with increased efficiency of the in-house effort, user participation may present different patterns. Because user participation is one of the most critical components for non-profit UGC platforms and influences their long-term success (Zhang et al. 2012), it is important to understand how the increased efficiency of in-house effort because of recent developments in generative AI and other technologies influences user participation. However, the literature on the non-profit UGC has yet to examine this issue formally. Therefore, we conduct the analyses based on our theoretical solution and discuss this topic that the practice deems important. Because the total donation and total content contributions from users are two important performance metrics of user participation and directly influence the overall content quality, and it is a debated topic in practice (Ackermann 2024), we first ask: *Does users' participation, at an aggregate level, increase if the platform becomes more efficient in its in-house quality improvement effort?* This question is addressed in Proposition 2.

Proposition 2. *If the platform is more efficient regarding in-house quality improvement effort (i.e., a_d is higher), interestingly, the total contribution from all users may actually sometimes reduce (i.e., $\frac{dC^*}{da_d} < 0$) even though the overall content quality (i.e., Π^*) and the total donation (i.e., D^*) always increase. This happens when the base number of users in the platform is higher than a threshold, i.e.,*

$$R_0 > \theta_{R_0}(a_d, .) = \frac{4a_d^2 c_e w(\overline{b_e} - \underline{b_e}) - c\Pi_0(1 - a_d w)^2(\overline{b_e} - q_e) - a_e c\Pi_0}{(1 - a_d w)^2(\overline{b_e} - q_e) + a_e}.$$

A higher efficiency for in-house quality improvement effort implies a higher level of content quality. Then, in response to higher quality, the literature suggests that users are expected to contribute more to the platform (Aaltonen and Seiler 2015). However, Proposition 2 reveals that if the in-house efficiency is higher, the total content contribution may actually be lower, although the total donation and overall content quality are higher. From the proofs of Lemma 1 and Proposition 2, it is evident that if the base number of users is higher than the given threshold (i.e., $R_0 > \theta_{R_0}(a_d, .)$) and the platform becomes more efficient in its in-house quality improvement effort, the platform prefers to reduce its community support effort (i.e., $\frac{dh^*}{da_d} < 0$). The rationale behind this behavior is that as the in-house effort becomes more efficient in improving the overall content quality, the platform strategically makes a higher level of fundraising effort in order to collect more funding to spend on the in-house quality improvement (i.e., $\frac{df^*}{da_d} > 0$). Furthermore, as the platform has a substantial number of users, the cost of serving users is a direct concern. These factors lead to a lower effort in supporting the community (i.e., $\frac{dh^*}{da_d} < 0$), and hence, a smaller total content contribution from users.

On the other hand, when the base number of users is lower than the given threshold (i.e., $R_0 < \theta_{R_0}(a_d, .)$), if the platform becomes more efficient in the in-house quality improvement effort, the total content contribution made by users increases. The proof of Proposition 2 implies that this result depends on the scale efficiency. In particular, because the base number of users in the platform is lower than the given threshold (i.e., $R_0 < \theta_{R_0}(a_d, .)$), the expenditure on maintaining the platform is lower (compared to a large platform). This reduced spending, along with the fact that the third-party funding increases as the overall content quality increases (because of the efficiency gain in the in-house development effort), enables the platform to allocate more funding to the in-house improvement effort by not necessarily reducing the community support (unlike the case for a platform with a substantial base number of users). Therefore, for a platform whose base number of users is lower than the given threshold (i.e., $R_0 < \theta_{R_0}(a_d, .)$), an increased level of effectiveness in the platform's in-house improvement effort results in a more pronounced level of total content contribution from users along with total donations and a higher level of content quality.

From a practical perspective, Proposition 2 reveals that as the advanced generative AI technology increases a non-profit UGC platform's efficiency of exerting in-house effort, if the size of the user community (i.e., R_0) is at a low level, the platform can achieve a higher level of total content contribution from its user base, as well as higher levels of total donations and total quality. However, if the platform has a large base number of users, it should be willing to take a reduction in the total content contribution to focus more on donations. This way, it is possible for the platform to increase its overall quality more effectively.

This finding is in contrast to the literature on UGC platforms (e.g., see Naeem and Ozuem 2021, Shen et al. 2017) and the perspective of the practice (e.g., see AdAlone 2022) that state that the users' content contribution is the most critical factor for the success of these platforms. In our model, we endogenize the platform's decisions, such as the in-house quality improvement effort, community support for content creation, and the fundraising effort to motivate users to make more donations. This fact enables us to report that, interestingly, a UGC platform might be willing to forgo the total content contribution from the users to generate more content rather than taking a knee-jerk reaction to increase the content contribution. This finding is one of our key contributions to the literature and is related to one of the issues we discuss in the motivation section.

Based on our discussions thus far, the threshold $\theta_{R_0}(a_d, .)$, as presented in Proposition 2, plays a vital role in the behavior of the UGC platform. In particular, we next discuss how this threshold is moderated by other factors in the business setting and the corresponding practical insights.

Corollary 1. *The loss in the total contributions from users owing to increased efficiency of in-house effort is less likely to occur if the platform is less efficient in utilizing users' content contribution (i.e.,* $\frac{d\theta_{R_0}(a_d,.)}{da_e} < 0$*).*

The practical implication of this result is that if the platform (i) has a large group of users (i.e., R_0 is high), but (ii) is not very efficient in translating users' content contribution to formal content that can be directly used to improve the platform's overall quality (i.e., a_e is low), then the increase of platforms' efficiency of in-house effort led by the advancement of generative AI technologies not only increases the overall quality of the platform but also increases the participation of users in content generation. Next, because the number of content contributors reflects the engagement of the user community in the UGC platform, in Corollary 2, we present the influence of efficiency in the platform's in-house effort on this measure.

Corollary 2. *If a UGC platform is more efficient in exerting in-house quality improvement effort (i.e., a_d is higher), interestingly, the number of contributors (i.e., $E^*[K_e]$) may sometimes be lower. This happens (i.e., $\frac{dE^*[K_e]}{da_d} < 0$) when the efficiency of in-house quality improvement effort is lower than a given threshold (i.e., $a_d < \theta_{a_d}(R_0, .)$). The expression for $\theta_{a_d}(R_0, .)$ is provided in APPENDIX A.3 of the Appendix.*

In particular, if the efficiency of in-house quality improvement effort (i.e., a_d) is at a low level (i.e., $a_d < \theta_{a_d}(R_0, .)$) and it increases, the number of content contributors decreases. This is because when the efficiency of the in-house quality improvement effort increases, the donations are relatively more important for the platform because a larger budget enables the platform to make a more profound in-house improvement of overall content quality. Therefore, the platform strategically reduces the community support effort and increases the fundraising effort (i.e., $\frac{df^*}{da_d} > 0, \frac{dh^*}{da_d} < 0$). Hence, the number of content contributors decreases. On the other hand, if the efficiency of the platform's in-house effort is already at a high level (i.e., $a_d > \theta_{a_d}(R_0, .)$) and it increases, the number of content contributors increases. The reason is that, in this case, the higher efficiency of the in-house quality improvement effort enables the platform to "leverage" content contributions from users better. Specifically, with the enhancement of overall quality owing to users' content contributions, the platform obtains a higher level of third-party funding. Therefore, the platform is able to increase the overall quality by exerting additional in-house effort. As a result, users who contribute to content experience higher levels of value from making a content contribution (see Expression (3)), and hence, more users prefer contributing content (i.e., $E[K_e]$ increases).

From a practical perspective, our results suggest that if generative AI technologies bring a UGC platform higher efficiency (or higher cost-effectiveness) in exerting in-house effort (i.e., a_d increases), the platform may also benefit from an increased willingness of users to contribute content. That is, the increased efficiency of a particular effort may result in an improvement in performance measures that are seemingly unrelated to this effort. Therefore, the managers of UGC platforms and policymakers should consider the synergy between "donations" and "content contributions." Furthermore, the result presented in Corollary 2 has implications for the theory of reciprocal altruism. Specifically, to the best of our knowledge, our study is the first to report that as the cost-effectiveness of in-house effort increases, users are more likely to contribute content because they expect the platform to be more successful. Another important finding is that, although the number of content contributors and the total content generated by users both measure the engagement of users in contributing to the platform, Proposition 2 and Corollary 2 reveal an interesting behavior if the platform becomes more efficient in exerting its in-house effort. These results are presented in Figure 2. In this figure, we observe an interesting phenomenon in the shaded area. Here, although the number of content contributors decreases, interestingly, the total contribution from them increases. This result is attributed to Proposition 2 and Corollary 2 and can be explained as follows.

As shown in the proof of Corollary 2, if the platform's base number of users and efficiency of in-house effort are both lower than thresholds $\theta_{R_0}(a_d, .)$ and $\theta_{a_d}(R_0, .)$ (i.e., the shaded area in Figure 2), the platform reduces its community support effort if it becomes more efficient in its in-house effort (i.e., $\frac{dh^*}{da_d} < 0$). Hence, the number of content contributors decreases. However, the higher efficiency of the in-house quality improvement effort indirectly encourages content contributors to exert higher levels of effort owing to the self-driven benefit explained before (see Expression (3)). Therefore, the average

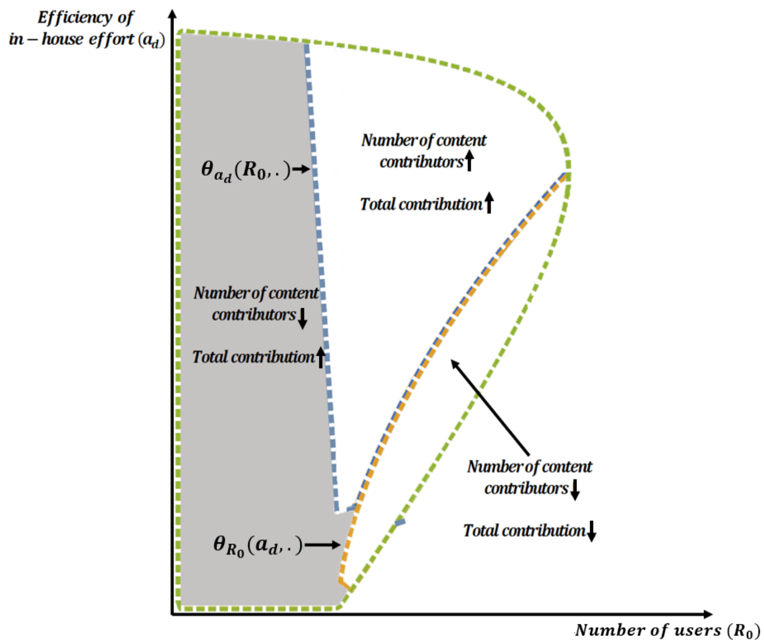

Figure 2. *Impact of a_d on The Number of Content Contributors and The Total Content Contributed by Users*

individual-level content contribution increases as well as the total contribution.

The contradictory patterns presented by the total content contribution and the number of content contributors suggest that although the platform sees a reduction in the number of content contributors, the total content contribution may be higher. Therefore, rather than focusing solely on content contributors, the platform should take into account the average level of content contribution as well. This is an important practical insight for policy-makers and managers of UGC platforms (especially during the rapid growth of generative AI technologies), as reports relevant to UGC platforms usually highlight the number of contributors but overlook the average level of user engagement in UGC platforms (Brandt 2013). We would like to note that the behaviors presented in this section are also observed in practice. For instance, although Wikipedia has experienced a reduction in the number of contributors, it presents a steady growth profile regarding the overall content contribution from users (Wikipedia 2024b).

In this section, we make an in-depth discussion on how advanced generative AI technologies change non-profit UGC platforms' community management strategy and overall performance (especially user participation) by enhancing their efficiency of exerting in-house effort. As discussed in CHAPTER 2.4, the literature does not incorporate the in-house efficiency of the platforms in their analysis. Generally, the sole focus is on the productivity of the UGC members. However, acknowledging that the overall quality of the content on the platform depends not only on the effort of the UGC members but also on the platform requires us to consider the in-house efficiency of the platform. By capturing this parameter, we are able to further analyze important issues in practice, such as the influence of generative AI on non-profit UGC platforms' efficiency in exerting their in-house effort. Although the literature has been myopic to this issue, it is gaining importance both in literature and practice. In effect, capturing this missing link in our model has allowed us to build two main findings: (1) the increased efficiency of the in-house quality improvement effort does not necessarily mean a decrease in other efforts (fundraising and community support), and (2) the increased efficiency of the in-house quality may actually reduce users' content contributions, although it helps the platform achieve higher overall content quality.

2.6.2. Should The Platform Make The "Stingy Generous Enough" or The "Generous More Generous?"

The results of past studies imply that users' generousness can present heterogeneity even on the same platform (e.g., Levina and Arriaga 2014, Marsh 2016). In our context, as non-profit UGC platforms are different from traditional UGC platforms as they rely on users' donations to maintain their operations, they must understand how user heterogeneity influences the aggregate behavior of users and, therefore, their performance. Notably, increasing the base level (i.e., the lower bound) $\underline{b_d}$ actually decreases the heterogeneity among users regarding generousness in making donations. In contrast, increasing the upper bound actually increases the heterogeneity. However, to the best of our knowledge, although the literature has examined the motivation of non-profit UGC contributors, no prior study has explored how the heterogeneity of user generousness influences platforms' financial sustainability and it is also a debated issue in practice (Philipose 2020), we ask: *Whether the* *platform should "make the stingy generous enough" so that they make donations, or "make the generous more generous" so that they make higher levels of donations?* Proposition 3 addresses this question.

Proposition 3. *Although the platform's total donation from users (i.e., D^*) increases in both the base level (i.e., $\underline{b_d}$) and upper level of user generousness (i.e., $\overline{b_d}$), interestingly, increasing the upper level of user generousness results in a more profound improvement in the total donation iff the barrier of users to make donations is at a high level. In other words,* $\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}} > 0$ *when* $q_d > \theta_{q_d}$.

Because users' generousness is directly related to non-profit UGCs' financial sustainability and business development, in practice, non-profit UGC platforms adopt various measures to increase users' generousness levels. These measures generally focus on inspiring non-donors to donate to the platform. For instance, Wikipedia asks for a moderate donation from its users: "the price of a cup of coffee is all we ask," that is, inspiring users that do not make donations to be generous enough to make donations (Wikipedia 2024c). Therefore, one may conjecture that the improvement in total donation is more remarkable if the platform focuses on making 'stingy' users more generous. However, Proposition 3 suggests a nuanced result that we elaborate on next.

On the one hand, when the barrier parameter is at a high level (i.e., $q_d > \theta_{q_d}$), not all users are eager to make donations. In this case, increasing the upper level of user generousness not only enhances the average generousness level of users (i.e., $\frac{\underline{b_d}+\overline{b_d}}{2}$ increases in $\overline{b_d}$) but also inspires users, who are already generous, to be even more generous in their donations. In contrast, increasing the base level of user generousness also increases the average generousness of users, but the focus is more on non-donors who only start to donate smaller amounts. Therefore, as evident from the proof of Proposition 3, increasing the upper level of user generousness, rather than increasing the base level, is more efficient in increasing the total donation from users (i.e., $\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$). Hence, compared with an increase in the base level, the same increase in the upper level of user generousness improves the total donation more significantly when the barrier to making donations is high. On the other hand, if the barrier effect for users to make donations is low (i.e., $q_d < \theta_{q_d}$), increasing the base level of user generousness is more efficient in enhancing the total donation from users because a denser distribution of user generousness enables the platform to inspire more users to make donations by exerting fundraising effort. Therefore, if the barrier for users to make donations is low, compared with increasing the upper level of user generousness, increasing the base level inspires users better and gathers a greater total donation for the platform.

Furthermore, the barrier parameter q_d influences (along with the fundraising effort) the portion of the users that donate and their generousness, hence, how much they donate. We denote the users who donate even without fundraising effort by inherent donors. Furthermore, the heterogeneity of inherent donors is high if the barrier is low, while the heterogeneity is low otherwise. Therefore, the practical implication is that making generous users more generous (hence, increasing the heterogeneity across users) is more efficient in raising donations if the barrier is high. In practice, this condition on the barrier implies that (i) the inherent donors have a low heterogeneity in their generousness, or (ii) inherent donors only make up a small portion of the entire user group. Otherwise, the focus should be on the users who are not already donors, hence reducing the heterogeneity.

In practice, a firm generally splits users into different groups and focuses more on a target group instead of giving the same attention to all groups (Ziliani 2006), to increase the return on its investment (Farahat and Bailey 2012). In our context, rather than striving to increase the generousness levels of all users, it can be more cost-effective to focus on users who are already generous if the barrier of making donations is low, or users who are non-donors otherwise. This insight is related to our motivation and is observed in practice. For instance, Wikipedia has a very small portion of users who make donations (i.e., a high barrier effect) (Wikipedia 2024f, Wikimedia 2022b). In line with our proposition, Wikipedia sends gifts to its outstanding donors and lists top donors (donors who donate more than one thousand dollars to Wikipedia) on its website (Wikimedia 2019). These practices help encourage the generous to be more generous and increase the total donations more significantly. Although the benefit of increasing the lower bound of user generousness is lower (compared to increasing the upper bound) in this case, Proposition 3 reveals that it is still useful. Therefore, the practice of showing the regular user the message "The price of a cup of coffee is all we ask" is also a wise business decision given that the cost of such an effort is minimal, as observed in practice.

Besides its practical importance, our finding makes a unique contribution to the literature. In particular, as discussed in CHAPTER 2.4, although earlier studies argue that user heterogeneity is an important factor in user participation in online platforms (Daniel et al. 2012, Ren et al. 2016, Singh and Tan 2010), unlike our study, the literature has not explicitly discussed the heterogeneity of user generousness and its impact on the financial sustainability of non-profit UGC platforms. We find that although "making stingy users generous enough" and "making generous users more generous" both increase the aggregate generousness level of users, they reflect opposite changes in the heterogeneity of generousness and improve the total donation at different degrees in different scenarios. This result, along with valuable insights derived from Proposition 3, allows us to uniquely contribute to the literature.

2.6.3. What Leads to "Failures" of UGC Platforms?

Wikipedia has succeeded in the non-profit UGC landscape and is one of the most visited websites worldwide (Similarweb 2024). In contrast, some special-interest platforms fare relatively poorly in attracting content contributions and building high-quality content. For instance, LocalWiki, a non-profit UGC platform founded in 2004, aims to host the entire world's local knowledge. However, as of March 2024, it has only about 100,000 articles from worldwide communities, far from achieving its announced goal. As successful non-profit UGC platforms are rare, it is important to understand why many platforms fail to thrive from both a theoretical and practical point of view.

Several reasons were proposed for the poor performance of special-interest UGC platforms. In particular, the literature and industry conventionally associate an increased number of users with better performance metrics (e.g., see Zhang et al. 2012). Therefore, special-interest UGC platforms with small audiences aim to become sustainable by expanding their user community. Furthermore, these platforms necessitate users to have a certain level of expertise on a given topic to make effective content contributions because many of the platforms that fail to thrive are tailored to special interest groups. Indeed, this "high barrier effect" that discourages users from contributing content is considered a critical factor that deteriorates the quality improvement on these platforms, yet the issue is still open to debate (Everipedia 2019, Torres 2016). Despite its practical importance, the issue of why many UGC platforms have failed to thrive has not been formally investigated in the literature. Therefore, we focus on these critical factors that the industry and literature deem the underlying reasons for the failures of some UGCs and ask: *Do a higher barrier to contributing content and a lower number of users always lead to worse platform performance?* We address this question in Proposition 4.

Proposition 4. *A higher barrier of contributing content (i.e., q_e) always leads to a lower overall content quality (i.e., Π^*) of a non-profit UGC platform. However, unlike the expectations of the industry, a lower number of users (i.e., R_0) may result in a higher overall content quality (i.e., $\frac{d\Pi^*}{dR_0} < 0$) if the barrier of contributing content is higher than a given threshold, i.e., $q_e > \theta_{R_0,q_e}$, where θ_{R_0,q_e} is provided in APPENDIX A.5.*

The practice and literature imply that small user communities and high barrier effects to contribute content are two likely causes of poor performance of special-interest platforms (Everipedia 2019, Torres 2016, Zhang et al. 2012). However, interestingly, our results suggest that while a high barrier to contributing content always leads to poor overall content quality, a small user community does not necessarily result in low overall content quality.

In particular, as the user community expands, it becomes more costly to maintain the user base (see Expression 4). Therefore, as we show in the proof, the platform's in-house effort to improve the overall content quality is reduced (i.e., $\frac{dv^*}{dR_0} < 0$), even though there is an increase in fundraising efforts to enhance sustainability (i.e., $\frac{df^*}{dR_0} > 0$) and there are more donations (i.e., $\frac{dD^*}{dR_0} > 0$). On the other hand, as the platform houses more users, the platform enhances its community support effort to take advantage of the expanded user base (i.e., $\frac{dh^*}{dR_0} > 0$), leading to an increased total content contribution from users. Evident from Lemma 1, this tradeoff between increased users' contributions and decreased in-house quality improvement depends on the barrier effect to contributing content. In particular, if the contribution barrier is low (i.e., $q_e < \theta_{R_0,q_e}$), the platform's community support effort increases considerably, resulting in an improvement in the platform's overall content quality. In contrast, if the contribution barrier is at a high level (i.e., $q_e > \theta_{R_0,q_e}$), the increase in the community support effort is less pronounced (i.e., $\frac{d^2h^*}{dR_0 dq_e} < 0$). Therefore, when the barrier to contributing content is high, having a smaller user base results in savings (due to more efficient community support and fundraising efforts) better utilized in in-house quality improvement effort, eventually increasing the overall content quality.

From a practical perspective, special-interest platforms, by their nature, usually necessitate users to have a certain level of expertise to contribute content. According to Proposition 4, this barrier effect does not only hinder the improvement of the platform's overall content quality but also deteriorates the benefits that the platform can get from a larger user community. Therefore, to overcome the obstacles in quality improvement and become more viable, special-interest platforms should prioritize the mitigation of high contribution barriers or better incentivize their users to contribute. In particular, beyond implementing a better community support effort, as explicitly captured in our model (see Expression 3), having a more generous community in contributing content can also mitigate the negative impact caused by the high contribution barrier. For example, by utilizing strategic advertisement campaigns and maintaining user relationships, non-profit UGC platforms can enhance users' feeling of belonging and sense of mission, resulting in a higher level of user generousness in making content contributions. The enhanced user generosity, as discussed above, can compensate for the barrier effect to contributing content, which may (i) lead to an improvement of the overall content quality, and (ii) allow the platform to benefit from a more effective community expansion strategy.

Next, instead of examining the role of population size and the actions or decisions of UGC platforms, the literature generally focuses on user community structure and aggregate features (e.g., see Zhang et al. 2012). Therefore, the earlier studies do not account for the strategic behavior of individual users in UGC platforms. Besides, the earlier studies (e.g., Zhang et al. 2012) that discuss the relationship between the population size and the platform's business performance usually adopt an empirical approach and focus on specific contexts based on data availability. In contrast, we adopt a comprehensive modeling approach by capturing the barrier effects to contribute and donate, the platform's quality improvement mechanism, and the strategic interaction between the UGC platform and users, and provide new and generalizable results and insights. In particular, our result suggests that a higher number of users does not necessarily mean a higher overall content quality. This finding is in contrast to the literature that assumes a positive relationship between the population size of users and the overall performance of platforms (e.g., see Zhang et al. 2012). Besides, Proposition 4 highlights the significant role of contribution barriers in the platforms' failures to thrive. These findings, along with other results that we have outlined so far, uniquely contribute to the literature and the practice.

2.7. Extensions

In this section, we provide several robustness checks by examining our results in several model specifications different from our main model.

2.7.1. Generousness Levels Regarding Content Contributions and Donations Are Correlated

Although we consider that users' generousness to contribute content and generousness to donate are independent in our main model, in different settings, it might be possible that users who prefer to contribute content are more likely to donate to the platform as well, especially in "small-scale or niche" platforms. For instance, a high proportion of donors of Citizendium also contributed content to help enhance its content quality (Citizendium 2024). Therefore, to examine this slightly different business setting, in this extension, we consider the specification that a user's generousness to contribute content (i.e., $b_{e,i}$) is positively correlated with their generousness to make donations (i.e., $b_{d,i}$). In particular, we consider that: $b_{e,i} = \rho b_{d,i} + m$, $\rho > 0$, and keep the other components of our model the same. This extension provides qualitatively similar results to those in Propositions 1$\sim$4 of the main study. Further details regarding this extension are provided in APPENDIX A.6.

2.7.2. Robustness to The Disutility Elasticity of Users

In this extension, we provide evidence that the results in our main model do not depend on the specification of users' disutility elasticity. In particular, we consider the following, more generalized version of the user utility function (i.e., Expression 3):

$$U_i = \Pi + ((b_{e,i} - (q_e - h^{1/2}))e_i - c_e e_i^{\eta}) + ((b_{d,i} + f^{1/2} - q_d)d_i - c_d d_i^{\zeta}),\ \forall i \in \{1, \ldots, R\}.$$

In this extension, we consider three other specifications, i.e., $(\eta, \zeta) \in \{(2,3), (3,2), (3,3)\}$, other than that utilized in the main model, i.e., $(\eta, \zeta) = (2,2)$. We analytically prove or numerically observe that, in each case, the key results in Propositions 1$\sim$4 are qualitatively the same. Further details are provided in APPENDIX A.7.

2.7.3. Robustness to The Cost Elasticity of The Platform's Efforts

In this extension, we provide evidence that the results in our main model do not depend on the specification of the cost elasticity of the platform's efforts (i.e., the community support and fundraising efforts exerted by the platform). In particular, we consider the following more generalized version of the user utility function (i.e., Expression 3):

$$U_i = \Pi + ((b_{e,i} - (q_e - h^{1/\beta}))e_i - c_e e_i^2) + ((b_{d,i} + f^{1/\alpha} - q_d)d_i - c_d d_i^2),\ \ \forall i \in 1, \ldots, R.$$

In this extension, we consider three other specifications, i.e., $(\alpha, \beta) \in \{(2,3), (3,2),$

$(3,3)\}$, other than that utilized in the main model, i.e., $(\alpha, \beta) = (2, 2)$. We analytically prove or numerically observe that, in each case, the key results in Propositions 1$\sim$4 are qualitatively similar. Further details are provided in APPENDIX A.8.

2.7.4. Negative Quality Contribution by Users

Although mainstream UGC platforms widely adopt tools such as rollback systems to prevent damage from users (Wikipedia 2024e), we explore this variation of our model to provide insights for platforms that do not have efficient rollback mechanisms or tools. Specifically, in this extension, we consider that the impact of content contribution on overall content quality is heterogeneous across content contributions (hence, we have individual-level parameters $a_{e,i}$ but not a_e), and it can be negative. We further consider that although $a_{e,i}$ is unobservable for all parties, the distribution is common knowledge. Therefore, we utilize a Hotelling line formulation to derive the impact of the content contributions made by each user on the overall content quality and denote the upper bound of $a_{e,i}$ with $\overline{a_e} > 0$ and the lower bound with $\underline{a_e} < 0$. We show that the key results in Propositions 1$\sim$4 are either the same or quantitatively similar to their counterparts in the main model. We provide further details and proofs in APPENDIX A.9 of the Appendix.

2.7.5. Robustness to The Distributional Assumptions for Users' Generousness

In this extension, we validate that the results in our Proposition 3 do not depend on the distributional assumptions for user generousness. In this extension, rather than considering uniformly distributed user generousness, we consider more realistic bimodal distributions where users are likelier to have low-level generousness than high-level generousness. We visualize a sample distribution of user generousness in contributing content (i.e., b_e) below. This extension also considers a bimodal distribution for donation side heterogeneity but we skip the discussion for brevity.

We denote the two bounds between the base and upper generousness levels of content contribution as b_{e,t_l} and b_{e,t_u}, where $b_{e,t_l} < b_{e,t_u}$. The portion of users who have low-

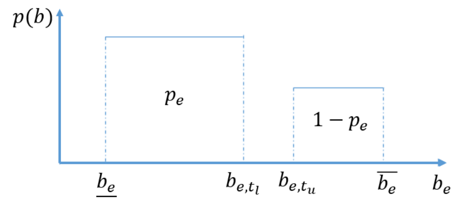

Figure 3. *A Sample Distribution of User Generousness in Contributing Content*

level generousness of contributing content (i.e., $\underline{b_e} < b_{e,i} < b_{e,t_l}$) is p_e. Based on the practical concern that only a small portion of UGC users are likely to contribute content or donate, we consider that $p_e > 1 - p_e$ and $\frac{p_e}{b_{e,t_l} - \underline{b_e}} > \frac{1-p_e}{\overline{b_e} - b_{e,t_u}}$. As explained in further detail in APPENDIX A.10 of the Appendix, we ensure that at least a portion of both user types contribute content, in addition to other practical considerations. We show that all our results in the main model are qualitatively similar to those in this extension.

2.7.6. Linear Disutility in Making Donations

In this extension, instead of considering a decreasing returns-to-scale setting, we consider that users induce a linear disutility when they make donations. In particular, user i's utility gain from donating d_i is $\Delta_{d,i}U = (b_{d,i} + f^{1/2} - q_d)d_i - c_d d_i$. We prove or numerically observe that the key results in Propositions 1$\sim$4 are qualitatively the same. Further details are provided in APPENDIX A.11.

2.7.7. Heterogeneous Platform Usage

In this extension, instead of considering the constant cost of serving each user, we consider a more traditional business setting where users' generousness level (which directly relates to their donations and content contributions) is positively correlated with the level of site usage (i.e., the serving cost) at the individual level. We define the cost of serving user i as $x_i = m_d b_{d,i} + m_e b_{e,i}$. Through proofs or numerical studies, we show that the key results in Propositions 1$\sim$4 are qualitatively the same as our main model. Further details are provided in APPENDIX A.12.

2.8. Conclusion

In this section, we summarize our findings and discuss practical insights. Following the success of Wikipedia, a growing number of platforms strive to be viable by adopting *Wikipedia's business model*. Specifically, they utilize users' content contributions to improve their overall quality and maintain financial sustainability through external party funding and user donations. As several important issues have not been addressed in the literature, we adopt a game-theoretical model to fill this gap and provide valuable findings and managerial insights.

2.8.1. Managerial and Theoretical Implications

First, although non-profit UGC platforms rely on users to create content and maintain financial sustainability, they can also attain higher content quality by exerting in-house effort (Wikimedia 2022a). The efficiency (or cost-effectiveness) in improving the content quality in-house is an important measure for the platform to maintain sustainability (Levin and Chisholm 2016, Mamatzakis et al. 2019). As the rapid growth in generative AI technologies brings platforms a considerable advantage to exert the in-house effort with greater efficiency, an important question arises: *How does a UGC platform's efficiency of in-house effort impact its community management strategy and performance?* The literature has overlooked these relationships and has yet to investigate this question. However, our findings reveal that (see Propositions 1 and 2) the answer to this question is nuanced and depends on the number of users on the platform and the base level of the cost-effectiveness of in-house effort. In particular, if a UGC platform is already efficient in exerting in-house effort to improve the content quality, becoming more efficient in exerting in-house effort (e.g., utilizing generative AI) does not necessarily make the platform even more self-reliant. This is because the platform does not necessarily reduce its efforts in managing its user community to make more content contributions (with its community support effort) and donations (with its fundraising effort).

This result is also observed in Wikipedia's effort-allocation strategy. Although Wikipedia has become more efficient in exerting various in-house efforts (Kanowitz 2020, Nasaw 2012), it has also been investing more in supporting the contributor community (Wikimedia 2024). Besides, if the platform serves a large population of users and becomes more efficient in utilizing its in-house quality improvement effort, while the platform experiences a decrease in the total content contribution from users, it benefits from a more significant improvement in the overall content quality. Therefore, we report that a UGC platform might be willing to forgo a level of content contribution from users to reach a higher overall content quality, as discussed in CHAPTER 2.6.1. This is in stark contrast to the literature's general focus on maximizing content contribution from users to improve the content quality. These results are missed in the earlier body of work since the literature overlooks the vital role of platforms in user community management and content management to achieve greater overall quality.

We further find that (see Corollary 2) if the platform is sufficiently efficient in exerting in-house effort, an increase in the same measure enables the platform to increase the number of content contributors. Another practical insight is that (see Proposition 2 and Corollary 2), interestingly, if the efficiency is at a low level and the platform serves a large number of users, although the number of content contributors might decrease, their average level of content contribution may increase, resulting in an increase in the total contribution from all users. Therefore, managers of UGC platforms should account for the average level of user engagement rather than focus on the number of contributors only, which is generally the case in practice (Brandt 2013).

Furthermore, because non-profit UGC platforms rely on user generousness to raise funds and help support their operations, heterogeneity of user generousness is a common concern for them (Levina and Arriaga 2014, Marsh 2016). However, unlike our study, the literature has yet to investigate how heterogeneity in user generousness affects the financial sustainability of UGC platforms. In particular, among other factors, we also investigate the change in generousness heterogeneity using a two-pronged approach by increasing the lower or upper bounds of user generousness levels. Our results imply that (see Proposition 3) if the generousness heterogeneity of inherent donors (i.e., users who are inherently altruistic and would donate even without the influence of the platform's fundraising effort) is at a low level, in contrast to increasing the lower bound of user generousness, enhancing the upper bound of user generousness allows the platform to achieve greater financial sustainability (although increasing both bounds are better for the platform). In contrast, if the inherent donors present high heterogeneity in their generousness, an increase in the lower bound of user generousness benefits the platform's financial sustainability more.

Moreover, although Wikipedia has been successful, many other non-profit UGC platforms, especially those that cater to niche user groups, have failed to thrive. Literature and practice imply that small user communities and high barriers to contributing content are likely the potential reasons. Because the literature has not formally examined these issues, we fill this gap to provide managerial insights to these platforms. Our results suggest that (see Proposition 4) while a low barrier to contributing content helps platforms improve the overall content quality. More interestingly, we also find that a larger user community does not improve the overall content quality if the base level of the barrier effect is high. Therefore, unlike the expectation in practice, expanding the user community cannot guarantee the business success of these special-interest platforms, especially when they have a high barrier effect to contributing content for users. Our findings, then, emphasize the critical role of the contribution barrier in deterring non-profit UGC platforms from thriving.

Furthermore, we examine different business settings that we do not focus on in our main model. We find that our key results are either the same or qualitatively similar to those of the main model, indicating that our key results are robust to the specification of different model components. The alternative specifications include: (i) user generousness to contribute content is positively correlated with user generousness to donate money (which may be the case in small-scale or niche UGC platforms); (ii) alternative cost elasticities of the platform's efforts; (iii) alternative disutility elasticities of users' donation and contribution behaviors; (iv) users' content contribution may deteriorate the overall content quality; (v) alternative distributions of user generousness to make donations and content contributions; (vi) the disutility of users' donation behavior is linear; and (vii) the hosting cost of users is positively correlated to their involvement in the platform.

Therefore, our results and findings yield several practicable managerial insights. Furthermore, through an all-inclusive modeling approach and analyzing user and platform behavior at equilibrium, we contribute to the literature on UGC platforms by revealing the mechanisms whereby non-profit UGC platforms can financially sustain and flourish their businesses. Our study also uniquely contributes to the literature by proposing a comprehensive modeling framework in settings with concurrent donation and content contribution aspects of non-profit UGC platforms. Furthermore, by investigating different practice-oriented phenomena in the context of UGC platforms that the literature has yet to focus on thus far, our study fills many gaps in the literature.

First, our study examines the influence of the enhanced efficiency of the platform's in-house quality improvement effort (e.g., because of its adoption of generative AI technologies) on the non-profit UGC platforms' community management and user participation efforts. Second, as the financial sustainability of non-profit UGCs is one of the most critical factors that influence their business success, our study examines the association between the heterogeneity of user generousness and the financial sustainability of non-profit UGC platforms. Third, we concentrate on the essential factors influencing the success of platforms that function similarly to Wikipedia, despite many of them struggling to become viable. All of these contribution are unique in the literature.

2.8.2. Future Research Directions

Although a small number of UGC platforms utilize it, no study (to our knowledge) has investigated a setting in which users can gain access to the platform only by contributing content or paying a subscription fee. In such a setting, the generousness aspect of our model is fully or partially replaced with utilitarian gains, and a comparison with our work may yield interesting results and managerial insights. Besides, a lab experiment that investigates user behavior while making donations or content contributions may yield practicable insights into UGCs. Furthermore, in business contexts of traditional UGC platforms and other types of online businesses, business entities may allocate more capacity to serve users that utilize their service more intensively or make more payments. We list modeling this interesting business setting as a future work possibility.

CHAPTER 3

TIMING MATTERS: CROWD-SOURCING WORKERS IN ON-DEMAND FREIGHT-MATCHING PLATFORMS

3.1. Abstract

The long-haul freight transportation industry has undergone significant changes with the advent of freight-matching platforms like Uber Freight and UShip. These platforms capitalize on the benefits of crowdsourced drivers but also encounter challenges in managing freight logistics, especially in driver sourcing. To tackle these issues, we utilize a comprehensive dataset from a leading U.S. freight-matching platform and conduct data-driven analyses. Our preliminary research highlights the significant impact of customers' order timing on drivers' preferences and behaviors in bidding for orders, prompting a deeper examination into the impact of customers' order timing on freight-matching performance. Therefore, we rigorously investigate how the request lead time, namely the interval between the shipper's order placement and the pickup date, affects sourcing costs and the probability of successful freight matches. Our findings highlight the significant impacts of request lead time on both business metrics. Specifically, sourcing costs exhibit a U-shaped relationship with lead time, while the probability of successful matches demonstrates an inverse U-shaped pattern.

The distinct impacts and optimal points of request lead time on sourcing costs and matching probability suggest a strategy to achieve optimal profitability by balancing increased matching probabilities with cost savings. For this initiative, and to provide a viable manner for the platform to apply the best practice using our empirical insights, we propose the policy innovation named "preordering policy," wherein platforms could predict shipping demand and initiate driver searches at the optimal timing to realize the best overall profit. We further validate the effectiveness of this policy through a well-designed counterfactual analysis. Our study holds practical significance and contributes to several literature

streams by offering a nuanced understanding of how strategic timing can optimize operations for digital logistics platforms.

3.2. Introduction and Motivations

Long-distance freight transportation for carrying large quantities of goods through commercial trucks plays a critical role in ensuring efficient movement and stable availability of production materials, products, and physical resources (Hall 2003). According to IBIS World (2022), long-haul trucking owns a market size of 331 billion dollars (measured by revenue) in 2022. Besides, Costello (2023) reports that truck transportation moves about 72.6% of the national freight by weight.

Among all types of long-haul transportation businesses, in recent years, freight-matching platforms, such as Uber Freight and UShip, emerged as an innovative and disruptive business model that allows transportation platforms to serve shippers using the crowdsourced workforce. Several online platforms adopt the crowdsourced transportation model by providing freight-matching services between their shippers and crowdsourced carriers.

In particular, unlike the traditional carrier businesses that have to own vehicles and hire full-time drivers, the crowdsourced model features transportation firms with (i) no fixed costs, as there are no expenditures necessary for purchasing vehicles and employing drivers, (ii) flexible capacities of serving shippers, as the capacity of serving shippers depends on the availability of carriers in the market and can be adjusted relying on monetary incentives (Kelley 2017), and (iii) low marginal cost, as drivers typically consider the freight-matching platform to search for backhaul orders to complement their headhaul transportation plans (LaGore 2020).[8]

Freight-matching platforms typically operate as a preorder system, where shippers place their requests to the platform ahead of the required pickup dates. This process allows the platform a window of time between the request time and pickup time to search

[8] Headhaul or Fronthaul describes the transportation from a driver's origin to the drop-off location, whereas backhaul indicates the journey of returning to the driver's origin spot.

for drivers. Drivers receive the shipment information shared by the platform and bid for these shipping requests by providing their desired payments. Notably, drivers' preferences on a shipping request (reflected in their offer rates) are characterized by high-level heterogeneity, especially as the pickup date of the request draws closer. An important reason for the increased heterogeneity is that, as the pickup date of the request approaches, drivers gradually obtain more information about their future routes and transportation schedules. In particular, drivers who have ensured that their future drop-off is near the pickup location would be willing to bid with low offer rates due to their low travel costs and availability. However, drivers who will be distant from the pickup location may bid a high offer rate to compensate for their high travel costs or inconvenient costs of picking up the order (referred to as pickup cost). Consequently, drivers' offer rates vary widely, reflecting the diverse costs they anticipate incurring to undertake the shipment. This heterogeneity in drivers' offer rates increases as the pickup date draws closer. For clarity, we define the time difference between offer arrivals and pickup dates as offer countdown time. As shown in Figure 4, the variance of drivers' offer rate per mile steadily increases as the pickup date draws closer, i.e., the offer countdown time, which is the time difference between offer made and pickup date, becomes shorter.[9]

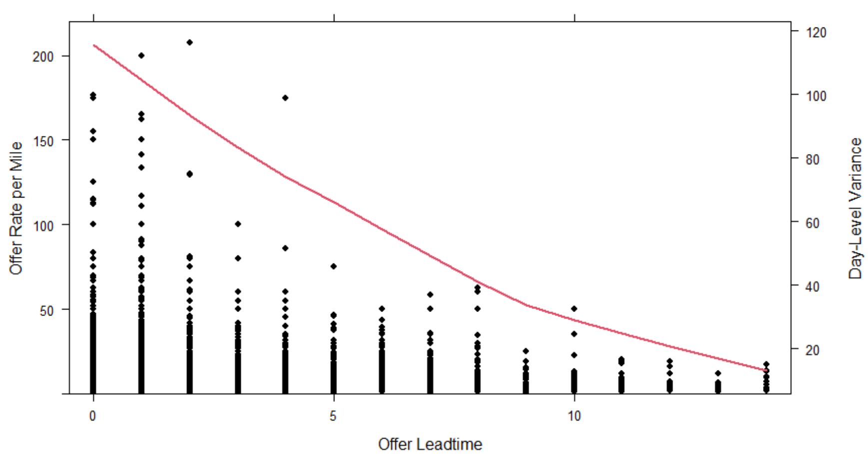

Figure 4. *Offer Rate versus Offer Countdown Time*

The changing heterogeneity in drivers' preferences over time highlights the critical role

[9]The red line depicts the variance of drivers' offer rate per mile at different offer countdown time.

of shippers' order timing in influencing the platform's profitability and matching probability. Specifically, on the one hand, an early search for drivers provides the platform with a longer time window, increasing the likelihood of successfully matching the request and gaining a low offer rate. On the other hand, a late search can yield a wider range of offer rates (due to the increase in drivers' preference heterogeneity over time). This may help the platform discover the lowest rate, particularly from drivers who realize their future locations will be near the pickup spot. While the shippers' order timing critically impacts the freight-matching outcomes and, thus, the business performance of freight-matching performance, the relationship has yet to be formally analyzed in the practice of freight matching. Therefore, we collect industrial data and investigate this issue.

In this paper, we focus on several performance metrics that are vital for freight-matching platforms. First, because the relatively low sourcing cost is important for sustaining the profitability of a freight-matching platform, we first ask: *Whether and, if so, how does shippers' order timing impact the sourcing cost of freight matching?* With a comprehensive understanding of this relationship, freight-matching platforms can reduce their cost by strategically planning driver searches and freight matching. Second, a high matching rate can benefit the freight-matching platform by meeting the expectations of shippers, enhancing overall satisfaction, and increasing the profitability of the freight-matching platform freight-matching platforms' profitability. Therefore, we next ask: *Whether and, if so, how shippers' order timing impacts the freight-matching probability?* Besides, to illustrate the business value of our empirical findings, we further ask: (iii) "*How, and to what extent, can our empirical findings benefit freight-matching platforms?*"

We collected a large and granular dataset from an online freight-matching platform that runs one of the largest transportation networks in the United States. We focus on order-level data (38,575 freight-matching orders) involving the platform's top ten shipment routes from December 2017 through February 2018. The dataset records order-level information on the shipment route, time stamps, the shipper's payment rate, sourcing costs, and freight-matching statuses. Leveraging the depth of our dataset, we measure shippers' order timing by defining *"request lead time"* as the time difference between when shippers send their requests and the pickup date of their shipping orders.

We implement a comprehensive data modeling approach, and our findings suggest that, first, shippers' order timing significantly impacts the freight-matching probability in an inverse-U-shaped pattern. In particular, as the request lead time increases, the freight-matching probability increases if the request lead time is low, and it may decrease otherwise. Secondly, we find that the influence of shippers' request lead time on the platform's freight-matching sourcing cost is nuanced and significant. Specifically, as the request lead time increases, the freight-matching sourcing cost reduces if the request lead time is low, and it may increase otherwise.

Furthermore, to answer our third research question (i.e., illustrating our empirical findings' business value), we propose the "preordering policy." Under this policy, the freight-matching platform predicts the demand for shipping services and searches for drivers even if shippers have yet to send their shipping requests. By making the order timing earlier, we conduct counterfactual analyses and simulate the freight-matching performance under this policy. In particular, we (1) develop a theoretical-based modeling work to account for drivers' abandonment behavior, and (2) adopt a data-driven manner for reproducing the freight-matching sourcing costs in the counterfactual case (i.e., the scenario where the platform adopts the preordering policy). Our simulation results suggest that the preordering policy, coupled with our empirical estimations, significantly improves the platform's profit and freight-matching probability.

3.3. Literature Review

Our paper is related to the following streams of literature: (1) traditional long-haul transportation services, (2) on-demand platforms, and (3) self-scheduling staffing. We highlight our contributions to each of these literature streams in this section.

First, the literature on traditional long-haul transportation services has dramatically expanded in past decades because of the increasing importance of long-haul transportation in the global economy. For an overview of the literature, we refer readers to Gorman et al. (2014) and Hall (2003). Align with the traditional business settings, this stream of the literature focuses on efficient driver dispatch policy (Gendreau et al. 1999, Hall 2003, Larsen et al. 2002, Powell 2007, Psaraftis 1995, Simao et al. 2009, Taylor et al. 2001) and design of transportation networks (Ali et al. 2002, Cabral et al. 2007, Campbell et al. 2005, Uster and Kewcharoenwong 2011) based on drivers' schedules determined by the firm. These studies highlight that through strategic decision-making, long-haul transportation firms can effectively align the supply level (i.e., transportation capacity) with shippers' demands while improving drivers' welfare. Our paper also focuses on reducing platforms' transportation expenses and better aligning the transportation capacity with the demand level but differs from this stream of literature in several perspectives. First, our work considers the business setting of freight-matching platforms where drivers' schedules and preferences are less predictable or controllable than the traditional settings. Second, in traditional settings, drivers receive per-mileage or hourly wage, leading to their homogeneous preferences on routes. In contrast, drivers exhibit heterogeneous preferences within freight-matching platforms as they need to account for pickup costs, which become even more pronounced as the pickup date approaches. The dynamics of heterogeneity highlight the importance of investigating how the timing of driver search, as a critical factor for drivers' preference heterogeneity, affects freight-matching performance.

Besides, our study is closely related to the literature on on-demand platforms, especially that of on-demand ride-hailing platforms. These platforms, such as Uber, match customers' riding requests with drivers and provide just-in-time services. The earlier studies in this stream cover pricing strategies (e.g., Bai et al. 2018, Banerjee et al. 2021, 2015, Chen et al. 2023, Feng et al. 2023) and the efficiency of matching (e.g., Braverman et al. 2019, Feng et al. 2020, Hong et al. 2020). Note that, in ride-hailing platforms, drivers' preferences are more homogeneous than those in freight-matching platforms, as drivers' payoffs are uniformly determined by hourly wages and their utilization. In contrast, in the context of freight-matching platforms, long-haul drivers' preferences are more heterogeneous as they have more notable differences in their availability and distance to pick up an order. This heterogeneity, as discussed, becomes more pronounced as it draws closer to the pickup date. This dynamics underlines the importance of the request lead time on the platform's performance. Our study aligns with this focus and manages to produce insightful discussions on this issue.

Moreover, because crowdsourced drivers generally decide their own work schedule, our study also relates to the literature on self-scheduling capacity. The literature focuses on adjusting the supply capacity to meet the demands of consumers in a business environment where workers can decide their work schedules (Allon et al. 2023, Banerjee et al. 2015, Gurvich et al. 2019, Lin and Zhou 2018, Lu et al. 2018, Cachon et al. 2017, Taylor 2018). To address the staffing issue in this context, the earlier studies focus on the effectiveness and implementation of pricing adjustment (including both dynamic and surge pricing) as an incentive strategy to drivers (e.g., Banerjee et al. 2015, Cachon et al. 2017, Gurvich et al. 2019, Taylor 2018), delay announcement (see Ibrahim 2017), caps on agent participation (see Gurvich et al. 2019, Yu et al. 2020), and blended workforce strategy (see Dong and Ibrahim 2020, Dong et al. 2021, 2022).

A stream of this literature specifically focuses on crowdsourced transportation. For example, several earlier studies investigate algorithmic matching of available drivers (e.g., Allahviranloo and Baghestani 2019, Arslan et al. 2019, Boysen et al. 2022, Dayarian and Savelsbergh 2020, Soto Setzke et al. 2017) with evaluation process relying on practical data (e.g., Soto Setzke et al. 2017), generated data (e.g., Boysen et al. 2022) or various scenario settings (e.g., Arslan et al. 2019). Other studies in this stream of literature also analytically investigate crowdshipping platforms' optimal pricing strategies (e.g., Fatehi and Wagner 2022, Wang and Xie 2021) and interactions and decision-making behaviors that exist between crowdshipping platforms and drivers (e.g., Fan et al. 2022, Zhang et al. 2020). Besides, a handful of studies empirically investigate the motivational factors of crowdshipping drivers, such as travel distance, geographic disparity, and pricing, and propose helpful suggestions to enhance drivers' participation on crowdshipping platforms (e.g., Ermagun and Stathopoulos 2018, Miller et al. 2017). However, unlike these existing studies that mostly focus on developing algorithms and motivational strategies to improve the self-scheduling capacity, our work focuses on the timing of capacity planning (i.e., driver searching in our context), which is also critically influential on the capacity control of self-scheduling workers (i.e., crowdsourced drivers in our context) but has yet to be formally investigated. Therefore, our study fills this gap in the literature and makes unique contributions by investigating this influence and generating several important insights.

Our study is also related to the literature on workforce adjustment in traditional business settings, such as just-in-time scheduling. Specifically, the literature focuses on the effects of schedule consistency and predictability on worker productivity (e.g., Kamalahmadi et al. 2018, Lambert et al. 2019, Lu et al. 2022) and work-life balance (e.g., Henly and Lambert 2014, Lambert 2008), the relationship between flexible labor resources and financial performance (e.g., Kesavan et al. 2014), and staffing models that allow dynamic adjustment of the staffing level (e.g., Bhandari et al. 2008, Hur et al. 2004, Slaugh et al. 2018). While the crowdsourced business model and just-in-time scheduling both offer companies the ability to adjust their operations and capacity according to need, a key distinction lies in the nature of the workforce: just-in-time scheduling depends on traditional employment forms, such as full-time, part-time, and on-call workers. These traditional workers generally receive predetermined levels of salary as defined by their contracts. In contrast, crowdsourced workers engage in a dynamic negotiation of their payment, with their payment expectations shifting over time. This flexibility inherent to the crowdsourced workforce highlights the importance of investigating the impact of order timing on drivers' participation and pricing strategy - an area not applicable to the rigid salary structures of traditional employment.

3.4. Study Context and Hypothesis Development

We consider a context where an online freight-matching platform looks for crowdsourced drivers to ship orders at pickup dates requested by shippers. In particular, a shipper sends a shipping request (i.e., freight-matching order) to the freight-matching platform before their requested pickup date. The period (in days) between requesting the service and picking up the freight is referred to as request lead time which varies for shippers and orders. After receiving freight-matching orders from shippers, the platform sends the order information to the pool of drivers. For each shipping request, interested drivers informed by the platform may provide an offer indicating their desired compensation for completing the shipment, referred to as an offer rate. The platform then matches the shipping request with the driver who provides the lowest offer rate. Nevertheless, if none of the offers prove profitable for the platform, it reserves the right to decline all offers, resulting in unsuccessful freight matching. Once a driver's offer is accepted and matched to the shipping request, the accepted offer rate is referred to as the sourcing cost for the platform for that order.

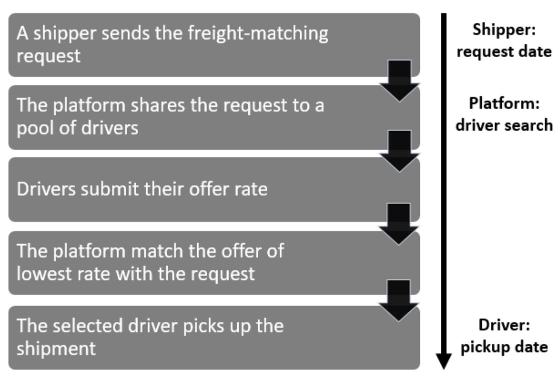

Figure 5. *The Process of Freight Matching*

In this study, we investigate the direct impacts of request lead time on two main performance measures: freight-matching probability (i.e., the probability of successfully matching shipping requests with drivers' offers) and sourcing costs (i.e., payment to crowdsourced drivers for completing shipping requests). We now elaborate on the positive and negative effects of request lead time on these performance metrics separately.

Positive Effects. A long request lead time may result in high freight-matching probability and low sourcing costs. The literature on service operation system optimization suggests that business entities usually consider a long service process time to increase their flexibility and variability, which helps better serve their customers (Hopp et al. 2007). If a shipper submits a request for the freight-matching service with a long request lead time, the platform can do an early search for drivers and potentially receive more offers from the crowdsourced drivers, resulting in a fair likelihood of successful freight matching (i.e., high freight-matching probability). The long request lead time also allows the platform to have more time to look for more offers from carriers, increasing the chance of low sourcing cost and high profitability (Hopp and Spearman 2000).

Negative Effects. Although a long request lead time may improve freight-matching performance due to greater flexibility, an extraordinarily long request lead time may result in drivers' low willingness to make offers and high offer rates. In particular, drivers typically use freight-matching platforms to find backhaul orders, which help them avoid returning empty after delivering their initial loads, known as headhaul shipments. As the pickup date of a shipping order on the platform draws closer, drivers become more aware of their upcoming dropoff locations for the headhaul orders. This information acquisition leads to increased heterogeneity in drivers' route preferences due to the broader range of pickup costs. This increased heterogeneity may allow the platform to receive lower offer rates from drivers who learned that their headhauls' dropoff locations are close to the pickup location of the shipping request. This proximity increases the chances of successfully matching shipping requests. Therefore, from the platform's perspective, a longer request lead time (i.e., an earlier-arrived freight-matching request) may not always be advantageous.

In summary, a longer request lead time may improve the platform's freight-matching performance. However, if the request lead time is excessively long, any additional increase could lessen or reverse its beneficial effects. Hence, we propose the following hypotheses regarding the effect of request lead time on freight-matching performance, such as the freight-matching probability and sourcing costs.

HYPOTHESIS 1. If the request lead time is low, as it increases, the freight-matching probability increases. Otherwise, the freight-matching probability decreases.

HYPOTHESIS 2. If the request lead time is low, as it increases, the freight-matching sourcing cost decreases. Otherwise, the freight-matching sourcing cost increases.

Furthermore, because we have informative industrial data that includes several critical factors, we may extend the hypotheses and empirical analyses to understand the moderating effects of these factors on the association between request lead time and freight-matching performance.

3.5. Data and Variable Definitions

We describe the data in CHAPTER 3.5.1 and define the critical variables in CHAPTER 3.5.2

3.5.1. Data Description

We collect data from a freight-matching platform that serves over 100,000 companies worldwide. The collected dataset involves orders (shipping requests) in the platform's transportation network (composed of over 10,000 cities, towns, and villages) in the U.S. The date of shipping requests accounts for four months, ranging from November 2017 to February 2018. Notably, the dataset provides us with order-level details regarding shipment, scheduling, and financial information.

While our data includes all nationwide freight-matching orders during the observation horizon, we specifically focus on the busiest 20 routes, i.e., pairs of states that have the most records of freight-matching orders, including 49,709 order records. This is because, as we discuss later in CHAPTER 3.6, we adopt route-level lagged instruments to address potential endogeneity issues, so we need to consider routes with adequate order records for the valid computation of instrumental variables. We present more details on our data in Table 2.

For each freight-matching order, we observe (1) the shipment information, including the transportation mileage, origin and destination, and the number of stops during delivery, (2) the transaction information, including the shipper's payment rate, whether the request was successfully matched with a driver (i.e., matching outcome), and the driver's requested payment rate (i.e., sourcing cost), and (3) the scheduling information, including the order placement date and the shipment pickup date.

In our study, we examine the influence of shippers' order timing on drivers' participation and bidding behaviors. This dynamic is particularly relevant for long-haul orders, as short-distance orders, which drivers can complete and return from within a day, generally do not significantly alter the drivers' existing transportation plans or notably increase their travel costs. According to our hypothesis development, the impact of order timing is less pronounced in these cases. Therefore, to maintain a clear focus and relevance, we specifically consider freight-matching requests that cannot be completed and travel back to the origin in a single day. According to CTA (2020), the average travel distance of a long-haul driver is 500 miles. Therefore, we only keep the orders whose transportation distance is above 250 miles. This includes 25,416 order records. We further remove 5% top of transportation mileage and the 2.5% top and bottom of shippers' payment rate to remove the effect of outliers (e.g., very long-distance or special orders). The final order records include 23,434 orders.

3.5.2. Performance Measurements and Variables Definitions

To examine the association between the request lead time and the freight-matching performance, we introduce two critical metrics of freight-matching performance. We also consider several critical factors (including focal and control variables) that have explanatory power on these performance metrics. We elaborate on their definition and summary statistics in this section.

Dependent Variables. In this study, we consider the freight-matching probability and sourcing costs as two critical metrics of the platform's freight-matching performance. First, to infer freight-matching probability (i.e., the likelihood that the platform can find a proper matching for a shipping request), we define a binary variable that equals one for successfully matched requests and zero otherwise, denoted as $Booked_i$ for shipping request i. Second, we adopt the freight-matching sourcing cost, i.e., the actual payment rate to drivers, as another critical performance metric and denote it as $Cost_i$ for request i.

Main Explanatory Variable (Focal Variable). Our main explanatory variable is the shippers' order timing. We quantify this factor using the concept of request lead time, defined as the time difference between the request submission and pickup dates of the shipment, and denote it as $Leadtime_i$ for shipping request i. Recall that the request lead time rules the earliest time for the platform's driver searching. Therefore, the request lead time determines the maximum period of time for the platform's driver searching and decision-making. Besides, in align with our hypotheses development in CHAPTER 3.4, to examine the nonlinear association between shippers' order timing and the freight-matching performance, We further introduce the second-order polynomial of the request lead time (i.e., $Leadtime_i^2$) in our empirical model to capture this nonlinearity.

Control Variables. Besides considering the impact of the main explanatory variables (i.e., $Leadtime_i$ and $Leadtime_i^2$), in this study, we also control for several critical variables that are influential to freight-matching performance. First, we control for several shipment-relevant features. In particular, to account for the variation in sourcing cost attributed to shipment distance, we control for the mileage of shipments, represented as $Mile_i$ for freight-matching request i. We also control the number of stops during delivery, denoted as $StopCount_i$, to capture its influence on drivers' inclination to accept orders and their anticipated payment rates. Since the data covers 10 distinct routes, we also control for the fixed effect of each route and denote it as $Route_i$ for the specific shipment route of order i.

In addition to the shipment-relevant characteristics, we also control for multiple features of shippers. In particular, we control for the shipper-level characteristics including each shipper's average unit payment rate per mile (denoted by $AvgUnitRate_{s_i}$) and their number of historical order records (denoted by $Freq_{s_i}$) during the observation horizon, where s_i is the unique identifier of the shipper who submit the order i. Considering these shipper-level variables in our model allows us to capture impacts of shippers' features, such as loyalty levels, knowledge of the market, business classes, and freight types, on freight-matching performance.

Besides, drivers' bidding strategy and the market thickness of transportation services heavily depend on whether freights are picked up on weekends or workdays. Therefore, we also control for the workday fixed effect of the pickup date through a dummy variable $PickupWD_i$, which equals one if the shipment's pickup date is a workday, and zero otherwise. Besides, We control for shippers' payment rate, denoted by $Rate_i$, because of its critical influence on whether the platform would accept an offer rate. We further capture the trend in the freight-matching market by accounting for the weekly fixed effect of the pickup date (denoted by $PickupWY_i$).

Table 2 reports the definition and summary statistics of the main variables defined in this section. Table 3 shows the correlation among quantitative variables. All of these variables are significantly correlated with one or both of our performance metrics.

Variable	**Definition**	**Obs.**	**Mean**	**Std. Dev.**	**Min**	**Max**
Dependent Variables (Performance Metrics):						
$Booked_i$	Dummy variable which equals 1 if order i was matched with a driver, 0 otherwise	23434	0.85	0.36	0.00	1.00
$Cost_i$	Sourcing cost (payment rate to driver) for order i	19960	975.60	442.19	1.00	5000.00
Explanatory Variables (Focal and Control Factors):						
$Leadtime_i$	Request lead time for order i	23434	6.20	6.18	0.03	53.31
$Rate_i$	Shipper payment rate for order i	23434	1145.80	492.94	450.00	3000.00
$StopCount_i$	Number of stops during the delivery of order i	23434	2.20	0.69	1.00	15.00
$Mile_i$	Transportation distance for order i	23434	458.30	231.71	250.00	1403.00
$AvgUnitRate_{s_i}$	Average historical unit payment rate per mile of the shipper who placed order i	23434	2.63	0.71	0.51	15.26
$Freq_{s_i}$	Total number of historical orders placed by the shipper who placed order i	23434	744.90	1219.33	1.00	5025.00
$PickupWD_i$	Dummy variable which equal to 1 if the pickup date of order i was a workday (from Monday to Friday), 0 otherwise	23434	0.94	0.24	0.00	1.00
$PickupWY_i$	Categorical variable that accounts for the weekly fixed effects of order i	23434	Levels: {1, 2, 3, 4, 5, 6}			
$Route_i$	Categorical variable that indicates the pair of the origin and destination of order i	23434	Levels: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}			

Table 2. *Definition and Summary Statistics of The Variables*

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
(a) $Booked_i$	1.00							
(b) $Cost_i$	0.63***	1.00						
(c) $LeadTime_i$	0.06***	0.05***	1.00					
(d) $Rate_i$	0.03***	0.66***	-0.04***	1.00				
(e) $StopCount_i$	0.08***	0.28***	0.07***	0.27***	1.00			
(f) $Mile_i$	-0.04***	0.43***	0.02***	0.62***	0.16***	1.00		
(g) $AvgUnitRate_i$	-0.004	0.13***	-0.05***	0.27***	0.10***	-0.14***	1.00	
(h) $Freq_i$	-0.22***	-0.19***	0.02***	-0.08***	-0.09***	-0.04***	0.05***	1.00

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table 3. *Correlation Matrix*

3.6. Model and Identification Strategy

In this section, we first introduce our empirical models to investigate the impacts of request lead time on freight-matching probability and sourcing costs. We then discuss the endogeneity and selection bias issues and our approach to address these issues.

3.6.1. Baseline Model on Freight-Matching Probability

We adopt a probit regression model as a baseline model to examine the impact of the request lead time on freight-matching probability.

$$Probit(Booked_i) = \alpha_0 + \alpha_1 Leadtime_i + \alpha_2 Leadtime_i^2 + \alpha_x X_i + \epsilon_{B,i}. \qquad \text{(Model 1)}$$

As we demonstrate in CHAPTER 3.5.1, the performance measure, $Booked_i$, is a binary indicator for freight-matching probability. Our main variables of interest are $Leadtime_i$ and $Leadtime_i^2$. X_i is a vector of control variables, as given below:

$$X_i = (Route_i, Mile_i, StopCount_i, PickupWD_i, PickupWY_i, Rate_i, AvgUnitRate_{s_i}, Freq_{s_i}). \qquad \text{(Control List)}$$

Lastly, we let $\epsilon_{B,i}$ be the idiosyncratic shock that captures the impacts of factors unobservable to us on the freight-matching probability.

3.6.2. Baseline Model on Freight-Matching Sourcing Cost

We consider a multivariate linear regression as our baseline model to investigate the impact of shippers' request lead time on the freight-matching sourcing cost.

$$SourcingCost_i = \beta_0 + \beta_1 Leadtime_i + \beta_2 Leadtime_i^2 + \beta_x X_i + \epsilon_{S,i}. \tag{Model 2}$$

The performance measure, $SourcingCost_i$, is the sourcing cost (i.e., the accepted offer rate) for shipping request i, and our key causal variables are $Leadtime_i$ and $Leadtime_i^2$. We consider variables in X_i to control several other important causal effects on the freight-matching performance and let $\epsilon_{S,i}$ be the idiosyncratic shock on the sourcing costs.

3.6.3. Econometric Challenges and Strategies

In this section, we demonstrate the sample selection bias and endogeneity issues that may deter us from producing unbiased model estimation. To simultaneously address both of them, we follow Webb et al. (2019) and cast our problem as a Heckman model with endogenous variables.

Selection bias

The first economic challenge of our model estimation lies in the issue of selection bias. Specifically, the freight-matching sourcing cost of a shipping request is not realized and remains unobservable if the platform fails to match it, and this failed matching may be attributed to factors that also contribute to the sourcing cost. For example, a shipping request to be picked up on a weekend is less appealing for drivers (implying a low likelihood of successful freight matching), and drivers generally require a high compensation to complete this request (indicating a high sourcing cost). Therefore, in this case, the matching result, which ties to the observability of sourcing costs, may relate to the factor (i.e., the weekday of the pickup date) that also affects the freight-matching sourcing costs. This fact may prevent us from obtaining an unbiased estimation of Model 2. For these reasons, in this study, we consider a Heckman procedure for correcting selection biases and ensuring unbiased model estimation (Wooldridge 2010).

To address the potential selection bias in our model estimation, we consider the Heckman Selection model, also known as the Heckman Two-Step Selection procedure, to mitigate the bias arising from non-random "sample selection", i.e., the factors that contribute to the outcome variable also influence whether that outcome can be observed. Specifically, the Heckman model corrects the sample selection bias using a two-stage procedure: in the first step, it explains the selection process of samples, i.e., observability of the dependent variable, based on explanatory variables that are influential to this dependent variable. Based on the estimation results of the first step, it generates a correction variable named inverse Mills ratio. Next, in the second step, the Heckman model estimates the base model (see Model 2) and corrects the selection bias by involving the inverse Mills ratio as an additional controlled factor. We now elaborate on more details of each step.

First, we characterize the selection process of samples in the first step using the following Probit model:

$$D_i = I\{h_v V_i + \zeta_i \geq 0\}, \tag{Heckman stage 1}$$

where D_i is a binary variable indicating whether freight order i has an observable sourcing cost ($D_i = 1$) or not ($D_i = 0$) in our data. Besides, V_i is a vector of variables that may impact the observability of sourcing costs, and h_v captures their influences on the observability. Our model considers that V_i includes all explanatory variables in the baseline model and an additional control variable that helps avoid potential singularities in applying the Heckman model. In this sense, we consider a lagged Heckman instrument, which is the number of drivers' offers received one week earlier, to address the singularity issue.

The error term ζ_i captures how unobservable factors (e.g., the category of freights) impact the observability of the dependent variable. To capture the correlation between the selection process (i.e., observability of sourcing costs) and the freight-matching sourcing cost, we follow Heckman (1979) and assume that (ϵ_i, ζ_i) follows a bivariate normal distribution, i.e.,

$$\begin{pmatrix} \epsilon \\ \zeta \end{pmatrix} = N \left(\begin{array}{c|cc} \mu_\epsilon & \sigma_\epsilon^2 & \sigma_\epsilon \sigma_\zeta \rho_{\epsilon,\zeta} \\ \mu_\zeta & \sigma_\epsilon \sigma_\zeta \rho_{\epsilon,\zeta} & \sigma_\zeta^2 \end{array} \right).$$

where $\mu_\epsilon = 0$ and $\mu_\zeta = 0$ denote the mean values of ϵ and ζ, and σ_ϵ and σ_ζ represent the standard deviations of ϵ and ζ respectively. Further, $\rho_{\epsilon,\zeta}$ is the correlation coefficient between ϵ and ζ.

Second, based on the estimation results of the first step and the mathematical representation above, we (i) compute the inverse Mills ratio, i.e., $\lambda(z) = \frac{\phi(z)}{\Phi(z)}$, where $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density function and the cumulative distribution function of the standard normal distribution respectively, and (ii) include $\lambda(h_v V_i)$ as independent variable in Model 2. This Heckman model corrects the selection bias in the estimation. We refer interested readers to Heckman (1979) for more details of the Heckman model.

Endogeneity issue

Furthermore, the endogeneity issue is also a key concern in our model estimation. In particular, shippers' request lead time is neither random nor exogenous. In particular, shippers' preference for order timing (i.e., request lead time) is subject to several factors. For example, first, in situations where a driver shortage occurs, shippers may opt to place their shipping orders earlier to allow a longer period for driver searches. Second, if a shipper requests to pick up the order that is less preferable for drivers, she may submit the freight-matching request earlier to leave the platform a longer time window for driver search. In sum, the factors that are crucial for the freight-matching performance may also affect our main variable of interest (i.e., request lead time), leading to endogeneity concerns. To address the issues, our model incorporates the Two-stage Residual Inclusion (2SRI) method (Wooldridge 2015).

The Two-Stage Residual Inclusion (2SRI) approach is a widely adopted statistical technique that relies on instrumental variables to address the issue of endogeneity within regression estimations. A proper instrumental variable should satisfy both (i) the relevance restriction, requiring instrumental variables to be correlated with endogenous focal variables, and (ii) the exclusion restriction, requiring instrumental variables to not directly impact the dependent variable apart from their effect through the endogenous explanatory variables. To satisfy these conditions, we construct a lagged instrumental variable, denoted as $LagLeadtime_i$, by calculating the average request lead time of shipping requests along the same route at a pickup date one week before that of shipping request i. Since lagged instrumental variables are computed based on the earlier-stage value of focal variables, they are less likely to be affected by the unobserved factors that influence the current-stage value of the outcome (i.e., sourcing cost). Therefore, the adoption of lagged instrumental variables can help satisfy the exclusion restriction. However, if there is a significant serial correlation in the estimation results, i.e., the error terms in the regression model are correlated with the counterparts of earlier time periods, it is possible that the instrumental variables may impact the dependent variable beyond their relationship with the endogenous focal variables. Specifically, because the lagged instrumental variables may be endogenous to the earlier-stage value of unobservable factors, they may affect the current value of the outcome through the serial correlation between the earlier-stage value and the current value of unobservable factors, which can violate the exclusion restriction. Therefore, we consider serial correlation tests to validate the effectiveness of our instrumental variables and present more details in CHAPTER 3.7.

3.6.4. Estimation Procedure

Based on the discussion in CHAPTER 3.6.3, we now present the estimation procedure of Model 2.

- **Heckman Stage I:** We estimate h_v in Heckman stage 1 and compute inverse Mills Ratio for order i: $\lambda(\hat{h}_v V_i) = \frac{\phi(\hat{h}_v V_i)}{\Phi(\hat{h}_v V_i)}$.
- **Heckman Stage II:** We incorporate the inverse Mills ratio (i.e., $\lambda(\hat{h}_v V_i)$) and 2SRI instrumental variable $LagLeadtime_i$ into our model as follows.
  - **2SRI-Stage I:** We regress the endogenous variable $Leadtime_i$ on the instrument $AveLeadtime_i$, the control variables in X_i, and the inverse Mills ratio $\lambda(\hat{h}_v V_i)$:

$$Leadtime_i = \gamma_{1,0} + \gamma_{1,1} LagLeadtime_i + \gamma_{1,2} LagLeadtime_i^2 + \gamma_{1,x} X_i + \gamma_{1,\lambda}\lambda(\hat{h}_v V_i) + \xi_{1,i}. \quad \text{(2SRI stage 1: } Leadtime_i\text{)}$$

Because $Leadtime_i^2$ is also endogenous, we further estimate:

$$Leadtime_i^2 = \gamma_{2,0} + \gamma_{2,1} LagLeadtime_i + \gamma_{2,2} LagLeadtime_i^2 + \gamma_{2,x} X_i + \gamma_{2,\lambda}\lambda(\hat{h}_v V_i) + \xi_{2,i}. \quad \text{(2SRI stage 1: } Leadtime_i^2\text{)}$$

As such, we obtain the fitted residuals $\hat{\xi}_{1,i}$ and $\hat{\xi}_{2,i}$.

– **2SRI-Stage II:** We control both fitted residuals $\xi_{1,i}$ and $\xi_{2,i}$ when estimating the main model (see Model 2):

$$\begin{aligned} SourcingCost_i =& b_0 + b_1 Leadtime_i + b_2 Leadtime_i^2 + b_x X_i \\ & + b_{r,1}\hat{\xi}_{1,i} + b_{r,2}\hat{\xi}_{2,i} + b_\lambda \lambda(\hat{h}_v V_i) + o_i. \quad \text{(2SRI stage 2: Sourcing Cost)} \end{aligned}$$

The coefficients b_1 and b_2 are unbiased estimators for β_1 and β_2 in Model 2.

It is important to note that when estimating coefficients of Model 1, we only apply the 2SRI estimation procedure.

- **2SRI-Stage I:** We regress the endogenous variable $Leadtime_i$ on the instrument $AveLeadtime_i$ and the control variables in X_i:

$$\begin{aligned} Leadtime_i &= \omega_{1,0} + \omega_{1,1} LagLeadtime_i + \omega_{1,2} LagLeadtime_i^2 + \omega_{1,x} X_i + \upsilon_{1,i}, \\ Leadtime_i^2 &= \omega_{2,0} + \omega_{2,1} LagLeadtime_i + \omega_{2,2} LagLeadtime_i^2 + \omega_{2,x} X_i + \upsilon_{2,i}. \end{aligned} \quad \text{(2SRI stage 1: Booked)}$$

  As such, we obtain the fitted residuals $\hat{\xi}_{1,i}$ and $\hat{\xi}_{2,i}$.

- **2SRI-Stage II:** We control both fitted residuals $\xi_{i,1}$ and $\xi_{i,2}$ when estimating the main model (see Model 1):

$$Probit(Booked_i) = a_0 + a_1 Leadtime_i + a_2 Leadtime_i^2 + a_x X_i + a_{v,1}\hat{v}_{1,i} + a_{v,2}\hat{v}_{2,i} + O_i. \tag{2SRI stage 2:Booked}$$

The coefficients a_1 and a_2 are unbiased estimators for α_1 and α_2 in Model 1.

3.7. Estimation Results and Discussions

We first present the impact of shippers' request lead time on freight-matching probability in CHAPTER 3.7.1. We then examine the impact of shippers' request lead time on freight-matching sourcing cost in CHAPTER 3.7.2.

3.7.1. How Shippers' Request Lead Time Impact FreightMatching Probability?

We present the estimation results of Model 1 characterized by Table 4. Column [1] reports the results of Model 1 by adopting a Probit estimator. The result indicates an inverse U-shaped association between the request lead time of orders and the probability of a successful order match. However, as noted in CHAPTER 3.6.3, the association may be due to the endogeneity of our main variable of interest (i.e., request lead time) rather than the direct impact of request lead time itself on the order matching.

In particular, as discussed in CHAPTER 3.6.3, our focal variables, $LeadTime_i$ and $LeadTime_i^2$, may still be correlated with the unobservable factors that are compounded in the error term $\epsilon_{B,i}$. Therefore, as detailed explained in CHAPTER 3.6.3 and CHAPTER 3.6.4, we further consider the instrument-based estimation strategy by following the 2SRI estimation procedure and adopting lagged instruments. We report the causal estimations using the 2SRI procedure in Column [2]. The findings indicate an inverse U-shaped pattern in the relationship between request lead time and order matching success, which is similar to that indicated by the traditional Probit estimator's findings. However, the analysis points to a peak request lead time (that maximizes the matching probability) of 9.28 days, which greatly diverges from the 20.37 days indicated by the Probit estimator. As such, our results support Hypothesis 1.

The distinct estimation results from the Probit estimator and 2SRI-Probit model, coupled with the significant role of 2SRI residuals (i.e., $\hat{v}_{1,i}$ and $\hat{v}_{2,i}$), indicate the presence of endogeneity in our focal variables and the validity of our instruments. However, note that as aforementioned in CHAPTER 3.6.3, while lagged instruments are generally effective in meeting the exclusion restriction, there is a risk they may not fulfill this condition due to serial correlation in unobservable factors. For these reasons, as a robustness check, we further examine the validity and effectiveness of our instrument variables by conducting a serial correlation test. We report the estimation result in Column [3], suggesting no evidence (p-value of Backward Residual > 0.1) of a significant serial correlation that may break the exclusion restriction. We relegate more details of this test but refer interested readers to Wooldridge (2015).

From a practical perspective, our results suggest that the platform can maximize the freight-matching probability at a specified request lead time. Given that the freight-matching probability represents the likelihood of a successful service sale, we conclude that the request lead time plays a critical role in enhancing the freight-matching platform's sales.

3.7.2. How Shippers' Request Lead Time Impact FreightMatching Sourcing Cost?

We present the estimation results of Model 2 characterized by Table 5. Column [1] presents an OLS estimation of Model 2 and suggests an inverse U-shaped association between the request lead time and the sourcing cost of freight matching. The estimation results are significant on focal variables. Nevertheless, as elaborated in CHAPTER 3.6.3, the observability of sourcing costs is subject to numerous explanatory factors that impact the outcome variable, namely sourcing costs. Therefore, we additionally present the OLS estimation results in Column [2] after addressing the selection bias using the Heckman model. This approach reveals variations in the coefficients and significance levels of certain explanatory variables, such as $Freq_i$, while maintaining consistency with other estimates from the standard OLS estimator. This consistency underscores the robustness of the Heckman model, which mitigates selection bias and meanwhile remains stable on other intricate

	Dependent variable:		
	$Booked_i$		
	(1)	(2)	(3)
$LeadTime_i$	0.055***	3.151***	3.107***
$LeadTime_i^2$	−0.001***	−0.170***	−0.167***
$Mile_i$	−0.001***	−0.001***	−0.001***
$StopCount_i$	0.238***	0.397***	0.387***
$Rate_i$	0.0004***	0.0004***	0.0004***
$AvgUnitRate_{s_i}$	−0.074***	−0.977***	−0.960***
$Freq_{s_i}$	−0.0002***	−0.002***	−0.002***
$PickupWD_i$	0.289***	2.118***	2.078***
$PickupWY_i$		INCLUDED	
$Route_i$		INCLUDED	
Constant	0.669***	7.616***	7.523***
$\hat{v}_{1,i}$		−6.234***	−6.186***
$\hat{v}_{2,i}$		6.434***	6.326***
Backward Residual			−0.363
Observations	23,434	23,434	23,434
Log Likelihood	−9,335.071	−9,319.767	−9,315.011
Akaike Inf. Crit.	18,714.140	18,687.530	18,680.020
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 4. *Impacts on Freight-Matching Probability*

dynamics influencing sourcing costs.

As discussed in CHAPTER 3.6.3 and CHAPTER 3.7.1, the unobservable factors involved in the residual terms $\hat{\xi}_{1,i}$ and $\hat{\xi}_{2,i}$ may correlate with the focal variables (i.e., $LeadTime_i$ and $LeadTime_i^2$). Therefore, the endogeneity may present in our model estimation and result in biased results. To address this issue, we organically integrate the 2SRI procedure and Heckman model into our estimation procedure. We formally report the estimation results in Column [3]. The results indicate a pattern opposite to our estimates from the OLS estimator. In particular, the 2SRI model suggests a U-shaped impact of the request lead time on the sourcing cost of freight matching, along with a minimum point at request lead time of 21 days. As such, our result from the Heckman-2SRI model supports Hypothesis 2.

As a robustness check, we further report the serial correlation test for Model 2 after adopting the 2SRI procedure and Heckman model. We report the results in Column [4]. Our analysis suggests that there is no evidence of serial correlation (p-value of Backward

	Dependent variable:			
	$log(Cost_i)$			
	(1)	(2)	(3)	(4)
$LeadTime_i$	0.006***	0.005***	−0.065*	−0.069*
$LeadTime_i^2$	−0.0001***	−0.0001***	0.002*	0.001
$Mile_i$	0.0004***	0.0004***	0.001***	0.001***
$StopCount_i$	0.014***	0.010***	−0.120*	−0.137*
$PickupWD_i$	−0.034***	−0.043***	−0.362**	−0.405**
$Rate_i$	0.001***	0.001***	0.0003	0.0002
$AvgUnitRate_{s_i}$	0.006**	0.008***	0.074**	0.081**
$Freq_{s_i}$	−0.00001***	−0.00000	0.0003*	0.0003*
$PickupWY_i$		INCLUDED		
$Route_i$		INCLUDED		
Constant	5.772***	5.801***	6.724***	6.872***
$\lambda(\hat{h}_v V_i)$		−0.079*	−2.748**	−3.125*
$\hat{\xi}_{1,i}$			0.306**	0.340*
$\hat{\xi}_{2,i}$			−0.064*	−0.059*
Backward Residual				−0.909
Observations	19,960	19,960	19,960	19,960
R^2	0.772	0.772	0.772	0.772
Adjusted R^2	0.771	0.771	0.771	0.772
Note:		*p<0.1; **p<0.05; ***p<0.01		

Table 5. *Impacts on Freight-Matching Sourcing Cost*

Residual > 0.1) in our model, supporting the effectiveness of the instrument selections.

Our analysis indicates that the platform can optimize/minimize its sourcing costs by strategically adjusting the request lead time. This insight, combined with our previous findings on matching probability presented in Column [2] of Table 4, suggests there is a sweet spot of request lead time that maximizes the platform's freight-matching profitability. Specifically, the request lead time influences both the platform's sourcing costs and the probability of successful freight matches in nuanced ways. Notably, the optimal points for minimizing sourcing costs and maximizing matching probability occur at different request lead times. Therefore, the platform can achieve a balance between increasing sales (through improved matching rates) and reducing expenses (by lowering sourcing costs), ultimately leading to increased overall profitability.

Based on this initiative and to provide freight-matching businesses with a viable way to apply our findings to enhance their business performance, in CHAPTER 3.8, we present a series of simulation and counterfactual case studies to help platforms realize better business profits by adopting our empirical findings.

3.8. Simulation and Counterfactual Analyses

We have empirically examined the impact of request lead time on freight-matching sourcing cost. To discover the business value of our findings, we conduct a counterfactual analysis in this section to examine: *How, and to what extent, our empirical findings can benefit freight-matching platforms?*

3.8.1. Theoretical Model Framework

When a shipping request is received by the platform with a request lead time of T, it promptly shares the order details with the driver pool to initiate a driver searching process. Interested drivers may submit their offer to the platform at any time during the driver searching phase (after the platform share order information and before it accepts any offer). These rates vary based on drivers' route preferences that are relevant to their pickup distance and transportation plan. For clarity and convenience, we define the countdown time t as the time remaining until the pickup, where t ranges from 0 to T. Simultaneously, the platform manages to pinpoint the most profitable offer, i.e., the one with the lowest offer rate. Notably, drivers might withdraw their offers or alter their transportation plans, leading to the expiration of their offers. Therefore, the platform is incentivized not to delay too much to make a decision on accepting an offer. Instead, the platform would accept offers at any countdown time $t \in [0, T]$, defined as the time remaining until the pickup, to secure the best offer available.

To formulate the freight-matching process, we adopt a multi-step modeling approach by (i) using an order statistics method to depict the distribution of minimum offer rate given a number of valid offers at countdown time t, (ii) utilizing a queue theoretical model to derive the expected minimum offer rate at a given countdown time t, and (iii) predicting the expected sourcing cost given the request lead time T. Next, we provide a brief overview of each step but relegate more details of the analysis in APPENDIX B.1 for brevity.

First, we consider an order statistics approach to determine the conditional expectation of the minimum offer rate for a specified countdown time t and its corresponding number of valid offers. Specifically, at countdown time t, we have a set of valid offers, denoted as N_t, with each offer i within N_t presenting an offer rate c_i. Therefore, the minimum offer rate at countdown time t is given by:

$$C_t = \min_{i \in N_t} c_i, \;\; t = 1, 2, ..., T.$$

We assume that drivers' offer rates (i.e., c_i) at countdown time t follow a uniform distribution, $Unif(\mu_t - \delta_t, \mu_t + \delta_t)$, where μ_t and δ_t captures the average level and variability in drivers' offer rates, resp. Next, utilizing the order statistic calculation, we are able to derive the expectation of the minimum offer rate at countdown time t conditional on the number of valid offers $n_t = |N_t|$, denoted as $E[C_t|n_t]$ and presented below:

$$E[C_t|n_t] = \frac{\mu_t n_t + \mu_t - n_t \delta_t + \delta_t}{n_t + 1}.$$

Second, as the number of valid offers at countdown time t, denoted as n_t, changes over time due to the new arrival and expiration of offers that are unobservable from data, we adopt an $M/M/\infty$ queue-theoretical approach to characterize it. In particular, we assume that the arrivals of drivers' offers follow a Poisson distribution, characterized by a mean arrival rate A_t that changes over countdown times t and represents the average number of offers received per day. Additionally, the expiration of offers is modeled using an exponential distribution with an average lasting duration of $\frac{1}{D_t}$ days. Therefore, the traffic intensity at each countdown time t is defined as $\rho_t = \frac{A_t}{D_t}$. Moreover, in this queuing system, the service time indicates the expiration period of offers. Upon arrival, offers commence their expiration countdown without delay for the platform's processing. This absence of waiting time, along with the earlier discussion on arrivals and expires of offers, aligns our model with the characteristics of an $M/M/\infty$ queue model. Given these specifications, we are able to derive the probability that n_t offers are valid at countdown time t (see Knessl and Yang 2001 and Forgo 2017), denoted as $P(n_t)$, as follows

$$P(n_t) = \frac{\rho_t^{n_t}}{n_t!} \frac{e^{-\rho_t}}{1 - e^{-\rho_t}}, \;\; n_t > 0.$$

This result enables us to further solve for the expected minimum offer rate at countdown time t as

$$E[C_t] = \sum_{n_t=1}^{\infty} E[C_t|n_t]P(n_t) = \mu_t - \frac{\delta_t \left(e^{\rho_t}(\rho_t - 2) + \rho_t + 2\right)}{\left(e^{\rho_t} - 1\right)\rho_t}.$$

Thirdly, it is crucial to understand the specific countdown time t when the platform reviews the offer pool and selects the cheapest offer for acceptance. Suppose that the platform determines the offer acceptance at a specific countdown time t^*, it chooses the lowest-priced offer available at t^* to match with the shipping request, leading to an expected sourcing cost as $E[C_{t^*}]$. Our data reveals that the countdown time for determining offer acceptance on the platform is stochastic, and its probability distribution $w(t|T)$ varies based on the request lead time of shipping requests. It follows that the expected sourcing cost can be derived as

$$E[S_T] = \sum_{t=0,1,\ldots,T} E[C_t]w(t|T)$$

Based on the discussions so far, we are able to solve for the expectations of the sourcing cost with request lead time T, i.e., $E[S_T]$, as presented in Lemma 2. We relegate more details of our analysis in APPENDIX B.1.

Lemma 2. *The expectation of the sourcing cost with request lead time T (i.e., $E[S_T]$) is:*

$$E[S_T] = \sum_{t=0,1,\ldots,T} E[C_t]w(t|T) = \sum_{t=0,1,\ldots,T} \left(\mu_t - \frac{\delta_t \left(e^{\rho_t}(\rho_t - 2) + \rho_t + 2\right)}{\left(e^{\rho_t} - 1\right)\rho_t}\right) w(t|T)$$

(Theoretical Model: Expected Sourcing Cost)

Based on the results in Lemma 2, we then conduct counterfactual analyses and investigate *how, and to what extent, our empirical findings can benefit freight-matching platforms* in the next section.

3.8.2. Counterfactual Analyses

Our empirical analyses indicate that the request lead time of shipping requests significantly impacts freight-matching performance, affecting both sourcing costs and matching probabilities. This observation suggests that managing the request lead time could enhance the platform's freight-matching profitability. Therefore, to explore the viability of this approach, in this section, we consider a platform that adopts a "preorder policy." That is, the freight-matching platform predicts the demand of freight matching and searches for drivers at the optimal timing, which maximizes the expected freight-matching profit, even if shippers have yet to send their shipping requests. In the counterfactual analysis, we employ a two-step approach: initially, we estimate the critical factors of the theoretical model using a hybrid of empirical estimation and least-square computation, and subsequently, we simulate the freight-matching outcomes using the theoretical framework and the derived parameter estimates.

Parameter estimation

To apply the theoretical model to the simulation, we estimate the factors used in our theoretical model, including the mean offer rate $\mu_{r,t}$ (the route-specified analog of μ_t for route r), variation of offer rate $\delta_{r,t}$, offer arrival rate $A_{r,t}$, offer expiration rate $D_{r,t}$, and the distribution of offer acceptance timing $w(t|r,T)$. In particular, while $A_{r,t}$ and $w(t|r,T)$ are observable and, thus, can be directly estimated from the data, others (i.e., $\mu_{r,t},\ \delta_{r,t}$, and $D_{r,t}$ are unobserved. Therefore, first, we derive the estimation of $A_{r,t}$ and $w(t|r,T)$ based on the data, as detailed explained in APPENDIX B.2. Secondly, given the estimations $\hat{A}_{r,t}$ and $\hat{w}(t|r,T)$, we estimate the remaining factors, i.e., $\mu_{r,t},\ \delta_{r,t}$, and $D_{r,t}$, by solving the following least-square problem. We also reproduce this model with detailed specifications in APPENDIX B.1.

$$\min_{\substack{\hat{a}_{r,\mu},\ \hat{c}_{r,\mu},\ \hat{a}_{r,\delta},\\ \hat{c}_{r,\delta},\ \hat{D}_{r,t}}} \sum_{T=0,\ldots,\overline{T}} \left(\hat{S}_{r,T} - \boxed{\sum_{\mathbf{t=0,\ldots,T}} \left(\hat{\mu}_{\mathbf{r,t}} - \frac{\hat{\delta}_{\mathbf{r,t}} \left(\mathbf{e}^{\hat{\rho}_{\mathbf{r,t}}} (\hat{\rho}_{\mathbf{r,t}} - \mathbf{2}) + \hat{\rho}_{\mathbf{r,t}} + \mathbf{2} \right)}{\left(\mathbf{e}^{\hat{\rho}_{\mathbf{r,t}}} - \mathbf{1} \right) \hat{\rho}_{\mathbf{r,t}}} \right) \hat{\mathbf{w}}(\mathbf{t}|\mathbf{r}, \mathbf{T})} \right)^2 \quad (7)$$

Note that in Expression 7, the expression part within the box is the expected sourcing cost as indicated in Lemma 2, and $\hat{S}_{r,T}$ is the estimation of sourcing cost with request lead time T predicted by the empirical model for route r (see Model 2SRI stage 2: Sourcing Cost in CHAPTER 3.6.4). Recall that the empirical model suggests that the association between the freight-matching sourcing cost and shippers' request lead time follows a U-shaped pattern (see 2SRI stage 2: Sourcing Cost):

$$\hat{S}_{r,T} = \hat{b}_1 T + \hat{b}_2 T^2 + c_{S_{r,T}},$$

where $\hat{b}_1$ and $\hat{b}_2$ are coefficient estimation of 2SRI stage 2: Sourcing Cost. Besides, the constant term $c_{S_{r,T}}$ accounts for the route-specified effect of control variables. We relegate more details of the estimation in APPENDIX B.2.1.

Simulation

We conduct the simulation to validate the business value of the empirical findings as well as the effectiveness of implementing a preorder policy. Specifically, for each pickup date, we decide on (1) the optimal request lead time (i.e., preorder timing) and (2) the number of shipping orders for drivers searching. We iterate on the pickup date forward from a specified starting point until the conclusion of our simulation period.

Step 1. Training data preparation: First, we dynamically form a training set for parameter estimation during the simulation, which includes historical data on load requests and driver offers. We consider that the platform has limited operational flexibility, meaning that for a specified pickup date U_P, it is restricted to setting a preorder strategy no more than $\overline{T}$ days in advance. Therefore, we consider that on the date $U_P - \overline{T}$, the platform assembles the training set and determines (1) the optimal preorder timing or request lead time $T^* \leq \overline{T}$ and (2) the number of shipping orders to preorder drivers for the specified pickup date U_P. Next, we introduce the parameter estimation and this decision-making mechanism in Steps 2 and 3, resp.

Step 2. Forecast of the upcoming demand and supply: Second, utilizing the training set, the platform estimates (1) the arrival rates of offers $\hat{A}_{r,t}$ for route r and countdown time t, (2) the demand for load requests expected to be picked up on U_P, denoted as $\overline{K}_{D|r,U_P}$, and (3) the freight-matching probability with request lead time T, denoted as $\overline{K}_{P|r,T}$. We provide further details of the estimation process in APPENDIX B.2.2.

Step 3. Strategic decision-making on the optimal request lead time: Third, with the estimations of $\hat{\mu}_{r,t}$, $\hat{\delta}_{r,t}$, $\hat{D}_{r,t}$, and $\hat{w}(t|r,T)$ from CHAPTER 3.8.2, alongside the revised estimation of $\hat{A}_{r,t}$ in Step 2, we are able to calculate the expected sourcing cost $E[S_{r,T}]$ utilizing our theoretical solution in Lemma 2. Building upon this foundation as well as the estimations of $\overline{K}_{D|r,U_P}$ and $\overline{K}_{P|r,T}$ from Step 2, we construct a newsvendor model to determine (1) the optimal quantity of shipping orders for driver searching and (2) the corresponding maximal profit for any specification of request lead time $T \leq \overline{T}$. We present the model as follows.

$$\begin{aligned}\max_{Q_{r,T}} \ \Pi_{r,T}(Q_{r,T}) =& min(K_{D|r,U_P}, K_{S|r,T,U_P})\overline{R}_r - K_{S|r,T,U_P}E[S_{r,T}] + (K_{S|r,T,U_P} - K_{D|r,U_P})^+ \\ & (1-c_O)E[S_{r,T}] - c_U(K_{D|r,U_P} - K_{S|r,T,U_P})^+.\end{aligned} \tag{8}$$

In our model, we define the random demand $K_{D|r,U_P}$ as the number of upcoming shipping requests, following a Poisson distribution with mean $\overline{K}_{D|r,U_P}$. We further account for the stochastic supply level $K_{S|r,T,U_P}$, defined as the number of matched drivers, and feature it using a truncated Poisson distribution with the mean parameter as $\overline{K}_{P|r,T}Q_{r,T}$ and an upper bound of $Q_{r,T}$. Meanwhile, We estimate the average revenue per successful match as $\overline{R}_r$ and include the oversupply penalty c_O, representing the payment for drivers accepted but not utilized, while the undersupply penalty c_U is set to zero. We relegate more details of this model in APPENDIX B.2.2.

Utilizing this newsvendor model, we are able to estimate the optimal order quantity $Q^*_{r,T}$ that maximizes the business profit for any specification of request lead time $T \leq \overline{T}$. Next, by comparing the maximum profit $\Pi_{r,T}(Q^*_{r,T})$ among all possible request lead time T, we are able to decide on the optimal request lead time T^* that results to the highest profit, i.e., $T^* = \arg\max_T \Pi_{r,T}(Q^*_{r,T})$.

Step 4. Driver assignment and outcome realization: Next, after deciding the optimal request lead time, we conclude the simulation of freight-matching for the pickup date U_P. We consider that the platform assigns the preordered drivers on a first-come-first-serve basis. In particular, the platform assigns preordered drivers to the earliest-arrived shipping requests that meet two criteria: (1) their unit revenue (shippers' payment rates) can at least compensate the sourcing cost $E[S_{T^*}]$, and (2) they are submitted after the optimal preorder timing, i.e., $T \leq T^*$. For all other shipping requests that arrive late (i.e., there are no unmatched preordered drivers) or cannot meet both criteria, we consider that their freight-matching outcomes are not different from the factual case. We iteratively execute Steps 1 through 4 for each pickup date moving forward until we reach the end of our simulation period. We relegate further details of the simulation process in APPENDIX B.2.

Results

Next, we summarize the simulation results by route. We present the overall profit and the number of matching within the chosen time horizon for both the counterfactual and actual cases in Table 6. This table specifically focuses on the outcomes from the simulation where an exceptionally high oversupply penalty of 50% was applied. This penalty, defined as the percentage of sourcing costs the platform must pay to preordered drivers when it cannot match them with sufficient shipping orders, illustrates the financial impact of unmet supply. We observe that given the 50% oversupply penalty, the total profit from the top 10 routes enhances by 46.24% (an increment of US\$ $341,425.6$ with respect to the original profit of US\$ $738,329.5$). Besides, we further illustrate route-level profit increment with oversupply penalty levels of $\{0.05,\ 0.2,\ 0.5\}$ in Figure 6.

Route	Counterfactual Profit	Factual Profit	Profit Increment (Percentage)	Counterfactual # of Matching	Factual # of Matching	# of Matching Increment (Percentage)
AZ-CA	78326.18	44507.42	76	319.55	336	-5
CA-AZ	161038.60	100130.14	61	411.49	400	3
CA-CA	267775.84	177931.66	50	932.23	882	6
FL-FL	44871.34	27203.62	65	211.19	204	4
GA-FL	109323.27	100802.85	8	416.56	414	1
GA-GA	24722.37	13624.09	81	66.83	66	1
IL-IL	5355.53	5355.53	0.00	33.00	33	0.00
MI-MI	15381.07	8015.16	92	78.56	79	-1
TX-CA	63616.80	37739.62	69	185.66	173	7
TX-TX	334110.02	223019.41	50	1415.93	1363	4

Table 6. *Case Study Results on Top 10 Busiest Routes with Oversupply Penalty of* 50%

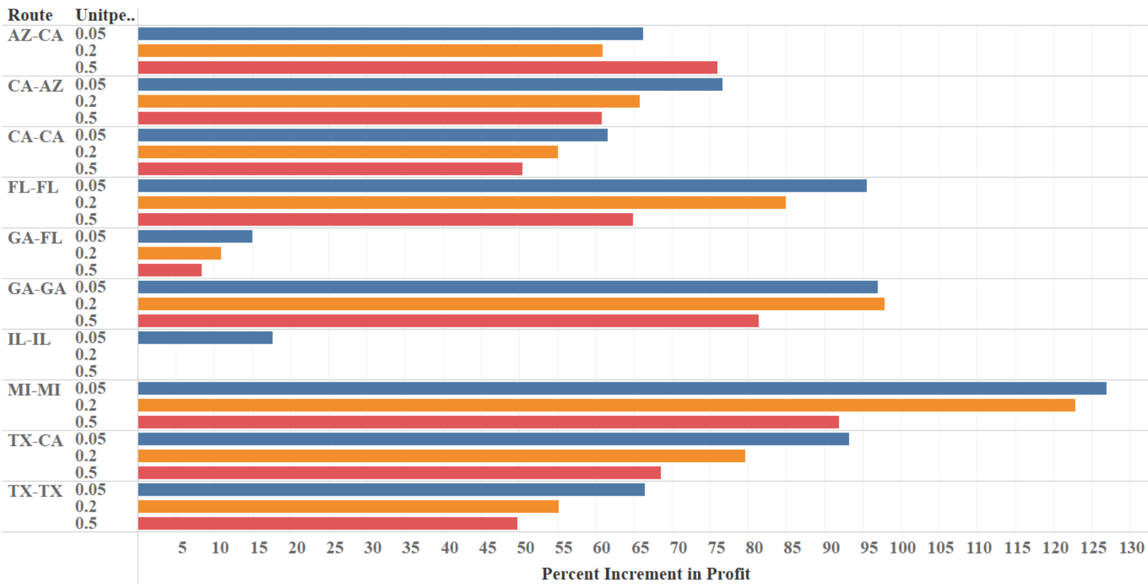

Figure 6. *Percentage Increment of Profits on Different Routes with Oversupply Penalty Levels of* $\{0.05,\ 0.2,\ 0.5\}$

To better examine the impact of route characteristics on the effectiveness of implementing the preorder policy, we summarize several route-level variables based on the historical orders of the routes of our interest among the whole observation horizon. In particular, for route r, we calculate the average level of control variables involving $Miles_i$, $StopCount_i$, $Rate_i$, $UnitRate_i$, and the original level of $LeadTime_i$, and we denote them as $RMiles_r$, $RStopCount_r$, $RRate_r$, $RUnitRate_r$, and RLT_r resp. We also consider (1) route r's type as either cross-state or inner-state, denoted as $RouteType_r$, (2) the demand level defined as the number of freight-matching requests and denoted as $RDemand_r$, and (3) the supply (or satisfaction) level measured by the number of actually matched requests and denoted as $RSupply_r$, and (4) the portion of requests with pickup dates on weekends, denoted as $RWeekend_r$. Lastly, we control for the oversupply penalty settings and denote it as $c_{O,r}$.

We consider the overall profit to be the measurement of performance improvement by adopting the preorder policy. In particular, for each route, we calculate the percentage increment of the total profit and that of the number of freight matching, denoted as $\Delta Profit$ and $\Delta NumMatch$, resp. Next, we formally investigate the impacts of route characteristics and oversupply penalty on the effectiveness of the preorder policy in Table 7.

	Dependent variable:	
	$\Delta Profit$	$\Delta NumMatch$
	(1)	(2)
c_O	-0.810^{***}	-0.020^{***}
$RMiles$	0.029^{***}	0.001^{***}
$RStopCount$	8.227^{***}	0.292^{***}
$RWeekend$	58.590^{***}	2.335^{***}
$RouteType$ (outer=1)	-1.102^{***}	-0.027^{***}
$RRate$	-0.013^{***}	-0.001^{***}
$RUnitRate$	4.316^{***}	0.219^{***}
$RDemand$	0.002^{***}	0.0001^{***}
$RSupply$	-0.006^{***}	-0.0001^{***}
RLT	-5.574^{***}	-0.208^{***}
$Constant$	-3.758^{***}	-0.319^{***}
Observations	110	110
Adjusted R^2	0.946	0.977
Note:	$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$	

Table 7. *Impacts of Route Characteristics and Oversupply Penalty on Performance Improvement Due to The Preorder Policy*

Our estimation results in Table 7 reveal several interesting findings. First, the significant and positive coefficients for $RMiles$, $RStopCount$, and $RWeekend$ suggest that the preorder policy can enhance the platform's profit and number of successful matching more effectively on routes that (1) have a greater portion of order pickups on weekends, (2) encompass a higher number of delivery stops, and/or (3) cover longer distances. Moreover, the positive coefficient for $RDemand$ alongside a negative coefficient for $RSupply$ indicates that the platform should prioritize routes with higher demand or lower supply for profit maximization through policy adoption. Additionally, the significantly positive coefficient for RouteType reveals that intra-state routes, those beginning and ending within the same state, show more improvement by adopting the preorder policy. Lastly, based on the coefficient estimation of $RUnitRate$, routes with a higher profit margin, can have a more remarkable improvement in profit under this policy.

3.9. Conclusion

Freight-matching platforms, such as Uber Freight and UShip, have gained a significant foothold in the long-haul transportation industry and have become increasingly ubiquitous. These platforms offer several advantages over traditional carrier businesses, including low fixed costs, flexible capacities, and low marginal costs. However, despite these benefits, freight-matching platforms still face challenges in managing freight logistics, particularly in dealing with driver sourcing. To address these challenges, we conducted preliminary analyses and noticed that one crucial factor of drivers' bidding behaviors, the dynamic heterogeneity of drivers' route preferences, significantly changed over time due to information acquisition. Recognizing this variability underscores the importance of timing as a crucial factor in freight-matching outcomes. Therefore, we investigate how the timing of shippers' orders, measured by "request lead time," impacts both the probability of successful matches and the associated sourcing costs.

Through the analysis of a comprehensive dataset from a leading freight-matching platform, we uncover optimal timing strategies that balance the benefits of early and late order placements. Our findings highlight the significant role of shippers' order timing in affecting the probability and sourcing cost of freight matching on the platform. In particular, our results suggest that as the request lead time increases, the freight-matching probability initially increases when the lead time is short but may decrease beyond an optimal point. Similarly, the influence of shippers' request lead time on the platform's sourcing cost is nuanced and significant; as the lead time increases, the sourcing cost tends to decrease initially when the lead time is short but may rise if the lead time becomes excessively long. This indicates that there is an optimal range for request lead times that maximizes matching success and minimizes costs.

To further demonstrate the practical implications of our findings, we proposed a pre-ordering policy. By predicting the demand for shipping services and initiating driver searches before receiving formal requests, platforms can enhance their matching performance. Our counterfactual analysis and simulations, considering drivers' abandonment behavior and data-driven cost estimations, indicate that this policy can substantially boost the platform's profitability and matching rates.

There are a number of interesting directions for future research, particularly in further exploring the nuances of timing strategies in freight-matching platforms. One potential avenue is to investigate the impact of varying economic conditions and market dynamics on optimal request lead times. Additionally, examining the role of advanced technologies, such as machine learning and predictive analytics, in enhancing the precision of timing strategies could provide valuable insights. Another interesting direction is to explore the effects of different incentive structures for drivers and shippers on platform efficiency and profitability. Lastly, expanding the research to include other types of transportation platforms and comparing their timing strategies could offer a broader understanding of best practices in the logistics industry.

CHAPTER 4

CONCLUSIONS

The advancement in information technology in the 21st century plays a critical role in driving many business innovations that rely on the quality and reliability of communication and data transfer. A notable trend in these innovations involves the active participation of crowds, a shift away from traditional businesses that solely rely on their in-house workforce. As evidenced by several successful cases in the business world, the crowd-based economy has considerable advantages in accelerating business progress and reducing operational costs for many innovative businesses. Therefore, in this dissertation, we focus on two prominent crowd-based business models, i.e., the non-profit UGC platforms and the freight-matching long-haul trucking, and examine several important research questions that the practice and literature deem critical.

We now elaborate on each of them and our main research questions. First, following the success of Wikipedia, a growing number of platforms strive to be successful or financially sustainable by adopting Wikipedia's business model. Specifically, they utilize users' content contributions to improve their overall quality and maintain financial sustainability through external parties' funding and users' donations. As several important issues have yet to be formally examined by the prior studies, in CHAPTER 2, we adopt a game-theoretical model to analytically investigate these issues and fill this gap.

In particular, although non-profit UGC platforms rely on users to create content and maintain financial sustainability, they can also attain higher content quality by exerting in-house effort (Wikimedia 2022a). The efficiency(or cost-effectiveness) in improving the content quality in-house is an important measure for the platform to maintain sustainability (Levin and Chisholm 2016, Mamatzakiset al. 2019). As the rapid growth in generative AI technologies brings platforms a considerable advantage to exert the in-house effort with greater efficiency, an important question arises: How does a UGC platform's efficiency of in-house effort impact its management strategy of the user community and performance in managing users to contribute content and make donations? The literature has overlooked this relationship and has yet to investigate this question.

Furthermore, because non-profit UGC platforms rely on user generousness to raise money and support their business operation, the heterogeneity of user generousness is a common concern for them (Levina and Arriaga 2014, Marsh 2015). However, unlike our study, the literature has yet to investigate how heterogeneity in user generousness affects the financial sustainability of UGC platforms. Therefore, we elucidate how heterogeneity of generousness affects the financial performance of UGC platforms. In particular, we investigate the change in generousness heterogeneity using a two-pronged approach by increasing the lower or upper bounds of user generousness levels.

In addition, although Wikipedia has achieved huge success, many other non-profit UGC platforms, especially those of special interest, failed to build high overall content quality. Based on the implications and evidence given by the literature and the industry, we notice that the small audience group (i.e., the user community) and the high barrier effect of contributing content are potential reasons for the obstacles faced by these special-interest platforms. Because the earlier studies have not formally examined this issue, our study focuses on these critical factors and examines the feasible solutions to help these platforms overcome these difficulties and achieve more significant improvement in their overall content quality. Therefore, our findings align with several practicable managerial insights.

Furthermore, through an all-inclusive modeling approach and analyzing user and platform behavior at equilibrium, we contribute to the literature on UGC platforms by revealing the mechanisms whereby non-profit UGC platforms can financially sustain and flourish their businesses. Our study also provides a unique contribution to the literature by proposing a comprehensive modeling framework in settings with concurrent donation and content contribution aspects of non-profit UGC platforms. Moreover, by investigating different practice-oriented phenomena in the context of UGC platforms that the literature has yet to focus on thus far, our study fills many gaps in the literature.

Second, as a new business type of long-haul trucking, freight-matching platforms rely on self-scheduled crowdsourced carriers to provide freight transportation services to shippers. Therefore, the freight-matching platform usually faces the risks of supply uncertainty, resulting in unexpected freight-matching performance (e.g., high sourcing cost, low freight-matching probability). While preliminary research suggests that shippers' order timing may contribute to or mitigate this problem, the literature has yet to examine this issue formally. Therefore, we consider a well-designed empirical schema to investigate the influence of shippers' order timing on two critical performance metrics of freight-matching (i.e., the freight-matching probability and sourcing costs).

Beyond examining the impact of shippers' order timing on the freight-matching performance, our work also considers an innovative design of counterfactual analyses to assess the practical value of our findings. In particular, by developing a theoretical framework that captures the freight-matching dynamics and considering a data-driven simulation process, we compare the freight-matching performance that the platform does or does not inspire shippers to send their shipping requests at the optimal timing. Our empirical findings, coupled with the counterfactual analytical framework, provide unique contributions to both the academic literature and the freight-matching industry.

In this dissertation, both of our studies contribute valuable insights to the understanding of crowd-based business models and address key questions that have yet to be thoroughly examined. The goal is to shed light on the role of crowds in the contemporary business landscape, offering practical value and inspiring future research in this evolving field.

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APPENDIX A

E-COMPANION OF CHAPTER 2

Figures and Tables

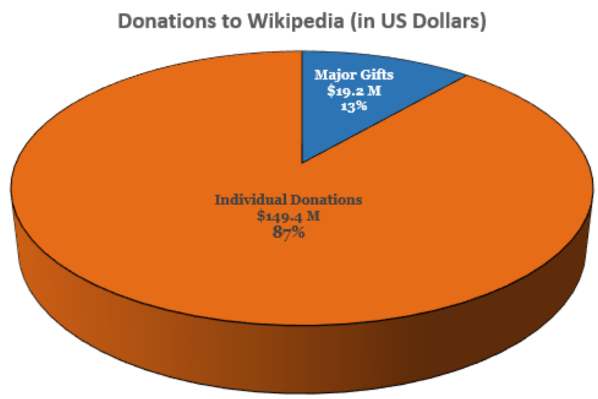


Figure 7. *Summary of Donations Received by Wikipedia in Fiscal Year 2022-23*

■

Proof of Lemma 1

As stated in the main paper, we consider that users' generousness levels $b_{e,i}$ and $b_{d,i}$ are uniformly distributed within the lower and upper bounds $(\underline{b_e}, \overline{b_e})$ and $(\underline{b_d}, \overline{b_d})$ respectively. We compute (i) the expected total donation (i.e., D) by multiplying the number of users (R_0) with the unconditional expectation of donations and (ii) the expected total contribution (i.e., C) in a similar way:

$$D = R \int_{\underline{b_d}}^{\overline{b_d}} d^*(b_d) \frac{1}{\overline{b_d} - \underline{b_d}} db_d = R \int_{b_d^*}^{\overline{b_d}} \frac{\frac{a_d}{1-a_d w} + b_d + \sqrt{f} - q_d}{(2c_d)(\overline{b_d} - \underline{b_d})} db_d = -\frac{(c\Pi_0 + R_0)\left(a_d w\left(-\left(\overline{b_d} + \sqrt{f} - q_d\right)\right) + a_d + \overline{b_d} + \sqrt{f} - q_d\right)^2}{-4c_d(1-a_d w)^2(\overline{b_d} - \underline{b_d})};$$

$$C = R \int_{\underline{b_e}}^{\overline{b_e}} e^*(b_e) \frac{1}{\overline{b_e} - \underline{b_e}} db_e = R \int_{b_e^*}^{\overline{b_e}} \frac{\frac{a_e}{1-a_d w} + b_e + \sqrt{h} - q_e}{(2c_e)(\overline{b_e} - \underline{b_e})} db_e = \frac{(c\Pi_0 + R_0)\left(a_e + (1-a_d w)\left(\overline{b_e} + \sqrt{h} - q_e\right)\right)^2}{4c_e(1-a_d w)^2(\overline{b_e} - \underline{b_e})}.$$

The recursive formula for the overall quality (see Equation 4 in the main model) is:

$$\Pi = \frac{1}{1 - a_d w} \left(\Pi_0 + a_d \left(w_0 + D - f - h - r - g - x(R_0 + c\Pi_0)\right) + a_e C\right).$$

By substituting C and D into the formula above and invoking the first-order conditions for the platform's decisions (i.e., f and h), we have

$$\frac{d\Pi}{df} = \frac{a_d\left(\frac{(c\Pi_0+R_0)\left(a_d\left(\overline{b_d}w+\sqrt{f}w-q_dw-1\right)-\overline{b_d}-\sqrt{f}+q_d\right)}{-c_d\sqrt{f}(\overline{b_d}-\underline{b_d})}+4a_dw-4\right)}{4(1-a_dw)^2}; \tag{9}$$

$$\frac{d\Pi}{dh} = \frac{-4a_d^2c_e\sqrt{h}w(\overline{b_e}-\underline{b_e})+a_da_ew(c\Pi_0+R_0)\left(\overline{b_e}+\sqrt{h}-q_e\right)+4a_dc_e\sqrt{h}(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0)\left(a_e+\overline{b_e}+\sqrt{h}-q_e\right)}{-4c_e\sqrt{h}(1-a_dw)^2(\overline{b_e}-\underline{b_e})}. \tag{10}$$

Next, we compute the equilibrium platform resource allocation strategy as the following:

$$f^* = \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}; \tag{11}$$

$$h^* = \frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}. \tag{12}$$

Given the non-negativity of users' content contributions and donations, the optimal contribution (i.e., e_i^*) and donation (i.e., d_i^*) decisions of an individual user i are

$$d_i^* = \max\left\{0, \frac{\frac{a_d}{1-a_dw}+b_{d,i}+\sqrt{f}-q_d}{2c_d}\right\};$$

$$e_i^* = \max\left\{0, \frac{\frac{a_e}{1-a_dw}+b_{e,i}+\sqrt{h}-q_e}{2c_e}\right\}.$$

By substituting the optimal decisions made by the platform (i.e., f^* and h^*), we have

$$e_i^* = \max\left\{0, \frac{-\frac{a_e(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))}{-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0)}+(1-a_dw)(b_{e,i}-q_e)+a_e}{2c_e(1-a_dw)}\right\};$$

$$d_i^* = \max\left\{0, \frac{-\frac{(c\Pi_0+R_0)(-a_d\overline{b_d}w+a_dq_dw+a_d+\overline{b_d}-q_d)}{(1-a_dw)(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)}+\frac{a_d}{1-a_dw}+b_{d,i}-q_d}{2c_d}\right\}.$$

Note that Equation 9 implies that for the solution to be an equilibrium, we need to have:

$$4a_dc_e\underline{b_e}-4a_dc_e\overline{b_e}+a_ec\Pi_0+a_eR_0<0 \text{ and } 4c_d\underline{b_d}-4c_d\overline{b_d}+cQ_0+R_0<0. \tag{13}$$

Further, the second-order conditions require:

$$\frac{d^2\Pi^*}{df^2} = -\frac{a_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{-8c_d(1-a_dw)^2(\overline{b_d}-\underline{b_d})\left(\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\right)^{3/2}} < 0,$$

and

$$\frac{d^2\Pi^*}{df^2}\frac{d^2\Pi^*}{dh^2} - \left(\frac{d^2\Pi^*}{dfdh}\right)^2 = \frac{a_d(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}{64a_ec_dc_e(1-a_dw)^2(\overline{b_d}-\underline{b_d})(\overline{b_e}-\underline{b_e})(-(1-a_dw)(\overline{b_e}-q_e)-a_e)\mathbf{E}_1} > 0,$$

where

$$\mathbf{E}_1 = \left(\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\right)^{3/2}\sqrt{\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}}.$$

Considering the boundary conditions $0 < \underline{b_d} < \overline{b_d}$ and $0 < \underline{b_e} < \overline{b_e}$, we use a simple algebra to refine the second-order conditions as shown below:

$$-\overline{b_d} + q_d + a_d(-1+\overline{b_d}w - q_dw) < 0 \text{ and } -a_e + (\overline{b_e} - q_e)(-1+a_dw) < 0. \tag{14}$$

Next, given the equilibrium levels of fundraising effort (i.e., f^*) and community support effort (i.e., h^*) in Expression 11, it is now straightforward to compute the following equilibrium levels:

$$\begin{aligned}
C^* =& \frac{4a_d^2c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2};\\
D^* =& \frac{4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2};\\
\Pi^* =& \frac{1}{1-a_dw}\left(-\frac{-4a_d^2a_ec_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}\right.\\
&+a_d\left(-\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2} - \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\right.\\
&\left.\left.-\frac{-4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} - x(c\Pi_0+R_0)-g-r+w_0\right)+\Pi_0\right);\\
v^* =& -\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2} - \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\\
&+\frac{4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} - x(c\Pi_0+R_0)-g-r+w_0;
\end{aligned}$$

$$
\begin{aligned}
b_e^* =& \frac{-4a_d^2 c_e q_e w(\overline{b_e} - \underline{b_e}) + a_d a_e \overline{b_e} w(c\Pi_0 + R_0) + 4a_d c_e (q_e - a_e)(\overline{b_e} - \underline{b_e}) - a_e \overline{b_e}(c\Pi_0 + R_0)}{(1 - a_d w)(4a_d c_e(\overline{b_e} - \underline{b_e}) - a_e(c\Pi_0 + R_0))};\\
b_d^* =& \frac{a_d \overline{b_d} w(c\Pi_0 + R_0) - 4a_d c_d(\overline{b_d} - \underline{b_d})(q_d w + 1) - c\overline{b_d}\Pi_0 - 4c_d \underline{b_d} q_d + 4c_d \overline{b_d} q_d - \overline{b_d} R_0}{-(1 - a_d w)(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)};\\
E[K_e^*] =& \frac{4a_d c_e(c\Pi_0 + R_0)(a_e + (1 - a_d w)(\overline{b_e} - q_e))}{(1 - a_d w)(4a_d c_e(\overline{b_e} - \underline{b_e}) - a_e(c\Pi_0 + R_0))};\\
E[K_d^*] =& \frac{4c_d(c\Pi_0 + R_0)(a_d(\overline{b_d} w - q_d w - 1) - \overline{b_d} + q_d)}{(1 - a_d w)(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)}.
\end{aligned}
$$

Furthermore, this solution is feasible when

$$a_d(\overline{b_d} w - q_d w - 1) - \overline{b_d} + q_d < 0 \text{ and } -(1 - a_d w)(\overline{b_e} - q_e) - a_e < 0, \tag{15}$$

$$4a_d c_e \underline{b_e} - 4a_d c_e \overline{b_e} + a_e c\Pi_0 + a_e R_0 < 0 \; and \; c\Pi_0 + 4c_d \underline{b_d} - 4c_d \overline{b_d} + R_0 < 0, \tag{16}$$

$$a_d > 0 \text{ and } w > 0 \text{ and } a_e > 0 \text{ and } \overline{b_e} > q_e > \underline{b_e} > 0 \text{ and } \overline{b_d} > q_d > \underline{b_d} > 0, \tag{17}$$

$$\Pi_0 > 0 \text{ and } r \geq 0 \text{ and } x \geq 0 \text{ and } g \geq 0 \text{ and } R_0 > 0 \text{ and } c > 0 \text{ and } c_d > 0 \text{ and } c_e > 0 \tag{18}$$

$$\overline{b_e} > -\frac{a_e}{1 - a_d w} + q_e > b_e^* > \underline{b_e} \text{ and } \overline{b_d} > -\frac{a_d}{1 - a_d w} + q_d > b_d^* > \underline{b_d} \tag{19}$$

$$v^* \geq 0 \text{ and } 1 - a_d w > 0 \text{ and } h^* < q_e^2. \tag{20}$$

Expressions 15 and 16 are the earlier feasibility constraints that we have already discussed. Furthermore, Expressions 17 and 18 list simple feasibility constraints based on the non-negativity of parameter values. In addition, based on practical concerns, Expression 19 ensures that some of the platform's users may either contribute content or donate to the platform even if the platform does not implement community support or fundraising efforts, and not all users contribute content or donate to the platform. Furthermore, Expression 20 ensures that the budget, the overall content quality, and the barrier of contributing content are non-negative.

Given all these constraints, we verify that the parameter space that denotes the feasible region is non-empty, e.g., at $\{a_d = 0.25, a_e = 1.5, q_d = 2, q_e = 8, w = 1, w_0 = 200, \overline{b_e} = 9, \underline{b_e} = 4.5, \overline{b_d} = 13, \underline{b_d} = 1, r = 1, g = 1, x = 1, \Pi_0 = 2.5, R_0 = 200, c = 1, c_d = 78, c_e = 204\}$, all feasibility constraints are satisfied, and the following equilibrium is realized: $\{h^* = 2.2, f^* = 0.42, C^* = 1.11, D^* = 7.76, \Pi^* = 5.76, v^* = 0.64, E^*[K_e] =$

$202, E^*[K_d] = 202, b_e^* = 4.52, b_d^* = 1.02\}$. Furthermore, we have also verified the feasibility (and not being redundant globally) of all the stated constraints and thresholds that are presented in the other propositions in our main model.

■

Proof of Propositions 1, 2, and Corollaries 1, 2

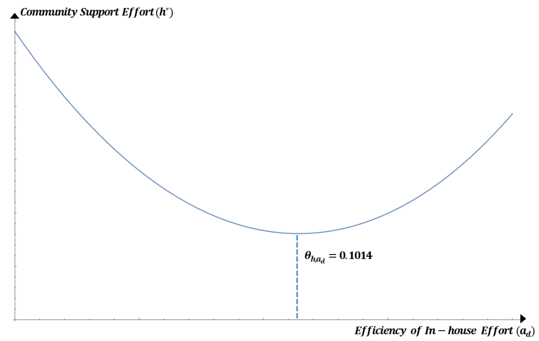
Let us first start with the proof of Proposition 2. We have:

$$\begin{aligned}\frac{dC^*}{da_d} =& -\frac{-8a_d a_e c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(-(1-a_d w)(\overline{b_e}-q_e)-a_e)}{-(1-a_d w)^3(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3}\big(4a_d^2 c_e \underline{b_e} w - 4a_d^2 c_e \overline{b_e} w + a_d^2 \overline{b_e} R_0 w^2 \\ & -a_d^2 q_e R_0 w^2 + c\Pi_0(1-a_d w)^2(\overline{b_e}-q_e) - 2a_d \overline{b_e} R_0 w + 2a_d q_e R_0 w + a_e(c\Pi_0+R_0) + \overline{b_e} R_0 - q_e R_0\big).\end{aligned}$$

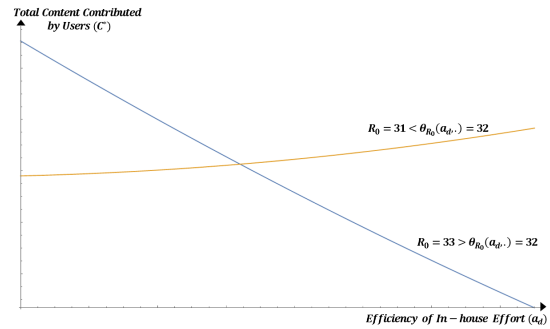
The above equality and the feasibility constraints (e.g., first-order, second-order, and non-negativity constraints that are presented in the proof of Lemma 1) reveal that if $R_0 > \theta_{R_0}(a_d, .) = \frac{4a_d^2 c_e w(\overline{b_e}-\underline{b_e})-c\Pi_0(1-a_d w)^2(\overline{b_e}-q_e)-a_e c\Pi_0}{(1-a_d w)^2(\overline{b_e}-q_e)+a_e}$ and a_d increases, then C^* decreases (i.e., $\frac{dC^*}{da_d} < 0$). Otherwise, C^* increases as a_d increases. Figure 8b further demonstrates the existence of threshold $\theta_{R_0}(a_d, .)$, such that, C^* is increasing in a_d when $R_0 < \theta_{R_0}(a_d, .)$, and C^* is decreasing in a_d otherwise. Besides, we also have

$$\begin{aligned}\frac{d\Pi^*}{da_d} =& \frac{1}{(1-a_d w)^2}\Bigg((1-a_d w)\Bigg(-\frac{32a_e c_e^2(\overline{b_e}-\underline{b_e})^2(c\Pi_0+R_0)(a_e+(\overline{b_e}-q_e)(1-a_d w))^2 a_d^2}{(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))^3(1-a_d w)^2} \\ & -\frac{8a_e c_e(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e)(c\Pi_0+R_0)w(a_e+(\overline{b_e}-q_e)(1-a_d w))a_d^2}{(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))^2(1-a_d w)^2} + \frac{8a_e c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)w(a_e+(\overline{b_e}-q_e)(1-a_d w))^2 a_d^2}{(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))^2(1-a_d w)^3} \\ & +\frac{8a_e c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(\overline{b_e}-q_e)(1-a_d w))^2 a_d}{(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))^2(1-a_d w)^2} - g - r - \frac{2a_d(c\Pi_0+R_0)}{(1-a_d w)^3}\Big(\frac{(c\Pi_0+R_0)w(a_e+(\overline{b_e}-q_e)(1-a_d w))^2 a_e^2}{(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))^2} \\ & -\frac{4c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(1-a_d w)(a_e+(\overline{b_e}-q_e)(1-a_d w))^2 a_e^2}{(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))^3} - \frac{(\overline{b_e}-q_e)(c\Pi_0+R_0)w(1-a_d w)(a_e+(\overline{b_e}-q_e)(1-a_d w))a_e^2}{(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))^2} \\ & -(\overline{b_d}-\underline{b_d})(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_d w-1))^2\frac{4c_d w}{(4c_d(\overline{b_d}-\underline{b_d})-c\Pi_0-R_0)^2} + \frac{(c\Pi_0+R_0)w(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_d w-1))^2}{(4c_d(\overline{b_d}-\underline{b_d})-c\Pi_0-R_0)^2} \\ & -\frac{4c_d(\overline{b_d}-\underline{b_d})(1-a_d w)(\overline{b_d}w-q_d w-1)(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_d w-1))}{(4c_d(\overline{b_d}-\underline{b_d})-c\Pi_0-R_0)^2} + (-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_d w-1))(\overline{b_d}w-q_d w-1) \\ & \frac{(c\Pi_0+R_0)(1-a_d w)}{(4c_d(\overline{b_d}-\underline{b_d})-c\Pi_0-R_0)^2}\Big) - \frac{(c\Pi_0+R_0)^2(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_d w-1))^2}{(4c_d(\overline{b_d}-\underline{b_d})-c\Pi_0-R_0)^2(1-a_d w)^2} + \frac{4c_d(\overline{b_d}-\underline{b_d})(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_d w-1))^2}{(4c_d(\overline{b_d}-\underline{b_d})-c\Pi_0-R_0)^2(1-a_d w)^2} \\ & (c\Pi_0+R_0) + w_0 - (c\Pi_0+R_0)x - \frac{a_e^2(c\Pi_0+R_0)^2(a_e+(\overline{b_e}-q_e)(1-a_d w))^2}{(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))^2(1-a_d w)^2}\Big) + w\big((\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(\overline{b_e}-q_e) \\ & \frac{4a_d^2 a_e c_e(1-a_d w))^2}{(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))^2(1-a_d w)^2} + \Pi_0 + a_d\Big(-\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(\overline{b_e}-q_e)(1-a_d w))^2}{(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))^2(1-a_d w)^2} - g - r + w_0 - (c\Pi_0+R_0) \\ & x - \frac{(c\Pi_0+R_0)^2(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_d w-1))^2}{(4c_d(\overline{b_d}-\underline{b_d})-c\Pi_0-R_0)^2(1-a_d w)^2} + \frac{4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_d w-1))^2}{(4c_d(\overline{b_d}-\underline{b_d})-c\Pi_0-R_0)^2(1-a_d w)^2}\Big)\big)\Big); \\ \frac{dD^*}{da_d} =& \frac{8c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(-a_d(\overline{b_d}w-q_d w-1)+\overline{b_d}-q_d)}{(1-a_d w)^3(-c\Pi_0+4c_d(\overline{b_d}-\underline{b_d})-R_0)^2}.\end{aligned}$$

Given feasibility constraints, we find that $\frac{d\Pi^*}{da_d} > 0$ and $\frac{dD^*}{da_d} > 0$.



(a) ***Illustration of Proposition 1***:h^* *vs.* a_d *at* $\{a_e = 0.731, q_d = 100, \underline{q_e} = 44.582, w = 6, w_0 = 20000, \overline{b_e} = 45, \underline{b_e} = 29, \overline{b_d} = 129, \underline{b_d} = 83, r = 0, g = 0, x = 0, R_0 = 152, \Pi_0 = 1, c = 21.269, c_d = 55, c_e = 92\}$.



(b) ***Illustration of Proposition 2***: C^* *vs.* a_d *at* $\{a_d = 0.25, a_e = 0.25, \underline{q_d} = 16, q_e = 1.5, w = 1, w_0 = 1000, \overline{b_e} = 2, \underline{b_e} = 1, \overline{b_d} = 17, \underline{b_d} = 1, r = 1, g = 1, x = 1, R_0 = 32, \Pi_0 = 1, c = 1, c_d = 1, c_e = 70.125\}$.

Figure 8. *Sensitivity of* h^* *and* C^* *to* a_d.

To support our mathematical arguments in CHAPTER 2.6.1 (and to prove Proposition 1), we next compute the first-order derivative of the equilibrium community support effort (i.e., h^*) and fundraising effort (i.e., f^*) exerted by the platform with respect to a_d as:

$$\frac{dh^*}{da_d} = -\frac{2a_e^2(c\Pi_0 + R_0)^2(a_e + (1 - a_d w)(\overline{b_e} - q_e))}{-(1 - a_d w)^3(-4a_d c_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))^3}\left(-4a_e c_e(2a_d w - 1)(\overline{b_e} - \underline{b_e})\right.$$
$$\left. + 4c_e(1 - a_d w)^2(\overline{b_e} - \underline{b_e})(\overline{b_e} - q_e) + a_e^2 w(c\Pi_0 + R_0)\right);$$
$$\frac{df^*}{da_d} = \frac{2(c\Pi_0 + R_0)^2(a_d(\overline{b_d} w - q_d w - 1) - \overline{b_d} + q_d)}{-(1 - a_d w)^3(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^2}.$$

Given the feasibility constraints (that are presented in the proof of Lemma 1), we find that, first, h^* decreases in a_d (i.e., $\frac{dh^*}{da_d} < 0$) if

$$R_0 > \theta_{R_0,h} = \frac{4a_e c_e(2a_d w - 1)(\overline{b_e} - \underline{b_e}) - 4c_e(1 - a_d w)^2(\overline{b_e} - \underline{b_e})(\overline{b_e} - q_e) - a_e^2 c\Pi_0 w}{a_e^2 w},$$

and it increases otherwise. Note that $\theta_{R_0,h} < \theta_{R_0}(a_d, .)$. Besides, f^* always increases in a_d regardless of the number of users served by the platform. Furthermore, the derivative of threshold $\theta_{R_0}(a_d, .)$ with respect to a_e is negative (i.e., $\frac{d\theta_{R_0}(a_d,.)}{da_e} = \frac{-4a_d^2 c_e w(\overline{b_e}-\underline{b_e})}{\left((1-a_d w)^2(\overline{b_e}-q_e)+a_e\right)^2} < 0$) proving the result provided in Corollary 1.

Besides, to support Proposition 1, we explore the first-order derivative of the equilibrium community support effort h^* with respect to a_d (i.e., $\frac{dh^*}{da_d}$). Given the feasibility constraints (that are presented in the proof of Lemma 1), we find that h^* increases in a_d if and only if

$$a_d > \theta_{h,a_d} = \Bigg(\sqrt{-a_e c_e w^2(\overline{b_e}-\underline{b_e})(a_e w(\overline{b_e}-q_e)(c\Pi_0+R_0)-4a_e c_e(\overline{b_e}-\underline{b_e})-4c_e(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e))}$$
$$-2a_e c_e w(\overline{b_e}-\underline{b_e})+2c_e\underline{b_e}\overline{b_e}w-2c_e\underline{b_e}q_e w-2c_e\overline{b_e}^2 w+2c_e\overline{b_e}q_e w\Bigg) \,/\, \left(-2c_e w^2(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e)\right).$$

Figure 8a further demonstrates the existence of such θ_{h,a_d}, such that, h^* is increasing in a_d when $a_d > \theta_{h,a_d}$ and it is decreasing in a_d otherwise.

Next, to support Corollary 2, given the derivative below:

$$\frac{dE^*[K_e]}{da_d} = -\frac{4a_e c_e(c\Pi_0+R_0)}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}\left(4a_d^2 c_e\underline{b_e}w-4a_d^2 c_e\overline{b_e}w+a_d^2\overline{b_e}R_0w^2-a_d^2 q_e R_0 w^2\right.$$
$$\left.+c\Pi_0(1-a_d w)^2(\overline{b_e}-q_e)-2a_d\overline{b_e}R_0 w+2a_d q_e R_0 w+a_e(c\Pi_0+R_0)+\overline{b_e}R_0-q_e R_0\right),$$

and the feasibility constraints (that are presented in the proof of Lemma 1), it is straightforward to show that if a_d increases, then $E^*[K_e]$ increases if and only if

$$a_d > \theta_{a_d}(R_0,.) = \Bigg(-\sqrt{w(c\Pi_0+R_0)^3(-a_e w(\overline{b_e}-q_e)(c\Pi_0+R_0)+4a_e c_e(\overline{b_e}-\underline{b_e})+4c_e(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e))}$$
$$+c^2\Pi_0^2 w(\overline{b_e}-q_e)+2c\Pi_0 R_0 w(\overline{b_e}-q_e)+\overline{b_e}R_0^2 w-q_e R_0^2 w\Big) \,/\, \big(w(c\Pi_0+R_0)(w(\overline{b_e}-q_e)(c\Pi_0+R_0)$$
$$-4c_e(\overline{b_e}-\underline{b_e}))\big).$$

Otherwise, we have $\frac{dE^*[K_e]}{da_d} < 0$.

■

Proof of Proposition 3

Note that increasing the upper or the lower bound by the same amount results in the same average level of generousness. Therefore, the change in the average level by increasing either the lower or upper bound does not affect the analysis. Then, we first have:

$$\frac{dD^*}{d\underline{b_d}} = -\frac{4c_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2(c\Pi_0-4c_d\underline{b_d}+4c_d\overline{b_d}+R_0)}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3};$$

$$\frac{dD^*}{d\overline{b_d}} = -\frac{4c_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}(a_d(c\Pi_0+R_0)(2\underline{b_d}w-3\overline{b_d}w+q_dw+1)$$
$$-4a_dc_d(\overline{b_d}-\underline{b_d})(2\underline{b_d}w-\overline{b_d}w-q_dw-1)-(c\Pi_0+R_0)(2\underline{b_d}-3\overline{b_d}+q_d)+4c_d(\overline{b_d}-\underline{b_d})(2\underline{b_d}-\overline{b_d}-q_d)).$$

Given the feasibility constraints (that are presented in the proof of Lemma 1), we have (i) the platform's total donation from users monotonously increases in the lower bound and upper bound of user generousness (i.e., $\frac{dD^*}{d\underline{b_d}} > 0$ and $\frac{dD^*}{d\overline{b_d}} > 0$), and (ii) the platform's total donation can be more efficiently increased by enhancing the upper bound of user generousness (i.e., $\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$) if and only if

$$q_d > \theta_{q_d} = \frac{a_d(c\Pi_0+R_0)(\underline{b_d}w-2\overline{b_d}w+1)-4a_dc_d(\overline{b_d}-\underline{b_d})(\underline{b_d}w-1)-(\underline{b_d}-2\overline{b_d})(c\Pi_0+R_0)+4c_d\underline{b_d}(\overline{b_d}-\underline{b_d})}{(1-a_dw)(c\Pi_0-4c_d\underline{b_d}+4c_d\overline{b_d}+R_0)}.$$

■

Proof of Proposition 4

The equilibrium overall content quality (i.e., Π^*) is provided in the proof of Lemma 1. Its derivatives with respect to q_e and R_0 are given below:

$$\frac{d\Pi^*}{dq_e} = \frac{2a_da_e(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))};$$

$$\frac{d\Pi^*}{dR_0} = \frac{a_d}{1-a_dw}\left(\frac{2a_e^3(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3} - \frac{2a_e^2(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}\right.$$
$$+\frac{-8a_da_e^2c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3} - \frac{-4a_da_ec_e(\overline{b_e}-\underline{b_e})(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}$$
$$+\frac{2(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3} + \frac{-8c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}$$
$$\left.-\frac{-4c_d(\overline{b_d}-\underline{b_d})(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} - \frac{2(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} - x\right).$$

Given the feasibility constraints, we have $\frac{d\Pi^*}{dq_e} < 0$. Besides, we find that

$$\frac{d^2\Pi^*}{dR_0dq_e} = \frac{-8a_d^2a_ec_e(\overline{b_e}-\underline{b_e})(a_e+(1-a_dw)(\overline{b_e}-q_e))}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2} < 0,$$

and

$$\frac{d\Pi^*}{dR_0} < 0, \text{ if } q_e > \theta_{R_0,q_e} = \frac{\frac{\sqrt{\mathbf{E}_2}}{-a_d a_e c_e(\overline{b_e}-\underline{b_e})(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}+2a_d^2\overline{b_e}w^2-2a_d w(a_e+2\overline{b_e})+2(a_e+\overline{b_e})}{2(1-a_d w)^2},$$

where

$$\begin{aligned}\mathbf{E}_2 =&a_d a_e c_e(1-a_d w)^2(\overline{b_e}-\underline{b_e})(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2\\ &\Big(a_d^2\Big(-4c_d(\overline{b_d}-\underline{b_d})\Big(2c\Pi_0 w^2 x+\overline{b_d}^2 w^2-2\overline{b_d}w(q_d w+1)+q_d^2 w^2+2q_d w+2R_0 w^2 x+1\Big)\\ &+w^2 x(c\Pi_0+R_0)^2+16c_d^2 w^2 x(\overline{b_d}-\underline{b_d})^2\Big)-2a_d\Big(-4c_d(\overline{b_d}-\underline{b_d})\Big(2wx(c\Pi_0+R_0)+\overline{b_d}^2 w\\ &-\overline{b_d}(2q_d w+1)+q_d^2 w+q_d\Big)+wx(c\Pi_0+R_0)^2+16c_d^2 wx(\overline{b_d}-\underline{b_d})^2\Big)+c^2\Pi_0^2 x+8cc_d\underline{b_d}\Pi_0 x\\ &-8cc_d\overline{b_d}\Pi_0 x+2c\Pi_0 R_0 x+16c_d^2\underline{b_d}^2 x-32c_d^2\underline{b_d}\,\overline{b_d}x+16c_d^2\overline{b_d}^2 x+4c_d\underline{b_d}\,\overline{b_d}^2-8c_d\underline{b_d}\,\overline{b_d}q_d\\ &+4c_d\underline{b_d}q_d^2+8c_d\underline{b_d}R_0 x-4c_d\overline{b_d}^3+8c_d\overline{b_d}^2 q_d-4c_d\overline{b_d}q_d^2-8c_d\overline{b_d}R_0 x+R_0^2 x\Big).\end{aligned}$$

Given the feasibility constraints, we numerically prove the existence of $\theta_{R_0,q_e} = 1.5$ at $\{a_d = 0.5, a_e = 0.016, q_d = 12, w = 1, w_0 = 2000, \overline{b_e} = 2, \underline{b_e} = 1, \overline{b_d} = 16, \underline{b_d} = 1, r = 1, g = 1, x = 2.062, \Pi_0 = 1, R_0 = 32, c = 1, c_d = 1, c_e = 1\}$. Therefore, $\frac{d\Pi^*}{dR_0} < 0$ if $q_e < \theta_{R_0,q_e}$, whereas $\frac{d\Pi^*}{dR_0} > 0$ otherwise.

To support our explanations in the main text regarding Proposition 4, we next present the first-order derivative of the equilibrium community support effort h^*, fundraising effort f^*, and in-house quality improvement effort v^* with respect to R_0. Given the feasibility constraints, we have

$$\begin{aligned}\frac{dh^*}{dR_0} =&\frac{-8a_d a_e^2 c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3} > 0;\\ \frac{df^*}{dR_0} =&\frac{-8c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3} > 0;\\ \frac{dv^*}{dR_0} =&\frac{2a_e^3(c\Pi_0+R_0)^2(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3}-\frac{2a_e^2(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}\\ &+\frac{2(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}+\frac{-8c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}\\ &-\frac{-4c_d(\overline{b_d}-\underline{b_d})(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}-\frac{2(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}-x < 0;\\ \frac{dD^*}{dR_0} =&\frac{4c_d(\overline{b_d}-\underline{b_d})(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2(-c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})-R_0)}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3} > 0.\end{aligned}$$

Besides, we also have

$$\frac{d^2h^*}{dR_0dq_e} = \frac{16a_da_e^2c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))}{(1-a_dw)(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3} < 0.$$

■

Proofs of The Extension: Generousness Levels Regarding Content Contributions and Donations Are Correlated

In this section, we provide proofs of results presented in CHAPTER 2.7.1. We derive the equilibrium results for this scenario similar to the main model. As discussed in CHAPTER 2.5, a user (say with index i without loss of generality) makes her decisions regarding which actions maximize her utility. In particular, these decisions depend on the individual generousness levels of users (i.e., $b_{d,i}$ and $b_{e,i}$) and can be listed as to whether to donate or not, and whether to make a content contribution or not; and further, how much donation and/or content contribution she makes if there is a donation and/or contribution.

Considering that user generousness in making content contributions and that in making donations are related to each other as $b_{d,i} = \rho b_{e,i} + m,\ \rho > 0$, we have users' generousness in contributing content and donations in the same order. Therefore, without loss of generality, we order $b_{d,i}$ such that $b_{d,i} \geq b_{d,i-1}$, and $b_{e,i} = \frac{b_{d,i}-m}{\rho} \geq b_{e,i-1} = \frac{b_{d,i-1}-m}{\rho}$. Hence, if user i decides to contribute content (resp., make a donation), user j ($\forall j > i$) will also contribute content (resp., make a donation). Utility maximization implies that the optimal content contribution effort of user i is $e_i^* = \frac{\frac{a_e}{1-a_dw}+b_d\rho+m+\sqrt{h}-q_e}{2c_e}$, and the optimal donation amount is $d_i^* = \frac{\frac{a_d}{1-a_dw}+b_d+\sqrt{f}-q_d}{2c_d}$. Denote b_e^* as the threshold of the user generousness that is indifferent between making a contribution or not, and b_d^* as the threshold of user generousness that is indifferent between making a donation or not. Then, we observe that:

$$b_d^* = -\frac{a_d}{1-a_dw} - \sqrt{f} + q_d \quad \text{and} \quad b_e^* = \frac{-\frac{a_e}{1-a_dw} - m - \sqrt{h} + q_e}{\rho}.$$

We further observe that users' generousness levels $b_{d,i}$ and $b_{e,i}$ are uniformly distributed within the intervals $(\underline{b_d}, \overline{b_d})$ and $(\rho\underline{b_d}+m, \rho\overline{b_d}+m)$ respectively, and we compute (i) the expected total donation (i.e., D) by multiplying the number of users (R_0) with the unconditional expectation of donations and (ii) the expected total contribution (i.e., C) in a similar way:

$$
\begin{aligned}
D &= R\int_{\underline{b_d}}^{\overline{b_d}} d^*(b_d)\frac{1}{\overline{b_d}-\underline{b_d}}db_d = R\int_{b_d^*}^{\overline{b_d}} \frac{\frac{a_d}{1-a_d w}+b_d+\sqrt{f}-q_d}{2c_d(\overline{b_d}-\underline{b_d})}db_d \\
&= \frac{(c\Pi_0+R_0)\left(a_d w\left(-\left(\overline{b_d}+\sqrt{f}-q_d\right)\right)+a_d+\overline{b_d}+\sqrt{f}-q_d\right)^2}{4c_d(1-a_d w)^2(\overline{b_d}-\underline{b_d})}; \\
C &= R\int_{\underline{b_e}}^{\overline{b_e}} e^*(b_e)\frac{1}{\overline{b_e}-\underline{b_e}}db_e = R\int_{b_e^*}^{\overline{b_e}} \frac{\frac{a_e}{1-a_d w}+b_d\rho+m+\sqrt{h}-q_e}{2c_e(\overline{b_e}-\underline{b_e})}db_e \\
&= \frac{(c\Pi_0+R_0)\left(a_e+(1-a_d w)\left(m+\overline{b_d}\rho+\sqrt{h}-q_e\right)\right)^2}{4c_e\rho^2(1-a_d w)^2(\overline{b_d}-\underline{b_d})}.
\end{aligned}
$$

The recursive formula for the overall quality (i.e., Equation 4 in the main model) reveals that:

$$
\Pi = \frac{1}{1-a_d w}\left(\Pi_0 + a_d\left(w_0 + D - f - h - r - g - x(c\Pi_0+R_0)\right) + a_e C\right).
$$

Substituting C and D into the formula above and by invoking the first-order conditions for the platform's decisions (i.e., f and h), we have:

$$
\begin{aligned}
\frac{d\Pi}{df} &= \frac{a_d\left(\frac{(c\Pi_0+R_0)\left(a_d\left(\overline{b_d}w+\sqrt{f}w-q_d w-1\right)-\overline{b_d}-\sqrt{f}+q_d\right)}{-c_d\sqrt{f}(\overline{b_d}-\underline{b_d})}+4a_d w-4\right)}{4(1-a_d w)^2} = 0; \\
\frac{d\Pi}{dh} &= \frac{-a_e(1-a_d w)(c\Pi_0+R_0)\left(m+\overline{b_d}\rho+\sqrt{h}-q_e\right)+4a_d c_e\sqrt{h}\rho^2(1-a_d w)(\overline{b_d}-\underline{b_d})-a_e^2(c\Pi_0+R_0)}{-4c_e\sqrt{h}\rho^2(1-a_d w)^2(\overline{b_d}-\underline{b_d})} = 0.
\end{aligned}
$$

Then, we compute the equilibrium platform resource allocation strategy as:

$$
\begin{aligned}
f^* &= \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}; \\
h^* &= \frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_d w)(m+\overline{b_d}\rho-q_e))^2}{(1-a_d w)^2\left(-4a_d c_e\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\right)^2}.
\end{aligned}
$$

Note that the first-order conditions necessitate that we have:

$$
-4a_d c_e\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)<0 \text{ and } c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0<0.
$$

Furthermore, the second-order conditions require that we have:

$$\frac{d^2\Pi^*}{df^2} = -\frac{a_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{-8c_d(1-a_dw)^2(\overline{b_d}-\underline{b_d})\left(\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\right)^{3/2}} < 0,$$

and

$$\frac{d^2\Pi^*}{df^2}\frac{d^2\Pi^*}{dh^2} - \left(\frac{d^2\Pi^*}{dfdh}\right)^2 = \frac{a_d(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)\left(-4a_dc_e\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\right)^2}{64a_ec_dc_e\rho^2(1-a_dw)^2(\overline{b_d}-\underline{b_d})^2(-(1-a_dw)(m+\overline{b_d}\rho-q_e)-a_e)\mathbf{E}_3\mathbf{E}_4} > 0,$$

where

$$\mathbf{E}_3 = \left(\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\right)^{3/2},$$

$$\text{and} \quad \mathbf{E}_4 = \sqrt{\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(m+\overline{b_d}\rho-q_e))^2}{(1-a_dw)^2\left(-4a_dc_e\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\right)^2}}.$$

Considering the boundary conditions $0 < \underline{b_d} < \overline{b_d}$, $0 < \underline{b_e} < \overline{b_e}$ and the fact that the quadratic forms in the expressions of the second-order conditions do not affect the inequality conditions, we derive the second-order conditions as:

$$a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d < 0 \text{ and } -(1-a_dw)(m+\overline{b_d}\rho-q_e)-a_e < 0.$$

We next summarize the equilibrium results relevant to our feasibility constraints below:

$$\begin{aligned}
b_d^* =& \frac{(c\Pi_0+R_0)(a_d(-\overline{b_d})w+a_dq_dw+a_d+\overline{b_d}-q_d)}{(1-a_dw)(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)} - \frac{a_d}{1-a_dw}+q_d;\\
b_e^* =& \frac{a_e(-c\overline{b_d}\Pi_0(1-a_dw)+4a_dc_e\underline{b_d}\rho-4a_dc_e\overline{b_d}\rho+a_d\overline{b_d}R_0w-\overline{b_d}R_0)-4a_dc_e\rho(1-a_dw)(m-q_e)(\overline{b_d}-\underline{b_d})}{(1-a_dw)\left(4a_dc_e\rho^2(\overline{b_d}-\underline{b_d})-a_e(c\Pi_0+R_0)\right)};\\
v^* =& -\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(c+\overline{b_d}\rho-q_e))^2}{(1-a_dw)^2\left(-4a_d\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\right)^2} - \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\\
&+\frac{4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} - x(c\Pi_0+R_0)-g-r+w_0.
\end{aligned}$$

Finally, we list below a set of constraints that implies that the presented solution is a feasible sustained equilibrium.

$$a_d(\overline{b_d}w - q_dw - 1) - \overline{b_d} + q_d < 0 \text{ and } -(1-a_dw)(m + \overline{b_d}\rho - q_e) - a_e < 0 \tag{21}$$

$$-4a_dc_e\rho^2(\overline{b_d} - \underline{b_d}) + a_e(c\Pi_0 + R_0) < 0 \text{ and } c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0 < 0 \tag{22}$$

$$a_d > 0 \text{ and } w > 0 \text{ and } a_e > 0 \text{ and } \overline{b_e} > q_e > \underline{b_e} > 0 \text{ and } \overline{b_d} > q_d > \underline{b_d} > 0 \text{ and } r \geq 0 \tag{23}$$

$$x \geq 0 \text{ and } \Pi_0 > 0 \text{ and } g \geq 0 \text{ and } R_0 > 0 \text{ and } c > 0 \text{ and } c_d > 0 \text{ and } c_e > 0 \text{ and } \rho > 0 \tag{24}$$

$$\overline{b_e} > \frac{-\frac{a_e}{1-a_dw} - m + q_e}{\rho} > b_e^* > \underline{b_e} \text{ and } \overline{b_d} > -\frac{a_d}{1-a_dw} + q_d > b_d^* > \underline{b_d} \tag{25}$$

$$v^* \geq 0 \text{ and } 1 - a_dw > 0 \text{ and } h^* < q_e^2. \tag{26}$$

We verify that the parameter space denoted by the above feasibility constraint is non-empty, and all of our results are valid in this parameter space. For example, at $\{a_d = 0.13, a_e = 0.06, q_d = 1.25, q_e = 1.39, w = 1, w_0 = 0, \overline{b_d} = 2, \underline{b_d} = 1, r = 0, g = 0, x = 0, \Pi_0 = 1, R_0 = 1, c = 1, c_d = 5, c_e = 114, \rho = 0.25, m = 1\}$, all of the above listed constraints are satisfied.

Proof of Proposition 1 and 2

In this extension, the equilibrium total content contribution from users and community support effort exerted by the platform are:

$$C^* = \frac{4a_d^2c_e\rho^2(\overline{b_d} - \underline{b_d})(c\Pi_0 + R_0)(a_e + (1-a_dw)(m + \overline{b_d}\rho - q_e))^2}{(1-a_dw)^2\left(-4a_dc_e\rho^2(\overline{b_d} - \underline{b_d}) + a_e(c\Pi_0 + R_0)\right)^2};$$
$$h^* = \frac{a_e^2(c\Pi_0 + R_0)^2(a_e + (1-a_dw)(m + \overline{b_d}\rho - q_e))^2}{(1-a_dw)^2\left(-4a_dc_e\rho^2(\overline{b_d} - \underline{b_d}) + a_e(c\Pi_0 + R_0)\right)^2}.$$

Next, we can derive the first-order derivatives of these measures with respect to a_d as:

$$\begin{aligned}\frac{dC^*}{da_d} =& -\frac{-8a_da_ec_e\rho^2(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)}{-(1-a_dw)^3\left(-4a_dc_e\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\right)^3}\left(-(1-a_dw)(m+\overline{b_d}\rho-q_e)-a_e\right)\left(a_d^2mR_0w^2\right.\\ &+4a_d^2c_e\underline{b_d}\rho^2w - 4a_d^2c_e\overline{b_d}\rho^2w + a_d^2\overline{b_d}R_0\rho w^2 - a_d^2q_eR_0w^2 + c\Pi_0(1-a_dw)^2(m+\overline{b_d}\rho-q_e) - 2a_dmR_0w\\ &\left.-2a_d\overline{b_d}R_0\rho w + 2a_dq_eR_0w + a_e(c\Pi_0+R_0) + mR_0 + \overline{b_d}R_0\rho - q_eR_0\right);\\ \frac{dh^*}{da_d} =& -\frac{2a_e^2(c\Pi_0+R_0)^2}{-(1-a_dw)^3\left(-4a_dc_e\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\right)^3}\left(a_e+(1-a_dw)(m+\overline{b_d}\rho-q_e)\right)\left(-4a_ec_e\rho^2\right.\\ &\left.(2a_dw-1)(\overline{b_d}-\underline{b_d}) + 4c_e\rho^2(1-a_dw)^2(\overline{b_d}-\underline{b_d})(m+\overline{b_d}\rho-q_e) + a_e^2w(c\Pi_0+R_0)\right).\end{aligned}$$

Given the feasibility constraints (that are presented in Expressions 21 to 25), we find that, first, h^* increases in a_d if and only if the following condition is satisfied:

$$a_d > \frac{\sqrt{\mathbf{E}_5} - 2a_e c_e \rho^2 w(\overline{b_d} - \underline{b_d}) + 2mc_e\underline{b_d}\rho^2 w - 2mc_e\overline{b_d}\rho^2 w + 2c_e\underline{b_d}\,\overline{b_d}\rho^3 w - 2c_e\underline{b_d}q_e\rho^2 w - 2c_e\overline{b_d}^2\rho^3 w + 2c_e\overline{b_d}q_e\rho^2 w}{-2c_e\rho^2 w^2(\overline{b_d} - \underline{b_d})(m + \overline{b_d}\rho - q_e)},$$

where

$$\mathbf{E}_5 = -a_e c_e \rho^2 w^2(\overline{b_d} - \underline{b_d})\left(a_e w(c\Pi_0 + R_0)(m + \overline{b_d}\rho - q_e) - 4a_e c_e\rho^2(\overline{b_d} - \underline{b_d}) - 4c_e\rho^2(\overline{b_d} - \underline{b_d})(m + \overline{b_d}\rho - q_e)\right).$$

Second, C^* decreases in a_d if and only if $R_0 > \frac{4a_d^2 c_e\rho^2 w(\overline{b_d} - \underline{b_d}) - c\Pi_0(1 - a_d w)^2(m + \overline{b_d}\rho - q_e) - a_e c\Pi_0}{(1 - a_d w)^2(m + \overline{b_d}\rho - q_e) + a_e}$. Therefore, in this extension, our main results in Propositions 1 and 2 are qualitatively the same. We have also checked the feasibility of these presented results numerically for when a_d is greater or less than the threshold provided above.

Proof of Proposition 3

We first have:

$$D^* = \frac{4c_d(\overline{b_d} - \underline{b_d})(c\Pi_0 + R_0)(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)^2}{(1 - a_d w)^2(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^2};$$

$$\frac{dD^*}{d\underline{b_d}} = -\frac{4c_d(c\Pi_0 + R_0)(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)^2(c\Pi_0 - 4c_d\underline{b_d} + 4c_d\overline{b_d} + R_0)}{(1 - a_d w)^2(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^3};$$

$$\frac{dD^*}{d\overline{b_d}} = -\frac{4c_d(c\Pi_0 + R_0)(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)}{(1 - a_d w)^2(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^3}(a_d(c\Pi_0 + R_0)(2\underline{b_d}w - 3\overline{b_d}w + q_d w + 1) - 4a_d c_d(\overline{b_d} - \underline{b_d})$$
$$(2\underline{b_d}w - \overline{b_d}w - q_d w - 1) - (c\Pi_0 + R_0)(2\underline{b_d} - 3\overline{b_d} + q_d) + 4c_d(\overline{b_d} - \underline{b_d})(2\underline{b_d} - \overline{b_d} - q_d)).$$

We then compute the differences between the first-order derivatives of the total donation from users with respect to upper bounds and lower bounds of user generousness parameters as follows:

$$\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}} = -\frac{8c_d(c\Pi_0 + R_0)(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)}{(1 - a_d w)^2(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^3}(a_d(c\Pi_0 + R_0)(\underline{b_d}w - 2\overline{b_d}w + q_d w + 1)$$
$$-4a_d c_d(\overline{b_d} - \underline{b_d})(\underline{b_d}w - q_d w - 1) - (c\Pi_0 + R_0)(\underline{b_d} - 2\overline{b_d} + q_d) + 4c_d(\overline{b_d} - \underline{b_d})(\underline{b_d} - q_d)).$$

Given the feasibility constraints, we have the total donation from users monotonously increases in the lower bound and upper bound of user generousness (i.e., $\frac{dD^*}{d\underline{b_d}} > 0$ and $\frac{dD^*}{d\overline{b_d}} > 0$), and (ii) the platform's overall content quality can be more efficiently increased by enhancing the upper bound of user generousness (i.e., $\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$ if and only if the barrier parameter q_d is at a high level, i.e.,

$$q_d > \frac{a_d(c\Pi_0+R_0)(\underline{b_d}w-2\overline{b_d}w+1)-4a_dc_d(\overline{b_d}-\underline{b_d})(\underline{b_d}w-1)-(\underline{b_d}-2\overline{b_d})(c\Pi_0+R_0)+4c_d\underline{b_d}(\overline{b_d}-\underline{b_d})}{(1-a_dw)(c\Pi_0-4c_d\underline{b_d}+4c_d\overline{b_d}+R_0)}.$$

Proof of Proposition 4

In this extension, the equilibrium overall content quality is

$$\Pi^* = \frac{1}{1-a_dw}\Bigg(-\frac{-4a_d^2a_ec_e\rho^2(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_e+(1-a_dw)(m+\overline{b_d}\rho-q_e))^2}{(1-a_dw)^2\big(-4a_dc_e\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\big)^2} + a_d\Bigg(-\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(c+\overline{b_d}\rho-q_e))^2}{(1-a_dw)^2\big(-4a_d\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\big)^2}$$
$$-\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} - \frac{-4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} - x(c\Pi_0+R_0)-g-r+w_0\Bigg)+\Pi_0\Bigg).$$

Therefore, we can derive its first-order derivatives with respect to q_e and R_0 as

$$\frac{d\Pi^*}{dq_e} = -\frac{2a_da_e(c\Pi_0+R_0)\left(-\frac{-4a_dc_e\rho^2(\overline{b_d}-\underline{b_d})(a_e+(1-a_dw)(m+\overline{b_d}\rho-q_e))}{\left(-4a_dc_e\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\right)^2}-\frac{a_e(c\Pi_0+R_0)(a_e+(1-a_dw)(c+\overline{b_d}\rho-q_e))}{\left(-4a_d\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\right)^2}\right)}{(1-a_dw)^2};$$

$$\frac{d\Pi^*}{dR_0} = \frac{a_d}{1-a_dw}\Bigg(\frac{2a_e^3(c\Pi_0+R_0)^2(a_e+(1-a_dw)(c+\overline{b_d}\rho-q_e))^2}{(1-a_dw)^2\big(-4a_d\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\big)^3} - \frac{2a_e^2(c\Pi_0+R_0)(a_e+(1-a_dw)(c+\overline{b_d}\rho-q_e))^2}{(1-a_dw)^2\big(-4a_d\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\big)^2}$$
$$+\frac{-8a_da_e^2c_e\rho^2(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_e+(1-a_dw)(m+\overline{b_d}\rho-q_e))^2}{(1-a_dw)^2\big(-4a_dc_e\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\big)^3} - \frac{-4a_da_ec_e\rho^2(\overline{b_d}-\underline{b_d})(a_e+(1-a_dw)(m+\overline{b_d}\rho-q_e))^2}{(1-a_dw)^2\big(-4a_dc_e\rho^2(\overline{b_d}-\underline{b_d})+a_e(c\Pi_0+R_0)\big)^2}$$
$$+\frac{2(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3} + \frac{-8c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}$$
$$-\frac{-4c_d(\overline{b_d}-\underline{b_d})(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} - \frac{2(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} - x\Bigg).$$

Given the feasibility constraints (that are presented in the Expressions 21 to 25), we numerically verify that the overall content quality decreases in q_e, and it decreases in R_0 if and only if

$$q_e > \frac{(1-a_dw)}{4a_da_e}\Bigg(\frac{a_e^2(c\Pi_0+R_0)^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3} - \frac{a_e(c\Pi_0+R_0)}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2} + \frac{-4a_da_ec_e(\overline{b_d}-\underline{b_d})\rho^2(c\Pi_0+R_0)}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}$$
$$-\frac{-2a_dc_e(\overline{b_d}-\underline{b_d})\rho^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}\Bigg)^{-1}\Bigg(\frac{-4a_da_e}{1-a_dw}\Bigg(-\frac{(c\Pi_0+R_0)^2a_e^3}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3(1-a_dw)} - (c\Pi_0+R_0)^2$$

$$\frac{ca_e^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}-\frac{\overline{b_d}(c\Pi_0+R_0)^2\rho a_e^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}+\frac{(c\Pi_0+R_0)a_e^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2(1-a_dw)}$$

$$-\frac{-4a_dc_e(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)\rho^2a_e^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3(1-a_dw)}+\frac{c(c\Pi_0+R_0)a_e}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}+\frac{\overline{b_d}(c\Pi_0+R_0)\rho a_e}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}$$

$$-\frac{-4a_dc_e(\overline{b_d}-\underline{b_d})\overline{b_d}(c\Pi_0+R_0)\rho^3a_e}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}-\frac{-4a_dmc_e(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)\rho^2a_e}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}+\frac{-2a_dc_e(\overline{b_d}-\underline{b_d})\rho^2a_e}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2(1-a_dw)}$$

$$+\frac{-2a_dc_e(\overline{b_d}-\underline{b_d})\overline{b_d}\rho^3}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}+\frac{-2a_dmc_e(\overline{b_d}-\underline{b_d})\rho^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}\Bigg)-2\sqrt{2}\sqrt{\mathbf{E}_6}\Bigg),$$

where

$$\mathbf{E}_6=\frac{a_d^2a_e}{(1-a_dw)^2}\Bigg(2a_e\Bigg(\frac{(c\Pi_0+R_0)^2a_e^3}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3(1-a_dw)}+\frac{c(c\Pi_0+R_0)^2a_e^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}$$

$$+\frac{\overline{b_d}(c\Pi_0+R_0)^2\rho a_e^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}-\frac{(c\Pi_0+R_0)a_e^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2(1-a_dw)}+\frac{-4a_dc_e(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)\rho^2a_e^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3(1-a_dw)}$$

$$-\frac{c(c\Pi_0+R_0)a_e}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}-\frac{\overline{b_d}(c\Pi_0+R_0)\rho a_e}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}+\frac{-4a_dc_e(\overline{b_d}-\underline{b_d})\overline{b_d}(c\Pi_0+R_0)\rho^3a_e}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}$$

$$+\frac{-4a_dmc_e(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)\rho^2a_e}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}-\frac{-2a_dc_e(\overline{b_d}-\underline{b_d})\rho^2a_e}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2(1-a_dw)}+(\overline{b_d}-\underline{b_d})\overline{b_d}\rho^3$$

$$\frac{2a_dc_e}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}+\frac{2a_dmc_e(\overline{b_d}-\underline{b_d})\rho^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}\Bigg)^2-\Bigg(\frac{a_e^2(c\Pi_0+R_0)^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}$$

$$-\frac{a_e(c\Pi_0+R_0)}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}+\frac{-4a_da_ec_e(\overline{b_d}-\underline{b_d})\rho^2(c\Pi_0+R_0)}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}-\frac{-2a_dc_e(\overline{b_d}-\underline{b_d})\rho^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}\Bigg)$$

$$\Bigg(\frac{2(c\Pi_0+R_0)^2a_e^5}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3(1-a_dw)^2}+\frac{4c(c\Pi_0+R_0)^2a_e^4}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3(1-a_dw)}+(c\Pi_0+R_0)^2$$

$$\frac{4\overline{b_d}\rho a_e^4}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3(1-a_dw)}-\frac{2(c\Pi_0+R_0)a_e^4}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2(1-a_dw)^2}-(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)$$

$$\frac{8a_dc_e\rho^2a_e^4}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3(1-a_dw)^2}+\frac{2c^2(c\Pi_0+R_0)^2a_e^3}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}+\frac{2\overline{b_d}^2(c\Pi_0+R_0)^2\rho^2a_e^3}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}$$

$$+\frac{4c\overline{b_d}(c\Pi_0+R_0)^2\rho a_e^3}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}-\frac{4c(c\Pi_0+R_0)a_e^3}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2(1-a_dw)}-\frac{4\overline{b_d}(c\Pi_0+R_0)\rho a_e^3}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2(1-a_dw)}$$

$$+\frac{-16a_dc_e(\overline{b_d}-\underline{b_d})\overline{b_d}(c\Pi_0+R_0)\rho^3a_e^3}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3(1-a_dw)}+\frac{-16a_dmc_e(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)\rho^2a_e^3}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3(1-a_dw)}+(\overline{b_d}-\underline{b_d})$$

$$\frac{4a_dc_e\rho^2a_e^3}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2(1-a_dw)^2}-\frac{2\overline{b_d}^2(c\Pi_0+R_0)\rho^2a_e^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}-\frac{2c^2(c\Pi_0+R_0)a_e^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}$$

$$-\frac{4c\overline{b_d}(c\Pi_0+R_0)\rho a_e^2}{\big(-4a_d(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}+\frac{-8a_dc_e(\overline{b_d}-\underline{b_d})\overline{b_d}^2(c\Pi_0+R_0)\rho^4a_e^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}+\frac{-16a_dmc_e(\overline{b_d}-\underline{b_d})\overline{b_d}(c\Pi_0+R_0)\rho^3a_e^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}$$

$$+\frac{-8a_dm^2c_e(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)\rho^2a_e^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^3}-\frac{-8a_dc_e(\overline{b_d}-\underline{b_d})\overline{b_d}\rho^3a_e^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2(1-a_dw)}-\frac{-8a_dmc_e(\overline{b_d}-\underline{b_d})\overline{b_d}\rho^3a_e}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}$$

$$-\frac{-8a_dmc_e(\overline{b_d}-\underline{b_d})\rho^2a_e^2}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2(1-a_dw)}-\frac{-4a_dc_e(\overline{b_d}-\underline{b_d})\overline{b_d}^2\rho^4a_e}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}-\frac{-4a_dm^2c_e(\overline{b_d}-\underline{b_d})\rho^2a_e}{\big(-4a_dc_e(\overline{b_d}-\underline{b_d})\rho^2+a_e(c\Pi_0+R_0)\big)^2}$$

$$+\frac{2(c\Pi_0+R_0)^2(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_dw-1))^2}{(-4c_d(\overline{b_d}-\underline{b_d})+c\Pi_0+R_0)^3(1-a_dw)^2}+\frac{-8c_d(c\Pi_0+R_0)(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_dw-1))^2(\overline{b_d}-\underline{b_d})}{(-4c_d(\overline{b_d}-\underline{b_d})+c\Pi_0+R_0)^3(1-a_dw)^2}-x$$

$$-\frac{-4c_d(\overline{b_d}-\underline{b_d})(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_dw-1))^2}{(-4c_d(\overline{\overline{b_d}}-\underline{b_d})+c\Pi_0+R_0)^2(1-a_dw)^2}-\frac{2(c\Pi_0+R_0)(-\overline{b_d}+q_d+a_d(\overline{b_d}w-q_dw-1))^2}{(-4c_d(\overline{b_d}-\underline{b_d})+c\Pi_0+R_0)^2(1-a_dw)^2}\Bigg)\Bigg).$$

We have checked the existence of this threshold and provide a feasible set of parameter values as: $\{a_d = 0.125, a_e = 0.063, q_d = 1.25, w = 1, w_0 = 100, \overline{b_d} = 2, \underline{b_d} = 1, r = 0, g = 0, \Pi_0 = 1, R_0 = 1, c = 1, c_d = 5, c_e = 114, \rho = 0.25, m = 1, x = 0.069\}$.

■

Proofs of The Extension: Robustness to The Disutility Elasticity of Users

In this section, we provide proofs of results presented in CHAPTER 2.7.2. In this extension, the equilibrium results are derived similarly to the main model, and hence, the details are omitted for brevity. However, the important variables of interest at the equilibrium are as follows:

$$
v^* = \begin{cases}
\begin{array}{l}
\text{if } \eta = 2 \text{ and } \zeta = 3: \\[6pt]
-\frac{a_e^2}{288 a_d^4 c_e^2 (1-a_d w)(\overline{b_e}-\underline{b_e})^4} \left(24 a_d^2 a_e c^2 c_e \Pi_0^2 (\overline{b_e} - \underline{b_e})^2 + 48 a_d^2 a_e c c_e \Pi_0 R_0 (\overline{b_e} - \underline{b_e})^2 + 24 a_d^2 a_e c_e R_0^2 (\overline{b_e} - \underline{b_e})^2 - 24 a_d^2 c^2\right. \\
c_e \Pi_0^2 q_e (1 - a_d w)(\overline{b_e} - \underline{b_e})^2 + 24 a_d^2 c^2 c_e \overline{b_e} \Pi_0^2 (1 - a_d w)(\overline{b_e} - \underline{b_e})^2 - 48 a_d^2 c c_e \Pi_0 q_e R_0 (1 - a_d w)(\overline{b_e} - \underline{b_e})^2 + 48 a_d^2 c c_e \\
\overline{b_e} \Pi_0 R_0 (1 - a_d w)(\overline{b_e} - \underline{b_e})^2 - 24 a_d^2 c_e q_e R_0^2 (1 - a_d w)(\overline{b_e} - \underline{b_e})^2 + 24 a_d^2 c_e \overline{b_e} R_0^2 (1 - a_d w)(\overline{b_e} - \underline{b_e})^2 - a_e (1 - a_d w) \\
(c\Pi_0 + R_0)^3 \sqrt{\frac{\mathbf{E}_8 c_e^4 (\overline{b_e}-\underline{b_e})^8}{a_e^6 (c\Pi_0+R_0)^6}} + a_e^2 c^4 \Pi_0^4 (1 - a_d w) + 4 a_e^2 c^3 \Pi_0^3 R_0 (1 - a_d w) + 6 a_e^2 c^2 \Pi_0^2 R_0^2 (1 - a_d w) + 4 a_e^2 c \Pi_0 R_0^3 (1 - a_d w) + a_e^2 R_0^4 \\
\left.(1 - a_d w)\right) - \frac{(c\Pi_0+R_0)^2 (a_d(\overline{b_d} w - q_d w - 1) - \overline{b_d} + q_d)^2}{(1-a_d w)^2 (c\Pi_0 - 4 c_d (\overline{b_d}-\underline{b_d}) + R_0)^2} - (a_d(\overline{b_d} w - q_d w - 1) - \overline{b_d} + q_d)^2 \frac{-4 c_d (\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)}{(1-a_d w)^2 (c\Pi_0 - 4 c_d (\overline{b_d}-\underline{b_d}) + R_0)^2} \\
-x(c\Pi_0 + R_0) - g - r + w_0, \\[6pt]
\text{if } \eta = 3 \text{ and } \zeta = 2: \\[6pt]
-g - r + w_0 - (c\Pi_0 + R_0)x - \frac{1}{288 c_d^2 (\overline{b_d}-\underline{b_d})^4 (1-a_d w)} \left(c_d^2 (1 - a_d w) \sqrt{\mathbf{E}_7} (\overline{b_d} - \underline{b_d})^4 + 24 a_d c^2 c_d \Pi_0^2 (\overline{b_d} - \underline{b_d})^2 + 24 a_d c_d R_0^2\right. \\
(\overline{b_d} - \underline{b_d})^2 + 48 a_d c c_d \Pi_0 R_0 (\overline{b_d} - \underline{b_d})^2 + 24 c^2 c_d \overline{b_d} \Pi_0^2 (1 - a_d w)(\overline{b_d} - \underline{b_d})^2 + 24 c_d \overline{b_d} R_0^2 (1 - a_d w)(\overline{b_d} - \underline{b_d})^2 - 24 c_d q_d R_0^2 \\
(1 - a_d w)(\overline{b_d} - \underline{b_d})^2 - 24 c^2 c_d \Pi_0^2 q_d (1 - a_d w)(\overline{b_d} - \underline{b_d})^2 + 48 c c_d \overline{b_d} \Pi_0 R_0 (1 - a_d w)(\overline{b_d} - \underline{b_d})^2 - 48 c c_d \Pi_0 q_d R_0 (1 - a_d w) \\
\left.(\overline{b_d} - \underline{b_d})^2 + 4 c \Pi_0 R_0^3 (1 - a_d w) + 6 c^2 \Pi_0^2 R_0^2 (1 - a_d w) + 4 c^3 \Pi_0^3 R_0 (1 - a_d w) + c^4 \Pi_0^4 (1 - a_d w) + R_0^4 (1 - a_d w)\right) + \frac{1}{3\sqrt{3}\sqrt{c_d}(\overline{b_d}-\underline{b_d})} \\
\left(2(c\Pi_0 + R_0)\left(\frac{a_d}{1-a_d w} + \overline{b_d} - q_d + \frac{\sqrt{\mathbf{E}_9}}{12\sqrt{2}}\right)^{3/2}\right) - \frac{a_e^2 (c\Pi_0+R_0)^2 (a_e - (\overline{b_e}-q_e)(1-a_d w))^2}{(-4 a_d c_e (\overline{b_e}-\underline{b_e}) + a_e (c\Pi_0+R_0))^2 (1-a_d w)^2}, \\[6pt]
\text{if } \eta = 3 \text{ and } \zeta = 3: \\[6pt]
-g - r + w_0 - (c\Pi_0 + R_0)x - \frac{a_e^2}{288 a_d^4 c_e^2 (\overline{b_e}-\underline{b_e})^4 (1-a_d w)} \left(a_e^2 c^4 (1 - a_d w)\Pi_0^4 + 4 a_e^2 c^3 R_0 (1 - a_d w)\Pi_0^3 + 24 a_d^2 a_e c^2 c_e (\overline{b_e} - \underline{b_e})^2 \Pi_0^2\right. \\
+6 a_e^2 c^2 R_0^2 (1 - a_d w)\Pi_0^2 + 24 a_d^2 c^2 c_e (\overline{b_e} - \underline{b_e})^2 \overline{b_e} (1 - a_d w)\Pi_0^2 - 24 a_d^2 c^2 c_e (\overline{b_e} - \underline{b_e})^2 q_e (1 - a_d w)\Pi_0^2 + 48 a_d^2 a_e c c_e (\overline{b_e} - \underline{b_e})^2 R_0 \Pi_0 \\
+4 a_e^2 c R_0^3 (1 - a_d w)\Pi_0 + 48 a_d^2 c c_e (\overline{b_e} - \underline{b_e})^2 \overline{b_e} R_0 (1 - a_d w)\Pi_0 - 48 a_d^2 c c_e (\overline{b_e} - \underline{b_e})^2 q_e R_0 (1 - a_d w)\Pi_0 + 24 a_d^2 a_e c_e (\overline{b_e} - \underline{b_e})^2 R_0^2 \\
\left.+24 a_d^2 c_e (\overline{b_e} - \underline{b_e})^2 \overline{b_e} R_0^2 (1 - a_d w) - 24 a_d^2 c_e (\overline{b_e} - \underline{b_e})^2 q_e R_0^2 (1 - a_d w) + a_e^2 R_0^4 (1 - a_d w) + a_e (c\Pi_0 + R_0)^3 (1 - a_d w) \sqrt{\frac{\mathbf{E}_8 c_e^4 (\overline{b_e}-\underline{b_e})^8}{a_e^6 (c\Pi_0+R_0)^6}}\right) \\
-\frac{1}{288 c_d^2 (\overline{b_d}-\underline{b_d})^4 (1-a_d w)} \left(c^4 (1 - a_d w)\Pi_0^4 + 4 c^3 R_0 (1 - a_d w)\Pi_0^3 + 24 a_d c^2 c_d (\overline{b_d} - \underline{b_d})^2 \Pi_0^2 + 6 c^2 R_0^2 (1 - a_d w)\Pi_0^2 + 24 c^2 c_d\right. \\
(\overline{b_d} - \underline{b_d})^2 \overline{b_d} (1 - a_d w)\Pi_0^2 - 24 c^2 c_d (\overline{b_d} - \underline{b_d})^2 q_d (1 - a_d w)\Pi_0^2 + 48 a_d c c_d (\overline{b_d} - \underline{b_d})^2 R_0 \Pi_0 + 4 c R_0^3 (1 - a_d w)\Pi_0 + 48 c c_d (\overline{b_d} - \underline{b_d})^2 \\
\overline{b_d} R_0 (1 - a_d w)\Pi_0 - 48 c c_d (\overline{b_d} - \underline{b_d})^2 q_d R_0 (1 - a_d w)\Pi_0 + 24 a_d c_d (\overline{b_d} - \underline{b_d})^2 R_0^2 + 24 c_d (\overline{b_d} - \underline{b_d})^2 \overline{b_d} R_0^2 (1 - a_d w) - 24 c_d (\overline{b_d} - \underline{b_d})^2 \\
\left.q_d R_0^2 (1 - a_d w) + R_0^4 (1 - a_d w) + (c\Pi_0 + R_0)^3 (1 - a_d w) \sqrt{\frac{\mathbf{E}_7 c_d^4 (\overline{b_d}-\underline{b_d})^8}{(c\Pi_0+R_0)^6}}\right) - \frac{2(c\Pi_0+R_0)}{-3\sqrt{3}\sqrt{c_d}(\overline{b_d}-\underline{b_d})} \left(\frac{a_d}{1-a_d w} + \overline{b_d} - q_d + \frac{\sqrt{\mathbf{E}_9}}{12\sqrt{2}}\right)^{3/2},
\end{array}
\end{cases}
$$

$$
b_d^* = \begin{cases}
-\frac{\sqrt{\mathbf{E}_9}}{12\sqrt{2}} - \frac{a_d}{1-a_d w} + q_d, & \text{if } \eta = 3, \\
\frac{(c\Pi_0+R_0)(-a_d \overline{b_d} w + a_d q_d w + a_d + \overline{b_d} - q_d)}{(1-a_d w)(c\Pi_0 - 4 c_d (\overline{b_d}-\underline{b_d}) + R_0)} - \frac{a_d}{1-a_d w} + q_d, & \text{if } \eta = 2,
\end{cases}
$$

$$
b_e^* = \begin{cases} -\frac{a_e\sqrt{\mathbf{E}_{10}}}{12\sqrt{2}a_d^2c_e(\overline{b_e}-\underline{b_e})^2} - \frac{a_e}{1-a_dw} + q_e, & \text{if } \zeta = 3, \\ \frac{\frac{a_e(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))}{-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0)}+q_e(1-a_dw)-a_e}{1-a_dw}, & \text{if } \zeta = 2, \end{cases}
$$

$$
f^* = \begin{cases} \frac{1}{288c_d^2(1-a_dw)(\overline{b_d}-\underline{b_d})^4}\big(c^4\Pi_0^4(1-a_dw) + 4c^3\Pi_0^3R_0(1-a_dw) - 24c^2c_d\Pi_0^2q_d(1-a_dw)(\overline{b_d}-\underline{b_d})^2 & \\ +24c^2c_d\overline{b_d}\Pi_0^2(1-a_dw)(\overline{b_d}-\underline{b_d})^2 + 24a_dc^2c_d\Pi_0^2(\overline{b_d}-\underline{b_d})^2 + 6c^2\Pi_0^2R_0^2(1-a_dw) + c_d^2(1-a_dw) & \\ (\overline{b_d}-\underline{b_d})^4\sqrt{\mathbf{E}_7} - 48cc_d\Pi_0q_dR_0(1-a_dw)(\overline{b_d}-\underline{b_d})^2 + 48cc_d\overline{b_d}\Pi_0R_0(1-a_dw)(\overline{b_d}-\underline{b_d})^2 + 48a_d & \\ cc_d\Pi_0R_0(\overline{b_d}-\underline{b_d})^2 + 4c\Pi_0R_0^3(1-a_dw) - 24c_dq_dR_0^2(1-a_dw)(\overline{b_d}-\underline{b_d})^2 + 24c_d\overline{b_d}R_0^2(1-a_dw) & \\ (\overline{b_d}-\underline{b_d})^2 + 24a_dc_dR_0^2(\overline{b_d}-\underline{b_d})^2 + R_0^4(1-a_dw)\big), & \text{if } \eta = 3, \\ \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}, & \text{if } \eta = 2, \end{cases}
$$

$$
h^* = \begin{cases} \frac{1}{288a_d^4c_e^2(1-a_dw)(\overline{b_e}-\underline{b_e})^4}\big(24a_d^2a_e^3c^2c_e\Pi_0^2(\overline{b_e}-\underline{b_e})^2 + 48a_d^2a_e^3cc_e\Pi_0R_0(\overline{b_e}-\underline{b_e})^2 + 24a_d^2a_e^3c_eR_0^2 & \\ (\overline{b_e}-\underline{b_e})^2 - 24a_d^2a_e^2c^2c_e\Pi_0^2q_e(1-a_dw)(\overline{b_e}-\underline{b_e})^2 + 24a_d^2a_e^2c^2c_e\overline{b_e}\Pi_0^2(1-a_dw)(\overline{b_e}-\underline{b_e})^2 - 48a_d^2 & \\ a_e^2cc_e\Pi_0q_eR_0(1-a_dw)(\overline{b_e}-\underline{b_e})^2 + 48a_d^2a_e^2cc_e\overline{b_e}\Pi_0R_0(1-a_dw)(\overline{b_e}-\underline{b_e})^2 - 24a_d^2a_e^2c_eq_eR_0^2 & \\ (1-a_dw)(\overline{b_e}-\underline{b_e})^2 + 24a_d^2a_e^2c_e\overline{b_e}R_0^2(1-a_dw)(\overline{b_e}-\underline{b_e})^2 + c_e^2(1-a_dw)(\overline{b_e}-\underline{b_e})^4\sqrt{\mathbf{E}_8} + a_e^4c^4\Pi_0^4(1-a_dw) & \\ +4a_e^4c^3\Pi_0^3R_0(1-a_dw) + 6a_e^4c^2\Pi_0^2R_0^2(1-a_dw) + 4a_e^4c\Pi_0R_0^3(1-a_dw) + a_e^4R_0^4(1-a_dw)\big), & \text{if } \zeta = 3, \\ \frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}, & \text{if } \zeta = 2, \end{cases}
$$

where

$$
\mathbf{E}_7 = \frac{(c\Pi_0+R_0)^6\big(a_dw(c\Pi_0+R_0)^2+48a_dc_d(\overline{b_d}-\underline{b_d})^2(\overline{b_d}w-q_dw-1)-(c\Pi_0+R_0)^2-48c_d(\overline{b_d}-\underline{b_d})^2(\overline{b_d}-q_d)\big)}{-c_d^4(1-a_dw)(\overline{b_d}-\underline{b_d})^8},
$$

$$
\mathbf{E}_8 = \frac{a_e^6(c\Pi_0+R_0)^6\big(48a_d^3c_ew(\overline{b_e}-\underline{b_e})^2(\overline{b_e}-q_e)-48a_d^2c_e(\overline{b_e}-\underline{b_e})^2(a_e+\overline{b_e}-q_e)+a_da_e^2w(c\Pi_0+R_0)^2-a_e^2(c\Pi_0+R_0)^2\big)}{-c_e^4(1-a_dw)(\overline{b_e}-\underline{b_e})^8},
$$

$$
\begin{aligned}
\mathbf{E}_9 = {} & \frac{1}{c_d^2(\overline{b_d}-\underline{b_d})^4(1-a_dw)}\big(c_d^2(1-a_dw)\sqrt{\mathbf{E}_7}(\overline{b_d}-\underline{b_d})^4 + 24a_dc^2c_d\Pi_0^2(\overline{b_d}-\underline{b_d})^2 + 24a_dc_dR_0^2(\overline{b_d}-\underline{b_d})^2 + 48a_dcc_d\Pi_0R_0 \\
& (\overline{b_d}-\underline{b_d})^2 + 24c^2c_d\overline{b_d}\Pi_0^2(1-a_dw)(\overline{b_d}-\underline{b_d})^2 + 24c_d\overline{b_d}R_0^2(1-a_dw)(\overline{b_d}-\underline{b_d})^2 - 24c_dq_dR_0^2(1-a_dw)(\overline{b_d}-\underline{b_d})^2 - 24c^2 \\
& c_d\Pi_0^2q_d(1-a_dw)(\overline{b_d}-\underline{b_d})^2 + 48cc_d\overline{b_d}\Pi_0R_0(1-a_dw)(\overline{b_d}-\underline{b_d})^2 - 48cc_d\Pi_0q_dR_0(1-a_dw)(\overline{b_d}-\underline{b_d})^2 + 4c\Pi_0R_0^3 \\
& (1-a_dw) + 6c^2\Pi_0^2R_0^2(1-a_dw) + 4c^3\Pi_0^3R_0(1-a_dw) + c^4\Pi_0^4(1-a_dw) + R_0^4(1-a_dw)\big),
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{E}_{10} = {} & \frac{1}{1-a_dw}\big(24a_d^2a_ec^2c_e\Pi_0^2(\overline{b_e}-\underline{b_e})^2 + 48a_d^2a_ecc_e\Pi_0R_0(\overline{b_e}-\underline{b_e})^2 + 24a_d^2a_ec_eR_0^2(\overline{b_e}-\underline{b_e})^2 - 24a_d^2c^2c_e\Pi_0^2q_e(1-a_dw) \\
& (\overline{b_e}-\underline{b_e})^2 + 24a_d^2c^2c_e\overline{b_e}\Pi_0^2(1-a_dw)(\overline{b_e}-\underline{b_e})^2 - 48a_d^2cc_e\Pi_0q_eR_0(1-a_dw)(\overline{b_e}-\underline{b_e})^2 + 48a_d^2cc_e\overline{b_e}\Pi_0R_0(1-a_dw) \\
& (\overline{b_e}-\underline{b_e})^2 - 24a_d^2c_eq_eR_0^2(1-a_dw)(\overline{b_e}-\underline{b_e})^2 + 24a_d^2c_e\overline{b_e}R_0^2(1-a_dw)(\overline{b_e}-\underline{b_e})^2 + a_e(1-a_dw)(c\Pi_0+R_0)^3\sqrt{\frac{\mathbf{E}_8c_e^4(\overline{b_e}-\underline{b_e})^8}{a_e^6(c\Pi_0+R_0)^6}}
\end{aligned}
$$

$$+a_e^2c^4\Pi_0^4(1-a_dw)+4a_e^2c^3\Pi_0^3R_0(1-a_dw)+6a_e^2c^2\Pi_0^2R_0^2(1-a_dw)+4a_e^2c\Pi_0R_0^3(1-a_dw)+a_e^2R_0^4(1-a_dw)\big).$$

The equilibrium solution in this extension is feasible when the following set of conditions is satisfied:

$$S_1 \text{ and } S_2 \text{ and } F_0 \; (that\; are\; provided\; in\; Table\; 8) \tag{27}$$

$$a_d > 0 \text{ and } a_e > 0 \text{ and } \overline{b_e} > q_e > \underline{b_e} > 0 \text{ and } \overline{b_d} > q_d > \underline{b_d} > 0 \text{ and } w > 0 \text{ and } \Pi_0 > 0 \tag{28}$$

$$R_0 > 0 \text{ and } c > 0 \text{ and } r \geq 0 \text{ and } x \geq 0 \text{ and } g \geq 0 \text{ and } c_d > 0 \text{ and } c_e > 0 \tag{29}$$

$$\overline{b_d} > -\frac{a_e}{1-a_dw} + q_e > b_d^* > \underline{b_d} \text{ and } \overline{b_d} > -\frac{a_d}{1-a_dw} + q_d > b_e^* > \underline{b_e} \tag{30}$$

$$v^* \geq 0 \text{ and } 1-a_dw > 0 \text{ and } h^* > q_e^2, \tag{31}$$

where S_1 and S_2 are second-order conditions and F_0 is implied by the first order conditions. The expressions for S_1, S_2, and F_0 are presented in Table 10. We numerically verify that the parameter space given the feasibility constraint set is not empty, and all of our results are feasible. For example, when $(\eta, \zeta) = (2, 3)$, the set of feasibility constraints is satisfied at $\{a_d = 0.25, \underline{b_e} = 1., \overline{b_e} = 8., a_e = 1.75, R_0 = 20., q_d = 1., q_e = 4.75, w = 0.5, w_0 = 100., \overline{b_d} = 2., \underline{b_d} = 0.375, r = 1., g = 1., x = 1., \Pi_0 = 1., c = 1., c_d = 16., c_e = 128\}$. When $(\eta, \zeta) = (3, 2)$, we have the set of feasibility constraints satisfied at $\{a_d = 0.25, a_e = 0.25, \underline{b_d} = 1, \overline{b_d} = 2, w = 2, w_0 = 1000, q_d = 1.75, q_e = 1, \overline{b_e} = 2, \underline{b_e} = 0.25, r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 1.75, c = 1, c_d = 16, c_e = 4\}$. Further, when $(\eta, \zeta) = (3, 3)$, at $\{a_d = 0.25, \underline{b_e} = 1, \overline{b_e} = 8, a_e = 1.75, q_d = 0.97, q_e = 5, w = 1, w_0 = 19.8, \overline{b_d} = 1.13, \underline{b_d} = 0.08, r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 4, c_e = 2\}$, the set of feasibility constraints is satisfied.

Proof Propositions 1 and 2

In this extension, we first consider the specification of $(\eta = 3, \zeta = 2)$. The equilibrium total content contribution effort and community support effort are:

	$\eta = 2,\ \zeta = 3$
F_0	$c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0 < 0$ and $h^* > 0$
S_1	$-\frac{a_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{-8c_d(1-a_dw)^2(\overline{b_d}-\underline{b_d})\left(\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\right)^{3/2}} < 0$
S_2	$\frac{a_da_e(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)\left(2a_e+(1-a_dw)\left(2\overline{b_e}+\sqrt{h}-2q_e\right)\right)}{-64\sqrt{3}c_d\sqrt{c_e}f^{3/2}h^{3/2}(1-a_dw)^4(\overline{b_d}-\underline{b_d})(\overline{b_e}-\underline{b_e})\sqrt{\frac{a_e}{1-a_dw}+\overline{b_e}+\sqrt{h}-q_e}} > 0$
	$\eta = 3,\ \zeta = 2$
F_0	$-4a_dc_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0) < 0$ and $f^* > 0$
S_1	$\frac{a_e(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))}{-8c_e(1-a_dw)^2(\overline{b_e}-\underline{b_e})\left(\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}\right)^{3/2}} < 0$
S_2	$\frac{a_da_e(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))\left(a_d\left(2\overline{b_d}w+\sqrt{f}w-2q_dw-2\right)-2\overline{b_d}-\sqrt{f}+2q_d\right)}{-64\sqrt{3}\sqrt{c_d}c_ef^{3/2}h^{3/2}(1-a_dw)^4(\overline{b_d}-\underline{b_d})(\overline{b_e}-\underline{b_e})\sqrt{\frac{a_d}{1-a_dw}+\overline{b_d}+\sqrt{f}-q_d}} > 0$
	$\eta = 3,\ \zeta = 3$
F_0	$f^* > 0$ and $h^* > 0$
S_1	$\frac{a_d(c\Pi_0+R_0)\left(a_d\left(2\overline{b_d}w+\sqrt{f}w-2q_dw-2\right)-2\overline{b_d}-\sqrt{f}+2q_d\right)}{8\sqrt{3}\sqrt{c_d}f^{3/2}(1-a_dw)^2(\overline{b_d}-\underline{b_d})\sqrt{\frac{a_d}{1-a_dw}+\overline{b_d}+\sqrt{f}-q_d}} < 0$
S_2	$\frac{a_da_e(c\Pi_0+R_0)^2\left(a_d\left(2\overline{b_d}w+\sqrt{f}w-2q_dw-2\right)-2\overline{b_d}-\sqrt{f}+2q_d\right)\left(-(1-a_dw)\left(2\overline{b_e}+\sqrt{h}-2q_e\right)-2a_e\right)}{192\sqrt{c_d}\sqrt{c_e}f^{3/2}h^{3/2}(1-a_dw)^4(\overline{b_d}-\underline{b_d})(\overline{b_e}-\underline{b_e})\sqrt{\frac{a_d}{1-a_dw}+\overline{b_d}+\sqrt{f}-q_d}\sqrt{\frac{a_e}{1-a_dw}+\overline{b_e}+\sqrt{h}-q_e}} > 0$

Table 8. *Constraints in Expression 27 under Different Specification of The Users' Disutility Elasticity (η, ζ)*

$$C^* = \frac{4a_d^2c_e(\overline{b_e} - \underline{b_e})(c\Pi_0 + R_0)(a_e + (1 - a_dw)(\overline{b_e} - q_e))^2}{(1 - a_dw)^2(-4a_dc_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))^2};$$
$$h^* = \frac{a_e^2(c\Pi_0 + R_0)^2(a_e + (1 - a_dw)(\overline{b_e} - q_e))^2}{(1 - a_dw)^2(-4a_dc_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))^2}.$$

The first-order derivatives of these measures with respect to a_d are

$$\begin{aligned}\frac{dC^*}{da_d} =& -\tfrac{-8a_da_ec_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(-(1-a_dw)(\overline{b_e}-q_e)-a_e)}{-(1-a_dw)^3(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3}\left(4a_d^2c_e\underline{b_e}w - 4a_d^2c_e\overline{b_e}w + a_d^2\overline{b_e}R_0w^2 - a_d^2q_eR_0w^2\right.\\ &\left.+c\Pi_0(1 - a_dw)^2(\overline{b_e} - q_e) - 2a_d\overline{b_e}R_0w + 2a_dq_eR_0w + a_e(c\Pi_0 + R_0) + \overline{b_e}R_0 - q_eR_0\right);\\ \frac{dh^*}{da_d} =& -\tfrac{2a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))}{-(1-a_dw)^3(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3}\left(-4a_ec_e(2a_dw - 1)(\overline{b_e} - \underline{b_e}) + 4c_e(1 - a_dw)^2(\overline{b_e} - \underline{b_e})(\overline{b_e} - q_e)\right.\\ &\left.+a_e^2w(c\Pi_0 + R_0)\right).\end{aligned}$$

Given the feasibility constraints (Expressions 32 to 36 when $\eta = 3$ and $\zeta = 2$), we find that, first, as a_d increases, C^* decreases if and only if

$$R_0 > \frac{a_d^2(-c)\overline{b_e}\Pi_0 w^2 + a_d^2 c\Pi_0 q_e w^2 - 4a_d^2 c_e \underline{b_e} w + 4a_d^2 c_e \overline{b_e} w + 2a_d c\overline{b_e}\Pi_0 w - 2a_d c\Pi_0 q_e w - a_e c\Pi_0 - c\overline{b_e}\Pi_0 + c\Pi_0 q_e}{a_d^2 \overline{b_e} w^2 - a_d^2 q_e w^2 - 2a_d \overline{b_e} w + 2a_d q_e w + a_e + \overline{b_e} - q_e},$$

and second, h^* increases in a_d if and only if the following condition is satisfied:

$$a_d > \frac{-\sqrt{\mathbf{E}_{11}} - 8a_e c_e \underline{b_e} w + 8a_e c_e \overline{b_e} w - 8c_e \underline{b_e}\overline{b_e} w + 8c_e \underline{b_e} q_e w + 8c_e \overline{b_e}^2 w - 8c_e \overline{b_e} q_e w}{2\left(-4c_e \underline{b_e}\overline{b_e} w^2 + 4c_e \underline{b_e} q_e w^2 + 4c_e \overline{b_e}^2 w^2 - 4c_e \overline{b_e} q_e w^2\right)},$$

where

$$\begin{aligned}\mathbf{E}_{11} =& \left(8a_e c_e \underline{b_e} w - 8a_e c_e \overline{b_e} w + 8c_e \underline{b_e}\overline{b_e} w - 8c_e \underline{b_e} q_e w - 8c_e \overline{b_e}^2 w + 8c_e \overline{b_e} q_e w\right)^2 - 4\left(-4c_e \underline{b_e}\overline{b_e} w^2 + 4c_e \underline{b_e} q_e w^2\right.\\ &\left.+4c_e \overline{b_e}^2 w^2 - 4c_e \overline{b_e} q_e w^2\right)\left(a_e^2 c\Pi_0 w + a_e^2 R_0 w - 4a_e c_e \underline{b_e} + 4a_e c_e \overline{b_e} - 4c_e \underline{b_e}\overline{b_e} + 4c_e \underline{b_e} q_e + 4c_e \overline{b_e}^2 - 4c_e \overline{b_e} q_e\right).\end{aligned}$$

Furthermore, note that the equilibrium total content contribution and its first-order derivative with respect to a_d for the other two specifications (i.e., $(\eta = 2, \zeta = 3)$ and $(\eta = 3, \zeta = 3)$) follow the same functional forms. These equilibrium measures are

$$\begin{aligned}C^* =& \frac{2(c\Pi_0 + R_0)\left(\frac{a_e\sqrt{\mathbf{E}_{10}}}{12\sqrt{2}a_d^2 c_e(\overline{b_e} - \underline{b_e})^2} + \frac{a_e}{1 - a_d w} + \overline{b_e} - q_e\right)^{3/2}}{3\sqrt{3}\sqrt{c_e}(\overline{b_e} - \underline{b_e})};\\ h^* =& \frac{1}{288 a_d^4 c_e^2 (1 - a_d w)(\overline{b_e} - \underline{b_e})^4}\left(24a_d^2 a_e^3 c^2 c_e \Pi_0^2 (\overline{b_e} - \underline{b_e})^2 + 48a_d^2 a_e^3 c c_e \Pi_0 R_0 (\overline{b_e} - \underline{b_e})^2 + 24a_d^2 a_e^3 c_e R_0^2 (\overline{b_e} - \underline{b_e})^2\right.\\ &-24a_d^2 a_e^2 c^2 c_e \Pi_0^2 q_e (1 - a_d w)(\overline{b_e} - \underline{b_e})^2 + 24a_d^2 a_e^2 c^2 c_e \overline{b_e}\Pi_0^2 (1 - a_d w)(\overline{b_e} - \underline{b_e})^2 - 48a_d^2 a_e^2 c c_e \Pi_0 q_e R_0 (1 - a_d w)\\ &(\overline{b_e} - \underline{b_e})^2 + 48a_d^2 a_e^2 c c_e \overline{b_e}\Pi_0 R_0 (1 - a_d w)(\overline{b_e} - \underline{b_e})^2 - 24a_d^2 a_e^2 c_e q_e R_0^2 (1 - a_d w)(\overline{b_e} - \underline{b_e})^2 + 24a_d^2 a_e^2 c_e \overline{b_e} R_0^2\\ &(1 - a_d w)(\overline{b_e} - \underline{b_e})^2 + c_e^2 (1 - a_d w)(\overline{b_e} - \underline{b_e})^4 \sqrt{\mathbf{E}_8} + a_e^4 c^4 \Pi_0^4 (1 - a_d w) + 4a_e^4 c^3 \Pi_0^3 R_0 (1 - a_d w) + 6a_e^4 c^2 \Pi_0^2 R_0^2\\ &\left.(1 - a_d w) + 4a_e^4 c\Pi_0 R_0^3 (1 - a_d w) + a_e^4 R_0^4 (1 - a_d w)\right).\end{aligned}$$

The first-order derivatives of these expressions with respect to a_d can be derived from the equilibrium measures presented above. However, for brevity, we do not present them in our paper as they are rather cumbersome. Because of this complication, we carry out an extensive numerical analysis and obtain qualitative results like those presented in Propositions 1 and 2. Representative numerical examples are provided in Table 9.

Proof Proposition 3

As the first-order derivatives are quite cumbersome for settings where $\eta = 3$, we only present the results of $\eta = 2$. However, we numerically verify the existence of each of the thresholds in all specifications. When $\eta = 2$, we have the following results

$$D^* = \frac{4c_d(\overline{b_d} - \underline{b_d})(c\Pi_0 + R_0)(a_d(\overline{b_d}w - q_dw - 1) - \overline{b_d} + q_d)^2}{(1 - a_dw)^2(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^2};$$

$$\frac{dD^*}{d\underline{b_d}} = -\frac{4c_d(c\Pi_0 + R_0)(a_d(\overline{b_d}w - q_dw - 1) - \overline{b_d} + q_d)^2(c\Pi_0 - 4c_d\underline{b_d} + 4c_d\overline{b_d} + R_0)}{(1 - a_dw)^2(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^3};$$

$$\frac{dD^*}{d\overline{b_d}} = -\frac{4c_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}(a_d(c\Pi_0 + R_0)(2\underline{b_d}w - 3\overline{b_d}w + q_dw + 1) - 4a_dc_d (\overline{b_d} - \underline{b_d})(2\underline{b_d}w - \overline{b_d}w - q_dw - 1) - (c\Pi_0 + R_0)(2\underline{b_d} - 3\overline{b_d} + q_d) + 4c_d(\overline{b_d} - \underline{b_d})(2\underline{b_d} - \overline{b_d} - q_d)).$$

We compute the differences between the first-order derivatives of the total donation from users with respect to upper bounds and lower bounds of user generousness parameters as follows:

$$\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}} = -\frac{8c_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}(a_d(c\Pi_0 + R_0)(\underline{b_d}w - 2\overline{b_d}w + q_dw + 1) -4a_dc_d(\overline{b_d} - \underline{b_d})(\underline{b_d}w - q_dw - 1) - (c\Pi_0 + R_0)(\underline{b_d} - 2\overline{b_d} + q_d) + 4c_d(\overline{b_d} - \underline{b_d})(\underline{b_d} - q_d)).$$

Given the feasibility constraints, we have the total donation from users monotonously increases in the lower bound and upper bound of user generousness (i.e., $\frac{dD^*}{d\underline{b_d}} > 0$ and $\frac{dD^*}{d\overline{b_d}} > 0$), and (ii) the platform's overall content quality can be more efficiently increased by enhancing the upper bound of user generousness (i.e., $\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$ if and only if the barrier parameter q_d is at a high level, i.e.,

$$q_d > \frac{a_d(c\Pi_0+R_0)(\underline{b_d}w-2\overline{b_d}w+1)-4a_dc_d(\overline{b_d}-\underline{b_d})(\underline{b_d}w-1)-(\underline{b_d}-2\overline{b_d})(c\Pi_0+R_0)+4c_d\underline{b_d}(\overline{b_d}-\underline{b_d})}{(1-a_dw)(c\Pi_0-4c_d\underline{b_d}+4c_d\overline{b_d}+R_0)}.$$

For the settings $(\eta = 3, \zeta = 2)$ and $(\eta = 3, \zeta = 3)$, we numerically verify the existence of the threshold of q_d and observe qualitatively the same patterns. Representative numerical examples are provided in Table 9.

Proof Proposition 4

In this extension, we consider three specifications of η and ζ. For the specification of $\eta = 2$ and $\zeta = 3$, the objective function of the platform is given as follows.

$$
\begin{aligned}
\Pi^* =& \tfrac{1}{1-a_d w} \Big(\Pi_0 + a_d \Big(-g - r + w_0 - (c\Pi_0 + R_0)x - \tfrac{a_e^2}{288 a_d^4 c_e^2 (\overline{b_e} - \underline{b_e})^4 (1-a_d w)} \big(a_e^2 c^4 (1 - a_d w) \Pi_0^4 + 4 a_e^2 c^3 R_0 (1 - a_d w) \\
& \Pi_0^3 + 24 a_d^2 a_e c^2 c_e (\overline{b_e} - \underline{b_e})^2 \Pi_0^2 + 6 a_e^2 c^2 R_0^2 (1 - a_d w) \Pi_0^2 + 24 a_d^2 c^2 c_e (\overline{b_e} - \underline{b_e})^2 \overline{b_e} (1 - a_d w) \Pi_0^2 - 24 a_d^2 c^2 c_e (\overline{b_e} - \underline{b_e})^2 q_e \\
& (1 - a_d w) \Pi_0^2 + 48 a_d^2 a_e c c_e (\overline{b_e} - \underline{b_e})^2 R_0 \Pi_0 + 4 a_e^2 c R_0^3 (1 - a_d w) \Pi_0 + 48 a_d^2 c c_e (\overline{b_e} - \underline{b_e})^2 \overline{b_e} R_0 (1 - a_d w) \Pi_0 - 48 a_d^2 c c_e \\
& (\overline{b_e} - \underline{b_e})^2 q_e R_0 (1 - a_d w) \Pi_0 + 24 a_d^2 a_e c_e (\overline{b_e} - \underline{b_e})^2 R_0^2 + 24 a_d^2 c_e (\overline{b_e} - \underline{b_e})^2 \overline{b_e} R_0^2 (1 - a_d w) - 24 a_d^2 c_e (\overline{b_e} - \underline{b_e})^2 q_e R_0^2 (1 - a_d w) \\
& + a_e^2 R_0^4 (1 - a_d w) - a_e (c\Pi_0 + R_0)^3 (1 - a_d w) \sqrt{\tfrac{\mathbf{E}_8 c_e^4 (\overline{b_e} - \underline{b_e})^8}{a_e^6 (c\Pi_0 + R_0)^6}} \Big) - \tfrac{(c\Pi_0 + R_0)^2 (-\overline{b_d} + q_d + a_d(\overline{b_d} w - q_d w - 1))^2}{(-4 c_d (\overline{b_d} - \underline{b_d}) + c\Pi_0 + R_0)^2 (1 - a_d w)^2} \\
& - \tfrac{-4 c_d (\overline{b_d} - \underline{b_d})(c\Pi_0 + R_0)}{(-4 c_d (\overline{b_d} - \underline{b_d}) + c\Pi_0 + R_0)^2 (1 - a_d w)^2} (-\overline{b_d} + q_d + a_d(\overline{b_d} w - q_d w - 1))^2 \Big) - \tfrac{2 a_e (c\Pi_0 + R_0)}{-3\sqrt{3}\sqrt{c_e}(\overline{b_e} - \underline{b_e})} \Big(\tfrac{a_e}{1 - a_d w} + \overline{b_e} - q_e \\
& + \frac{a_e \sqrt{\mathbf{E}_{12}}}{12\sqrt{2} a_d^2 c_e (\overline{b_e} - \underline{b_e})^2} \Big)^{3/2} \Bigg),
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{E}_{12} =& \tfrac{1}{1-a_d w} \big(a_e^2 c^4 (1 - a_d w) \Pi_0^4 + 4 a_e^2 c^3 R_0 (1 - a_d w) \Pi_0^3 + 24 a_d^2 a_e c^2 c_e (\overline{b_e} - \underline{b_e})^2 \Pi_0^2 + 6 a_e^2 c^2 R_0^2 (1 - a_d w) \Pi_0^2 + 24 a_d^2 c^2 c_e \\
& (\overline{b_e} - \underline{b_e})^2 \overline{b_e} (1 - a_d w) \Pi_0^2 - 24 a_d^2 c^2 c_e (\overline{b_e} - \underline{b_e})^2 q_e (1 - a_d w) \Pi_0^2 + 48 a_d^2 a_e c c_e (\overline{b_e} - \underline{b_e})^2 R_0 \Pi_0 + 4 a_e^2 c R_0^3 (1 - a_d w) \Pi_0 \\
& + 48 a_d^2 c c_e (\overline{b_e} - \underline{b_e})^2 \overline{b_e} R_0 (1 - a_d w) \Pi_0 - 48 a_d^2 c c_e (\overline{b_e} - \underline{b_e})^2 q_e R_0 (1 - a_d w) \Pi_0 + 24 a_d^2 a_e c_e (\overline{b_e} - \underline{b_e})^2 R_0^2 + 24 a_d^2 c_e \\
& (\overline{b_e} - \underline{b_e})^2 \overline{b_e} R_0^2 (1 - a_d w) - 24 a_d^2 c_e (\overline{b_e} - \underline{b_e})^2 q_e R_0^2 (1 - a_d w) + a_e^2 R_0^4 (1 - a_d w) + a_e (c\Pi_0 + R_0)^3 (1 - a_d w) \\
& \sqrt{\frac{\mathbf{E}_8 c_e^4 (\overline{b_e} - \underline{b_e})^8}{a_e^6 (c\Pi_0 + R_0)^6}} \Bigg).
\end{aligned}
$$

For the specification of $\eta = 3$ and $\zeta = 2$:

$$
\begin{aligned}
\Pi^* =& \tfrac{1}{1-a_d w} \Big(-\tfrac{-4 a_d^2 a_e c_e (\overline{b_e} - \underline{b_e})(c\Pi_0 + R_0)(a_e - (\overline{b_e} - q_e)(1 - a_d w))^2}{(-4 a_d c_e (\overline{b_e} - \underline{b_e}) + a_e (c\Pi_0 + R_0))^2 (1 - a_d w)^2} + \Pi_0 + a_d \big(-g - r + w_0 - (c\Pi_0 + R_0)x - \big(c_d^2 (1 - a_d w) \\
& \sqrt{\mathbf{E}_7} (\overline{b_d} - \underline{b_d})^4 + 24 a_d c^2 c_d \Pi_0^2 (\overline{b_d} - \underline{b_d})^2 + 24 a_d c_d R_0^2 (\overline{b_d} - \underline{b_d})^2 + 48 a_d c c_d \Pi_0 R_0 (\overline{b_d} - \underline{b_d})^2 + 24 c^2 c_d \overline{b_d} \Pi_0^2 (1 - a_d w)(\overline{b_d} - \underline{b_d})^2 \\
& + 24 c_d \overline{b_d} R_0^2 (1 - a_d w)(\overline{b_d} - \underline{b_d})^2 - 24 c_d q_d R_0^2 (1 - a_d w)(\overline{b_d} - \underline{b_d})^2 - 24 c^2 c_d \Pi_0^2 q_d (1 - a_d w)(\overline{b_d} - \underline{b_d})^2 + 48 c c_d \overline{b_d} \Pi_0 R_0 (1 - a_d w) \\
& (\overline{b_d} - \underline{b_d})^2 - 48 c c_d \Pi_0 q_d R_0 (1 - a_d w)(\overline{b_d} - \underline{b_d})^2 + 4 c \Pi_0 R_0^3 (1 - a_d w) + 6 c^2 \Pi_0^2 R_0^2 (1 - a_d w) + 4 c^3 \Pi_0^3 R_0 (1 - a_d w) + c^4 \Pi_0^4 (1 - a_d w) \\
& + R_0^4 (1 - a_d w) \big) \tfrac{1}{288 c_d^2 (\overline{b_d} - \underline{b_d})^4 (1 - a_d w)} - \tfrac{2 (c\Pi_0 + R_0)}{-3\sqrt{3}\sqrt{c_d}(\overline{b_d} - \underline{b_d})} \Big(\tfrac{a_d}{1 - a_d w} + \overline{b_d} - q_d + \tfrac{\sqrt{\mathbf{E}_9}}{12\sqrt{2}} \Big)^{3/2} - (a_e - (\overline{b_e} - q_e)(1 - a_d w))^2 \\
& \frac{a_e^2 (c\Pi_0 + R_0)^2}{(-4 a_d c_e (\overline{b_e} - \underline{b_e}) + a_e (c\Pi_0 + R_0))^2 (1 - a_d w)^2} \Big) \Big).
\end{aligned}
$$

For the specification of $\eta = 3$ and $\zeta = 3$:

$$
\begin{aligned}
\Pi^* =& \tfrac{1}{1-a_d w}\Big(\Pi_0 + a_d\Big(-g - r + w_0 - (c\Pi_0 + R_0)x - \tfrac{a_e^2}{288a_d^4c_e^2(\overline{b_e}-\underline{b_e})^4(1-a_dw)}\big(a_e^2c^4(1-a_dw)\Pi_0^4 + 4a_e^2c^3R_0(1-a_dw)\Pi_0^3 \\
&+24a_d^2a_ec^2c_e(\overline{b_e}-\underline{b_e})^2\Pi_0^2 + 6a_e^2c^2R_0^2(1-a_dw)\Pi_0^2 + 24a_d^2c^2c_e(\overline{b_e}-\underline{b_e})^2\overline{b_e}(1-a_dw)\Pi_0^2 - 24a_d^2c^2c_e(\overline{b_e}-\underline{b_e})^2q_e(1-a_dw)\Pi_0^2 \\
&+48a_d^2a_ecc_e(\overline{b_e}-\underline{b_e})^2R_0\Pi_0 + 4a_e^2cR_0^3(1-a_dw)\Pi_0 + 48a_d^2cc_e(\overline{b_e}-\underline{b_e})^2\overline{b_e}R_0(1-a_dw)\Pi_0 - 48a_d^2cc_e(\overline{b_e}-\underline{b_e})^2q_eR_0(1-a_dw)\Pi_0 \\
&+24a_d^2a_ec_e(\overline{b_e}-\underline{b_e})^2R_0^2 + 24a_d^2c_e(\overline{b_e}-\underline{b_e})^2\overline{b_e}R_0^2(1-a_dw) - 24a_d^2c_e(\overline{b_e}-\underline{b_e})^2q_eR_0^2(1-a_dw) + a_e^2R_0^4(1-a_dw) + a_e(c\Pi_0+R_0)^3 \\
&(1-a_dw)\sqrt{\tfrac{\mathbf{E}_8c_e^4(\overline{b_e}-\underline{b_e})^8}{a_e^6(c\Pi_0+R_0)^6}}\Big) - \tfrac{1}{288c_d^2(\overline{b_d}-\underline{b_d})^4(1-a_dw)}\big(c^4(1-a_dw)\Pi_0^4 + 4c^3R_0(1-a_dw)\Pi_0^3 + 24a_dc^2c_d(\overline{b_d}-\underline{b_d})^2\Pi_0^2 \\
&+6c^2R_0^2(1-a_dw)\Pi_0^2 + 24c^2c_d(\overline{b_d}-\underline{b_d})^2\overline{b_d}(1-a_dw)\Pi_0^2 - 24c^2c_d(\overline{b_d}-\underline{b_d})^2q_d(1-a_dw)\Pi_0^2 + 48a_dcc_d(\overline{b_d}-\underline{b_d})^2R_0\Pi_0 + 4cR_0^3 \\
&(1-a_dw)\Pi_0 + 48cc_d(\overline{b_d}-\underline{b_d})^2\overline{b_d}R_0(1-a_dw)\Pi_0 - 48cc_d(\overline{b_d}-\underline{b_d})^2q_dR_0(1-a_dw)\Pi_0 + 24a_dc_d(\overline{b_d}-\underline{b_d})^2R_0^2 + 24c_d(\overline{b_d}-\underline{b_d})^2\overline{b_d} \\
&R_0^2(1-a_dw) - 24c_d(\overline{b_d}-\underline{b_d})^2q_dR_0^2(1-a_dw) + R_0^4(1-a_dw) + (c\Pi_0+R_0)^3(1-a_dw)\sqrt{\tfrac{\mathbf{E}_7c_d^4(\overline{b_d}-\underline{b_d})^8}{(c\Pi_0+R_0)^6}}\Big) - \Big(\tfrac{a_d}{1-a_dw} + \overline{b_d} \\
&-q_d + \tfrac{\sqrt{\mathbf{E}_9}}{12\sqrt{2}}\Big)^{3/2}\tfrac{2(c\Pi_0+R_0)}{-3\sqrt{3}\sqrt{c_d}(\overline{b_d}-\underline{b_d})}\Big) - \frac{2a_e(c\Pi_0+R_0)\left(\frac{a_e}{1-a_dw}+\overline{b_e}-q_e+\frac{a_e\sqrt{(E)_{12}}}{12\sqrt{2}a_d^2c_e(\overline{b_e}-\underline{b_e})^2}\right)^{3/2}}{-3\sqrt{3}\sqrt{c_e}(\overline{b_e}-\underline{b_e})}\Big).
\end{aligned}
$$

Next, as the first-order derivatives are quite cumbersome, for brevity, we only present one of them. However, we numerically verify the existence of each threshold in all specifications. Specifically, the first-order derivatives of the equilibrium overall content quality with respect to q_e and R_0 in the specification of $\eta = 3, \zeta = 2$ can be written as the following:

$$
\begin{aligned}
\frac{d\Pi^*}{dq_e} =& \frac{2a_da_e(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))}, \\
\frac{d\Pi^*}{dR_0} =& \tfrac{1}{1-a_dw}\Big(-\tfrac{-4a_d^2a_ec_e(\overline{b_e}-\underline{b_e})(a_e+(\overline{b_e}-q_e)(1-a_dw))^2}{(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2(1-a_dw)^2} + \tfrac{-8a_d^2a_e^2c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(\overline{b_e}-q_e)(1-a_dw))^2}{(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3(1-a_dw)^2} \\
&+a_d\Big(\tfrac{2a_e^3(c\Pi_0+R_0)^2(a_e+(\overline{b_e}-q_e)(1-a_dw))^2}{(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3(1-a_dw)^2} - x - \tfrac{1}{288c_d^2(\overline{b_d}-\underline{b_d})^4(1-a_dw)}\big(48a_dcc_d(\overline{b_d}-\underline{b_d})^2\Pi_0 \\
&-\frac{4\left(-36c_d(\overline{b_d}-q_d)(\overline{b_d}-\underline{b_d})^2+36a_dc_d(\overline{b_d}w-q_dw-1)(\overline{b_d}-\underline{b_d})^2-(c\Pi_0+R_0)^2+a_d(c\Pi_0+R_0)^2w\right)(c\Pi_0+R_0)^5}{c_d^2(\overline{b_d}-\underline{b_d})^4\sqrt{\frac{(c\Pi_0+R_0)^6\left(48c_d(\overline{b_d}-q_d)(\overline{b_d}-\underline{b_d})^2-48a_dc_d(\overline{b_d}w-q_dw-1)(\overline{b_d}-\underline{b_d})^2+(c\Pi_0+R_0)^2-a_d(c\Pi_0+R_0)^2w\right)}{c_d^4(\overline{b_d}-\underline{b_d})^8(1-a_dw)}}} \\
&+48a_dc_d(\overline{b_d}-\underline{b_d})^2R_0 + 4c^3\Pi_0^3(1-a_dw) - 4R_0^3(1-a_dw) + 12c\Pi_0R_0^2(1-a_dw) + 48cc_d(\overline{b_d}-\underline{b_d})^2\overline{b_d}\Pi_0(1-a_dw) - 48cc_d \\
&(\overline{b_d}-\underline{b_d})^2\Pi_0q_d(1-a_dw) + 12c^2\Pi_0^2R_0(1-a_dw) + 48c_d(\overline{b_d}-\underline{b_d})^2\overline{b_d}R_0(1-a_dw) - 48c_d(\overline{b_d}-\underline{b_d})^2q_dR_0(1-a_dw)\big) \\
&+\tfrac{1}{3\sqrt{3}\sqrt{c_d}(\overline{b_d}-\underline{b_d})}\Big(2\Big(\tfrac{a_d}{1-a_dw} + \overline{b_d} - q_d + \tfrac{\sqrt{\mathbf{E}_9}}{12\sqrt{2}}\Big)^{3/2}\Big) - \tfrac{2a_e^2(c\Pi_0+R_0)(a_e+(\overline{b_e}-q_e)(1-a_dw))^2}{(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2(1-a_dw)^2} - (c\Pi_0+R_0) \\
&\tfrac{\sqrt{\frac{a_d}{1-a_dw}+\overline{b_d}-q_d+\frac{\sqrt{\mathbf{E}_9}}{12\sqrt{2}}}}{-24\sqrt{6}c_d^{5/2}(\overline{b_d}-\underline{b_d})^5(1-a_dw)\sqrt{\mathbf{E}_9}}\big(48a_dcc_d(\overline{b_d}-\underline{b_d})^2\Pi_0 + 48a_dc_d(\overline{b_d}-\underline{b_d})^2R_0 + 4c^3\Pi_0^3(1-a_dw) + 4R_0^3(1-a_dw) \\
&+12c\Pi_0R_0^2(1-a_dw) + 48cc_d(\overline{b_d}-\underline{b_d})^2\overline{b_d}\Pi_0(1-a_dw) - 48cc_d(\overline{b_d}-\underline{b_d})^2\Pi_0q_d(1-a_dw) + 12c^2\Pi_0^2R_0(1-a_dw)
\end{aligned}
$$

$$+48c_d(\overline{b_d}-\underline{b_d})^2\overline{b_d}R_0(1-a_dw)-48c_d(\overline{b_d}-\underline{b_d})^2q_dR_0(1-a_dw)-\big(4\,\big(-36c_d(\overline{b_d}-q_d)(\overline{b_d}-\underline{b_d})^2+36a_dc_d(\overline{b_d}w-q_dw-1)$$

$$(\overline{b_d}-\underline{b_d})^2-(c\Pi_0+R_0)^2+a_d(c\Pi_0+R_0)^2w\big)\,(c\Pi_0+R_0)^5\big)\,\Big(c_d^2(\overline{b_d}-\underline{b_d})^4\sqrt{\mathbf{E}_7}\Big)^{-1}\Big)\Big)\Big).$$

For every setting in this extension, given the corresponding feasibility constraints (see Expressions 27 to 31 and Table 8), we numerically verify the existence of the threshold of q_e and observe qualitatively the same patterns as the main model. Representative numerical examples are provided in Table 9.

With specification $\eta = 2$ and $\zeta = 3$	
The numerical evidence for $\frac{dC^*}{da_d}$	
Base specification: $a_d = 0.25, \underline{b_e} = 1, \overline{b_e} = 8, a_e = 1.75, R_0 = 20, q_d = 1, q_e = 4.75, w = 0.5,$ $w_0 = 100, \overline{b_d} = 2, \underline{b_d} = 0.38, r = 1, g = 1, x = 1, \Pi_0 = 1, c = 1, c_d = 16, c_e = 128$	
$R_0 < 3.76$	$R_0 > 3.76$
C^* monotonously increases in a_d	C^* monotonously decreases in a_d
The numerical evidence for $\frac{dh^*}{da_d}$	
Base specification: $\underline{b_e} = 1, a_e = 1.75, R_0 = 20, q_e = 20, w = 0.5, w_0 = 100, \underline{b_d} = 0.38, r = 1,$ $g = 1, x = 1, q_d = 8, \overline{b_d} = 16, \overline{b_e} = 20.75, \Pi_0 = 1, c = 1, c_d = 2, c_e = 2.59$	
$a_d < 1.42$	$a_d > 1.42$
h^* monotonously decreases in a_d	h^* monotonously increases in a_d
The numerical evidence for $\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}}$	
Base specification: $a_d = 0.25, \underline{b_e} = 1, \overline{b_e} = 8, a_e = 1.75, q_e = 5, w = 1, w_0 = 19.8, \underline{b_d} = 0.078,$ $r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 4, c_e = 2, \overline{b_d} = 8$	
$q_d < 2.655$	$q_d > 2.655$
$\frac{dD^*}{d\overline{b_d}} < \frac{dD^*}{d\underline{b_d}}$	$\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$
The numerical evidence for $\frac{d\Pi^*}{dR_0}$	

Base specification: $\underline{b_e} = 1, a_e = 1.75, R_0 = 20, q_e = 20, w = 0.5, w_0 = 100, \underline{b_d} = 0.375, r = 1,$ $g = 1, q_d = 8, a_d = 1.422, \overline{b_d} = 16, \overline{b_e} = 20.75, \Pi_0 = 1, c = 1, c_d = 2, c_e = 2.588, x = 2.241$	
$q_e < 20$	$q_e > 20$
Π^* monotonously increases in R_0	Π^* monotonously decreases in R_0
With specification $\eta = 3$ **and** $\zeta = 2$	
The numerical evidence for $\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}}$	
Base specification: $a_d = 0.25, a_e = 0.25, \underline{b_d} = 1, w = 2, w_0 = 1000, q_e = 1, \overline{b_e} = 2, \underline{b_e} = 0.25,$ $r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 1.75, c = 1, c_d = 16, c_e = 4, \overline{b_d} = 2.109$	
$q_d < 2.073$	$q_d > 2.073$
$\frac{dD^*}{d\overline{b_d}} < \frac{dD^*}{d\underline{b_d}}$	$\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$
The numerical evidence for $\frac{d\Pi^*}{dR_0}$	
Base specification: $a_d = 0.25, a_e = 0.25, \underline{b_d} = 1, \overline{b_d} = 20, \overline{b_e} = 15, \underline{b_e} = 0.25, w = 2,$ $w_0 = 1000, q_d = 6, r = 1, g = 1, \Pi_0 = 1, R_0 = 32, c = 0.625, c_d = 1, c_e = 64, x = 1.416$	
$q_e < 1$	$q_e > 1$
Π^* monotonously increases in R_0	Π^* monotonously decreases in R_0
With specification $\eta = 3$ **and** $\zeta = 3$	
The numerical evidence for $\frac{dC^*}{da_d}$	
Base specification: $a_d = 0.25, \underline{b_e} = 1, \overline{b_e} = 8, a_e = 1.75, q_d = 1, q_e = 4.75, w = 0.5,$ $w_0 = 100, \overline{b_d} = 2, \underline{b_d} = 0.38, r = 1, g = 1, x = 1, \Pi_0 = 1, c = 1, c_d = 16, c_e = 128$	
$R_0 < 3.76$	$R_0 > 3.76$
C^* monotonously increases in a_d	C^* monotonously decreases in a_d
The numerical evidence for $\frac{dh^*}{da_d}$	
Base specification: $\underline{b_e} = 1, a_e = 1.75, R_0 = 20, q_e = 20, w = 0.5, w_0 = 100, \underline{b_d} = 0.38, r = 1, g = 1,$ $x = 1, q_d = 8, \overline{b_d} = 16, \overline{b_e} = 20.75, \Pi_0 = 1, c = 1, c_d = 2, c_e = 2.59$	
$a_d < 1.42$	$a_d > 1.42$

h^* monotonously decreases in a_d	h^* monotonously increases in a_d
The numerical evidence for $\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}}$	
Base specification: $a_d = 0.25, \underline{b_e} = 1, \overline{b_e} = 8, a_e = 1.75, q_e = 5, w = 1, w_0 = 19.8, \underline{b_d} = 0.078,$ $r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 4, c_e = 2, \overline{b_d} = 8$	
$q_d < 2.655$	$q_d > 2.655$
$\frac{dD^*}{d\overline{b_d}} < \frac{dD^*}{d\underline{b_d}}$	$\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$
The numerical evidence for $\frac{d\Pi^*}{dR_0}$	
Base specification: $a_d = 0.25, \underline{b_e} = 1, \overline{b_e} = 8, a_e = 1.75, q_d = 0.969, w = 1, w_0 = 19.8,$ $\overline{b_d} = 1.125, \underline{b_d} = 0.078, r = 1, g = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 4, c_e = 2, x = 5.135$	
$q_e < 5$	$q_e > 5$
Π^* monotonously increases in R_0	Π^* monotonously decreases in R_0

The selections of specification and monotonicity test are based on the feasibility constraints.

Table 9. *Existence Proofs for Different Settings of Disutility Elasticity of Users*

■

Proofs of The Extension: Robustness to The Cost Elasticity of The Platform's Efforts

In this section, we provide proofs of results presented in CHAPTER 2.7.3. In this extension, the equilibrium results are derived similarly to the main model, and hence, the details are omitted for brevity. However, the important variables of interest at the equilibrium are as follows:

$$b_d^* = \begin{cases} -\frac{\sqrt[3]{\sqrt{\mathbf{E}_{13}}+\mathbf{E}_{15}}}{6\sqrt[3]{2}} - \frac{a_d}{1-a_d w} + q_d, & \text{if } \alpha = 3, \\ \frac{(c\Pi_0+R_0)(a_d(-\overline{b_d})w+a_d q_d w+a_d+\overline{b_d}-q_d)}{(1-a_d w)(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)} - \frac{a_d}{1-a_d w} + q_d, & \text{if } \alpha = 2, \end{cases}$$

$$b_e^* = \begin{cases} -\frac{\sqrt[3]{\frac{\sqrt{\mathbf{E}_{14}}-\mathbf{E}_{16}}{a_d^3}}}{6\sqrt[3]{2}} - \frac{a_e}{1-a_d w} + q_e, & \text{if } \beta = 3, \\ \frac{\frac{a_e(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))}{-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0)}+q_e(1-a_d w)-a_e}{1-a_d w}, & \text{if } \beta = 2, \end{cases}$$

$$
f^* = \begin{cases} \frac{1}{432}\left(\sqrt{\mathbf{E}_{13}} + \mathbf{E}_{15}\right), & \text{if } \alpha = 3, \\ \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}, & \text{if } \alpha = 2, \end{cases}
$$

$$
h^* = \begin{cases} \frac{\sqrt{\mathbf{E}_{14}}-\mathbf{E}_{16}}{432a_d^3}, & \text{if } \beta = 3, \\ \frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}, & \text{if } \beta = 2, \end{cases}
$$

$$
v^* = \begin{cases} \text{if } \alpha = 2 \text{ and } \beta = 3: \\ -\frac{\sqrt{\mathbf{E}_{14}}-\mathbf{E}_{16}}{432a_d^3} - \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} + \frac{4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} \\ -x(c\Pi_0+R_0) - g - r + w_0, \\ \text{if } \alpha = 3 \text{ and } \beta = 2: \\ -\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2} + \frac{(c\Pi_0+R_0)\left(\frac{\sqrt[3]{\sqrt{\mathbf{E}_{13}}+\mathbf{E}_{15}}}{6\sqrt[3]{2}} - a_dw\left(\frac{\sqrt[3]{\sqrt{\mathbf{E}_{13}}+\mathbf{E}_{15}}}{6\sqrt[3]{2}}+\overline{b_d}-q_d\right)+a_d+\overline{b_d}-q_d\right)^2}{4c_d(1-a_dw)^2(\overline{b_d}-\underline{b_d})} \\ +\frac{1}{432}\left(-\sqrt{\mathbf{E}_{13}}-\mathbf{E}_{15}\right) - x(c\Pi_0+R_0) - g - r + w_0, \\ \text{if } \alpha = 3 \text{ and } \beta = 3: \\ \frac{(c\Pi_0+R_0)\left(\frac{\sqrt[3]{\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}}}{6\sqrt[3]{2}} - a_dw\left(\frac{\sqrt[3]{\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}}}{6\sqrt[3]{2}}+\overline{b_d}-q_d\right)+a_d+\overline{b_d}-q_d\right)^2}{4c_d(\overline{b_d}-\underline{b_d})(1-a_dw)^2} - g - r + \frac{1}{432}\left(-\mathbf{E}_{15}-\sqrt{\mathbf{E}_{13}}\right) + w_0 \\ -(c\Pi_0+R_0)x - \frac{\sqrt{\mathbf{E}_{14}}-\mathbf{E}_{16}}{432a_d^3}, \end{cases}
$$

where

$$
\begin{aligned}
\mathbf{E}_{13} = & \frac{(c\Pi_0+R_0)^3}{-c_d^6(1-a_dw)^3(\overline{b_d}-\underline{b_d})^6}(a_dw(c\Pi_0+R_0) + 24a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1) - c\Pi_0 - 24c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d) \\
& -R_0)(-a_dw(c\Pi_0+R_0) - 6a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1) + c\Pi_0 + 6c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d) + R_0)^2, \\
\mathbf{E}_{14} = & \frac{a_e^3(c\Pi_0+R_0)^3}{-c_e^6(1-a_dw)^3(\overline{b_e}-\underline{b_e})^6}\left(+6a_d^2c_ew(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e) + a_da_e(w(c\Pi_0+R_0) - 6c_e(\overline{b_e}-\underline{b_e})) - 6a_dc_e(\overline{b_e}-\underline{b_e})\right. \\
& \left.(\overline{b_e}-q_e) - a_e(c\Pi_0+R_0)\right)^2\left(24a_d^2c_ew(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e) + a_d(a_ew(c\Pi_0+R_0) - 24a_ec_e(\overline{b_e}-\underline{b_e})\right. \\
& \left.-24c_e(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e)) - a_e(c\Pi_0+R_0)\right), \\
\mathbf{E}_{15} = & \frac{(c\Pi_0+R_0)^2(-a_dw(c\Pi_0+R_0) - 18a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1) + c\Pi_0 + 18c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d) + R_0)}{c_d^3(1-a_dw)(\overline{b_d}-\underline{b_d})^3}, \\
\mathbf{E}_{16} = & \frac{a_e^2(c\Pi_0+R_0)^2}{c_e^3(1-a_dw)(\overline{b_e}-\underline{b_e})^3}\left(18a_d^2c_ew(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e) + a_d(a_ew(c\Pi_0+R_0) - 18a_ec_e(\overline{b_e}-\underline{b_e}) - 18c_e(\overline{b_e}-\underline{b_e})\right. \\
& \left.(\overline{b_e}-q_e)) - a_e(c\Pi_0+R_0)\right).
\end{aligned}
$$

The equilibrium solution in this extension is feasible when the following set of conditions is satisfied:

$$S_1 \text{ and } S_2 \text{ and } F_0 \ \ (see\, Table\, 10) \tag{32}$$

$$a_d > 0 \text{ and } a_e > 0 \text{ and } \overline{b_e} > q_e > \underline{b_e} > 0 \text{ and } \overline{b_d} > q_d > \underline{b_d} > 0 \text{ and } w > 0 \text{ and } \Pi_0 > 0 \tag{33}$$

$$R_0 > 0 \text{ and } c > 0 \text{ and } r \geq 0 \text{ and } x \geq 0 \text{ and } g \geq 0 \text{ and } c_d > 0 \text{ and } c_e > 0 \tag{34}$$

$$\overline{b_d} > -\frac{a_e}{1-a_d w} + q_e > b_d^* > \underline{b_d} \text{ and } \overline{b_d} > -\frac{a_d}{1-a_d w} + q_d > b_e^* > \underline{b_e} \tag{35}$$

$$v^* \geq 0 \text{ and } 1 - a_d w > 0 \text{ and } h^* > q_e^{\beta}, \tag{36}$$

where S_1 and S_2 are second-order conditions and F_0 is implied by the first order conditions. The expressions for S_1, S_2, and F_0 are presented in Table 10. We numerically verify that the parameter space given the feasibility constraint set is not empty, and all of our results are feasible. For example, when $(\alpha, \beta) = (2, 3)$, the set of feasibility constraints is satisfied at $\{a_d = 0.25, \underline{b_e} = 1, \overline{b_e} = 8, a_e = 1.75, q_d = 1, q_e = 3.63, w = 0.63, w_0 = 19.8, \overline{b_d} = 2, \underline{b_d} = 0.38, r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 4, c_e = 16\}$. When $(\alpha, \beta) = (3, 2)$, we have the set of feasibility constraints satisfied at $\{a_d = 0.25, a_e = 0.25, \underline{b_d} = 1, \overline{b_d} = 2, w = 2, w_0 = 1000, q_d = 1.75, q_e = 1, \overline{b_e} = 2, \underline{b_e} = 0.25, r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 1.75, c = 1, c_d = 8, c_e = 4\}$. Further, when $(\alpha, \beta) = (3, 3)$, at $\{a_d = 0.25, \underline{b_e} = 1, \overline{b_e} = 8, a_e = 1.75, q_d = 0.97, q_e = 5, w = 1, w_0 = 1000, \overline{b_d} = 1, \underline{b_d} = 0.5, r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 32, c_e = 2\}$, the set of feasibility constraints is satisfied.

Proof Propositions 1 and 2

In this extension, we first consider the specification of $(\alpha = 3, \beta = 2)$. The equilibrium total content contribution effort and community support effort are:

$$C^* = \frac{4a_d^2 c_e(\overline{b_e} - \underline{b_e})(c\Pi_0 + R_0)(a_e + (1 - a_d w)(\overline{b_e} - q_e))^2}{(1 - a_d w)^2(-4a_d c_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))^2};$$
$$h^* = \frac{a_e^2(c\Pi_0 + R_0)^2(a_e + (1 - a_d w)(\overline{b_e} - q_e))^2}{(1 - a_d w)^2(-4a_d c_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))^2}.$$

The first-order derivatives of these measures with respect to a_d are

	$\alpha = 2,\ \beta = 3$
F_0	$c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0 < 0$ and $h^* > 0$
S_1	$-\frac{a_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{-8c_d(1-a_dw)^2(\overline{b_d}-\underline{b_d})\left(\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\right)^{3/2}} < 0$
S_2	$\frac{a_da_e(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)\left(-(1-a_dw)\left(2\overline{b_e}+\sqrt[3]{h^*}-2q_e\right)-2a_e\right)}{144c_dc_e(f^*)^{3/2}(h^*)^{5/3}(1-a_dw)^4(\overline{b_d}-\underline{b_d})(\overline{b_e}-\underline{b_e})} > 0$
	$\alpha = 3,\ \beta = 2$
F_0	$-4a_dc_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0) < 0$ and $f^* > 0$
S_1	$\frac{a_e(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))}{-8c_e(1-a_dw)^2(\overline{b_e}-\underline{b_e})\left(\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}\right)^{3/2}} < 0$
S_2	$-\frac{a_da_e(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))\left(a_d\left(2\overline{b_d}w+\sqrt[3]{f^*}w-2q_dw-2\right)-2\overline{b_d}-\sqrt[3]{f^*}+2q_d\right)}{144c_dc_e(f^*)^{5/3}(h^*)^{3/2}(1-a_dw)^4(\overline{b_d}-\underline{b_d})(\overline{b_e}-\underline{b_e})} > 0$
	$\alpha = 3,\ \beta = 3$
F_0	$f^* > 0$ and $h^* > 0$
S_1	$-\frac{a_d(c\Pi_0+R_0)\left(a_d\left(2\overline{b_d}w+\sqrt[3]{f^*}w-2q_dw-2\right)-2\overline{b_d}-\sqrt[3]{f^*}+2q_d\right)}{-18c_d(f^*)^{5/3}(1-a_dw)^2(\overline{b_d}-\underline{b_d})} < 0$
S_2	$\frac{a_da_e(c\Pi_0+R_0)^2\left(a_d\left(2\overline{b_d}w+\sqrt[3]{f^*}w-2q_dw-2\right)-2\overline{b_d}-\sqrt[3]{f^*}+2q_d\right)\left(-(1-a_dw)\left(2\overline{b_e}+\sqrt[3]{h^*}-2q_e\right)-2a_e\right)}{324c_dc_e(f^*)^{5/3}(h^*)^{5/3}(1-a_dw)^4(\overline{b_d}-\underline{b_d})(\overline{b_e}-\underline{b_e})} > 0$

Table 10. *Constraints in Expression 32 under Different Specification of The Platform's Cost Elasticity* (α, β)

$$\frac{dC^*}{da_d} = -\frac{-8a_da_ec_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(-(1-a_dw)(\overline{b_e}-q_e)-a_e)}{-(1-a_dw)^3(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3}\big(4a_d^2c_e\underline{b_e}w - 4a_d^2c_e\overline{b_e}w + a_d^2\overline{b_e}R_0w^2 - a_d^2q_eR_0w^2$$
$$+c\Pi_0(1-a_dw)^2(\overline{b_e}-q_e) - 2a_d\overline{b_e}R_0w + 2a_dq_eR_0w + a_e(c\Pi_0+R_0) + \overline{b_e}R_0 - q_eR_0\big)\,;$$
$$\frac{dh^*}{da_d} = -\frac{2a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))}{-(1-a_dw)^3(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3}\big(-4a_ec_e(2a_dw-1)(\overline{b_e}-\underline{b_e}) + 4c_e(1-a_dw)^2(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e)$$
$$+a_e^2w(c\Pi_0+R_0)\big)\,.$$

Given the feasibility constraints (Expressions 32 to 36 when $\alpha = 3$ and $\beta = 2$), we find that, first, as a_d increases, C^* decreases if and only if

$$R_0 > \frac{-a_d^2c\overline{b_e}\Pi_0w^2+a_d^2c\Pi_0q_ew^2-4a_d^2c_e\underline{b_e}w+4a_d^2c_e\overline{b_e}w+2a_dc\overline{b_e}\Pi_0w-2a_dc\Pi_0q_ew-a_ec\Pi_0-c\overline{b_e}\Pi_0+c\Pi_0q_e}{a_d^2\overline{b_e}w^2-a_d^2q_ew^2-2a_d\overline{b_e}w+2a_dq_ew+a_e+\overline{b_e}-q_e},$$

and second, h^* increases in a_d if and only if the following condition is satisfied:

$$a_d > \frac{-\sqrt{\mathbf{E}_{17}} - 8a_ec_e\underline{b_e}w + 8a_ec_e\overline{b_e}w - 8c_e\underline{b_e}\overline{b_e}w + 8c_e\underline{b_e}q_ew + 8c_e\overline{b_e}^2w - 8c_e\overline{b_e}q_ew}{2\left(-4c_e\underline{b_e}\overline{b_e}w^2 + 4c_e\underline{b_e}q_ew^2 + 4c_e\overline{b_e}^2w^2 - 4c_e\overline{b_e}q_ew^2\right)},$$

where

$$\mathbf{E}_{17} = \Big(8a_e c_e \underline{b_e} w - 8a_e c_e \overline{b_e} w + 8c_e \underline{b_e}\,\overline{b_e} w - 8c_e \underline{b_e} q_e w - 8c_e \overline{b_e}^2 w + 8c_e \overline{b_e} q_e w\Big)^2 - 4\left(-4c_e \underline{b_e}\,\overline{b_e} w^2 + 4c_e \underline{b_e} q_e w^2 \right.$$
$$\left. + 4c_e \overline{b_e}^2 w^2 - 4c_e \overline{b_e} q_e w^2\right)\left(a_e^2 c\Pi_0 w + a_e^2 R_0 w - 4a_e c_e \underline{b_e} + 4a_e c_e \overline{b_e} - 4c_e \underline{b_e}\,\overline{b_e} + 4c_e \underline{b_e} q_e + 4c_e \overline{b_e}^2 - 4c_e \overline{b_e} q_e\right).$$

Furthermore, note that the equilibrium total content contribution and its first-order derivative with respect to a_d for the other two specifications (i.e., $(\alpha = 2, \beta = 3)$ and $(\alpha = 3, \beta = 3)$) follow the same functional forms. These equilibrium measures are

$$C^* = \frac{(c\Pi_0 + R_0)\left(a_e + (1 - a_d w)\left(\frac{\sqrt[3]{\frac{\sqrt{\mathbf{E}_{14}} - \mathbf{E}_{16}}{a_d^3}}}{6\sqrt[3]{2}} + \overline{b_e} - q_e\right)\right)^2}{4c_e(1 - a_d w)^2(\overline{b_e} - \underline{b_e})};$$

$$h^* = \frac{\sqrt{\mathbf{E}_{14}} - \mathbf{E}_{16}}{432 a_d^3}.$$

The first-order derivatives of these expressions with respect to a_d can be derived from the equilibrium measures presented above. However, we do not present them in our paper for brevity as they are rather cumbersome. Because of this complication, we carry out an extensive numerical analysis and obtain results that are qualitatively the same as those presented in Propositions 1 and 2. Representative numerical examples are provided in Table 11.

Proof Proposition 3

As the first-order derivatives are quite cumbersome for settings where $\alpha = 3$, we only present the results of $\alpha = 2$. However, we numerically verify the existence of each of the thresholds in all specifications. When $\alpha = 2$, we have the following results

$$D^* = \frac{4c_d(\overline{b_d} - \underline{b_d})(c\Pi_0 + R_0)(a_d(\overline{b_d} w - q_d w - 1) - \overline{b_d} + q_d)^2}{(1 - a_d w)^2(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^2};$$

$$\frac{dD^*}{d\underline{b_d}} = -\frac{4c_d(c\Pi_0 + R_0)(a_d(\overline{b_d} w - q_d w - 1) - \overline{b_d} + q_d)^2(c\Pi_0 - 4c_d\underline{b_d} + 4c_d\overline{b_d} + R_0)}{(1 - a_d w)^2(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^3};$$

$$\frac{dD^*}{d\overline{b_d}} = -\frac{4c_d(c\Pi_0 + R_0)(a_d(\overline{b_d} w - q_d w - 1) - \overline{b_d} + q_d)}{(1 - a_d w)^2(c\Pi_0 - 4c_d(\overline{b_d} - \underline{b_d}) + R_0)^3}(a_d(c\Pi_0 + R_0)(2\underline{b_d} w - 3\overline{b_d} w + q_d w + 1) - 4a_d c_d(\overline{b_d} - \underline{b_d})$$
$$(2\underline{b_d} w - \overline{b_d} w - q_d w - 1) - (c\Pi_0 + R_0)(2\underline{b_d} - 3\overline{b_d} + q_d) + 4c_d(\overline{b_d} - \underline{b_d})(2\underline{b_d} - \overline{b_d} - q_d)).$$

We compute the differences between the first-order derivatives of the total donation from users with respect to upper bounds and lower bounds of user generousness parameters as follows:

$$\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}} = -\frac{8c_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}(a_d(c\Pi_0+R_0)(\underline{b_d}w-2\overline{b_d}w+q_dw+1)$$
$$-4a_dc_d(\overline{b_d}-\underline{b_d})(\underline{b_d}w-q_dw-1)-(c\Pi_0+R_0)(\underline{b_d}-2\overline{b_d}+q_d)+4c_d(\overline{b_d}-\underline{b_d})(\underline{b_d}-q_d)).$$

Given the feasibility constraints, we have the total donation from users monotonously increases in the lower bound and upper bound of user generousness (i.e., $\frac{dD^*}{d\underline{b_d}} > 0$ and $\frac{dD^*}{d\overline{b_d}} > 0$), and (ii) the platform's overall content quality can be more efficiently increased by enhancing the upper bound of user generousness (i.e., $\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$) if and only if the barrier parameter q_d is at a high level, i.e.,

$$q_d > \frac{a_d(c\Pi_0+R_0)(\underline{b_d}w-2\overline{b_d}w+1)-4a_dc_d(\overline{b_d}-\underline{b_d})(\underline{b_d}w-1)-(\underline{b_d}-2\overline{b_d})(c\Pi_0+R_0)+4c_d\underline{b_d}(\overline{b_d}-\underline{b_d})}{(1-a_dw)(c\Pi_0-4c_d\underline{b_d}+4c_d\overline{b_d}+R_0)}.$$

For the settings $(\alpha=3,\beta=2)$ and $(\alpha=3,\beta=3)$, we numerically verify the existence of the threshold of q_d and observe qualitatively the same patterns. Representative numerical examples are provided in Table 11.

Proof Proposition 4

In this extension, we consider three specifications of α and β. For the specification of $\alpha=2$ and $\beta=3$, the objective function of the platform is given as follows.

$$\Pi^* = \frac{1}{1-a_dw}\left(-\frac{a_e(c\Pi_0+R_0)\left(a_e+(1-a_dw)\left(\frac{\sqrt[3]{\sqrt{\mathbf{E}_{14}}-\mathbf{E}_{16}}}{6\sqrt[3]{2}a_d}+\overline{b_e}-q_e\right)\right)^2}{-4c_e(1-a_dw)^2(\overline{b_e}-\underline{b_e})} + a_d\left(-\frac{\sqrt{\mathbf{E}_{14}}-\mathbf{E}_{16}}{432a_d^3} - x(c\Pi_0+R_0) - g - r\right.\right.$$
$$\left.\left.-\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} - \frac{-4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2} + w_0\right) + \Pi_0\right).$$

For the specification of $\alpha=3$ and $\beta=2$:

$$\Pi^* = \frac{1}{1-a_dw}\left(-\frac{-4a_d^2a_ec_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e-(\overline{b_e}-q_e)(1-a_dw))^2}{(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2(1-a_dw)^2} + \Pi_0 + a_d\left(-\frac{a_e^2(c\Pi_0+R_0)^2(a_e-(\overline{b_e}-q_e)(1-a_dw))^2}{(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2(1-a_dw)^2}\right.\right.$$

$$
-g-r+\tfrac{1}{432}\left(-\mathbf{E}_{15}-\sqrt{\mathbf{E}_{13}}\right)-\frac{(c\Pi_0+R_0)\left(\frac{\sqrt[3]{\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}}}{6\sqrt[3]{2}}+a_d+\overline{b_d}-q_d-a_dw\left(\frac{\sqrt[3]{\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}}}{6\sqrt[3]{2}}+\overline{b_d}-q_d\right)\right)^2}{-4c_d(\overline{b_d}-\underline{b_d})(1-a_dw)^2}+w_0-(c\Pi_0+R_0)x\Bigg)\Bigg).
$$

For the specification of $\alpha=3$ and $\beta=3$:

$$
\Pi^* =\frac{1}{1-a_dw}\Bigg(-\frac{a_e(c\Pi_0+R_0)\left(a_e+(1-a_dw)\left(\frac{\sqrt[3]{\frac{\sqrt{\mathbf{E}_{14}}-\mathbf{E}_{16}}{a_d^3}}}{6\sqrt[3]{2}}+\overline{b_e}-q_e\right)\right)^2}{-4c_e(\overline{b_e}-\underline{b_e})(1-a_dw)^2}+\Pi_0-g-r+\tfrac{1}{432}\left(-\mathbf{E}_{15}-\sqrt{\mathbf{E}_{13}}\right)+w_0
$$

$$
+a_d\Bigg(-\frac{(c\Pi_0+R_0)\left(\frac{\sqrt[3]{\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}}}{6\sqrt[3]{2}}-a_dw\left(\frac{\sqrt[3]{\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}}}{6\sqrt[3]{2}}+\overline{b_d}-q_d\right)+a_d+\overline{b_d}-q_d\right)^2}{-4c_d(\overline{b_d}-\underline{b_d})(1-a_dw)^2}-(c\Pi_0+R_0)x-\frac{\sqrt{\mathbf{E}_{14}}-\mathbf{E}_{16}}{432a_d^3}\Bigg)\Bigg).
$$

Next, as the first-order derivatives are quite cumbersome, for brevity, we only present one of them. However, we numerically verify the existence of each of the thresholds in all specifications. Specifically, the first-order derivatives of the equilibrium overall content quality with respect to q_e and R_0 in the specification of $\alpha=3,\beta=2$ can be written as the following:

$$
\frac{d\Pi^*}{dq_e}=\frac{2a_da_e(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))};
$$

$$
\frac{d\Pi^*}{dR_0}=\frac{1}{1-a_dw}\Bigg(-\frac{-4a_d^2a_ec_e(\overline{b_e}-\underline{b_e})(a_e+(\overline{b_e}-q_e)(1-a_dw))^2}{(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2(1-a_dw)^2}+\frac{-8a_d^2a_e^2c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(\overline{b_e}-q_e)(1-a_dw))^2}{(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3(1-a_dw)^2}
$$

$$
+a_d\Bigg(\frac{2(c\Pi_0+R_0)^2(a_e+(\overline{b_e}-q_e)(1-a_dw))^2a_e^3}{(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3(1-a_dw)^2}-\frac{2(c\Pi_0+R_0)(a_e+(\overline{b_e}-q_e)(1-a_dw))^2a_e^2}{(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2(1-a_dw)^2}-\frac{1}{-4c_d(\overline{b_d}-\underline{b_d})(1-a_dw)^2}
$$

$$
\left(\frac{\sqrt[3]{\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}}}{6\sqrt[3]{2}}-a_dw\left(\frac{\sqrt[3]{\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}}}{6\sqrt[3]{2}}+\overline{b_d}-q_d\right)+a_d+\overline{b_d}-q_d\right)^2-x+\frac{(c\Pi_0+R_0)}{864c_d^6(\overline{b_d}-\underline{b_d})^6}\Big(-2c_d^3(c\Pi_0+R_0)(\overline{b_d}-\underline{b_d})^3
$$

$$
+\frac{-4c_d^3(c\Pi_0+18c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)+R_0-a_d(c\Pi_0+R_0)w-18a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))(\overline{b_d}-\underline{b_d})^3}{1-a_dw}+(c\Pi_0+R_0)
$$

$$
\frac{(c\Pi_0+6c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)+R_0-a_d(c\Pi_0+R_0)w-6a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))}{(1-a_dw)^3\sqrt{\mathbf{E}_{13}}}(-(c\Pi_0+R_0)(1-a_dw)(c\Pi_0+6c_d(\overline{b_d}-\underline{b_d})
$$

$$
(\overline{b_d}-q_d)+R_0-a_d(c\Pi_0+R_0)w-6a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))+3(-c\Pi_0-24c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)-R_0+a_d(c\Pi_0+R_0)w
$$

$$
+24a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))(c\Pi_0+6c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)+R_0-a_d(c\Pi_0+R_0)w-6a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))
$$

$$
+2(c\Pi_0+R_0)(1-a_dw)(-c\Pi_0-24c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)-R_0+a_d(c\Pi_0+R_0)w+24a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))))\Big)
$$

$$
-\frac{(c\Pi_0+R_0)^2(1-a_dw)}{-72\sqrt[3]{2}c_d^7(\overline{b_d}-\underline{b_d})^7(1-a_dw)^2\left(\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}\right)^{2/3}}\left(\frac{\sqrt[3]{\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}}}{6\sqrt[3]{2}}-a_dw\left(\frac{\sqrt[3]{\mathbf{E}_{15}+\sqrt{\mathbf{E}_{13}}}}{6\sqrt[3]{2}}+\overline{b_d}-q_d\right)+a_d+\overline{b_d}-q_d\right)\Big(2c_d^3
$$

$$
(c\Pi_0+R_0)(\overline{b_d}-\underline{b_d})^3+\frac{4c_d^3(c\Pi_0+18c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)+R_0-a_d(c\Pi_0+R_0)w-18a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))(\overline{b_d}-\underline{b_d})^3}{1-a_dw}
$$

$$
-\frac{(c\Pi_0+R_0)(c\Pi_0+6c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)+R_0-a_d(c\Pi_0+R_0)w-6a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))}{(1-a_dw)^3\sqrt{\mathbf{E}_{13}}}(-(c\Pi_0+R_0)(1-a_dw)
$$

$$
(c\Pi_0+6c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)+R_0-a_d(c\Pi_0+R_0)w-6a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))+3(-c\Pi_0-24c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)-R_0
$$

$$+a_d(c\Pi_0+R_0)w+24a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))(c\Pi_0+6c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)+R_0-a_d(c\Pi_0+R_0)w-6a_dc_d(\overline{b_d}-\underline{b_d})$$

$$(\overline{b_d}w-q_dw-1))+2(c\Pi_0+R_0)(1-a_dw)(-c\Pi_0-24c_d(\overline{b_d}-\underline{b_d})(\overline{b_d}-q_d)-R_0+a_d(c\Pi_0+R_0)w$$

$$+24a_dc_d(\overline{b_d}-\underline{b_d})(\overline{b_d}w-q_dw-1))))\Big)\Big).$$

For every setting in this extension, given the corresponding feasibility constraints (see Expressions 32 to 36 and Table 10), we numerically verify the existence of the threshold of q_e and observe qualitatively the same patterns as the main model. Representative numerical examples are provided in Table 11.

With specification $\alpha = 2$ **and** $\beta = 3$	
The numerical evidence for $\frac{dC^*}{da_d}$	
Base specification: $a_d = 0.25, \underline{b_e} = 0.33, \overline{b_e} = 8, a_e = 1.75, q_d = 1, q_e = 3.63, w = 0.63, w_0 = 19.8,$ $\overline{b_d} = 2, \underline{b_d} = 0.13, r = 1, g = 1, x = 1, \Pi_0 = 1, c = 1, c_d = 4, c_e = 16$	
$R_0 < 7.60$	$R_0 > 7.60$
C^* monotonously increases in a_d	C^* monotonously decreases in a_d
The numerical evidence for $\frac{dh^*}{da_d}$	
Base specification: $\underline{b_e} = 0.33, \overline{b_e} = 8, a_e = 1.75, q_d = 3.5, q_e = 6.5, w = 0.63, w_0 = 19.8, \overline{b_d} = 4,$ $\underline{b_d} = 0.13, r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 4, c_e = 16$	
$a_d < 0.94$	$a_d > 0.94$
h^* monotonously decreases in a_d	h^* monotonously increases in a_d
The numerical evidence for $\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}}$	
Base specification: $\underline{b_e} = 0.333, \overline{b_e} = 8, a_e = 1.75, q_e = 6.5, w = 0.625, w_0 = 19.8,$ $\overline{b_d} = 4, \underline{b_d} = 0.125, r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 4, c_e = 16, a_d = 0.94$	
$q_d < 2.761$	$q_d > 2.761$
$\frac{dD^*}{d\overline{b_d}} < \frac{dD^*}{d\underline{b_d}}$	$\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$
The numerical evidence for $\frac{d\Pi^*}{dR_0}$	

Base specification: $\underline{b_e} = 0.333, \overline{b_e} = 8, a_e = 1.75, q_d = 3.5, w = 0.625, w_0 = 19.8,$

$\overline{b_d} = 4, \underline{b_d} = 0.125, r = 1, g = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 4, c_e = 16, a_d = 0.94, x = 0.272$

$q_e < 6.5$	$q_e > 6.5$
Π^* monotonously increases in R_0	Π^* monotonously decreases in R_0

With specification $\alpha = 3$ and $\beta = 2$

The numerical evidence for $\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}}$

Base specification: $\underline{b_e} = 0.333, \overline{b_e} = 8, a_e = 1.75, q_e = 6.5, w = 0.625, w_0 = 19.8,$

$\overline{b_d} = 4, \underline{b_d} = 0.125, r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 4, c_e = 16, a_d = 0.94$

$q_d < 2.922$	$q_d > 2.922$
$\frac{dD^*}{d\overline{b_d}} < \frac{dD^*}{d\underline{b_d}}$	$\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$

The numerical evidence for $\frac{d\Pi^*}{dR_0}$

Base specification: $a_d = \frac{1}{4}, a_e = 0.25, \underline{b_d} = 1, \overline{b_d} = 2, \overline{b_e} = 2, \underline{b_e} = 0.25, w = 2.039, w_0 = 1000,$

$q_d = 1.520, r = 1, g = 1, \Pi_0 = 1, R_0 = 2, c = 34, c_d = 65536, c_e = 258.460, x = 0.0002$

$q_e < 1.990$	$q_e > 1.990$
Π^* monotonously increases in R_0	Π^* monotonously decreases in R_0

With specification $\alpha = 3$ and $\beta = 3$

The numerical evidence for $\frac{dC^*}{da_d}$

Base specification: $a_d = 0.25, \underline{b_e} = 1, \overline{b_e} = 8, a_e = 1.75, q_d = 0.97, q_e = 5, w = 1, w_0 = 1000, \overline{b_d} = 1,$

$\underline{b_d} = 0.13, r = 1, g = 1, x = 1, \Pi_0 = 1, c = 1, c_d = 32, c_e = 2$

$R_0 < 3.22$	$R_0 > 3.22$
C^* monotonously increases in a_d	C^* monotonously decreases in a_d

The numerical evidence for $\frac{dh^*}{da_d}$

Base specification: $\underline{b_e} = 0.13, \overline{b_e} = 8, a_e = 1.75, q_d = 2, q_e = 6, w = 1, w_0 = 1000, \overline{b_d} = 1,$

$\underline{b_d} = 0.13, r = 1, g = 1, x = 1, \Pi_0 = 1, c = 1, c_d = 32, c_e = 2, R_0 = 3$

$a_d < 0.62$	$a_d > 0.62$

h^* monotonously decreases in a_d	h^* monotonously increases in a_d
The numerical evidence for $\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}}$	
Base specification: $\underline{b_e} = 0.333, \overline{b_e} = 8, a_e = 1.75, q_e = 6.5, w = 0.625, w_0 = 19.8,$ $\overline{b_d} = 4, \underline{b_d} = 0.125, r = 1, g = 1, x = 1, \Pi_0 = 1, R_0 = 2, c = 1, c_d = 4, c_e = 16, a_d = 0.94$	
$q_d < 2.922$	$q_d > 2.922$
$\frac{dD^*}{d\overline{b_d}} < \frac{dD^*}{d\underline{b_d}}$	$\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$
The numerical evidence for $\frac{d\Pi^*}{dR_0}$	
Base specification: $\underline{b_e} = 1, \overline{b_e} = 8, a_e = 1.75, q_d = 0.422, w = 1, w_0 = 1000,$ $\overline{b_d} = 1, \underline{b_d} = 0.047, r = 1, g = 1, \Pi_0 = 1, c = 1, c_d = 4096, c_e = 16, R_0 = 32, a_d = 0.25, x = 0.738$	
$q_e < 5$	$q_e > 5$
Π^* monotonously increases in R_0	Π^* monotonously decreases in R_0

The selections of specification and monotonicity test are based on the feasibility constraints.

Table 11. *Existence Proofs for Different Settings of Effort Elasticity of The Platform*

■

Proofs of The Extension: Negative Quality Contribution by Users

In this section, we provide the proofs of the results presented in CHAPTER 2.7.4. The equilibrium results for this extension are derived similar to those in the main model. Here, we consider the setting where no user knows the exact level of efficiency in improving the overall content quality based on her content contribution effort (i.e., $a_{e,i}$) before she finalizes the content contribution.[10] However, users know the average level (among all users) of such efficiency (i.e., $E[a_{e,i}]$). Further, the UGC platform knows the distribution of this efficiency.

In particular, a user (say with index i without loss of generality) makes her decisions regarding which actions maximize her utility. In particular, these decisions can be listed as

[10] Otherwise, user i will never contribute content if she knows that $a_{e,i} < 0$.

to whether to make a donation or not, whether to make a content contribution or not, and further, how much donation and/or content contribution she makes if there is a donation and/or contribution. In this extension, we consider that these decisions depend on the individual generousness levels of users (i.e., $b_{d,i}$ and $b_{e,i}$) and the average level of efficiency of utilizing users' content contributions (i.e., $E[a_{e,i}]$) to improve the overall content quality.

Therefore, without loss of generality, we sort $b_{e,i}$ so that $b_{e,i} \geq b_{e,i-1}$. Hence, in this group, if user i decides to contribute content, user j ($\forall j > i$) will also contribute. Solving the utility maximization of users, the optimal contribution effort of user i is $e_i^* = \frac{2(1-a_d w)\left(b_e+\sqrt{h}-q_e\right)+\underline{a_e}+\overline{a_e}}{4c_e(1-a_d w)}$. Therefore, the threshold value of user generousness that is indifferent between making a contribution or not is equal to:

$$b_e^* = \frac{2a_d\sqrt{h}w - 2a_d q_e w - \underline{a_e} - \overline{a_e} - 2\sqrt{h} + 2q_e}{2(1-a_d w)}.$$

To compute the expected aggregate quality improvement due to content contribution from users (i.e., $E[a_e C]$), we compute the unconditional expectation of quality improvement due to content contribution. The derivation is provided below:

$$\begin{aligned}
E[a_e C] &= R\int_{\underline{a_e}}^{\overline{a_e}} a_e \int_{\underline{b_e}}^{\overline{b_e}} e^*(b_e)\frac{1}{\overline{b_e}-\underline{b_e}}db_e \frac{1}{\overline{a_e}-\underline{a_e}}da_e \\
&= R\int_{\underline{a_e}}^{\overline{a_e}} a_e \int_{b_e^*}^{\overline{b_e}} + \frac{2(1-a_d w)\left(b_e+\sqrt{h}-q_e\right)+\underline{a_e}+\overline{a_e}}{4c_e(1-a_d w)}db_e \frac{1}{\overline{a_e}-\underline{a_e}}da_e \\
&= \frac{(c\Pi_0+R_0)\left(2(1-a_d w)\left(\overline{b_e}+\sqrt{h}-q_e\right)+\underline{a_e}+\overline{a_e}\right)^2}{16c_e(1-a_d w)^2}.
\end{aligned}$$

Next, we order $b_{d,i}$ such that $b_{d,i} \geq b_{d,i-1}$. As such, if user i decides to make a donation, user j ($\forall j > i$) will also donate. Solving the utility maximization problems for users, the optimal donation for user i is $d_i^* = \frac{\frac{a_d}{1-a_d w}+b_d+\sqrt{f}-q_d}{2c_d}$. Denoting b_d^* as the threshold of the user generousness that is indifferent between making a donation or not, we observe that

$$b_d^* = -\frac{a_d}{1-a_d w} - \sqrt{f} + q_d.$$

Given that users' generousness level $b_{d,i}$ is uniformly distributed within the range $(\underline{b_d}, \overline{b_d})$, we compute the expected total donation(i.e., D) by:

$$
\begin{aligned}
D =& R\int_{\underline{b_d}}^{\overline{b_d}} d^*(b_d)\frac{1}{\overline{b_d}-\underline{b_d}}db_d = R\int_{b_d^*}^{\overline{b_d}} \frac{\frac{a_d}{1-a_d w}+b_d+\sqrt{f}-q_d}{2c_d(\overline{b_d}-\underline{b_d})}db_d \\
=& \frac{(c\Pi_0+R_0)\left(a_d w\left(-\left(\overline{b_d}+\sqrt{f}-q_d\right)\right)+a_d+\overline{b_d}+\sqrt{f}-q_d\right)^2}{4(1-a_d w)^2(\overline{b_d}-\underline{b_d})}.
\end{aligned}
$$

The recursive formula for the overall quality (i.e., Equation 4 in the main model) reveals that:

$$
\Pi = \frac{1}{1-a_d w}\left(\Pi_0 + a_d\left(w_0 + D - f - h - r - g - xR\right) + E[a_e C]\right).
$$

By substituting C and D into the formula above and invoking the first-order conditions for the platform's decisions (i.e., f and h), we have:

$$
\begin{aligned}
\frac{d\Pi}{df} =& \frac{a_d\left(\overline{b_d}(1-a_d w)\left(-c\Pi_0+4\sqrt{f}-R_0\right)+(c\Pi_0+R_0)\left(a_d\left(\sqrt{f}w-q_d w-1\right)-\sqrt{f}+q_d\right)-4\underline{b_d}\sqrt{f}(1-a_d w)\right)}{-4\sqrt{f}(1-a_d w)^2(\overline{b_d}-\underline{b_d})} = 0; \\
\frac{d\Pi}{dh} =& \frac{16a_d^2 c_e\sqrt{h}w+2a_d\left(w\left(\underline{a_e}^2-\overline{a_e}^2\right)(c\Pi_0+R_0)\left(\overline{b_e}+\sqrt{h}-q_e\right)-8c_e\sqrt{h}\right)-\left(\underline{a_e}^2-\overline{a_e}^2\right)(c\Pi_0+R_0)\left(\underline{a_e}+\overline{a_e}+2\left(\overline{b_e}+\sqrt{h}-q_e\right)\right)}{16c_e\sqrt{h}(1-a_d w)^2} = 0.
\end{aligned}
$$

We compute the equilibrium platform resource allocation strategy as the following:

$$
\begin{aligned}
f^* =& \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)^2}; \\
h^* =& \frac{\left(\underline{a_e}^2-\overline{a_e}^2\right)^2(c\Pi_0+R_0)^2(2(1-a_d w)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})^2}{4(1-a_d w)^2\left(8a_d c_e+\left(\underline{a_e}^2-\overline{a_e}^2\right)(c\Pi_0+R_0)\right)^2}.
\end{aligned}
$$

Note that the first-order conditions imply that we have:

$$
c\Pi_0 + 4\underline{b_d} - 4\overline{b_d} + R_0 < 0 \;\text{ and }\; 8a_d c_e + \left(\underline{a_e}^2 - \overline{a_e}^2\right)(c\Pi_0 + R_0) > 0. \tag{37}
$$

Further, the second-order conditions are

$$\frac{d^2\Pi^*}{df^2} = -\frac{a_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{-8(1-a_dw)^2(\overline{b_d}-\underline{b_d})\left(\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)^2}\right)^{3/2}} < 0;$$

$$\frac{d^2\Pi^*}{df^2}\frac{d^2\Pi^*}{dh^2}-\left(\frac{d^2\Pi^*}{dfdh}\right)^2 = -\frac{a_d\left(\underline{a_e}^2-\overline{a_e}^2\right)^2(c\Pi_0+R_0)^2}{32c_e(1-a_dw)^4(\overline{a_e}-\underline{a_e})(\underline{a_e}+\overline{a_e})(\overline{b_d}-\underline{b_d})\left(\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)^2}\right)^{3/2}}$$
$$\frac{(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)(2(1-a_dw)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})}{\left(\frac{\left(\underline{a_e}^2-\overline{a_e}^2\right)^2(c\Pi_0+R_0)^2(2(1-a_dw)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})^2}{(1-a_dw)^2\left(8a_dc_e+\left(\underline{a_e}^2-\overline{a_e}^2\right)(c\Pi_0+R_0)\right)^2}\right)^{3/2}} > 0.$$

The second-order conditions can be simplified as below, given the feasibility constraints:

$$\begin{aligned} &a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d<0 \\ \text{and }\ &2(1-a_dw)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e}>0. \end{aligned} \tag{38}$$

Below are the equilibrium levels of important variables:

$$\begin{aligned}
h^* =& \frac{\left(\underline{a_e}^2-\overline{a_e}^2\right)^2(c\Pi_0+R_0)^2(2(1-a_dw)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})^2}{4(1-a_dw)^2\left(8a_dc_e+\left(\underline{a_e}^2-\overline{a_e}^2\right)(c\Pi_0+R_0)\right)^2}, \\
f^* =& \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)^2}, \\
b_d^* =& \frac{\frac{(c\Pi_0+R_0)(a_d(-\overline{b_d})w+a_dq_dw+a_d+\overline{b_d}-q_d)}{c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0}-a_dq_dw-a_d+q_d}{1-a_dw}, \\
b_e^* =& \frac{8a_d^2c_eq_ew-a_d\overline{b_e}w(\overline{a_e}-\underline{a_e})(\underline{a_e}+\overline{a_e})(c\Pi_0+R_0)+4a_dc_e(\underline{a_e}+\overline{a_e}-2q_e)+\overline{b_e}(\overline{a_e}-\underline{a_e})(\underline{a_e}+\overline{a_e})(c\Pi_0+R_0)}{-(1-a_dw)(8a_dc_e-(\overline{a_e}-\underline{a_e})(\underline{a_e}+\overline{a_e})(c\Pi_0+R_0))}, \\
v^* =& -\frac{\left(\underline{a_e}^2-\overline{a_e}^2\right)^2(c\Pi_0+R_0)^2(2(1-a_dw)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})^2}{4(1-a_dw)^2\left(8a_dc_e+\left(\underline{a_e}^2-\overline{a_e}^2\right)(c\Pi_0+R_0)\right)^2} - \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)^2} \\
&+\frac{4(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)^2} - x(c\Pi_0+R_0)-g-r+w_0.
\end{aligned}$$

Given the same definitions and notations of equilibrium results, the equilibrium solution in this extension is feasible if the following set of feasibility constraints is satisfied:

$$c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0<0 \text{ and } 8a_dc_e+\left(\underline{a_e}^2-\overline{a_e}^2\right)(c\Pi_0+R_0)>0 \tag{39}$$

$$a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d<0 \text{ and } 2(1-a_dw)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e}>0 \tag{40}$$

$$a_d>0 \text{ and } w>0 \text{ and } \overline{a_e}>0>\underline{a_e} \text{ and } \underline{a_e}+\overline{a_e}>0 \text{ and } \overline{b_e}>q_e>\underline{b_e}>0 \text{ and } \overline{b_d}>q_d>\underline{b_d}>0 \tag{41}$$

$$\Pi_0>0 \text{ and } r\geq 0 \text{ and } x\geq 0 \text{ and } g\geq 0 \text{ and } R_0>0 \text{ and } c>0 \text{ and } c_d>0 \text{ and } c_e>0 \tag{42}$$

$$\overline{b_e}>\frac{2a_dq_ew+\underline{a_e}+\overline{a_e}-2q_e}{-2(1-a_dw)}>b_e^*>\underline{b_e}\ and\ \overline{b_d}>-\frac{a_d}{1-a_dw}+q_d>b_d^*>\underline{b_d} \tag{43}$$

$$v^* \geq 0 \text{ and } 1 - a_d w > 0 \text{ and } h^* < q_e^2. \tag{44}$$

Given the set of feasibility constraints, we verify that the parameter space is not empty, and all of our results are feasible. For example, at $\{a_d = \frac{1}{4}, \underline{a_e} = -2, \overline{a_e} = 3, q_d = 15, q_e = 2, w = 1, w_0 = 0, \overline{b_e} = 3, \underline{b_e} = 1, \overline{b_d} = 16, \underline{b_d} = 1, r = 0, g = 0, x = 0, \Pi_0 = 1, R_0 = 51, c = 1, c_d = 1, c_e = 793\}$, all feasibility constraints are satisfied.

Proofs of Propositions 1 and 2

In this specification, we obtain the equilibrium total content contribution from users and community support effort exerted by the platform as:

$$\begin{aligned} E^*[a_e C] =& \frac{4a_d^2 c_e (c\Pi_0 + R_0)(2(1 - a_d w)(\overline{b_e} - q_e) + \underline{a_e} + \overline{a_e})^2}{(1 - a_d w)^2 \left(8a_d c_e + \left(\underline{a_e}^2 - \overline{a_e}^2\right)(c\Pi_0 + R_0)\right)^2}; \\ h^* =& \frac{\left(\underline{a_e}^2 - \overline{a_e}^2\right)^2 (c\Pi_0 + R_0)^2 (2(1 - a_d w)(\overline{b_e} - q_e) + \underline{a_e} + \overline{a_e})^2}{4(1 - a_d w)^2 \left(8a_d c_e + \left(\underline{a_e}^2 - \overline{a_e}^2\right)(c\Pi_0 + R_0)\right)^2}. \end{aligned}$$

The first-order derivatives of these measures with respect to a_d are given as:

$$\begin{aligned} \frac{dE^*[a_e C]}{da_d} =& \frac{8a_d c_e(\underline{a_e} + \overline{a_e})(c\Pi_0 + R_0)(-2(1 - a_d w)(\overline{b_e} - q_e) - \underline{a_e} - \overline{a_e})}{-(1 - a_d w)^3 \left(8a_d c_e + \left(\underline{a_e}^2 - \overline{a_e}^2\right)(c\Pi_0 + R_0)\right)^3} \left(8a_d^2 c_e w + 2\underline{a_e}(1 - a_d w)^2 (\overline{b_e} - q_e)(c\Pi_0 + R_0)\right. \\ & \left. -2\overline{a_e}(1 - a_d w)^2 (\overline{b_e} - q_e)(c\Pi_0 + R_0) + \underline{a_e}^2 (c\Pi_0 + R_0) - \overline{a_e}^2 (c\Pi_0 + R_0)\right); \\ \frac{dh^*}{da_d} =& -\frac{\left(\underline{a_e}^2 - \overline{a_e}^2\right)^2 (c\Pi_0 + R_0)^2 (2(1 - a_d w)(\overline{b_e} - q_e) + \underline{a_e} + \overline{a_e})}{-2(1 - a_d w)^3 \left(8a_d c_e + \left(\underline{a_e}^2 - \overline{a_e}^2\right)(c\Pi_0 + R_0)\right)^3} \left(8\underline{a_e} c_e (2a_d w - 1) + 8\overline{a_e} c_e (2a_d w - 1)\right. \\ & \left. -16c_e(1 - a_d w)^2 (\overline{b_e} - q_e) + \underline{a_e}^3 w(c\Pi_0 + R_0) + \underline{a_e}^2 \overline{a_e} w(c\Pi_0 + R_0) - \underline{a_e}\,\overline{a_e}^2 w(c\Pi_0 + R_0) - \overline{a_e}^3 w(c\Pi_0 + R_0)\right). \end{aligned}$$

To support our discussions in CHAPTER 2.7.4, given the feasibility constraints (that are presented in Expressions 39 to 44), we first show that $E^*[a_e C]$ increases in a_d if and only if R_0 is greater than a threshold, i.e.,

$$R_0 > \frac{8a_d^2 c_e w + 2\underline{a_e} c\Pi_0 (1 - a_d w)^2 (\overline{b_e} - q_e) - 2\overline{a_e} c\Pi_0 (1 - a_d w)^2 (\overline{b_e} - q_e) + \underline{a_e}^2 c\Pi_0 - \overline{a_e}^2 c\Pi_0}{(\overline{a_e} - \underline{a_e})\left(2(1 - a_d w)^2 (\overline{b_e} - q_e) + \underline{a_e} + \overline{a_e}\right)}.$$

Second, h^* increases in a_d if and only if the efficiency of in-house effort (i.e., a_d) is greater than a threshold

$$a_d > \frac{-\sqrt{c_e w^2(\underline{a_e}+\overline{a_e})\left(\underline{a_e}^2 w(\overline{b_e}-q_e)(c\Pi_0+R_0)+4\underline{a_e}c_e-\overline{a_e}^2 w(\overline{b_e}-q_e)(c\Pi_0+R_0)+4\overline{a_e}c_e+8c_e(\overline{b_e}-q_e)\right)}+2\underline{a_e}c_e w+2\overline{a_e}c_e w+4c_e\overline{b_e}w-4c_e q_e w}{4c_e w^2(\overline{b_e}-q_e)}.$$

Proof Proposition 3

We first have:

$$\begin{aligned}
D^* =& \frac{4(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)^2};\\
\frac{dD^*}{d\underline{b_d}} =& \frac{4(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2(-c\Pi_0+4\underline{b_d}-4\overline{b_d}-R_0)}{(1-a_d w)^2(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)^3};\\
\frac{dD^*}{d\overline{b_d}} =& \tfrac{4(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)}{(1-a_d w)^2(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)^3}((a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)\\
&-2(1-a_d w)(\overline{b_d}-\underline{b_d})(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)+8(\overline{b_d}-\underline{b_d})(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)).
\end{aligned}$$

We compute the differences between the first-order derivatives of the total donation from users with respect to upper bounds and lower bounds of user generousness parameters as follows:

$$\begin{aligned}
\frac{dD^*}{d\overline{b_d}}-\frac{dD^*}{d\underline{b_d}} =& -\tfrac{8(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)}{(1-a_d w)^2(c\Pi_0+4\underline{b_d}-4\overline{b_d}+R_0)^3}\left(a_d\left(\underline{b_d}(c\Pi_0 w-4\overline{b_d}w-4q_d w+R_0 w-4)\right.\right.\\
&-2\overline{b_d}(c\Pi_0 w-2q_d w+R_0 w-2)+(q_d w+1)(c\Pi_0+R_0)+4\underline{b_d}^2 w\big)+\underline{b_d}(-c\Pi_0+4\overline{b_d}\\
&\left.+4q_d-R_0)+2c\overline{b_d}\Pi_0-c\Pi_0 q_d-4\underline{b_d}^2-4\overline{b_d}q_d+2\overline{b_d}R_0-q_d R_0\right).
\end{aligned}$$

Given the feasibility constraints, we have the total donation from users monotonously increases in the lower bound and upper bound of user generousness (i.e., $\frac{dD^*}{d\underline{b_d}} > 0$ and $\frac{dD^*}{d\overline{b_d}} > 0$), and (ii) the platform's overall content quality can be more efficiently increased by enhancing the upper bound of user generousness (i.e., $\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$ if and only if the barrier parameter q_d is at a high level, i.e.,

$$
q_d > \frac{1}{(1-a_d w)(c\Pi_0 - 4\underline{b_d} + 4\overline{b_d} + R_0)} \big(a_d \big(\underline{b_d}(c\Pi_0 w - 4\overline{b_d} w + R_0 w - 4) - 2\overline{b_d}(c\Pi_0 w + R_0 w - 2)
$$

$$
+ c\Pi_0 + 4\underline{b_d}^2 w + R_0\big) + \underline{b_d}(-c\Pi_0 + 4\overline{b_d} - R_0) + 2\overline{b_d}(c\Pi_0 + R_0) - 4\underline{b_d}^2\big) .
$$

Proof Proposition 4

In this extension, the equilibrium overall content quality and its first-order derivatives with respect to q_e and R_0 are given as:

$$
\Pi^* = \frac{1}{1-a_d w} \Bigg(-\frac{2a_d^2 c_e \big(\underline{a_e}^2 - \overline{a_e}^2\big)(c\Pi_0+R_0)(2(1-a_d w)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})^2}{(1-a_d w)^2\big(8a_d c_e + \big(\underline{a_e}^2-\overline{a_e}^2\big)(c\Pi_0+R_0)\big)^2} - \frac{a_d(c\Pi_0+R_0)^2(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)^2}{(1-a_d w)^2(c\Pi_0 + 4\underline{b_d} - 4\overline{b_d} + R_0)^2}
$$

$$
+ a_d \Bigg(-\frac{\big(\underline{a_e}^2-\overline{a_e}^2\big)^2(c\Pi_0+R_0)^2(2(1-a_d w)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})^2}{4(1-a_d w)^2\big(8a_d c_e + \big(\underline{a_e}^2-\overline{a_e}^2\big)(c\Pi_0+R_0)\big)^2} - \frac{-4(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)^2}{(1-a_d w)^2(c\Pi_0 + 4\underline{b_d} - 4\overline{b_d} + R_0)^2}
$$

$$
- x(c\Pi_0 + R_0) - g - r + w_0\Bigg) + \Pi_0\Bigg) ;
$$

$$
\frac{d\Pi^*}{dq_e} = \frac{a_d\left(\overline{a_e}^2 - \underline{a_e}^2\right)(c\Pi_0 + R_0)(-2(1-a_d w)(\overline{b_e} - q_e) - \underline{a_e} - \overline{a_e})}{(1-a_d w)^2\left(8a_d c_e - \left(\overline{a_e}^2 - \underline{a_e}^2\right)(c\Pi_0 + R_0)\right)} ;
$$

$$
\frac{d\Pi^*}{dR_0} = \frac{a_d}{1-a_d w} \Bigg(\frac{\big(\underline{a_e}^2-\overline{a_e}^2\big)^3(c\Pi_0+R_0)^2(2(1-a_d w)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})^2}{2(1-a_d w)^2\big(8a_d c_e + \big(\underline{a_e}^2-\overline{a_e}^2\big)(c\Pi_0+R_0)\big)^3} - \frac{2a_d c_e\big(\underline{a_e}^2-\overline{a_e}^2\big)(2(1-a_d w)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})^2}{(1-a_d w)^2\big(8a_d c_e + \big(\underline{a_e}^2-\overline{a_e}^2\big)(c\Pi_0+R_0)\big)^2}
$$

$$
+ \frac{4a_d c_e\big(\underline{a_e}^2-\overline{a_e}^2\big)^2(c\Pi_0+R_0)(2(1-a_d w)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})^2}{(1-a_d w)^2\big(8a_d c_e + \big(\underline{a_e}^2-\overline{a_e}^2\big)(c\Pi_0+R_0)\big)^3} - \frac{\big(\underline{a_e}^2-\overline{a_e}^2\big)^2(c\Pi_0+R_0)(2(1-a_d w)(\overline{b_e}-q_e)+\underline{a_e}+\overline{a_e})^2}{2(1-a_d w)^2\big(8a_d c_e + \big(\underline{a_e}^2-\overline{a_e}^2\big)(c\Pi_0+R_0)\big)^2}
$$

$$
+ \frac{2(c\Pi_0+R_0)^2(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)^2}{(1-a_d w)^2(c\Pi_0 + 4\underline{b_d} - 4\overline{b_d} + R_0)^3} + \frac{-8(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)^2}{(1-a_d w)^2(c\Pi_0 + 4\underline{b_d} - 4\overline{b_d} + R_0)^3}
$$

$$
- \frac{-4(\overline{b_d}-\underline{b_d})(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)^2}{(1-a_d w)^2(c\Pi_0 + 4\underline{b_d} - 4\overline{b_d} + R_0)^2} - \frac{2(c\Pi_0+R_0)(a_d(\overline{b_d}w - q_d w - 1) - \overline{b_d} + q_d)^2}{(1-a_d w)^2(c\Pi_0 + 4\underline{b_d} - 4\overline{b_d} + R_0)^2} - x\Bigg).
$$

Given the feasibility constraints (that are presented in Expressions 39 to 44), we find that the overall content quality decreases in q_e. Besides, it decreases in R_0 if and only if

$$
q_e > \frac{1}{4(1-a_d w)^2} \Bigg(\frac{\sqrt{2}}{a_d c_e\big(\underline{a_e}^2-\overline{a_e}^2\big)(c\Pi_0 + 4\underline{b_d} - 4\overline{b_d} + R_0)^2} \left(-a_d c_e (1-a_d w)^2 \left(\underline{a_e}^2 - \overline{a_e}^2\right)\left(c\Pi_0 + 4\underline{b_d} - 4\overline{b_d} + R_0\right)^2\right.
$$

$$
\left(8a_d c_e + \left(\underline{a_e}^2 - \overline{a_e}^2\right)(c\Pi_0 + R_0)\right)^2 \Big(a_d^2 \Big(4\underline{b_d} \Big(2c\Pi_0 w^2 x + \overline{b_d}^2 w^2 - 2\overline{b_d} w(q_d w + 4wx + 1) + q_d^2 w^2 + 2q_d w
$$

$$
+ 2R_0 w^2 x + 1) - 4\overline{b_d}\left(2c\Pi_0 w^2 x + q_d^2 w^2 + 2q_d w + 2R_0 w^2 x + 1\right) + w^2 x(c\Pi_0 + R_0)^2 + 16\underline{b_d}^2 w^2 x - 4\overline{b_d}^3 w^2
$$

$$
+ 8\overline{b_d}^2 w(q_d w + 2wx + 1)\Big) - 2a_d\Big(4\underline{b_d}\Big(2wx(c\Pi_0 + R_0) + \overline{b_d}^2 w - \overline{b_d}(2q_d w + 8wx + 1) + q_d^2 w + q_d\Big) - 4\overline{b_d}
$$

$$
(2wx(c\Pi_0 + R_0) + q_d^2 w + q_d) + wx(c\Pi_0 + R_0)^2 + 16\underline{b_d}^2 wx - 4\overline{b_d}^3 w + 4\overline{b_d}^2(2q_d w + 4wx + 1)\Big) + c^2\Pi_0^2 x
$$

$$+8c\underline{b_d}\Pi_0 x - 8c\overline{b_d}\Pi_0 x + 2c\Pi_0 R_0 x + 16\underline{b_d}^2 x + 4\underline{b_d}\overline{b_d}^2 - 8\underline{b_d}\overline{b_d}q_d - 32\underline{b_d}\overline{b_d}x + 4\underline{b_d}q_d^2 + 8\underline{b_d}R_0 x - 4\overline{b_d}^3 + 8\overline{b_d}^2 q_d$$

$$+16\overline{b_d}^2 x - 4\overline{b_d}q_d^2 - 8\overline{b_d}R_0 x + R_0^2 x\Big)\Big)^{1/2} + 4a_d^2\overline{b_e}w^2 - 2a_d w(\underline{a_e} + \overline{a_e} + 4\overline{b_e}) + 2(\underline{a_e} + \overline{a_e} + 2\overline{b_e})\Bigg).$$

We have also verified the feasibility of this condition numerically at $\{a_d = 0.25, \underline{a_e} = -2, \overline{a_e} = 3, q_d = 15, q_e = 2, w = 1, w_0 = 80, \overline{b_e} = 3, \underline{b_e} = 1, \overline{b_d} = 16, \underline{b_d} = 1, r = 0, g = 0, \Pi_0 = 1, R_0 = 51, c = 1, c_d = 1, c_e = 793, x = 1.679\}$, given all the other constraints.

■

Proofs of The Extension: Robustness to The Distributional Assumptions for Users' Generousness

In this section, we provide the proofs of the results presented in CHAPTER 2.7.5. We solve for the equilibrium results for this extension by adopting a model setting where we consider a bimodal distribution where users are more likely to have low-level generousness (i.e., $\underline{b_e} < b_e < b_{e,t_l}$, $\underline{b_d} < b_d < b_{d,t_l}$) than high-level generousness (i.e., $b_{e,t_u} < b_e < \overline{b_e}$, $b_{d,t_u} < b_d < \overline{b_d}$). Because in the real world, even people with low-level generousness may contribute content (or make donations) to non-profit UGC platforms, we suppose that b_{e,t_l} is above b_e^* (i.e., the minimum generousness level enables a user to make content contributions) and b_{d,t_l} is above b_d^*. Based on a similar solving process as the main model, we derive the optimal content contribution and donation for user i as:

$$\begin{aligned} d_i^* &= \max\left\{0, \frac{\frac{a_d}{1-a_d w} + b_{d,i} + \sqrt{f} - q_d}{2c_d}\right\}; \\ e_i^* &= \max\left\{0, \frac{\frac{a_e}{1-a_d w} + b_{e,i} + \sqrt{h} - q_e}{2c_e}\right\}. \end{aligned}$$

Based on users' optimal behaviors, we compute the expected total donation (i.e., D) and content contribution (i.e., C) from users as:

$$D^* = \frac{(c\Pi_0 + R_0)}{4c_d(1-a_d w)^2(b_{d,t_l} - \underline{b_d})}\left(a_d^2\left(p_d w^2\left(\underline{b_d}\left(b_{d,t_u} + \overline{b_d} + 2\sqrt{f} - 2q_d\right) + b_{d,t_l}^2 - b_{d,t_l}(b_{d,t_u} + \overline{b_d}) + \left(\sqrt{f} - q_d\right)^2\right)\right.\right.$$

$$+w(\underline{b_d} - b_{d,t_l})\left(2 - w\left(b_{d,t_u} + \overline{b_d} + 2\sqrt{f} - 2q_d\right)\right) - 2p_d w\left(\underline{b_d} + \sqrt{f} - q_d\right) + p_d\Big) + 2a_d\left(\underline{b_d}(1 - p_d)\left(w\left(b_{d,t_u} + \overline{b_d}\right.\right.\right.$$

$$+2\sqrt{f} - 2q_d\big) - 1\big) - w\left(b_{d,t_l}^2 p_d + b_{d,t_l}\left(-p_d(b_{d,t_u} + \overline{b_d}) + b_{d,t_u} + \overline{b_d} + 2\sqrt{f} - 2q_d\right) + p_d\left(\sqrt{f} - q_d\right)^2\right) + b_{d,t_l}$$

$$
\begin{aligned}
&+\sqrt{f}p_d - p_d q_d) - \underline{b_d}(1-p_d)\left(b_{d,t_u} + \overline{b_d} + 2\sqrt{f} - 2q_d\right) + b_{d,t_l}^2 p_d - b_{d,t_l} b_{d,t_u} p_d + b_{d,t_l} b_{d,t_u} - b_{d,t_l}\overline{b_d}p_d + b_{d,t_l}\overline{b_d} - 2q_d \\
&\left(b_{d,t_l} + \sqrt{f}p_d\right) + 2b_{d,t_l}\sqrt{f} + fp_d + p_d q_d^2\Big);
\end{aligned}
$$

$$
\begin{aligned}
C^* =& \tfrac{(c\Pi_0+R_0)}{4c_e(1-a_d w)^2(b_{e,t_l}-\underline{b_e})}\left(2a_e(1-a_d w)\left(-\underline{b_e}(1-p_e) + b_{e,t_l} + p_e\left(\sqrt{h} - q_e\right)\right) + (1-a_d w)^2\left(-\underline{b_e}(1-p_e)\left(b_{e,t_u} + \overline{b_e}\right.\right.\right. \\
&\left.+2\sqrt{h} - 2q_e\right) + b_{e,t_l}^2 p_e + b_{e,t_l}\left(-p_e(b_{e,t_u} + \overline{b_e}) + b_{e,t_u} + \overline{b_e} + 2\sqrt{h} - 2q_e\right) + p_e\left(\sqrt{h} - q_e\right)^2\Big) + a_e^2 p_e\Big).
\end{aligned}
$$

The recursive formula for the overall quality reveals that:

$$
\Pi = \frac{1}{1-a_d w}\left(\Pi_0 + a_d\left(w_0 + D - f - h - r - g - xR\right) + a_e C\right).
$$

By substituting C and D into the formula above and invoking the first-order conditions for the platform's decisions (i.e., f and h), we compute the platform's optimal effort allocation as:

$$
\begin{aligned}
f^* &= \frac{(c\Pi_0+R_0)^2(\underline{b_d}(1-p_d)(1-a_d w) - b_{d,t_l}(1-a_d w) - p_d(a_d q_d w + a_d - q_d))^2}{(1-a_d w)^2(p_d(c\Pi_0+R_0) + 4c_d(\underline{b_d} - b_{d,t_l}))^2}; \\
h^* &= \frac{a_e^2(c\Pi_0+R_0)^2(p_e(a_d q_e w + a_e - q_e) - \underline{b_e}(1-p_e)(1-a_d w) - a_d b_{e,t_l} w + b_{e,t_l})^2}{(1-a_d w)^2(4a_d c_e(\underline{b_e} - b_{e,t_l}) + a_e p_e(c\Pi_0+R_0))^2}.
\end{aligned}
$$

Based on the first-order and the second-order conditions, we consider the following constraints:

$$
\begin{aligned}
&\underline{b_d}(1-p_d)(1-a_d w) - b_{d,t_l}(1-a_d w) - p_d(a_d q_d w + a_d - q_d) < 0; \\
&p_e(a_d q_e w + a_e - q_e) - \underline{b_e}(1-p_e)(1-a_d w) - a_d b_{e,t_l} w + b_{e,t_l} > 0; \\
&p_d(c\Pi_0+R_0) + 4c_d(\underline{b_d} - b_{d,t_l}) < 0; \\
&4a_d c_e(\underline{b_e} - b_{e,t_l}) + a_e p_e(c\Pi_0+R_0) < 0.
\end{aligned}
$$

Next, based on the platform's optimal effort allocation, we obtain the equilibrium levels of important variables:

$$
\begin{aligned}
h^* =& \frac{a_e^2(c\Pi_0+R_0)^2(p_e(a_d q_e w + a_e - q_e) - \underline{b_e}(1-p_e)(1-a_d w) - a_d b_{e,t_l} w + b_{e,t_l})^2}{(1-a_d w)^2(4a_d c_e(\underline{b_e} - b_{e,t_l}) + a_e p_e(c\Pi_0+R_0))^2}, \\
f^* =& \frac{(c\Pi_0+R_0)^2(\underline{b_d}(1-p_d)(1-a_d w) - b_{d,t_l}(1-a_d w) - p_d(a_d q_d w + a_d - q_d))^2}{(1-a_d w)^2(p_d(c\Pi_0+R_0) + 4c_d(\underline{b_d} - b_{d,t_l}))^2}, \\
d^* =& \frac{\frac{(c\Pi_0+R_0)(\underline{b_d}(1-p_d)(1-a_d w) - b_{d,t_l}(1-a_d w) - p_d(a_d q_d w + a_d - q_d))}{(1-a_d w)(p_d(c\Pi_0+R_0) + 4c_d(\underline{b_d} - b_{d,t_l}))} + \frac{a_d}{1-a_d w} + b_d - q_d}{2c_d},
\end{aligned}
$$

$$
e^* =\frac{\frac{a_e(c\Pi_0+R_0)(p_e(a_dq_ew+a_e-q_e)-\underline{b_e}(1-p_e)(1-a_dw)-a_db_{e,t_l}w+b_{e,t_l})}{(1-a_dw)(4a_dc_e(b_{e,t_l}-\underline{b_e})-a_ep_e(c\Pi_0+R_0))}+\frac{a_e}{1-a_dw}+b_e-q_e}{2c_e},
$$

$$
\begin{aligned}
v^* ={}& -\frac{(c\Pi_0+R_0)^2(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))^2}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))^2(1-a_dw)^2}-g-r-\frac{(c\Pi_0+R_0)}{4c_d(\underline{b_d}-b_{d,t_l})(1-a_dw)^2}\Big(\Big(p_d\Big(b_{d,t_l}^2\\
& -(b_{d,t_u}+\overline{b_d})b_{d,t_l}+\Big(q_d+\frac{(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_dw)}\Big)^2+\underline{b_d}\Big(b_{d,t_u}+\overline{b_d}-2q_d\\
& +\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)\Big)w^2+(\underline{b_d}-b_{d,t_l})\Big(2-w\Big(b_{d,t_u}+\overline{b_d}-2q_d\\
& +\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)\Big)w-(4c_d(\underline{b_d}-b_{d,t_l})(q_d-\underline{b_d})(1-a_dw)+(c\Pi_0\\
& +R_0)(-a_dwb_{d,t_l}+b_{d,t_l}+a_dp_d-\underline{b_d}(1-a_dw)))\frac{2p_dw}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_dw)}+p_d\Big)a_d^2+2\,(b_{d,t_l}-p_dq_d+(-b_{d,t_l}\\
& (1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))\frac{p_d(c\Pi_0+R_0)}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}-w\Big(p_db_{d,t_l}^2+\Big(b_{d,t_u}+\overline{b_d}\\
& -(b_{d,t_u}+\overline{b_d})p_d-2q_d+\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)b_{d,t_l}+p_d\,(q_d+(-b_{d,t_l}\\
& (1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))\frac{(c\Pi_0+R_0)}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_dw)}\Big)^2\Big)+\underline{b_d}(1-p_d)\,(w\\
& \Big(b_{d,t_u}+\overline{b_d}-2q_d+\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)-1\Big)\Big)a_d+p_dq_d^2+(-b_{d,t_l}\\
& (1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))^2\frac{p_d(c\Pi_0+R_0)^2}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))^2(1-a_dw)^2}+b_{d,t_l}b_{d,t_u}+b_{d,t_l}\overline{b_d}\\
& +b_{d,t_l}^2p_d-b_{d,t_l}b_{d,t_u}p_d-b_{d,t_l}\overline{b_d}p_d+\frac{2b_{d,t_l}(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}-\underline{b_d}(1-p_d)\\
& \Big(b_{d,t_u}+\overline{b_d}-2q_d+\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)-2q_d\,(b_{d,t_l}+(-b_{d,t_l}(1-a_dw)\\
& +\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))\frac{p_d(c\Pi_0+R_0)}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)\Big)+w_0-(c\Pi_0+R_0)x-(-a_dwb_{e,t_l}\\
& +b_{e,t_l}-\underline{b_e}(1-p_e)(1-a_dw)+p_e(a_e-q_e+a_dq_ew))^2\frac{a_e^2(c\Pi_0+R_0)^2}{(4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0))^2(1-a_dw)^2}.
\end{aligned}
$$

Given the same definitions and notations of equilibrium results as the main model, the equilibrium solution in this extension is feasible if the following set of feasibility constraints is satisfied:

$$0<1-p_e<p_e<1 \text{ and } 0<1-p_d<p_d<1 \text{ and } \frac{p_e}{b_{e,t_l}-\underline{b_e}}>\frac{1-p_e}{\overline{b_e}-b_{e,t_u}} \text{ and } \frac{p_d}{b_{d,t_l}-\underline{b_d}}>\frac{1-p_d}{\overline{b_d}-b_{d,t_u}} \tag{45}$$

$$p_d(c\Pi_0+R_0)+4c_d(\underline{b_d}-b_{d,t_l})<0 \text{ and } 4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0)<0 \tag{46}$$

$$\underline{b_d}(1-p_d)(1-a_dw)-b_{d,t_l}(1-a_dw)-p_d(a_dq_dw+a_d-q_d)<0 \tag{47}$$

$$p_e(a_dq_ew+a_e-q_e)-\underline{b_e}(1-p_e)(1-a_dw)-a_db_{e,t_l}w+b_{e,t_l}>0 \tag{48}$$

$$a_d>0 \text{ and } w>0 \text{ and } a_e>0 \text{ and } \overline{b_e}>q_e>\underline{b_e}>0 \text{ and } \overline{b_d}>q_d>\underline{b_d}>0 \tag{49}$$

$$\Pi_0>0 \text{ and } r\geq 0 \text{ and } x\geq 0 \text{ and } g\geq 0 \text{ and } R_0>0 \text{ and } c>0 \text{ and } c_d>0 \text{ and } c_e>0 \tag{50}$$

$$-\frac{a_d}{1-a_dw}+q_d<\overline{b_d} \text{ and } -\frac{a_e}{1-a_dw}+q_e<\overline{b_e} \text{ and } \overline{b_e}>b_{e,t_u}>b_{e,t_l}>b_e^*>\underline{b_e} \text{ and } \overline{b_d}>b_{d,t_u}>b_{d,t_l}>b_d^*>\underline{b_d} \tag{51}$$

$$v^*\geq 0 \text{ and } 1-a_dw>0 \text{ and } h^*<q_e^2. \tag{52}$$

We verify that the parameter space given the set of feasibility constraints is not empty, and all of our results are feasible. For example, at $\{a_e = 0.14, q_e = 1, w = 1.76, w_0 = 0, \overline{b_e} = 8.33, \underline{b_e} = 0.5, \overline{b_d} = 16, \underline{b_d} = 1.88, r = 0, g = 0, x = 0, \Pi_0 = 1, c = 1, c_d = 4.77, c_e = 17.88, b_{e,t_l} = 1, b_{e,t_u} = 4, b_{d,t_l} = 4, b_{d,t_u} = 5.65, p_d = 0.75, p_e = 0.75, q_d = 4, a_d = 0.25, R_0 = 15\}$, all feasibility constraints are satisfied.

Proofs of Propositions 1 and 2

In this specification, we obtain the equilibrium total content contribution from users and community support effort exerted by the platform as:

$$
\begin{aligned}
C^* =& \tfrac{(c\Pi_0+R_0)}{4c_e(1-a_dw)^2(b_{e,t_l}-\underline{b_e})} \Big(-\big(4a_d^2c_ew(\underline{b_e}-b_{e,t_l})(-\underline{b_e}(1-p_e)+b_{e,t_l}-p_eq_e) - 4a_dc_e(\underline{b_e}-b_{e,t_l})(-\underline{b_e}(1-p_e)+b_{e,t_l}-p_eq_e) + a_e^2p_e^2 \\
& (c\Pi_0+R_0)\big) \tfrac{2a_e}{4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0)} + (1-a_dw)^2 \Big(b_{e,t_l}\Big(\big(p_e(a_dq_ew+a_e-q_e) - \underline{b_e}(1-p_e)(1-a_dw) - a_db_{e,t_l}w + b_{e,t_l}\big) \\
& \tfrac{2a_e(c\Pi_0+R_0)}{-(1-a_dw)(4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0))} - p_e(b_{e,t_u}+\overline{b_e}) + b_{e,t_u} + \overline{b_e} - 2q_e\Big) - \underline{b_e}(1-p_e)\Big(\big(p_e(a_dq_ew+a_e-q_e) - \underline{b_e}(1-p_e) \\
& (1-a_dw) - a_db_{e,t_l}w + b_{e,t_l}\big)\tfrac{2a_e(c\Pi_0+R_0)}{-(1-a_dw)(4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0))} + b_{e,t_u} + \overline{b_e} - 2q_e\Big) + p_e\big(p_e(a_dq_ew+a_e-q_e) - \underline{b_e} \\
& (1-p_e)(1-a_dw) - a_db_{e,t_l}w + b_{e,t_l}\big)\Big(q_e - \tfrac{a_e(c\Pi_0+R_0)}{-(1-a_dw)(4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0))}\Big)^2 + b_{e,t_l}^2p_e\Big) + a_e^2p_e\Big); \\
h^* =& \frac{a_e^2(c\Pi_0+R_0)^2(p_e(a_dq_ew+a_e-q_e) - \underline{b_e}(1-p_e)(1-a_dw) - a_db_{e,t_l}w + b_{e,t_l})^2}{(1-a_dw)^2(4a_dc_e(\underline{b_e}-b_{e,t_l}) + a_ep_e(c\Pi_0+R_0))^2}.
\end{aligned}
$$

The first-order derivatives of these measures with respect to a_d are given as:

$$
\begin{aligned}
\frac{dC^*}{da_d} =& -\tfrac{8a_da_ec_e(\underline{b_e}-b_{e,t_l})(c\Pi_0+R_0)(-p_e(a_dq_ew+a_e-q_e)+\underline{b_e}(1-p_e)(1-a_dw)-b_{e,t_l}(1-a_dw))}{-(1-a_dw)^3(4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0))^3}\big(4a_d^2c_e\underline{b_e}w - 4a_d^2c_eb_{e,t_l}w \\
& + a_d^2b_{e,t_l}R_0w^2 - a_d^2p_eq_eR_0w^2 + c\Pi_0\big(p_e\big(a_e - q_e(1-a_dw)^2\big) - \underline{b_e}(1-p_e)(1-a_dw)^2 + b_{e,t_l}(1-a_dw)^2\big) - \underline{b_e}(1-p_e)R_0 \\
& (1-a_dw)^2 - 2a_db_{e,t_l}R_0w + 2a_dp_eq_eR_0w + a_ep_eR_0 + b_{e,t_l}R_0 - p_eq_eR_0\big); \\
\frac{dh^*}{da_d} =& -\tfrac{2(c\Pi_0+R_0)^2(p_e(a_dq_ew+a_e-q_e)-\underline{b_e}(1-p_e)(1-a_dw)-a_db_{e,t_l}w+b_{e,t_l})}{-(1-a_dw)^3(4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0))^3}\big(a_e^4p_e^2w(c\Pi_0+R_0) - 4a_e^2c_e(\underline{b_e}-b_{e,t_l}) \\
& \big(-p_e\big(a_e(2a_dw-1) + q_e(1-a_dw)^2\big) - \underline{b_e}(1-p_e)(1-a_dw)^2 + b_{e,t_l}(1-a_dw)^2\big)\big).
\end{aligned}
$$

To support our discussions in CHAPTER 2.7.5, given the feasibility constraints (that are presented in Expressions 45 to 52), we first show that C^* increases in a_d if and only if R_0 is greater than a threshold (i.e., $R_0 > \frac{4a_d^2c_ew(b_{e,t_l}-\underline{b_e})}{p_e(a_e-q_e(1-a_dw)^2)-\underline{b_e}(1-p_e)(1-a_dw)^2+b_{e,t_l}(1-a_dw)^2} - c\Pi_0$). Second, h^* increases in a_d if and only if

$$
\begin{aligned}
a_d >& \frac{1}{2c_e w^2(\underline{b_e} - b_{e,t_l})(-\underline{b_e}(1-p_e)+b_{e,t_l}-p_e q_e)} \big(2c_e w(\underline{b_e}-b_{e,t_l})(p_e(a_e-q_e)-\underline{b_e}(1-p_e)+b_{e,t_l}) \\
&+\sqrt{a_e c_e p_e w^2(\underline{b_e}-b_{e,t_l})(a_e p_e w(c\Pi_0+R_0)(-\underline{b_e}(1-p_e)+b_{e,t_l}-p_e q_e)+4c_e(\underline{b_e}-b_{e,t_l})(p_e(a_e-q_e)-\underline{b_e}(1-p_e)+b_{e,t_l}))}\big).
\end{aligned}
$$

Proof Proposition 3

We first have:

$$
\begin{aligned}
D^* =& -\frac{(c\Pi_0+R_0)}{4c_d(\underline{b_d}-b_{d,t_l})(1-a_d w)^2}\bigg(\bigg(p_d\Big(b_{d,t_l}^2 - (b_{d,t_u}+\overline{b_d})b_{d,t_l} + \Big(q_d + (-b_{d,t_l}(1-a_d w)+\underline{b_d}(1-p_d)(1-a_d w) \\
&-p_d(q_d w a_d + a_d - q_d))\frac{(c\Pi_0+R_0)}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)}\Big)^2 + \underline{b_d}\Big(b_{d,t_u}+\overline{b_d}-2q_d-(-b_{d,t_l}(1-a_d w) \\
&+\underline{b_d}(1-p_d)(1-a_d w) - p_d(q_d w a_d+a_d-q_d))\frac{2(c\Pi_0+R_0)}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)}\Big)\Big) w^2 + (\underline{b_d}-b_{d,t_l}) \\
&\bigg(2 - w\bigg(b_{d,t_u}+\overline{b_d}-2q_d-\frac{2(-b_{d,t_l}(1-a_d w)+\underline{b_d}(1-p_d)(1-a_d w)-p_d(q_d w a_d+a_d-q_d))(c\Pi_0+R_0)}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)}\bigg)\bigg) w \\
&-\frac{2p_d(4c_d(\underline{b_d}-b_{d,t_l})(q_d-\underline{b_d})(1-a_d w)+(c\Pi_0+R_0)(-a_d w b_{d,t_l}+b_{d,t_l}+a_d p_d-\underline{b_d}(1-a_d w)))w}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)} + p_d\bigg) a_d^2 \\
&+2\bigg(b_{d,t_l} - p_d q_d - w\bigg(p_d b_{d,t_l}^2 + \Big(b_{d,t_u}+\overline{b_d}-(b_{d,t_u}+\overline{b_d})p_d - 2q_d - (-b_{d,t_l}(1-a_d w)+\underline{b_d}(1-p_d)(1-a_d w) \\
&-p_d(q_d w a_d+a_d-q_d))\frac{2(c\Pi_0+R_0)}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)}\Big) b_{d,t_l} + p_d\Big(q_d + (-b_{d,t_l}(1-a_d w)+\underline{b_d} \\
&(1-p_d)(1-a_d w)-p_d(q_d w a_d+a_d-q_d))\frac{(c\Pi_0+R_0)}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)}\Big)^2\bigg) + \underline{b_d}(1-p_d) \\
&\bigg(w\bigg(b_{d,t_u}+\overline{b_d}-2q_d-\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_d w)+\underline{b_d}(1-p_d)(1-a_d w)-p_d(q_d w a_d+a_d-q_d))}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)}\bigg)-1\bigg) \\
&-\frac{p_d(c\Pi_0+R_0)(-b_{d,t_l}(1-a_d w)+\underline{b_d}(1-p_d)(1-a_d w)-p_d(q_d w a_d+a_d-q_d))}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)}\bigg) a_d + p_d q_d^2 + (-b_{d,t_l}(1-a_d w) \\
&+\underline{b_d}(1-p_d)(1-a_d w)-p_d(q_d w a_d+a_d-q_d))^2\frac{p_d(c\Pi_0+R_0)^2}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))^2(1-a_d w)^2} + b_{d,t_l}b_{d,t_u} + b_{d,t_l}\overline{b_d} \\
&+b_{d,t_l}^2 p_d - b_{d,t_l}b_{d,t_u}p_d - b_{d,t_l}\overline{b_d}p_d - \underline{b_d}(1-p_d)\Big(b_{d,t_u}+\overline{b_d}-2q_d-(-b_{d,t_l}(1-a_d w)+\underline{b_d}(1-p_d)(1-a_d w)-p_d \\
&(q_d w a_d+a_d-q_d))\frac{2(c\Pi_0+R_0)}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)}\Big) - 2q_d\Big(b_{d,t_l} - (-b_{d,t_l}(1-a_d w)+\underline{b_d}(1-p_d) \\
&(1-a_d w)-p_d(q_d w a_d+a_d-q_d))\frac{p_d(c\Pi_0+R_0)}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)}\Big) - (-b_{d,t_l}(1-a_d w)+\underline{b_d}(1-p_d) \\
&(1-a_d w)-p_d(q_d w a_d+a_d-q_d))\frac{2b_{d,t_l}(c\Pi_0+R_0)}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_d w)}\bigg); \\
\frac{dD^*}{d\underline{b_d}} =& \frac{p_d(c\Pi_0+R_0)(a_d(1-p_d)w(c\Pi_0+R_0)+4a_d c_d(b_{d,t_l}w-q_d w-1)-(1-p_d)(c\Pi_0+R_0)-4c_d(b_{d,t_l}-q_d))}{4c_d(1-a_d w)^2(p_d(c\Pi_0+R_0)+4c_d(\underline{b_d}-b_{d,t_l}))^3} \\
&\big(a_d\big(-4c_d(c\Pi_0+R_0)(p_d(3\underline{b_d}w-2b_{d,t_l}w-q_d w-1)+3w(b_{d,t_l}-\underline{b_d}))+(1-p_d)p_d w(c\Pi_0+R_0)^2+16c_d^2(\underline{b_d}-b_{d,t_l}) \\
&(b_{d,t_l}w-q_d w-1)\big)-4c_d(c\Pi_0+R_0)(3\underline{b_d}(1-p_d)+b_{d,t_l}(2p_d-3)+p_d q_d)-(1-p_d)p_d(c\Pi_0+R_0)^2-16c_d^2 \\
&(\underline{b_d}-b_{d,t_l})(b_{d,t_l}-q_d)\big);
\end{aligned}
$$

$$\frac{dD^*}{d\overline{b_d}} = -\frac{-(1-p_d)(c\Pi_0+R_0)}{4c_d}.$$

We compute the differences between the first-order derivatives of the total donation from users with respect to upper bounds and lower bounds of user generousness parameters as follows:

$$\begin{aligned}\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}} =& \frac{(c\Pi_0+R_0)}{4c_d}\Big(-\frac{p_d}{(1-a_dw)^2(p_d(c\Pi_0+R_0)+4c_d(\underline{b_d}-b_{d,t_l}))^3}(a_d(1-p_d)w(c\Pi_0+R_0)\\ &+4a_dc_d(b_{d,t_l}w-q_dw-1)-(1-p_d)(c\Pi_0+R_0)-4c_d(b_{d,t_l}-q_d))\,(a_d\,(-4c_d(c\Pi_0+R_0)\\ &(p_d(3\underline{b_d}w-2b_{d,t_l}w-q_dw-1)+3w(b_{d,t_l}-\underline{b_d}))+(1-p_d)p_dw(c\Pi_0+R_0)^2+16c_d^2\\ &(\underline{b_d}-b_{d,t_l})(b_{d,t_l}w-q_dw-1)\big)-4c_d(c\Pi_0+R_0)(3\underline{b_d}(1-p_d)+b_{d,t_l}(2p_d-3)+p_dq_d)\big)\\ &-(1-p_d)p_d(c\Pi_0+R_0)^2-16c_d^2(\underline{b_d}-b_{d,t_l})(b_{d,t_l}-q_d)-p_d+1\Big).\end{aligned}$$

Given the feasibility constraints, we have the total donation from users monotonously increases in the lower bound and upper bound of user generousness (i.e., $\frac{dD^*}{d\underline{b_d}} > 0$ and $\frac{dD^*}{d\overline{b_d}} > 0$), and (ii) the platform's overall content quality can be more efficiently increased by enhancing the upper bound of user generousness (i.e., $\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$ if and only if the barrier parameter q_d is at a high level, i.e.,

$$\begin{aligned}q_d >& \frac{1}{4(4c_d(\underline{b_d}-b_{d,t_l})-p_d(c\Pi_0+R_0))}\Big((a_d(c\Pi_0+R_0)(p_d(2\underline{b_d}w-b_{d,t_l}w-1)+2w(b_{d,t_l}-\underline{b_d}))\\ &-4a_dc_d(\underline{b_d}-b_{d,t_l})(b_{d,t_l}w-1)+(c\Pi_0+R_0)(b_{d,t_l}(p_d-2)+2\underline{b_d}(1-p_d))+4c_db_{d,t_l}(\underline{b_d}-b_{d,t_l}))\\ &\frac{4}{1-a_dw}-\frac{\sqrt{\frac{-(1-p_d)p_d(c\Pi_0+R_0)^2\left(p_d(2p_d-1)(c\Pi_0+R_0)^2-16c_d^2(\underline{b_d}-b_{d,t_l})^2\right)}{(p_d(c\Pi_0+R_0)+4c_d(\underline{b_d}-b_{d,t_l}))^4}}(p_d(c\Pi_0+R_0)+4c_d(\underline{b_d}-b_{d,t_l}))^3}{c_dp_d(c\Pi_0+R_0)}\Bigg).\end{aligned}$$

We have also numerically checked the feasibility of this condition at $\{R_0 = 20, a_d = 0.25, a_e = 0.25, q_e = 1, w = 1.5, \overline{b_e} = 2, \underline{b_e} = 0.5, \overline{b_d} = 2, \underline{b_d} = 0.5, b_{e,t_l} = 1, b_{d,t_l} = 1, c = 1, q_d = 0.986, w_0 = 0, r = 1, g = 1, x = 1, \Pi_0 = 13, c_d = 222, c_e = 128, p_d = 0.516, p_e = 0.75, b_{d,t_u} = 1.25, b_{e,t_u} = 1.25\}$.

Proof Proposition 4

In this extension, the equilibrium overall content quality and its first-order derivatives with respect to q_e and R_0 are given as:

$$\Pi^* = \tfrac{1}{1-a_dw} \big(\Pi_0 - \big(p_e a_e^2 - \big(4c_e(\underline{b_e} - b_{e,t_l})(b_{e,t_l} - \underline{b_e}(1-p_e) - p_e q_e) w a_d^2 - 4c_e(\underline{b_e} - b_{e,t_l})(b_{e,t_l} - \underline{b_e}(1-p_e) - p_e q_e) a_d$$

$$+ a_e^2 p_e^2 (c\Pi_0 + R_0)\big) \tfrac{2a_e}{4a_d c_e(\underline{b_e} - b_{e,t_l}) + a_e p_e (c\Pi_0 + R_0)} + (1-a_dw)^2 \big(p_e b_{e,t_l}^2 + \big(b_{e,t_u} + \overline{b_e} - (b_{e,t_u} + \overline{b_e}) p_e - 2q_e$$

$$+ \tfrac{2a_e(c\Pi_0+R_0)(-a_dwb_{e,t_l} + b_{e,t_l} - \underline{b_e}(1-p_e)(1-a_dw) + p_e(a_e - q_e + a_d q_e w))}{(4a_dc_e(-\underline{b_e} + b_{e,t_l}) - a_e p_e (c\Pi_0+R_0))(1-a_dw)}\Big) b_{e,t_l} + p_e \big(q_e - (-a_dwb_{e,t_l}$$

$$+ b_{e,t_l} - \underline{b_e}(1-p_e)(1-a_dw) + p_e(a_e - q_e + a_d q_e w)) \tfrac{a_e(c\Pi_0+R_0)}{(4a_dc_e(-\underline{b_e} + b_{e,t_l}) - a_e p_e (c\Pi_0+R_0))(1-a_dw)}\Big)^2 + \underline{b_e}(p_e$$

$$-1) \Big(b_{e,t_u} + \overline{b_e} - 2q_e + \frac{2a_e(c\Pi_0+R_0)(-a_dwb_{e,t_l} + b_{e,t_l} - \underline{b_e}(1-p_e)(1-a_dw) + p_e(a_e - q_e + a_d q_e w))}{(4a_dc_e(-\underline{b_e} + b_{e,t_l}) - a_e p_e (c\Pi_0+R_0))(1-a_dw)}\Big)\Big)\Big)$$

$$\frac{a_e(c\Pi_0+R_0)}{4c_e(\underline{b_e} - b_{e,t_l})(1-a_dw)^2} + a_d \Big(-\frac{(c\Pi_0+R_0)^2(-b_{d,t_l}(1-a_dw) + \underline{b_d}(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d))^2}{(4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0))^2(1-a_dw)^2}$$

$$-g - r - \Big(\Big(p_d \Big(\Big(q_d - \tfrac{c\Pi_0+R_0}{4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0)} \tfrac{(-b_{d,t_l}(1-a_dw) + \underline{b_d}(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d))}{(1-a_dw)}\Big)^2 + b_{d,t_l}^2$$

$$-(b_{d,t_u} + \overline{b_d}) b_{d,t_l} + \underline{b_d} \Big(b_{d,t_u} + \overline{b_d} - 2q_d + \tfrac{2(c\Pi_0+R_0)}{(4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0))(1-a_dw)} (-b_{d,t_l}(1-a_dw) + \underline{b_d}$$

$$(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d)))\big) w^2 + (\underline{b_d} - b_{d,t_l}) \big(2 - w \big(b_{d,t_u} + \overline{b_d} - 2q_d + (-b_{d,t_l}(1-a_dw) + \underline{b_d}$$

$$(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d)) \tfrac{2(c\Pi_0+R_0)}{(4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0))(1-a_dw)}\Big)\Big) w - (4c_d(\underline{b_d} - b_{d,t_l})$$

$$(q_d - \underline{b_d})(1-a_dw) + (c\Pi_0 + R_0)(-a_dwb_{d,t_l} + b_{d,t_l} + a_d p_d - \underline{b_d}(1-a_dw))) \tfrac{2p_dw}{(4c_d(-\underline{b_d} + b_{d,t_l}) - p_d(c\Pi_0+R_0))(1-a_dw)}$$

$$+p_d\big) a_d^2 + 2 \Big(b_{d,t_l} - p_d q_d + \frac{p_d(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw) + \underline{b_d}(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d))}{(4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0))(1-a_dw)} - w$$

$$\Big(p_d b_{d,t_l}^2 + \Big(b_{d,t_u} + \overline{b_d} - (b_{d,t_u} + \overline{b_d}) p_d - 2q_d + \tfrac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw) + \underline{b_d}(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d))}{(4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0))(1-a_dw)}\Big)$$

$$b_{d,t_l} + p_d \Big(q_d + \tfrac{(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw) + \underline{b_d}(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d))}{(4c_d(-\underline{b_d} + b_{d,t_l}) - p_d(c\Pi_0+R_0))(1-a_dw)}\Big)^2\Big) + \underline{b_d}(1-p_d)\,(w\,(b_{d,t_u}$$

$$+\overline{b_d} - 2q_d + \tfrac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw) + \underline{b_d}(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d))}{(4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0))(1-a_dw)}\Big) - 1\Big)\Big) a_d + p_d q_d^2 + (-b_{d,t_l}$$

$$(1-a_dw) + \underline{b_d}(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d))^2 \tfrac{p_d(c\Pi_0+R_0)^2}{(4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0))^2(1-a_dw)^2} + b_{d,t_l} b_{d,t_u} + b_{d,t_l}\overline{b_d}$$

$$+b_{d,t_l}^2 p_d - b_{d,t_l} b_{d,t_u} p_d - b_{d,t_l}\overline{b_d} p_d + \tfrac{2b_{d,t_l}(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw) + \underline{b_d}(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d))}{(4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0))(1-a_dw)}$$

$$-\underline{b_d}(1-p_d) \Big(b_{d,t_u} + \overline{b_d} - 2q_d + \tfrac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw) + \underline{b_d}(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d))}{(4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0))(1-a_dw)}\Big) - 2q_d$$

$$\Big(b_{d,t_l} + \tfrac{p_d(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw) + \underline{b_d}(1-p_d)(1-a_dw) - p_d(q_dwa_d + a_d - q_d))}{(4c_d(\underline{b_d} - b_{d,t_l}) + p_d(c\Pi_0+R_0))(1-a_dw)}\Big)\Big) \tfrac{(c\Pi_0+R_0)}{4c_d(\underline{b_d} - b_{d,t_l})(1-a_dw)^2}$$

$$+w_0 - (c\Pi_0 + R_0)x - \tfrac{a_e^2(c\Pi_0+R_0)^2(-a_dwb_{e,t_l} + b_{e,t_l} - \underline{b_e}(1-p_e)(1-a_dw) + p_e(a_e - q_e + a_d q_e w))^2}{(4a_dc_e(\underline{b_e} - b_{e,t_l}) + a_e p_e(c\Pi_0+R_0))^2(1-a_dw)^2}\Big)\Big);$$

$$\frac{d\Pi^*}{dq_e} = -\frac{2a_d a_e(c\Pi_0 + R_0)(-p_e(a_d q_e w + a_e - q_e) + \underline{b_e}(1-p_e)(1-a_dw) - b_{e,t_l}(1-a_dw))}{(1-a_dw)^2(4a_dc_e(\underline{b_e} - b_{e,t_l}) + a_e p_e(c\Pi_0 + R_0))};$$

$$\frac{d\Pi^*}{dR_0} = \tfrac{1}{1-a_dw} \Big(8a_d^2 a_e^2 c_e(\underline{b_e} - b_{e,t_l})(c\Pi_0 + R_0) \tfrac{(-b_{e,t_l}(1-a_dw) + \underline{b_e}(1-p_e)(1-a_dw) - p_e(a_e - q_e + a_d q_e w))^2}{(4a_dc_e(\underline{b_e} - b_{e,t_l}) + a_e p_e(c\Pi_0+R_0))^3(1-a_dw)^2}$$

$$-\tfrac{a_e}{4c_e(\underline{b_e} - b_{e,t_l})(1-a_dw)^2} \Big(p_e a_e^2 - \tfrac{2a_e}{4a_dc_e(\underline{b_e} - b_{e,t_l}) + a_e p_e(c\Pi_0+R_0)} \big(4c_e(\underline{b_e} - b_{e,t_l})(b_{e,t_l} - \underline{b_e}(1-p_e)$$

$$-p_eq_e)wa_d^2-4c_e(\underline{b_e}-b_{e,t_l})(b_{e,t_l}-\underline{b_e}(1-p_e)-p_eq_e)a_d+a_e^2p_e^2(c\Pi_0+R_0)\Big)+(1-a_dw)^2\Big(p_eb_{e,t_l}^2+(b_{e,t_u}$$

$$+\overline{b_e}-(b_{e,t_u}+\overline{b_e})p_e-2q_e+\frac{2a_e(c\Pi_0+R_0)(-a_dwb_{e,t_l}+b_{e,t_l}-\underline{b_e}(1-p_e)(1-a_dw)+p_e(a_e-q_e+a_dq_ew))}{(4a_dc_e(-\underline{b_e}+b_{e,t_l})-a_ep_e(c\Pi_0+R_0))(1-a_dw)}\Big)$$

$$b_{e,t_l}+p_e\Big(q_e-\frac{a_e(c\Pi_0+R_0)(-a_dwb_{e,t_l}+b_{e,t_l}-\underline{b_e}(1-p_e)(1-a_dw)+p_e(a_e-q_e+a_dq_ew))}{(4a_dc_e(-\underline{b_e}+b_{e,t_l})-a_ep_e(c\Pi_0+R_0))(1-a_dw)}\Big)^2-\underline{b_e}(1-p_e)$$

$$\Big(b_{e,t_u}+\overline{b_e}-2q_e+\frac{2a_e(c\Pi_0+R_0)(-a_dwb_{e,t_l}+b_{e,t_l}-\underline{b_e}(1-p_e)(1-a_dw)+p_e(a_e-q_e+a_dq_ew))}{(4a_dc_e(-\underline{b_e}+b_{e,t_l})-a_ep_e(c\Pi_0+R_0))(1-a_dw)}\Big)\Big)\Big)+a_d$$

$$\Big(\frac{2p_e(c\Pi_0+R_0)^2(-a_dwb_{e,t_l}+b_{e,t_l}-\underline{b_e}(1-p_e)(1-a_dw)+p_e(a_e-q_e+a_dq_ew))^2a_e^3}{(4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0))^3(1-a_dw)^2}-(-a_dwb_{e,t_l}+b_{e,t_l}-\underline{b_e}$$

$$(1-p_e)(1-a_dw)+p_e(a_e-q_e+a_dq_ew))^2\frac{2(c\Pi_0+R_0)a_e^2}{(4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0))^2(1-a_dw)^2}+(-b_{d,t_l}(1-a_dw)$$

$$+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))^2\frac{2p_d(c\Pi_0+R_0)^2}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))^3(1-a_dw)^2}+(-b_{d,t_l}(1-a_dw)$$

$$+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))^2\frac{8c_d(\underline{b_d}-b_{d,t_l})(c\Pi_0+R_0)}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))^3(1-a_dw)^2}-\Big(\Big(p_d\Big(b_{d,t_l}^2$$

$$-(b_{d,t_u}+\overline{b_d})b_{d,t_l}+\Big(q_d+\frac{(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_dw)}\Big)^2+\underline{b_d}$$

$$\Big(b_{d,t_u}+\overline{b_d}-2q_d+\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)\Big)w^2+(\underline{b_d}$$

$$-b_{d,t_l})\Big(2-w\Big(b_{d,t_u}+\overline{b_d}-2q_d+\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)\Big)$$

$$w-\frac{2p_d(4c_d(\underline{b_d}-b_{d,t_l})(q_d-\underline{b_d})(1-a_dw)+(c\Pi_0+R_0)(-a_dwb_{d,t_l}+b_{d,t_l}+a_dp_d-\underline{b_d}(1-a_dw)))w}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_dw)}+p_d\Big)a_d^2$$

$$+2\Big(b_{d,t_l}-p_dq_d+\frac{p_d(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}-w\Big(p_db_{d,t_l}^2$$

$$+\Big(b_{d,t_u}+\overline{b_d}-(b_{d,t_u}+\overline{b_d})p_d-2q_d+\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)$$

$$b_{d,t_l}+p_d\Big(q_d+\frac{(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(-\underline{b_d}+b_{d,t_l})-p_d(c\Pi_0+R_0))(1-a_dw)}\Big)^2\Big)+\underline{b_d}(1-p_d)(w$$

$$\Big(b_{d,t_u}+\overline{b_d}-2q_d+\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)-1\Big)\Big)a_d+p_dq_d^2$$

$$+\frac{p_d(c\Pi_0+R_0)^2(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))^2}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))^2(1-a_dw)^2}+b_{d,t_l}b_{d,t_u}+b_{d,t_l}\overline{b_d}+b_{d,t_l}^2p_d$$

$$-b_{d,t_l}b_{d,t_u}p_d-b_{d,t_l}\overline{b_d}p_d+\frac{2b_{d,t_l}(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}-\underline{b_d}$$

$$(1-p_d)\Big(b_{d,t_u}+\overline{b_d}-2q_d+\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)-2q_d$$

$$\Big(b_{d,t_l}+\frac{p_d(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))(1-a_dw)}\Big)\Big)\frac{1}{4c_d(\underline{b_d}-b_{d,t_l})(1-a_dw)^2}$$

$$-\frac{2(c\Pi_0+R_0)(-b_{d,t_l}(1-a_dw)+\underline{b_d}(1-p_d)(1-a_dw)-p_d(q_dwa_d+a_d-q_d))^2}{(4c_d(\underline{b_d}-b_{d,t_l})+p_d(c\Pi_0+R_0))^2(1-a_dw)^2}\Big)-a_dx\Big).$$

Given the feasibility constraints (that are presented in Expressions 39 to 44), we find that the overall content quality decreases in q_e. Besides, we numerically verify that the overall content quality decreases in R_0 if and only if

$$q_e>\frac{1}{4a_d^2a_ec_dc_e^2p_e(1-a_dw)^2(\underline{b_e}-b_{e,t_l})(p_d(c\Pi_0+R_0)+4c_d(\underline{b_d}-b_{d,t_l}))^2}\Big((-a_d^2a_ec_dc_e^2(1-a_dw)^2(\underline{b_e}-b_{e,t_l})(p_d(c\Pi_0+R_0)+4c_d$$

$$(\underline{b_d}-b_{d,t_l}))^2(4a_dc_e(\underline{b_e}-b_{e,t_l})+a_ep_e(c\Pi_0+R_0))^2\Big(a_dc_ep_e\Big(a_d^2\Big(-16c_d^2(\underline{b_d}-b_{d,t_l})(p_d(w(-2wx(c\Pi_0+R_0)+\underline{b_d}(2-w(b_{d,t_u}+\overline{b_d}$$

$$-2q_d)) + b_{d,t_l}w(-b_{d,t_l} + b_{d,t_u} + \overline{b_d}) - q_d(q_d w + 2)) - 1) + w(\underline{b_d} - b_{d,t_l})(w(b_{d,t_u} + \overline{b_d} - 2q_d) - 2)) + 4c_d w^2(c\Pi_0 + R_0) (p_d^2 x$$

$$(c\Pi_0 + R_0) + 2(1 - p_d)(\underline{b_d} - b_{d,t_l})(p_d(\underline{b_d} + b_{d,t_l} - b_{d,t_u} - \overline{b_d}) - \underline{b_d} + b_{d,t_l})) + (1 - p_d)p_d w^2(c\Pi_0 + R_0)^2(p_d(\underline{b_d} + b_{d,t_l} - b_{d,t_u} - \overline{b_d})$$

$$-\underline{b_d} + b_{d,t_l}) + 64c_d^3 w^2 x(\underline{b_d} - b_{d,t_l})^2) - 2a_d (-16c_d^2(\underline{b_d} - b_{d,t_l}) (-p_d (2wx(c\Pi_0 + R_0) + q_d^2 w + q_d) + \underline{b_d}(1 - p_d)(w(b_{d,t_u} + \overline{b_d} - 2q_d)$$

$$-1) + b_{d,t_l}^2(-p_d)w + b_{d,t_l}w(-(1 - p_d)(b_{d,t_u} + \overline{b_d}) + 2q_d) + b_{d,t_l}) + 4c_d w(c\Pi_0 + R_0) (p_d^2 x(c\Pi_0 + R_0) + 2(1 - p_d)(\underline{b_d} - b_{d,t_l})(-\underline{b_d}(1 - p_d)$$

$$+b_{d,t_l}(p_d + 1) - p_d(b_{d,t_u} + \overline{b_d}))) + (1 - p_d)p_d w(c\Pi_0 + R_0)^2(-\underline{b_d}(1 - p_d) + b_{d,t_l}(p_d + 1) - p_d(b_{d,t_u} + \overline{b_d})) + 64c_d^3 wx(\underline{b_d} - b_{d,t_l})^2)$$

$$+16c_d^2(\underline{b_d} - b_{d,t_l}) (p_d (2x(c\Pi_0 + R_0) + q_d^2) - \underline{b_d}(1 - p_d)(b_{d,t_u} + \overline{b_d} - 2q_d) + b_{d,t_l}^2 p_d + b_{d,t_l}(1 - p_d)(b_{d,t_u} + \overline{b_d}) - 2b_{d,t_l}q_d)$$

$$+4c_d(c\Pi_0 + R_0) (p_d^2 x(c\Pi_0 + R_0) + 2(1 - p_d)(\underline{b_d} - b_{d,t_l})(-\underline{b_d}(1 - p_d) + b_{d,t_l}(p_d + 1) - p_d(b_{d,t_u} + \overline{b_d}))) + (1 - p_d)p_d(c\Pi_0 + R_0)^2$$

$$(-\underline{b_d}(1 - p_d) + b_{d,t_l}(p_d + 1) - p_d(b_{d,t_u} + \overline{b_d})) + 64c_d^3 x(\underline{b_d} - b_{d,t_l})^2) + a_e c_d(1 - p_e)(1 - a_d w)^2(-\underline{b_e}(1 - p_e) + b_{e,t_l}(p_e + 1) - p_e$$

$$(b_{e,t_u} + \overline{b_e}))(p_d(c\Pi_0 + R_0) + 4c_d(\underline{b_d} - b_{d,t_l}))^2))^{1/2} - 4a_d^2 a_e c_d c_e^2(1 - a_d w)(\underline{b_e} - b_{e,t_l})(\underline{b_e}(1 - p_e)(1 - a_d w) - b_{e,t_l}(1 - a_d w) - a_e p_e)$$

$$(p_d(c\Pi_0 + R_0) + 4c_d(\underline{b_d} - b_{d,t_l}))^2) \,.$$

We have also checked the feasibility of this condition numerically at $\{a_e = 0.140, q_e = 1, w = 1.765, w_0 = 0, \overline{b_e} = 8.333, \underline{b_e} = 0.5, \overline{b_d} = 16, \underline{b_d} = 1.875, r = 0, g = 0, \Pi_0 = 1, c = 1, c_d = 4.769, c_e = 17.882, b_{e,t_l} = 1, b_{e,t_u} = 4, b_{d,t_l} = 4, b_{d,t_u} = 5.646, p_d = 0.75, p_e = 0.75, q_d = 4, a_d = 0.25, R_0 = 15, x = 0.242\}$, given all the other constraints.

■

Proofs of The Extension: Linear Disutility in Making Donations

In this section, we provide proof of results presented in CHAPTER 2.7.6. In this extension, as we consider a linear disutility term for making donations in users' utility functions (i.e., $(b_{d,i} + f^{1/2} - q_d)d_i - c_d d_i$ for user i), our model yields a bang-bang solution. This implies that users who donate would donate all their budgets. Therefore, based on this practical concern, we capture individual-level budget heterogeneity and denote the budget of user i as k_i. We consider a uniform distribution $Unif(\underline{k}, \overline{k})$ to capture the heterogeneity and make our results traceable. Next, the solution process differs only in the way we derive the optimal donation amount of user i as:

$$d_i^* = \begin{cases} 0, & if\ b_{d,i} < b_d^*, \\ k_i, & otherwise. \end{cases}$$

Next, the equilibrium results are derived similarly to the main model, and hence, the details are omitted for brevity. The equilibrium levels of the key variables are shown below:

$$h^* = \frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2};$$

$$f^* = \frac{\left(\underline{k}^2-\overline{k}^2\right)^2(c\Pi_0+R_0)^2}{16(\overline{b_d}-\underline{b_d})^2};$$

$$C^* = \frac{4a_d^2c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2};$$

$$D^* = \frac{(\overline{k}-\underline{k})(\underline{k}+\overline{k})(c\Pi_0+R_0)}{8(1-a_dw)(\overline{b_d}-\underline{b_d})}\left(\frac{a_dw\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\overline{b_d}-\underline{b_d}}+4a_dc_dw-4a_d\overline{b_d}w+4a_dq_dw\right.$$
$$\left.+4a_d+\frac{\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}}-4c_d+4\overline{b_d}-4q_d\right);$$

$$\Pi^* = \frac{1}{1-a_dw}\left(-\frac{-4a_d^2a_ec_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}+a_d\left(-(a_e-(a_dw\right.\right.$$
$$-1)(\overline{b_e}-q_e))^2\frac{a_e^2(c\Pi_0+R_0)^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}+\left(\frac{a_dw\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}}\right.$$
$$\left.-4a_dc_dw+4a_d\overline{b_d}w-4a_dq_dw-4a_d-\frac{\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}}+4c_d-4\overline{b_d}+4q_d\right)(c\Pi_0+R_0)$$
$$\left.\left.\frac{-(\overline{k}-\underline{k})(\underline{k}+\overline{k})}{8(1-a_dw)(\overline{b_d}-\underline{b_d})}-\frac{\left(\underline{k}^2-\overline{k}^2\right)^2(c\Pi_0+R_0)^2}{16(\overline{b_d}-\underline{b_d})^2}-x(c\Pi_0+R_0)-g-r+w_0\right)+\Pi_0\right);$$

$$v^* = -\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}+\left(\frac{a_dw\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}}-4a_dc_dw\right.$$
$$\left.+4a_d\overline{b_d}w-4a_dq_dw-4a_d-\frac{\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}}+4c_d-4\overline{b_d}+4q_d\right)\frac{-(\overline{k}-\underline{k})(\underline{k}+\overline{k})(c\Pi_0+R_0)}{8(1-a_dw)(\overline{b_d}-\underline{b_d})}$$
$$-\frac{\left(\underline{k}^2-\overline{k}^2\right)^2(c\Pi_0+R_0)^2}{16(\overline{b_d}-\underline{b_d})^2}-x(c\Pi_0+R_0)-g-r+w_0;$$

$$b_e^* = \frac{4a_d^2c_eq_ew(\overline{b_e}-\underline{b_e})-a_da_e\overline{b_e}w(c\Pi_0+R_0)-4a_dc_e(q_e-a_e)(\overline{b_e}-\underline{b_e})+a_e\overline{b_e}(c\Pi_0+R_0)}{(1-a_dw)(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))};$$

$$
\begin{aligned}
b_d^* =& -\frac{a_d}{1-a_d w} - \frac{(\overline{k}-\underline{k})(\underline{k}+\overline{k})(c\Pi_0+R_0)}{4(\overline{b_d}-\underline{b_d})} + c_d + q_d;\\
E[K_e^*] =& \frac{4a_d c_e(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))}{(1-a_d w)(4a_d c_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))};\\
E[K_d^*] =& \frac{(c\Pi_0+R_0)\left(\frac{a_d}{1-a_d w}+\frac{\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{-4(\overline{b_d}-\underline{b_d})}-c_d+\overline{b_d}-q_d\right)}{\overline{b_d}-\underline{b_d}}.
\end{aligned}
$$

Furthermore, this solution is feasible when

$$4a_d c_e \underline{b_e} - 4a_d c_e \overline{b_e} + a_e c\Pi_0 + a_e R_0 < 0 \text{ and } a_e + (1-a_d w)(\overline{b_e}-q_e) > 0, \tag{53}$$

$$a_d > 0 \text{ and } w > 0 \text{ and } a_e > 0 \text{ and } \overline{b_e} > q_e > \underline{b_e} > 0 \text{ and } \overline{b_d} > q_d > \underline{b_d} > 0 \text{ and } \overline{k} > \underline{k} > 0, \tag{54}$$

$$\Pi_0 > 0 \text{ and } r \geq 0 \text{ and } x \geq 0 \text{ and } g \geq 0 \text{ and } R_0 > 0 \text{ and } c > 0 \text{ and } c_d > 0 \text{ and } c_e > 0 \tag{55}$$

$$\overline{b_e} > -\frac{a_e}{1-a_d w} + q_e > b_e^* > \underline{b_e} \text{ and } \overline{b_d} > -\frac{a_d}{1-a_d w} + c_d + q_d > b_d^* > \underline{b_d} \tag{56}$$

$$v^* \geq 0 \text{ and } 1 - a_d w > 0 \text{ and } h^* < q_e^2. \tag{57}$$

We numerically verify that the parameter space given the feasibility constraint set is not empty, e.g., at $\{a_d = 0.03, a_e = 3.69, q_d = 99, q_e = 73, w = 29, w_0 = 4.1, \overline{b_d} = 136, \underline{b_d} = 24, r = 27, g = 9, x = 0, c = 5, c_d = 6.67, c_e = 218, \overline{b_e} = 94, \underline{b_e} = 33, \Pi_0 = 15, R_0 = 35, \overline{k} = 18, \underline{k} = 1\}$, and all of our results are feasible.

Proof Propositions 1 and 2

In this extension, the equilibrium total content contribution from users and community support effort exerted by the platform are:

$$
\begin{aligned}
C^* &= \frac{4a_d^2 c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2};\\
h^* &= \frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}.
\end{aligned}
$$

Next, we can derive the first-order derivatives of these measures with respect to a_d as:

$$\frac{dC^*}{da_d} = -\frac{-8a_d a_e c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(-(1-a_d w)(\overline{b_e}-q_e)-a_e)}{-(1-a_d w)^3(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3}\left(4a_d^2 c_e\underline{b_e}w - 4a_d^2 c_e\overline{b_e}w + a_d^2\overline{b_e}R_0 w^2\right.$$
$$\left.-a_d^2 q_e R_0 w^2 + c\Pi_0(1-a_d w)^2(\overline{b_e}-q_e) - 2a_d\overline{b_e}R_0 w + 2a_d q_e R_0 w + a_e(c\Pi_0+R_0) + \overline{b_e}R_0 - q_e R_0\right);$$
$$\frac{dh^*}{da_d} = -\frac{2a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_d w)(\overline{b_e}-q_e))}{-(1-a_d w)^3(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3}\left(-4a_e c_e(2a_d w-1)(\overline{b_e}-\underline{b_e}) + 4c_e(1-a_d w)^2(\overline{b_e}-\underline{b_e})\right.$$
$$\left.(\overline{b_e}-q_e) + a_e^2 w(c\Pi_0+R_0)\right).$$

Given the feasibility constraints (that are presented in Expressions 53 to 56), we find that, first, h^* increases in a_d if and only if the following condition is satisfied:

$$a_d > \frac{1}{-2c_e w^2(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e)}\left(\sqrt{-a_e c_e w^2(\overline{b_e}-\underline{b_e})(a_e w(\overline{b_e}-q_e)(c\Pi_0+R_0) - 4a_e c_e(\overline{b_e}-\underline{b_e}) - 4c_e(\overline{b_e}-\underline{b_e})(\overline{b_e}-q_e))}\right.$$
$$\left.-2a_e c_e w(\overline{b_e}-\underline{b_e}) + 2c_e\underline{b_e}\overline{b_e}w - 2c_e\underline{b_e}q_e w - 2c_e\overline{b_e}^2 w + 2c_e\overline{b_e}q_e w\right).$$

Second, C^* decreases in a_d if and only if $R_0 > \frac{4a_d^2 c_e w(\overline{b_e}-\underline{b_e}) - c\Pi_0(1-a_d w)^2(\overline{b_e}-q_e) - a_e c\Pi_0}{(1-a_d w)^2(\overline{b_e}-q_e)+a_e}$. Therefore, in this extension, our main results in Propositions 1 and 2 are qualitatively the same. We have also checked the feasibility of these presented results numerically for when a_d is greater or less than the thresholds provided above.

Proof Proposition 3

We first have:

$$D^* = \frac{(\overline{k}-\underline{k})(\underline{k}+\overline{k})(c\Pi_0+R_0)}{8(1-a_d w)(\overline{b_d}-\underline{b_d})}\left(-\frac{a_d w\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}} + 4a_d c_d w - 4a_d\overline{b_d}w + 4a_d q_d w + 4a_d - 4c_d + 4\overline{b_d}\right.$$
$$\left.+\frac{\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}} - 4q_d\right);$$
$$\frac{dD^*}{d\underline{b_d}} = \frac{(\overline{k}-\underline{k})(\underline{k}+\overline{k})(c\Pi_0+R_0)}{4(1-a_d w)(\overline{b_d}-\underline{b_d})^3}\left(a_d\left(w\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0) - 2\underline{b_d}(c_d w - \overline{b_d}w + q_d w + 1) + 2\overline{b_d}(c_d w\right.\right.$$
$$\left.\left.+q_d w + 1) - 2\overline{b_d}^2 w\right) - c\underline{k}^2\Pi_0 + c\overline{k}^2\Pi_0 - 2c_d(\overline{b_d}-\underline{b_d}) - 2\underline{b_d}\overline{b_d} + 2\underline{b_d}q_d + 2\overline{b_d}^2 - 2\overline{b_d}q_d - \underline{k}^2 R_0 + \overline{k}^2 R_0\right);$$
$$\frac{dD^*}{d\overline{b_d}} = \frac{-(\overline{k}-\underline{k})(\underline{k}+\overline{k})(c\Pi_0+R_0)}{4(1-a_d w)(\overline{b_d}-\underline{b_d})^3}\left(a_d\left(w\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0) - 2\underline{b_d}(c_d w + \overline{b_d}w + q_d w + 1) + 2\overline{b_d}(c_d w + q_d w\right.\right.$$
$$\left.\left.+1) + 2\underline{b_d}^2 w\right) - c\underline{k}^2\Pi_0 + c\overline{k}^2\Pi_0 - 2c_d(\overline{b_d}-\underline{b_d}) - 2\underline{b_d}^2 + 2\underline{b_d}\overline{b_d} + 2\underline{b_d}q_d - 2\overline{b_d}q_d - \underline{k}^2 R_0 + \overline{k}^2 R_0\right).$$

We compute the differences between the first-order derivatives of the total donation from users with respect to upper bounds and lower bounds of user generousness parameters as follows:

$$\frac{dD^*}{d\overline{b_d}} - \frac{dD^*}{d\underline{b_d}} = \frac{-(\overline{k}-\underline{k})(\underline{k}+\overline{k})(c\Pi_0+R_0)}{2(1-a_d w)(\overline{b_d}-\underline{b_d})^3} \left(a_d \left(w \left(\underline{k}^2 - \overline{k}^2 \right) (c\Pi_0 + R_0) - 2\underline{b_d}(c_d w + q_d w + 1) + 2\overline{b_d} (c_d w + q_d w + 1) + \underline{b_d}^2 w - \overline{b_d}^2 w \right) - c\underline{k}^2 \Pi_0 + c\overline{k}^2 \Pi_0 - 2c_d(\overline{b_d} - \underline{b_d}) - \underline{b_d}^2 + 2\underline{b_d} q_d + \overline{b_d}^2 - 2\overline{b_d} q_d - \underline{k}^2 R_0 + \overline{k}^2 R_0 \right).$$

Given the feasibility constraints, we have the total donation from users monotonously increases in the lower bound and upper bound of user generousness (i.e., $\frac{dD^*}{d\underline{b_d}} > 0$ and $\frac{dD^*}{d\overline{b_d}} > 0$), and (ii) the platform's overall content quality can be more efficiently increased by enhancing the upper bound of user generousness (i.e., $\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$) if and only if the barrier parameter q_d is at a high level, i.e.,

$$q_d > \frac{1}{2(1-a_d w)(\overline{b_d}-\underline{b_d})} \left(a_d \left(w \left(\underline{k}^2 - \overline{k}^2 \right) (c\Pi_0 + R_0) - 2\underline{b_d}(c_d w + 1) + 2\overline{b_d}(c_d w + 1) + \underline{b_d}^2 w - \overline{b_d}^2 w \right) - c\underline{k}^2 \Pi_0 + c\overline{k}^2 \Pi_0 - 2c_d(\overline{b_d} - \underline{b_d}) - \underline{b_d}^2 + \overline{b_d}^2 - \underline{k}^2 R_0 + \overline{k}^2 R_0 \right).$$

We have numerically checked the feasibility of this condition at $\{a_d = 0.029, a_e = 3.688, q_d = 99, q_e = 73, R_0 = 20, w = 1500, \overline{b_d} = 128, w_0 = 0, \underline{b_d} = 98.417, r = 0, g = 0, x = 0, c = 5.125, c_d = 15.594, c_e = 16, \overline{b_e} = 74, \underline{b_e} = 1, \Pi_0 = 1, \overline{k} = 2, \underline{k} = 1\}$.

Proof Proposition 4

In this extension, the equilibrium overall content quality is

$$\Pi^* = \frac{1}{1-a_d w} \left(-\frac{-4a_d^2 a_e c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2} + a_d \left(-\frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2} + \frac{-(\overline{k}-\underline{k})(\underline{k}+\overline{k})(c\Pi_0+R_0)\left(\frac{a_d w\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}} - 4a_d c_d w + 4a_d \overline{b_d} w - 4a_d q_d w - 4a_d - \frac{\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}} + 4c_d - 4\overline{b_d} + 4q_d \right)}{8(1-a_d w)(\overline{b_d}-\underline{b_d})} - \frac{\left(\underline{k}^2 - \overline{k}^2\right)^2 (c\Pi_0 + R_0)^2}{16(\overline{b_d} - \underline{b_d})^2} - x(c\Pi_0 + R_0) - g - r + w_0 \right) + \Pi_0 \right).$$

Therefore, we can derive its first-order derivatives with respect to q_e and R_0 as

$$
\begin{aligned}
\frac{d\Pi^*}{dq_e} =& \frac{2a_d a_e(c\Pi_0 + R_0)(a_e + (1 - a_d w)(\overline{b_e} - q_e))}{(1 - a_d w)^2(-4a_d c_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))};\\
\frac{d\Pi^*}{dR_0} =& \tfrac{a_d}{1-a_d w} \left(\tfrac{2a_e^3(c\Pi_0+R_0)^2(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3} + \tfrac{-8a_d a_e^2 c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3} \right.\\
&- \tfrac{2a_e^2(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2} - \tfrac{-4a_d a_e c_e(\overline{b_e}-\underline{b_e})(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2} + (-4a_d c_d w\\
&\left. +4a_d\overline{b_d}w - 4a_d q_d w - 4a_d + \tfrac{a_d w\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}} - \tfrac{\left(\underline{k}^2-\overline{k}^2\right)(c\Pi_0+R_0)}{\underline{b_d}-\overline{b_d}} + 4c_d - 4\overline{b_d} + 4q_d\right) \tfrac{-(\overline{k}-\underline{k})(\underline{k}+\overline{k})}{8(1-a_d w)(\overline{b_d}-\underline{b_d})}\\
&\left. - \frac{\left(\underline{k}^2 - \overline{k}^2\right)^2 (c\Pi_0 + R_0)}{8(\overline{b_d} - \underline{b_d})^2} + \frac{(\overline{k} - \underline{k})^2(\underline{k} + \overline{k})^2(c\Pi_0 + R_0)}{8(\overline{b_d} - \underline{b_d})^2} - x \right).
\end{aligned}
$$

Given the feasibility constraints (that are presented in the Expressions 53 to 56), we find that the overall content quality of the platform decreases in q_e. Besides, it decreases in R_0 if and only if

$$
\begin{aligned}
q_e >& \tfrac{1}{8(1-a_d w)^2} \left(8a_d^2\overline{b_e}w^2 - \tfrac{\sqrt{2}}{a_d a_e c_e(\overline{b_d}-\underline{b_d})^2(\overline{b_e}-\underline{b_e})} \right. (-a_d a_e c_e(1 - a_d w)^3(\overline{b_d} - \underline{b_d})^2(\overline{b_e} - \underline{b_e})(-4a_d c_e(\overline{b_e} - \underline{b_e}) + a_e\\
&(c\Pi_0 + R_0))^2 \left(a_d \left(-w \left(\underline{k}^2 - \overline{k}^2\right)^2 (c\Pi_0 + R_0) + 4\underline{b_d} \left(\underline{k}^2(c_d w - \overline{b_d}w + q_d w + 1) - \overline{k}^2(c_d w - \overline{b_d}w + q_d w + 1)\right.\right.\right.\\
&\left.\left. -4\overline{b_d}wx\right) - 4\overline{b_d} \left(\underline{k}^2 - \overline{k}^2\right) (c_d w + q_d w + 1) + 8\underline{b_d}^2 wx + 4\overline{b_d}^2 w \left(\underline{k}^2 - \overline{k}^2 + 2x\right)\right) + c\underline{k}^4\Pi_0 - 2c\underline{k}^2\overline{k}^2\Pi_0 + c\overline{k}^4\Pi_0\\
&-4c_d\underline{b_d}\underline{k}^2 + 4c_d\underline{b_d}\overline{k}^2 + 4c_d\overline{b_d}\underline{k}^2 - 4c_d\overline{b_d}\overline{k}^2 - 8\underline{b_d}^2 x + 4\underline{b_d}\overline{b_d}\underline{k}^2 - 4\underline{b_d}\overline{b_d}\overline{k}^2 + 16\underline{b_d}\overline{b_d}x - 4\underline{b_d}\underline{k}^2 q_d + 4\underline{b_d}\overline{k}^2 q_d - 4\overline{b_d}^2\underline{k}^2\\
&\left.\left.+4\overline{b_d}^2\overline{k}^2 - 8\overline{b_d}^2 x + 4\overline{b_d}\underline{k}^2 q_d - 4\overline{b_d}\overline{k}^2 q_d + \underline{k}^4 R_0 - 2\underline{k}^2\overline{k}^2 R_0 + \overline{k}^4 R_0\right)\right) - 8a_d w(a_e + 2\overline{b_e}) + 8(a_e + \overline{b_e})\Big).
\end{aligned}
$$

We have also checked the feasibility of this condition numerically at $\{a_d = 0.029, a_e = 3.688, q_d = 99, q_e = 73, w = 22, w_0 = 15000, \overline{b_d} = 128, \underline{b_d} = 49, r = 1, g = 1, c = 1, c_d = 15, c_e = 50845.278, \overline{b_e} = 77, \underline{b_e} = 30, \Pi_0 = 5258, R_0 = 10516, \overline{k} = 1, \underline{k} = 0.5, x = 0.249\}$.

■

Proofs of The Extension: Heterogeneous Platform Usage

In this section, we provide proof of results presented in CHAPTER 2.7.7. In this extension, as we consider the level of generousness might be positively correlated with the level of site usage at the individual level, we define the cost of serving user i as $x_i = m_d b_{d,i} + m_e b_{e,i}$. By following the same solution process as our main model, we compute the equilibrium level of some important variables as below:

$$
\begin{aligned}
h^* =& \frac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2};\\
f^* =& \frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2};\\
C^* =& \frac{4a_d^2c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2};\\
D^* =& \frac{4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2};\\
\Pi^* =& \tfrac{1}{1-a_dw}\Big(-\tfrac{-4a_d^2a_ec_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}+a_d\big(-(a_e+(1-a_dw)(\overline{b_e}-q_e))^2\\
&\tfrac{a_e^2(c\Pi_0+R_0)^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}-\tfrac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\\
&-\tfrac{-4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}-\tfrac{1}{2}c_x(\overline{b_d}-\underline{b_d})(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)\\
&(\underline{b_d}m_d+\overline{b_d}m_d+m_e(\underline{b_e}+\overline{b_e}))-g-r+w_0\big)+\Pi_0\Big);\\
v^* =& -\tfrac{a_e^2(c\Pi_0+R_0)^2(a_e+(1-a_dw)(\overline{b_e}-q_e))^2}{(1-a_dw)^2(-4a_dc_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}-\tfrac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\\
&+\frac{4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}-\frac{1}{2}c_x(\overline{b_d}-\underline{b_d})(\overline{b_e}-\underline{b_e})\\
&(c\Pi_0+R_0)(\underline{b_d}m_d+\overline{b_d}m_d+m_e(\underline{b_e}+\overline{b_e}))-g-r+w_0;\\
b_e^* =& \frac{-4a_d^2c_eq_ew(\overline{b_e}-\underline{b_e})+a_da_e\overline{b_e}w(c\Pi_0+R_0)+4a_dc_e(q_e-a_e)(\overline{b_e}-\underline{b_e})-a_e\overline{b_e}(c\Pi_0+R_0)}{(1-a_dw)(4a_dc_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))};\\
b_d^* =& \frac{-a_d\overline{b_d}w(c\Pi_0+R_0)+4a_dc_d(\overline{b_d}-\underline{b_d})(q_dw+1)+c\overline{b_d}\Pi_0+4c_d\underline{b_d}q_d-4c_d\overline{b_d}q_d+\overline{b_d}R_0}{(1-a_dw)(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)};\\
E[K_e^*] =& \frac{4a_dc_e(c\Pi_0+R_0)(a_e+(1-a_dw)(\overline{b_e}-q_e))}{(1-a_dw)(4a_dc_e(\overline{b_e}-\underline{b_e})-a_e(c\Pi_0+R_0))};\\
E[K_d^*] =& \frac{4c_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{(1-a_dw)(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)}.
\end{aligned}
$$

Furthermore, this solution is feasible when

$$4a_dc_e\underline{b_e}-4a_dc_e\overline{b_e}+a_ec\Pi_0+a_eR_0<0 \text{ and } a_e+(1-a_dw)(\overline{b_e}-q_e)>0, \tag{58}$$

$$a_d>0 \text{ and } w>0 \text{ and } a_e>0 \text{ and } \overline{b_e}>q_e>\underline{b_e}>0 \text{ and } \overline{b_d}>q_d>\underline{b_d}>0 \text{ and } m_e>0 \text{ and } m_d>0, \tag{59}$$

$$\Pi_0>0 \text{ and } r\geq 0 \text{ and } x\geq 0 \text{ and } g\geq 0 \text{ and } R_0>0 \text{ and } c>0 \text{ and } c_d>0 \text{ and } c_e>0 \text{ and } c_x>0, \tag{60}$$

$$\overline{b_e}>-\frac{a_e}{1-a_dw}+q_e>b_e^*>\underline{b_e} \text{ and } \overline{b_d}>-\frac{a_d}{1-a_dw}+c_d+q_d>b_d^*>\underline{b_d}, \tag{61}$$

$$v^* \geq 0 \text{ and } 1 - a_d w > 0 \text{ and } h^* < q_e^2. \tag{62}$$

We numerically verify that the parameter space given the feasibility constraint set is not empty, e.g., at $\{a_d = 0.25, a_e = 1.5, q_d = 2, q_e = 8, w = 1, w_0 = 10000, \overline{b_e} = 9, \underline{b_e} = 4.5, \overline{b_d} = 13, \underline{b_d} = 1, r = 1, g = 1, x = 1, \Pi_0 = 2.5, R_0 = 200, c = 1, c_d = 78, c_e = 204, m_d = 1, m_e = 1, c_x = 0.05\}$, and all of our results are feasible.

Proof Propositions 1 and 2

In this extension, the equilibrium total content contribution from users and community support effort exerted by the platform are:

$$C^* = \frac{4a_d^2 c_e(\overline{b_e} - \underline{b_e})(c\Pi_0 + R_0)(a_e + (1 - a_d w)(\overline{b_e} - q_e))^2}{(1 - a_d w)^2(-4a_d c_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))^2};$$
$$h^* = \frac{a_e^2(c\Pi_0 + R_0)^2(a_e + (1 - a_d w)(\overline{b_e} - q_e))^2}{(1 - a_d w)^2(-4a_d c_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))^2}.$$

Next, we can derive the first-order derivatives of these measures with respect to a_d as:

$$\frac{dC^*}{da_d} = -\frac{-8a_d a_e c_e(\overline{b_e} - \underline{b_e})(c\Pi_0 + R_0)(-(1 - a_d w)(\overline{b_e} - q_e) - a_e)}{-(1 - a_d w)^3(-4a_d c_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))^3}\left(4a_d^2 c_e \underline{b_e} w - 4a_d^2 c_e \overline{b_e} w + a_d^2 \overline{b_e} R_0 w^2 - a_d^2 q_e R_0 w^2\right.$$
$$\left. + c\Pi_0(1 - a_d w)^2(\overline{b_e} - q_e) - 2a_d \overline{b_e} R_0 w + 2a_d q_e R_0 w + a_e(c\Pi_0 + R_0) + \overline{b_e} R_0 - q_e R_0\right);$$
$$\frac{dh^*}{da_d} = -\frac{2a_e^2(c\Pi_0 + R_0)^2(a_e + (1 - a_d w)(\overline{b_e} - q_e))\left(-4a_e c_e(2a_d w - 1)(\overline{b_e} - \underline{b_e}) + 4c_e(1 - a_d w)^2(\overline{b_e} - \underline{b_e})(\overline{b_e} - q_e) + a_e^2 w(c\Pi_0 + R_0)\right)}{-(1 - a_d w)^3(-4a_d c_e(\overline{b_e} - \underline{b_e}) + a_e(c\Pi_0 + R_0))^3}.$$

Given the feasibility constraints (that are presented in Expressions 58 to 61), we find that, first, h^* increases in a_d if and only if the following condition is satisfied:

$$a_d > \frac{1}{-2c_e w^2(\overline{b_e} - \underline{b_e})(\overline{b_e} - q_e)}\left(\sqrt{-a_e c_e w^2(\overline{b_e} - \underline{b_e})(a_e w(\overline{b_e} - q_e)(c\Pi_0 + R_0) - 4a_e c_e(\overline{b_e} - \underline{b_e}) - 4c_e(\overline{b_e} - \underline{b_e})(\overline{b_e} - q_e))}\right.$$
$$\left. -2a_e c_e w(\overline{b_e} - \underline{b_e}) + 2c_e \underline{b_e}\overline{b_e} w - 2c_e \underline{b_e} q_e w - 2c_e \overline{b_e}^2 w + 2c_e \overline{b_e} q_e w\right).$$

Second, C^* decreases in a_d if and only if $R_0 > \frac{4a_d^2 c_e w(\overline{b_e} - \underline{b_e}) - c\Pi_0(1 - a_d w)^2(\overline{b_e} - q_e) - a_e c\Pi_0}{(1 - a_d w)^2(\overline{b_e} - q_e) + a_e}$. Therefore, in this extension, our main results in Propositions 1 and 2 are qualitatively the same. We have also checked the feasibility of these presented results numerically for when a_d is greater or less than the thresholds provided above.

Proof Proposition 3

We first have:

$$
\begin{aligned}
D^* =& \frac{4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2};\\
\frac{dD^*}{d\underline{b_d}} =& -\frac{4c_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2(c\Pi_0-4c_d\underline{b_d}+4c_d\overline{b_d}+R_0)}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3};\\
\frac{dD^*}{d\overline{b_d}} =& -\frac{4c_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}(a_d(c\Pi_0+R_0)(2\underline{b_d}w-3\overline{b_d}w+q_dw+1)-4a_dc_d(\overline{b_d}-\underline{b_d})\\
&(2\underline{b_d}w-\overline{b_d}w-q_dw-1)-(c\Pi_0+R_0)(2\underline{b_d}-3\overline{b_d}+q_d)+4c_d(\overline{b_d}-\underline{b_d})(2\underline{b_d}-\overline{b_d}-q_d)).
\end{aligned}
$$

We compute the differences between the first-order derivatives of the total donation from users with respect to upper bounds and lower bounds of user generousness parameters as follows:

$$
\frac{dD^*}{d\overline{b_d}}-\frac{dD^*}{d\underline{b_d}} = -\frac{4c_d(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_dw-1)-\overline{b_d}+q_d)^2(c\Pi_0-4c_d\underline{b_d}+4c_d\overline{b_d}+R_0)}{(1-a_dw)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}.
$$

Given the feasibility constraints, we have the total donation from users monotonously increases in the lower bound and upper bound of user generousness (i.e., $\frac{dD^*}{d\underline{b_d}} > 0$ and $\frac{dD^*}{d\overline{b_d}} > 0$), and (ii) the platform's overall content quality can be more efficiently increased by enhancing the upper bound of user generousness (i.e., $\frac{dD^*}{d\overline{b_d}} > \frac{dD^*}{d\underline{b_d}}$) if and only if the barrier parameter q_d is at a high level, i.e.,

$$
q_d > \frac{a_d(c\Pi_0+R_0)(\underline{b_d}w-2\overline{b_d}w+1)-4a_dc_d(\overline{b_d}-\underline{b_d})(\underline{b_d}w-1)-(\underline{b_d}-2\overline{b_d})(c\Pi_0+R_0)+4c_d\underline{b_d}(\overline{b_d}-\underline{b_d})}{(1-a_dw)(c\Pi_0-4c_d\underline{b_d}+4c_d\overline{b_d}+R_0)}.
$$

Proof Proposition 4

As we consider a heterogeneous cost of serving users, the insight cannot be derived based on x. Thus, we consider c_x instead. In this extension, the equilibrium overall content quality is

$$
\begin{aligned}
\Pi^* =&\tfrac{1}{1-a_d w}\Big(-\tfrac{-4a_d^2 a_e c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}+a_d\big(-(a_e+(1-a_d w)(\overline{b_e}-q_e))^2\\
&\frac{a_e^2(c\Pi_0+R_0)^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}-\frac{(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\\
&-\tfrac{-4c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}-\tfrac{1}{2}c_x(\overline{b_d}-\underline{b_d})(q_e-a_e)(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)\\
&(\underline{b_d}m_d+\overline{b_d}m_d+m_e(\underline{b_e}+\overline{b_e}))-g-r+w_0\big)+\Pi_0\Big)\,.
\end{aligned}
$$

Therefore, we can derive its first-order derivatives with respect to q_e and R_0 as

$$
\begin{aligned}
\frac{d\Pi^*}{dq_e} =&\frac{2a_d a_e(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))};\\
\frac{d\Pi^*}{dR_0} =&\frac{a_d}{1-a_d w}(\theta_{x,e_7}-\frac{1}{2}(\overline{b_d}-\underline{b_d})(q_e-a_e)(\overline{b_e}-\underline{b_e})(\underline{b_d}m_d+\overline{b_d}m_d+m_e(\underline{b_e}+\overline{b_e}))c_x)\\
=&\tfrac{a_d}{1-a_d w}\Big(\tfrac{2a_e^3(c\Pi_0+R_0)^2(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3}+\tfrac{-8a_d a_e^2 c_e(\overline{b_e}-\underline{b_e})(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^3}\\
&-\frac{2a_e^2(c\Pi_0+R_0)(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}-\frac{-4a_d a_e c_e(\overline{b_e}-\underline{b_e})(a_e+(1-a_d w)(\overline{b_e}-q_e))^2}{(1-a_d w)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2}\\
&+\frac{2(c\Pi_0+R_0)^2(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}+\frac{-8c_d(\overline{b_d}-\underline{b_d})(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^3}\\
&-\frac{-4c_d(\overline{b_d}-\underline{b_d})(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}-\frac{2(c\Pi_0+R_0)(a_d(\overline{b_d}w-q_d w-1)-\overline{b_d}+q_d)^2}{(1-a_d w)^2(c\Pi_0-4c_d(\overline{b_d}-\underline{b_d})+R_0)^2}\\
&-\frac{1}{2}c_x(\overline{b_d}-\underline{b_d})(q_e-a_e)(\overline{b_e}-\underline{b_e})(\underline{b_d}m_d+\overline{b_d}m_d+m_e(\underline{b_e}+\overline{b_e}))\Big)\,.
\end{aligned}
$$

Given the feasibility constraints (that are presented in the Expressions 53 to 56), we find that the overall content quality of the platform decreases in q_e. Besides, it decreases in R_0 if and only if

$$
\begin{aligned}
q_e >&\tfrac{1}{4(1-a_d w)^2}\Big(4a_d^2\overline{b_e}w^2-4a_d(a_e+2\overline{b_e})w+4(a_e+\overline{b_e})+\tfrac{\sqrt{2}}{-a_d a_e c_e(\overline{b_e}-\underline{b_e})(-4c_d(\overline{b_d}-\underline{b_d})+c\Pi_0+R_0)^2}\big(-a_d a_e c_e(\overline{b_d}-\underline{b_d})(\overline{b_e}-\underline{b_e})\\
&(-4c_d(\overline{b_d}-\underline{b_d})+c\Pi_0+R_0)^2(-4a_d c_e(\overline{b_e}-\underline{b_e})+a_e(c\Pi_0+R_0))^2(1-a_d w)^2\big(16c_d^2c_x\underline{b_e}m_d\underline{b_d}^3-16c_d^2c_x\overline{b_e}m_d\underline{b_d}^3-16c_d^2c_x\overline{b_d}\underline{b_e}m_d\underline{b_d}^2\\
&+16c_d^2c_x\overline{b_d b_e}m_d\underline{b_d}^2+16c_d^2c_x\underline{b_e}^2m_e\underline{b_d}^2-16c_d^2c_x\overline{b_e}^2m_e\underline{b_d}^2+8cc_dc_x\underline{b_e}m_d\Pi_0\underline{b_d}^2-8cc_dc_x\overline{b_e}m_d\Pi_0\underline{b_d}^2+8c_dc_x\underline{b_e}m_dR_0\underline{b_d}^2-8c_dc_x\overline{b_e}\\
&m_dR_0\underline{b_d}^2+c^2c_x\underline{b_e}m_d\Pi_0^2\underline{b_d}-c^2c_x\overline{b_e}m_d\Pi_0^2\underline{b_d}+c_x\underline{b_e}m_dR_0^2\underline{b_d}-c_x\overline{b_e}m_dR_0^2\underline{b_d}-16c_d^2c_x\overline{b_d}^2\underline{b_e}m_d\underline{b_d}+16c_d^2c_x\overline{b_d}^2\overline{b_e}m_d\underline{b_d}-32c_d^2c_x\overline{b_d}\underline{b_e}^2\\
&m_e\underline{b_d}+32c_d^2c_x\overline{b_d b_e}^2m_e\underline{b_d}+8cc_dc_x\underline{b_e}^2m_e\Pi_0\underline{b_d}-8cc_dc_x\overline{b_e}^2m_e\Pi_0\underline{b_d}+8c_dc_x\underline{b_e}^2m_eR_0\underline{b_d}-8c_dc_x\overline{b_e}^2m_eR_0\underline{b_d}+2cc_x\underline{b_e}m_d\Pi_0R_0\underline{b_d}\\
&-2cc_x\overline{b_e}m_d\Pi_0R_0\underline{b_d}+8c_d\overline{b_d}^2+c^2c_x\overline{b_d}\underline{b_e}m_d\Pi_0^2-c^2c_x\overline{b_d b_e}m_d\Pi_0^2+c^2c_x\underline{b_e}^2m_e\Pi_0^2-c^2c_x\overline{b_e}^2m_e\Pi_0^2+8c_dq_d^2+c_x\overline{b_d}\underline{b_e}m_dR_0^2\\
&-c_x\overline{b_d b_e}m_dR_0^2+c_x\underline{b_e}^2m_eR_0^2-c_x\overline{b_e}^2m_eR_0^2+16c_d^2c_x\overline{b_d}^3\underline{b_e}m_d-16c_d^2c_x\overline{b_d}^3\overline{b_e}m_d+16c_d^2c_x\overline{b_d}^2\underline{b_e}^2m_e-16c_d^2c_x\overline{b_d}^2\overline{b_e}^2m_e-8cc_dc_x\\
&\overline{b_d}^2\underline{b_e}m_d\Pi_0+8cc_dc_x\overline{b_d}^2\overline{b_e}m_d\Pi_0-8cc_dc_x\overline{b_d}\underline{b_e}^2m_e\Pi_0+8cc_dc_x\overline{b_d b_e}^2m_e\Pi_0-16c_d\overline{b_d}q_d-8c_dc_x\overline{b_d}^2\underline{b_e}m_dR_0+8c_dc_x\overline{b_d}^2\overline{b_e}m_dR_0
\end{aligned}
$$

$$-8c_dc_x\overline{b_d}\underline{b_e}^2m_eR_0+8c_dc_x\overline{b_d\overline{b_e}}^2m_eR_0+2cc_x\overline{b_d}\underline{b_e}m_d\Pi_0R_0-2cc_x\overline{b_d\overline{b_e}}m_d\Pi_0R_0+2cc_x\underline{b_e}^2m_e\Pi_0R_0-2cc_x\overline{b_e}^2m_e\Pi_0R_0-2a_d$$

$$\big(-16c_d^2c_x(\overline{b_e}-\underline{b_e})(\underline{b_d}m_d+\overline{b_d}m_d+(\underline{b_e}+\overline{b_e})m_e)w(\overline{b_d}-\underline{b_d})^2-c_x(\overline{b_e}-\underline{b_e})(\underline{b_d}m_d+\overline{b_d}m_d+(\underline{b_e}+\overline{b_e})m_e)(c\Pi_0+R_0)^2w-8c_d$$

$$\Big((-cc_x(\overline{b_e}-\underline{b_e})m_d\Pi_0-c_x(\overline{b_e}-\underline{b_e})m_dR_0-1)w\overline{b_d}^2+\Big(c_xm_eR_0w\underline{b_e}^2+cc_x\Big(\underline{b_e}^2-\overline{b_e}^2\Big)m_e\Pi_0w+2q_dw-c_x\overline{b_e}^2m_eR_0w+1\Big)\overline{b_d}$$

$$-q_d-q_d^2w+c_x\underline{b_d}(\overline{b_e}-\underline{b_e})(\underline{b_d}m_d+(\underline{b_e}+\overline{b_e})m_e)(c\Pi_0+R_0)w)\big)+a_d^2\big(-c_x(\overline{b_e}-\underline{b_e})(\underline{b_d}m_d+\overline{b_d}m_d+(\underline{b_e}+\overline{b_e})m_e)(c\Pi_0+R_0)^2w^2$$

$$-16c_d^2c_x(\overline{b_d}-\underline{b_d})^2(\overline{b_e}-\underline{b_e})(\underline{b_d}m_d+\overline{b_d}m_d+(\underline{b_e}+\overline{b_e})m_e)w^2-8c_d\big(-q_d^2w^2-cc_x\underline{b_d}^2\underline{b_e}m_d\Pi_0w^2+cc_x\underline{b_d}^2\overline{b_e}m_d\Pi_0w^2-cc_x\underline{b_d\underline{b_e}}^2m_e$$

$$\Pi_0w^2+cc_x\underline{b_d}\overline{b_e}^2m_e\Pi_0w^2-c_x\underline{b_d}^2\underline{b_e}m_dR_0w^2+c_x\underline{b_d}^2\overline{b_e}m_dR_0w^2-c_x\underline{b_d\underline{b_e}}^2m_eR_0w^2+c_x\underline{b_d}\overline{b_e}^2m_eR_0w^2+\overline{b_d}^2(-cc_x(\overline{b_e}-\underline{b_e})m_d\Pi_0$$

$$-c_x(\overline{b_e}-\underline{b_e})m_dR_0-1)w^2-2q_dw+\overline{b_d}\Big(c_xm_eR_0w\underline{b_e}^2+cc_x\Big(\underline{b_e}^2-\overline{b_e}^2\Big)m_e\Pi_0w+2q_dw-c_x\overline{b_e}^2m_eR_0w+2\Big)w-1\Big)\Big)\Big)\Big)^{1/2}\Big).$$

We have also checked the feasibility of this condition numerically at $\{a_d = 0.25, a_e = 1.5, q_d = 2, q_e = 8, w = 1, w_0 = 10000, \overline{b_e} = 9, \underline{b_e} = 4.5, \overline{b_d} = 13, \underline{b_d} = 1, r = 1, g = 1, x = 1, \Pi_0 = 2.5, R_0 = 200, c = 1, c_d = 78, c_e = 204, m_d = 1, m_e = 1, c_x = 0.0001\}$.

■

APPENDIX B

E-COMPANION OF CHAPTER 3

Proof of Lemma 2

We prove Lemma 2 by (1) applying an order statistics method to represent the distribution of the minimum offer rate $C_t = \min_{i \in N_t} \ c_i$ given $n_t = |N_t|$, the number of valid offers at countdown time t, (2) using a queue-theoretical model to determine the expected minimum offer rate $E[C_t]$ for a given countdown time t, and (3) solving for the expected sourcing cost $E[S_T]$ when the request lead time is T. We now elaborate each step of this process.

Step 1: We calculate the probability distribution of the minimum offer rate at lead t, i.e., C_t. Given the cumulative distribution function of the offer rate $G(\cdot)$ and the corresponding density function $g_t(\cdot)$, we have the density function of minimum offer rate at countdown time t as

$$f_t(C_t|n_t) = n_t[1 - G_t(C_t)]^{n_t - 1} g_t(C_t).$$

Next, because we consider that drivers' offer rates (i.e., c_i) at countdown time t follow a uniform distribution $Unif(\mu_t - \delta_t, \mu_t + \delta_t)$, it follows that $G_t(C_t) = \frac{C_t - \mu_t + \delta_t}{2\delta_t}$ and $g_t(C_t) = \frac{1}{2\delta_t}$. Thus, the conditional P.D.F. and C.D.F. of the minimum offer rate at countdown time t are

$$\begin{aligned} f_t(C_t|n_t) =& \frac{n_t}{2\delta_t}\left[1 - \frac{C_t - (\mu_t - \delta_t)}{2\delta_t}\right]^{n_t - 1}, \\ F_t(C_t|n_t) =& \int_{\mu_t - \delta_t}^{\mu_t + \delta_t} f_t(C_t|n_t) dC_t = 1 - \left(\frac{\mu_t + \delta_t - C_t}{2\delta_t}\right)^{n_t}. \end{aligned}$$

Hence, the conditional expectation of minimum offer rate at countdown time t given the availability of drivers (i.e., n_t) as

$$E[C_t|n_t] = \int_{\mu_t-\delta_t}^{\mu_t+\delta_t} C_t f_t(C_t|n_t) dC_t = \int_{\mu_t-\delta_t}^{\mu_t+\delta_t} C_t \, \frac{n_t}{2\delta_t} \left(1 - \frac{C_t - (\mu_t - \delta_t)}{2\delta_t}\right)^{n_t-1} \, dC_t$$

Utilizing a simple transformation of $u = \frac{C_t-(\mu_t-\delta_t)}{2\delta_t}$, we have $C_t = 2\delta_t u + (\mu_t - \delta_t)$ and $dC_t = 2\delta_t du$. It follows that

$$\begin{aligned}
&\int_{\mu_t-\delta_t}^{\mu_t+\delta_t} C_t \, \frac{n_t}{2\delta_t} \left(1 - \frac{C_t - (\mu_t - \delta_t)}{2\delta_t}\right)^{n_t-1} \, dC_t \\
=&n_t \int_0^1 (2\delta_t u + \mu_t - \delta_t)(1-u)^{n_t-1} du \\
=&2n_t\delta_t \int_0^1 u(1-u)^{n_t-1} du + n_t(\mu_t - \delta_t) \int_0^1 (1-u)^{n_t-1} du \\
=&2n_t\delta_t B(2, n_t) + n_t(\mu_t - \delta_t) B(1, n_t),
\end{aligned}$$

where $B(x, y) = \int_0^1 u^{x-1}(1-u)^{y-1} du = \frac{(x-1)!(y-1)!}{(x+y-1)!}$ is the Beta function. By replacing u with $\frac{C_t-(\mu_t-\delta_t)}{2\delta_t}$, it follows that

$$E[C_t|n_t] = \frac{\mu_t n_t + \mu_t - n_t\delta_t + \delta_t}{n_t + 1}. \tag{63}$$

Step 2: Given that $P(n_t)$ is the probability that the number of valid offers at countdown time t is n_t, and given $E[C_t|n_t]$ from Expression 63, the unconditional expectation of minimum offer rate at countdown time t is

$$E[C_t] = \sum_{n_t=0}^{\infty} E[C_t|n_t] P(n_t) = \sum_{n_t=0}^{\infty} \frac{\mu_t n_t + \mu_t - n_t\delta_t + \delta_t}{n_t + 1} P(n_t)$$

We utilize a queuing model to derive $P(n_t)$. Given that the arrival and service completion follow Poisson (with a mean arrival rate A_t) and exponential (with a mean lasting period of $\frac{1}{D_t}$ days) distributions, respectively, the probability that n_t offers ($n_t \geq 1$) are valid in a $M/M/\infty$ queue model (see Knessl and Yang 2001 and Forgo 2017) at countdown time t is

$$P(n_t) = \frac{\rho_t^{n_t}}{n_t!} \frac{e^{-\rho_t}}{1 - e^{-\rho_t}}, \;\; n_t > 0.$$

Therefore, we have the unconditional expectation of the minimum offer rate at countdown time t, i.e., C_t, as

$$
\begin{aligned}
E[C_t] &= \sum_{n_t=1}^{\infty} \frac{\rho_t^{n_t}}{n_t!} e^{-\rho_t} \int_0^{+\infty} C_t \, n_t [1 - G_t(C_t)]^{n_t - 1} g_t(C_t) dC_t \\
&= \sum_{n_t=1}^{\infty} \frac{\mu_t n_t + \mu_t - n_t \delta_t + \delta_t}{n_t + 1} \frac{\rho_t^{n_t}}{n_t!} \frac{e^{-\rho_t}}{1 - e^{-\rho_t}} \\
&= \left(\left(\mu_t - \delta_t \left(2 \frac{e^{-\rho_t}}{\rho_t} + 1 - \frac{2}{\rho_t} \right) \right) - (\mu_t + \delta_t) e^{-\rho_t} \right) / (1 - e^{-\rho_t}) \\
&= \left((1 - e^{-\rho_t}) \mu_t - \delta_t \left(2 \frac{e^{-\rho_t}}{\rho_t} + e^{-\rho_t} + 1 - \frac{2}{\rho_t} \right) \right) / (1 - e^{-\rho_t}) \\
&= \mu_t - \frac{\delta_t \left(e^{\rho_t} (\rho_t - 2) + \rho_t + 2 \right)}{\left(e^{\rho_t} - 1 \right) \rho_t}.
\end{aligned}
$$

Step 3: The sourcing cost of a shipping request is determined by the minimum offer rate at the countdown time when the platform decides on offer acceptance. Therefore, given the request lead time T, and suppose that the platform determines offer acceptance at countdown time $t^* \leq T$, then the sourcing cost $S_T = C_{t^*}$. Since the decision time for offer acceptance is random, we factor in its distribution $w(t|T)$ in our calculation of the expected sourcing cost $E[S_T]$:

$$
\begin{aligned}
E[S_T] &= \sum_{t=0,1,\ldots,T} E[C_t] w(t|T) \\
&= \sum_{t=0,1,\ldots,T} \left(\mu_t - \frac{\delta_t \left(e^{\rho_t} (\rho_t - 2) + \rho_t + 2 \right)}{\left(e^{\rho_t} - 1 \right) \rho_t} \right) w(t|T)
\end{aligned}
$$

Simulation Analysis

We conduct a simulation based on the theoretical model in CHAPTER 3.8.1 and empirical estimation in CHAPTER 3.6. In particular, we consider the following steps in this simulation process: (1) parameter estimation, (2) counterfactual case study, and (3) result summary. In this section, we elaborate on the factor estimation of our theoretical model.

Pre-Simulation Parameter Estimation

We elaborate on the parameter estimations of our theoretical model. Recall that we have five critical factors in our theoretical model, i.e., μ_t, δ_t, A_t, D_t, as well as $w(t|T)$.

While the arrival rate A_t and the distribution of offer-acceptance countdown time $w(t|T)$ are computable based on the data, the other factors are not directly available. Therefore, we consider a two-pronged manner in our parameter estimation by, first, estimating A_t and $w(t|T)$ directly based on the data, and second, estimating μ_t, δ_t, and D_t by solving a least-square problem.

Estimation of A_t and $w(t|T)$: Given a route r, we estimate the arrival rate $A_{r,t}$ (the route-specified analog of A_t) directly based on data records. Given a specification of countdown time t and route r, we firstly gather (1) load requests that are open for driver matching at t, i.e., load requests that are neither booked nor canceled by countdown time t, and (2) corresponding offers that arrive at countdown time t. Next, based on these selected records, we compute the average number of offer arrivals per load request and denote it as $\overline{A}_{r,t}$. We observe a reciprocal association between $\overline{A}_{r,t}$ and t. Therefore, to scale our parameter space, we further consider the following regression to catch the dynamic of $A_{r,t}$ by estimating the coefficient term $C_{r,A}$:

$$A_{r,t} = \frac{C_{r,A}}{t} + u_{r,t}, \tag{64}$$

where $u_{r,t}$ represents the regression residual.

Moreover, we adopt a kernel density estimator for $w(t|r,T)$, i.e., the route-level analog of $w(t|T)$. In particular, for each pair of route r and request lead time T, we first empirically estimate the distribution $w(t|r,T)$. Next, based on the estimated empirical distribution, we implement *density* function in R package - *stats (version 3.6.2)* for a kernel density estimation $\hat{w}(t|r,T)$. We relegate more details but refer interested readers to RDocumentation (2024).

Estimation of $\mu_{r,t}$, $\delta_{r,t}$, and $D_{r,t}$: In order to estimate the other critical factors, i.e., $\mu_{r,t}$, $\delta_{r,t}$, and $D_{r,t}$ (route-specified analogs of μ_t, δ_t, and D_t), we adopt the earlier estimations $A_{r,t}$ and $w(t|r,T)$, as well as solving a least-square problem that incorporates both our empirical findings and theoretical model. In particular, first, we estimate the expected sourcing cost $S_{r,T}$ on route r and with request lead time T based on our empirical model estimation. For convenience, we reproduce the simplified model here:

$$\hat{S}_{r,T} = \hat{b}_1 T + \hat{b}_2 T^2 + c_{S_{r,T}}, \tag{65}$$

where $\hat{b}_1$ and $\hat{b}_2$ are coefficient estimation of 2SRI stage 2: Sourcing Cost, while $c_{S_{r,T}}$ contains representative values of control variables in 2SRI stage 2: Sourcing Cost for route r, specifically a list featuring the fixed effect $ROUTE_i$ for route r and the averages of other control variables over historical requests on route r. Next, upon this foundation as well as the estimations of A_t and $w(t|T)$, we solve the following least-square problem to estimate $\mu_{r,t}$, $\delta_{r,t}$, and $D_{r,t}$.

$$\min_{\substack{\hat{a}_{r,\mu},\, \hat{c}_{r,\mu},\, \hat{a}_{r,\delta},\\ \hat{c}_{r,\delta},\, \hat{D}_{r,t}}} \sum_{T=0,\ldots,\overline{T}} \left(\hat{S}_{r,T} - \sum_{t=0,\ldots,T} \left(\hat{\mu}_{r,t} - \frac{\hat{\delta}_{r,t} \left(e^{\hat{\rho}_{r,t}} (\hat{\rho}_{r,t} - 2) + \hat{\rho}_{r,t} + 2 \right)}{\left(e^{\hat{\rho}_{r,t}} - 1 \right) \hat{\rho}_{r,t}} \right) \hat{w}(t|r,T) \right)^2 \tag{66}$$

$$S.T. \quad \hat{\mu}_{r,t} = \hat{a}_{r,\mu} t + \hat{c}_{r,\mu}, \;\; \hat{\delta}_{r,t} = \hat{a}_{r,\delta} t + \hat{c}_{r,\delta}, \;\; \hat{\rho_{r,t}} = \frac{\hat{A}_{r,t}}{\hat{D}_{r,t}} = \frac{\hat{C}_{r,A}}{\hat{D}_{r,t} t}, \tag{67}$$

$$\hat{D}_{r,t} > 0, \; \hat{\mu}_{r,t} > 0, \; \hat{\delta}_{r,t} > 0 \;\; \forall t, \tag{68}$$

$$\hat{a}_{r,\mu} > 0, \; \hat{a}_{r,\delta} > 0. \tag{69}$$

Note that to ensure the scalability of the computation, rather than estimating the exact values of $\mu_{r,t}$ and $\delta_{r,t}$ for every pair (r,t), we estimate their linear function forms (i.e, $\hat{a}_{r,\mu}$, $\hat{c}_{r,\mu}$, $\hat{a}_{r,\delta}$, $\hat{c}_{r,\delta}$) instead, as indicated in Expression 67. This way, we are able to capture the trend of offer rates and their heterogeneity and, at the same time, make the solution scalable.

Simulation Process

With these estimations of $\mu_{r,t}$, $\delta_{r,t}$, and $D_{r,t}$ from APPENDIX B.2.1, we perform the simulation for large-scale counterfactual analyses. As outlined in CHAPTER 3.8.2, for a given pickup date U_P within our simulation window, we implement a series of estimations and decision-making mechanisms on the business date $U_P - \overline{T}$ to determine the optimal preorder timing (i.e., the request lead time that maximizes the overall profit) and preorder quantity for driver searching. We next elaborate on the simulation process and provide a brief overview of the simulation procedure below.

Procedure 1 Large-Scale Case Study for Counterfactual Analyses

Require: Parameter estimation set: $\hat{a}_{r,\mu}, \hat{c}_{r,\mu}, \hat{a}_{r,\delta}, \hat{c}_{r,\delta}, \hat{D}_{r,t}$; lookahead window $\overline{T} = 10$;
Require: Testing set of pickup dates: H_{te} : $01/30/2018 \leq U_P \leq 02/04/2018$;
Loop 1: **for** each route r **do** ▷ Perform route-level case analyses in a large-scale study
  Loop 2: **for** each pickup date $U_P \in H_{te}$ **do** ▷ Analyze each pickup date
    Define the training set H_{tr} by incorporating requests until $U_P - \overline{T}$ ▷ Dynamically update training set
    Estimate arrival rates $\hat{A}_{r,t}$ and demand $\overline{K}_{D|r,U_P}$ based on H_{tr}
    Loop 3: **for** each request lead time $T \leq \overline{T}$ **do** ▷ Decide on the optimal preorder timing and quantity
      Calculate expected sourcing costs $E[S_T]$ and forecast freight-matching probability $\overline{K}_{P|r,T}$
      Using Newsvendor model, determine the optimal preorder quantity $Q^*_{r,T}$ and corresponding profit $\Pi_{r,T}(Q^*_{r,T})$
    end Loop 2
    Decide on the optimal preorder timing $T^* = \arg\max_T \Pi_{r,T}(Q^*_{r,T})$
    Assign preordered drivers to meet high-priority requests as per established criteria
    Evaluate and summarize freight-matching outcomes for route r on pickup date U_P
  end Loop 2
  Summarize overall performance metrics for route r
end Loop 1

In this simulation, we consider that, in a most realistic setting, the platform has to decide the optimal request lead time for requests to be picked up on day U_P at the business date $U_P - \overline{T}$, where $\overline{T}$ represents the operational flexibility for the platform as explained in CHAPTER 3.8.2. Therefore, the highest possible value of the optimal request lead time is $\overline{T}$. In this simulation, we consider the first pickup date for decision-making to be $01/30/2018$ and the operational flexibility $\overline{T} = 10$ days. This requires us to finalize the preorder strategy by $01/20/2018$. For this purpose, we establish an initial training set that includes shipping requests with pickup dates on or before $01/20/2018$, and we run the following simulation steps to determine the optimal preorder timing and quantity for driver searching. We proceed by iterating through each pickup date sequentially, repeatedly constructing the training set and executing subsequent simulation steps.

In-simulation parameter estimation

Given $CurDate$, the platform compiles a dataset of historical load requests and drivers' offers as the training set. Utilizing this dataset, the platform (1) estimates the arrival rate of offers $A_{r,t}$ for every pair of route r and countdown time t to make offers, (2) forecasts the volume of requests on route r and expected to be picked up on U_P, denoted as $\overline{K}_{D|r,U_P}$, and

(3) estimate the freight-matching probability for every pair of route r and request lead time T, denoted as $\overline{K}_{P|r,T}$. In particular, first, based on the training data, the platform estimates the regression model 64 and predicts the arrival rate of offers $A_{r,t}$ for every pair of (r,t). Second, the platform forecasts the volume of requests to be picked up on U_P. In this simulation, we consider a simple rolling-average-styled forecast model. Specifically, we calculate the average number of requests for each weekday, determined by their designated pickup dates. The average for the same weekday as U_P serves as our forecast for the number of requests on U_P. Third, based on the model estimation of 2SRI stage 2:Booked, the platform predicts the expected freight-matching probability $\overline{K}_{P|r,T}$ for every pair of route r and request lead time T:

$$\overline{K}_{P|r,T} = \hat{a}_0 + \hat{a}_1 T + \hat{a}_2 T^2 + \hat{a}_x \overline{X}_r + \hat{a}_{v,1}\overline{v}_{1,r} + \hat{a}_{v,2}\overline{v}_{2,r}.$$

Here, $\overline{X}_r$ contains representative values for route r of the control variables, specifically a list featuring the fixed effect $ROUTE_i$ for route r and the averages of other control variables (as indicated by Control List) over historical requests on route r. Besides, $\overline{v}_{1,r}$ and $\overline{v}_{2,r}$ are averages of $\hat{v}_{1,i}$ and $\hat{v}_{2,i}$ over historical requests on route r.

Decision-making on the optimal request lead time and preorder quantity

Next, based on the estimation of the offer arrival rate $A_{r,t}$ from APPENDIX B.2.2, along with other parameter estimation (i.e., $\hat{\mu}_{r,\mu},\ \hat{\delta}_{r,t},\ \hat{D}_{r,t},$ and $\hat{w}(t|T)$) from APPENDIX B.2.1, the platform is able to forecast the sourcing cost $E[S_{r,T}]$ using the theoretical result in Lemma 2. Given the forecasts of sourcing cost $E[S_{r,T}]$, freight-matching probability $\overline{K}_{P|r,T}$, and volume of requests $\overline{K}_{D|r,U_P}$, the platform considers a newsvendor model (see Model 8) to decide on the optimal order quantity $Q^*_{r,T}$ (i.e., the optimal number of requests for driver searching) and freight-matching profit $\Pi_{r,T}(Q^*_{r,T})$. For convenience, we reproduce the newsvendor model as below

$$\max_{Q_{r,T}} \ \Pi_{r,T}(Q_{r,T}) = min(K_{D|r,U_P}, K_{S|r,T,U_P})\overline{R}_r - K_{S|r,T,U_P} E[S_{r,T}] + (K_{S|r,T,U_P} - K_{D|r,U_P})^+$$

$$(1-c_O)E[S_{r,T}] - c_U(K_{D|r,U_P} - K_{S|r,T,U_P})^+.$$

As explained in CHAPTER 3.8.2, we consider that the demand level $K_{D|r,U_P}$ follow the Poisson distribution $Pois(\overline{K}_{D|r,U_P})$ and a truncated Poisson distribution, while the supply level $K_{S|r,T,U_P}$ is subject to a truncated Poisson distribution with a mean parameter of $\overline{K}_{P|r,T}Q_{r,T}$ and an upper bound $Q_{r,T}$.. Furthermore, we calculate the unit revenue of pairing drivers and load requests, i.e., $\overline{R}_r$, as the average shippers' payment rate (i.e., $\overline{R}_r$) for their requests on route r in the training set. Additionally, in scenarios where the platform lacks a sufficient number of pre-ordered drivers to fulfill shipping requests, it can still recruit drivers on demand, as observed in real-world situations. Consequently, the supply shortage penalty term, c_U, is set to 0. We also explore various levels of oversupply penalty (c_O) in our analyses to assess their impact on the platform's decision-making and performance, with values ranging from $0.00,\ 0.05,\ ...,\ 0.50$.

By solving the newsvendor model, we are able to determine the ideal preorder quantity Q^*_T and its corresponding profit level $\Pi_{r,T}(Q^*_{r,T})$ for every pair of route r and request lead time T. We further decide on the optimal request lead time T^* that satisfies:

$$T^* = \arg\max_T \Pi_{r,T}(Q^*_{r,T}).$$

Therefore, we are now able to decide the preorder strategy for the pickup date U_P based on the optimal preorder timing T^* and its corresponding optimal preorder quantity Q^*_{r,T^*}.

Assignment of preordered drivers

When implementing this preorder strategy, the platform follows a first-come-first-serve policy. It allocates preordered drivers to the earliest arriving shipping requests that meet two criteria: (1) they offer a unit revenue (i.e., the shippers' payment rates) that exceeds the sourcing cost ($E[S_T]$), and (2) they are submitted after the established preorder timing, meaning their request lead time is less than the optimal time. For the remaining orders that (1) do not meet these conditions or (2) arrive after all preordered drivers are assigned, we consider that they do not benefit from the preorder strategy and eventually reach the same matching status and sourcing costs as in the factual cases. In other words, these remaining shipping requests are subjected to the standard matching procedure that was in place before introducing the new policy.