

**THE EFFECTS OF A CONCRETE, REPRESENTATIONAL, ABSTRACT (CRA)
INSTRUCTIONAL MODEL ON TIER 2 FIRST-GRADE MATH STUDENTS IN
A RESPONSE TO INTERVENTION MODEL: EDUCATIONAL IMPLICATIONS
FOR NUMBER SENSE AND COMPUTATIONAL FLUENCY**

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ABSTRACT

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This study was designed to evaluate the effects of a Concrete Representational Abstract (CRA) instructional model on Tier 2 first-grade mathematics students in a Response to Intervention model. Twelve students were instructed three times a week using the Expeditions to Numeracy program. The Test of Early Mathematics Ability-3rd edition results overwhelmingly indicated dramatic student growth. A *t-test*, which included all twelve students in the study, was found to be statistically significant ($t=5.79$, $p<.01$). The effects of a CRA instructional model on students' computational fluency were measured through the use of the curriculum based assessments given weekly to all first-grade students. Only those students who had not yet met the first-grade benchmark were included in the analysis of growth over time. All intervention students showed significant growth on their CBM scores throughout the study, exceeding the recommended weekly growth of 0.35. Two-thirds of the students exceed the recommended growth by two or more points. A single-subject analysis of the CBM data all indicated the strong student growth. The analyses of the student growth on the CBMs,

as well as the variables that affect this change, were also analyzed using Growth Curve Modeling. The final model of analysis indicated the treatment group's slope was statistically significantly greater than the slope for other members of the study, including those students progressing at a typical first-grade level and therefore not eligible for services. These results indicate a statistically significant affect of a CRA instructional model when used with Tier 2 students in a Response to Intervention Model on students' computational fluency and mathematical achievement.

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CHAPTER 1 INTRODUCTION

The Research Problem: Background and Need

Instructional models which assist students who are struggling to gain access to mathematics are crucial to the development of an effective educational system. Meeting the needs of such students has long been the focus of research in education. However, many believe that educators have demonstrated little success in improving the American student's education. No Child Left Behind (NCLB, 2001), signed into law by President George Bush in 2002, brought a dramatic increase in reporting and accountability measurements for K-12 schools in the United States. NCLB established four pillars to help educators deliver a more valuable education. The first pillar provides stronger accountability for student achievement. States must close the achievement gap for all students, including those who are disadvantaged. Annual reports to local communities in each state must show adequate yearly gains. If insufficient progress is attained, then supplemental services, such as tutoring and after-school programs, must be provided to help students make significant progress. The second pillar of NCLB provides more local control and decision making for states and communities. School districts may use their federal funding with less regulation, allowing them to direct funds to areas where needs are most critical, including teacher salaries, professional development, or recruitment. The third pillar emphasizes the importance of using curriculum derived from solid scientific research base. Federal funding for school districts is provided to develop and support such programs. The final pillar of NCLB provides choices for parents in low

performing schools. Students attending a school that has not made adequate yearly progress over two consecutive years may transfer to better achieving schools in their district. If a school fails to meet state standards for three years, students may be eligible for free supplemental services (US Department of Education, 2008).

In addition, the reauthorization of the Individuals with Disabilities Education Act (IDEA) was signed into law in December 2004. This reauthorization shared alignment with NCLB and facilitated a change in the identification of students with specific learning disabilities. Effective July 2005, states could no longer require school districts to use a discrepancy model for the identification of students with a specific learning disability. The discrepancy model required a severe difference between a student's intellectual ability (as measured on an intelligence quotient test) and educational achievement when determining if the child needed special education services. Educators waited until there were sufficient gaps to provide students with special services. This model placed a disproportionate number of minority students into special education classrooms (wrightlaw.com). IDEA 2004 permits the use of a process that relies on scientific, researched-based interventions to determine whether a child qualifies for special education services. In fact, IDEA 2004 requires that students be provided with appropriate instruction, delivered by a qualified teacher who formally documented the student's progress at reasonable intervals of time, prior to any student being referred for an evaluation. Qualified teachers must be deemed as such through the NCLB teacher certification regulations (IDEA, 2004).

Rationale

IDEA and NCLB have heightened awareness regarding the need for reform in the way students are identified for special education. Historically, there has been a disproportionate number of minority students placed in special education, including classes for specific learning disabilities (Zhang & Katsiyannis, 2002). African Americans account for 14.8% of the general population; however, they represent 20% of special education classifications (Losen & Orfield, 2002). Blanchett (2006) asserts that placements are usually made by school personnel using subjective referral and eligibility criteria, despite the original the intent of special education to provide objective evaluation, instruction, and assessment for students (Blanchett & Shealey, 2005). When African American students are placed into special education classes they are typically segregated from the regular education population, have higher dropout rates, lower standardized assessment scores, and watered-down curriculum (Ferri & Connor, 2005). The National Research Council (2002) asserts that improving the regular education instruction students receive can reduce the placement of minority students in special education, and increase the likelihood of success for all students.

Response to Intervention

The mandates of IDEA and NCLB have led some states to adopt a new model for identifying students with specific learning disabilities. This model is called Response to Intervention (RtI). RtI is divided into three tiers and requires educators to screen all students, not just those students that a teacher may notice having difficulty in the curriculum. The National Research Center on Learning Disabilities describes RtI in the following way:

RtI is an assessment and intervention process for systematically monitoring student progress and making decisions about the need for instructional modifications or increasingly intensified services using progress monitoring data. The following is the fundamental question of RtI procedures: Under what conditions will a student successfully demonstrate a response to the curriculum? Thus interventions are selected and implemented under rigorous conditions to determine what will work for the student (Retrieved August 28, 2008).

The first tier of RtI calls for universal screening of all students. At this tier, all students are receiving standard research-based curriculum prescribed by the district as part of their comprehensive education. Research-based curricula are curricula that have been tested through experimental measures and whose results have been published in a peer reviewed journal (Kovaleski, 2007). Many advocates of RtI recommend that all students have a one-time screening to determine eligibility. However, short-term continuous curriculum-based assessment is the ideal model for identifying students with potential learning challenges since some students may “recover” or catch up to their peers on their own (Fuchs, Fuchs & Hollenbeck, 2007). This provides many data points to determine eligibility for more specialized services.

Those students who are labeled as non-responders in the regular curriculum progress to Tier 2 instruction. At this tier students are taught in small groups, ideally for one-half hour, three times a week, over an eight-week period (Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005). These students are provided with a researched-based program that has been proven successful with learners of similar need (wrightslaw.com). The program is delivered by a qualified individual, such as a teacher or instructional aide, and weekly progress monitoring tracks student progress. Best practice for evaluation at

the end of the cycle is determined by adequate progress based on either criterion-referenced or norm-referenced estimates of weekly improvement (Fuchs et al., 2005). Children who are identified as responders in Tier 2 are placed again in Tier 1 with progress monitoring to ensure continued success. However, if a student is still determined to be a non-responder, not meeting the goals of progress monitoring then the child is sent to Tier 3. In Tier 3 students are either tested for a specific learning disability and/or given even more intensive interventions in groups of one or two to help them overcome their learning challenges, thus increasing student success in their academic endeavors.

In recent years, classrooms have become more inclusive, including students with diverse needs. Meeting the needs of all students has become a challenge for classroom teachers, as some research indicates the need for direct instruction and other literature indicates the need for creative problem-solving ability. The National Council of Teachers of Mathematics (NCTM), in their document *Principles and Standards for School Mathematics* (2000), called for a mathematics education that permits all students access to an education which enables them to solve problems creatively. Such education will assist students in gaining access to careers previously not available to those who do not have an understanding of mathematics. According to the *Principles and Standards* (2000) students must learn mathematics with understanding, actively building upon prior mathematical knowledge. Teachers need to provide mathematical instruction that allows students to construct knowledge based on prior skills and understandings. The current research in the area of diverse learning needs indicates that teachers need to focus on explicit teaching (Hudson, Miller, & Butler, 2006). Hudson et al. suggest that instruction

for struggling learners should contain explicit instruction, and have students actively constructing their mathematical knowledge.

Theoretical Framework

This study will draw on Jerome Bruner's theory of Interactional Cognitive Development. Bruner believes that knowledge is developed through three modes of representation that allows learners to construct an understanding of their world. The first mode is the Enactive Representation. This mode refers to "representing past events through appropriate motor response" (Driscoll, 2005). Students learning at this level need to have concrete examples that they can physically manipulate through their own actions. The use of manipulatives in mathematics education is important in the development of student's enactive representations necessary to develop number sense, which can then subsequently be applied to more advanced mathematical skills. The Iconic Representation mode refers to the development of skills through the use of images. This would include diagrams and pictures of concepts that allow the learner to identify the important features of a concept. Once the student has shown a mastery of this level of representation, he/she is ready to move to the third mode called Symbolic Representation. At this phase of development learners are able to use symbols, such as language and mathematical notations, to represent concepts being developed. Bruner cautioned that pushing learners to the symbolic level quickly, or skipping the enactive and iconic modes all together, can impede the development of a concept (Driscoll, 2005).

According to Bruner (1966) culture plays a significant role in the development of knowledge. Bruner believed that learners progressed through various stages of

development, but these stages were not age dependent. Any student could learn a concept, at any age or developmental level, as long as he/she was given the proper prerequisite knowledge and experiences. Bruner believes that any idea can be presented to students via a discovery process where children are led through experiences that allow them to use their prior knowledge to gain an understanding, facilitated by the use of active dialogue.

K-12 learning should be an active, engaging process not haphazard in nature, but rather the result of careful planning. Bruner believes schools should equip students with the skills and tools necessary for the culture in which they live. Bruner believes that if the education system did a better job helping students transfer their skills to their culture, schools would be more successful (Driscoll, 2005).

Instructional Model

The Concrete-Representational-Abstract (CRA) approach offers students explicit instruction in mathematics and assists the learner in constructing his/her knowledge through the use of multiple representations of a concept (Hudson et al., 2006). This approach contains three stages of instruction, similar to Bruner's stages of knowledge development. The first stage is the Concrete Stage, where the teacher models the concept with concrete materials such as base ten blocks, digi-blocks, or fraction bars. The use of manipulatives has been proven useful in assisting struggling learners to acquire and maintain various mathematical skills (Cass, Cates, Smith, & Jackson, 2003). In the Representation Stage, the teacher changes the concrete representation to a semi-concrete level that may include drawings or tallies. This stage prevents the distortion of concepts by connecting the pictorial with the concrete, which has been proven instrumental for

students with learning challenges in mathematics (Witzel, Mercer & Miller, 2003). In the final stage, called the Symbolic Stage, a model is used which includes numerals and/or symbols as the sole representation of a number sentence or concept. Witzel et al. (2003) believe that the value of the use of symbols in mathematics is to work beyond what one can see or touch and establish the connections to other situations. Meaningful connections between these representations are created for students to assist them in the development of a mathematical concept (Access Center, n.d.).

This meaningful development of concepts is crucial for young children. Research indicates that 66% of the variance in first-grade student math achievement can be attributed to a student's number sense (Jordan, Kaplan, Locuniak, & Ramineni, 2007). Number Sense is defined as "abilities related to counting, number patterns, magnitude comparisons, estimating and number transformations" (Berch, as quoted in Jordan et al., 2007). Research conducted by Jordan et al. (2007) indicates that number sense is a predictor of mathematics achievement by the end of first-grade. Their study also reveals a strong and significant correlation between number combination retrieval and story problems, suggesting that these skills are fundamental to learning conventional mathematics. It is therefore crucial that children experiencing early difficulty in mathematics be given explicit instruction in number sense in order to increase their success in mathematics throughout later grades.

Study Purpose and Design

The purpose of this study is to examine the effects of explicit instruction on a child's ability to develop number sense and computational fluency. The following research questions were addressed:

1. What are the effects of a concrete-representational-abstract instructional model on students' achievement as measured on a mathematical achievement test?
2. What are the effects of a concrete-representational-abstract instructional model on Tier 2 students' development of computational fluency?
3. What are the differences in student growth, measured via a progress monitoring tool, when students receive concrete-representational-abstract instruction?

These questions were investigated in a single subject design study, as is common in many RtI studies due to the small number of participants qualifying for Tier 2 interventions. This method has a behaviorist background that allows the researcher to observe behavioral changes over time, where each participant acts as his/her own control. All first-grade students at the study sites were screened using: 1.) the Early Numeracy Indicators, a curriculum based assessment developed by Lembke and Foegen (2005), 2.) Fuchs and Fuchs Curriculum Based Assessments (CBM) and 3.) teacher informal assessments. Winter norms developed by Lembke and Foegen were used to identify students performing at or below the 35th percentile. The Lembke and Foegen instrument is based on research studies indicating that items assessing number magnitude judgments, reading numerals, and quantity discrimination are early predictors of mathematical achievement (Baker, Gersten, Flojo, Katz, Chard, & Clark, 2002; Clarke & Shinn, 2004; Mazzocco & Thompson, 2005).

Curriculum based assessment has been used for more than 25 years in special education as a mechanism to both monitor students' progress toward IEP goals, and identify students at-risk (Deno, 2003; Shinn, 2007; Stecker, Fuchs, & Fuchs, 2005). Stecker, Fuchs and Fuchs (2005) highlight three distinguishing features of CBM. First,

the CBM must be used to assess students' progress toward long term goals that are general, rather than specific in nature. The second feature is frequent monitoring of student's progress at least once per week, as it is a formative assessment that reflects performance over time. To be used effectively, data obtained on the assessments must be used to drive the curriculum by making instructional decisions based upon these assessment results (Stecker, et al., 2005). The third crucial feature of CBM is the use of a sound instrument which is technically adequate. The instruments typically include a set of standardized and validated assessments in mathematics computation, mathematics applications, and other reading skills that take between one and four minutes to administer (Shinn, 2007). There are many documented studies that support the use of progress monitoring as part of the RTI model of evaluating student's response to an intervention (Deno, 2003; Fuchs & Fuchs, 1986; Fuchs & Vaughan, 2005). During each week of the small group instruction all first-grade students were assessed using Monitoring Basic Skills Progress (MBSP).

Students who are identified as at-risk in both the regular curriculum by teacher observation and the Lembke-Foegen instrument, or the CBM were given the Test of Early Mathematic Abilities (TEMA-3) assessment to measure their mathematics achievement of concepts and skills. It provides norm-referenced scores, including scale and standard scores, and can be used to report progress and analyze growth. If a student was labeled at-risk by these initial screenings and the classroom teacher, he/she was identified as a Tier 2 student. These students will receive thirty instructional sessions using the Expeditions to Numeracy program, which has a CRA-based approach

embedded into its lessons. At the conclusion of the intervention, the Lembke-Foegen and TEMA-3 assessments were re-administered again to assess student growth.

Definition of Terms

In order to clearly understand the research questions, the following definitions are provided to ensure that the researcher and reader are clear about the concepts to be studied:

Number sense is defined as the ability to understand the base-ten number system, estimate, make sense of numbers, and recognize the relative and absolute magnitude of numbers (NCTM, 2000, p.32)

Place value is defined as the value of a digit in the base-ten number system. This value is based upon its location in relation to other digits in the number.

Computational fluency is a student's ability to have efficient, flexible and accurate methods for computing (NCTM, 2000).

Explicit instruction systematically breaks concepts down into small sub-skills and provides experiences for students to develop conceptual understanding through guided discovery.

Response to Intervention (RtI) is an assessment and intervention model which systematically monitors student progress to make instructional decisions which allow for increasingly intensified services as student need indicates (National Research Center on Learning Disabilities, 2008).

Limitations

This study has several limitations. The first limitation is the use of a single subject design. Although this is the most common model used to study special education

students, this design limits the ability to extrapolate the results to other populations.

However, it does provide the basis for further research. Another limitation is the small sample size. Approximately 10 to 15% of the student population should qualify for Tier 2 services. If the population size is greater in a Tier 2 intervention, the instruction offered to all regular education students needs to be examined. The small sample size lowers the power of the study. Finally, the population of students participating in the study is not representative of the general population of the United States, as they are predominately upper-middle class children.

Significance of Study

The research on RtI and Tier 2 interventions is limited and in its infancy. While many schools are implementing many reading programs for Tier 2 students, mathematics interventions have not been fully studied or implemented. Current research indicates the rigor of mathematics courses students enroll in are powerful predictors of school mathematics achievement in later education (Tate, 2005). In order for all students to reach their full potential, educators must provide students with a variety of opportunities to be successful. Berry (2008) indicates that the affective connection that teachers make with their students influences their academic outcomes. The use of small group instruction can facilitate this connection. This study looked at a CRA instructional model that offers explicit instruction in computational fluency. This model of instruction provided the students with lessons which were broken down into small sub-skills, but still involved teacher directed discovery of the concepts underlying the development of addition and subtraction strategies. Through rich discourse and conceptual activities in the classroom, students developed strategies to increase their computational fluency and

over-all mathematics achievement. Selection of students for this study was based on research indicating that young students' ability to identify numbers, make quantity comparisons, and identify the missing number in patterns are predictors of later success in mathematics. Designing and implementing targeted mathematics programs which include small group instruction, should enable students to learn prerequisite skills, which will contribute to success in future grades.

CHAPTER 2 LITERATURE REVIEW

Number Sense

This work is grounded in the developmental stages of number sense, a topic discussed widely in the field of mathematics education. Berch (2005) identified thirty different definitions. According to his research the definition can encompass “awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process conceptual structure, or mental number line” (p. 333). Berch further indicates that number sense can be attributed to natural ability or acquired skills. The National Council of Teachers of Mathematics (NCTM), in their *Principles and Standards for School Mathematics* (2000) defines number sense as:

“The ability to decompose numbers naturally, use particular numbers like 100 or $\frac{1}{2}$ as referents, use of the relationship among arithmetic operations to solve problems, understand the base ten number system, estimate, make sense of numbers, and recognize the relative and absolute magnitude of numbers.” (p.32)

The NCTM states that number sense is the cornerstone of elementary mathematics and should be developed between grades kindergarten and two. Certainly performing number combinations such as single digit addition facts is essential for success in mathematics. However, developing computational fluency must go hand-in-hand with an understanding of the meaning of the operation (NCTM, 2000).

Berch and Mazzocco (2007) claim that number sense cannot be taught; rather it is an innate ability. Engaging in mathematical games and thinking can develop these skills (Berch et al., 2007). In contrast, this nativist position is refuted by researchers who

believe number sense is linked to the cerebral part of the brain, which under normal circumstances can be developed spontaneously. Other researchers such as Gersten, Jordan, and Flojo (2005) have discovered that informal knowledge of numbers is linked to number sense in young children. These researchers believe that the link between mathematics relationships, principles and procedures can be enhanced by gaining informal knowledge before a child enters school.

Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, and Van de Rijt (2009) studied the relationship between domain-general and domain-specific factors and early numeracy. The researchers utilized 115 randomly chosen students in the Netherlands to study executive functions, fluid intelligence, subitizing, and language. Their results indicate 45% of the variance in early numeracy scores can be explained by measures which assess planning, updating, and inhibition, with updating showing the highest correlation. A second finding indicates that subitizing accounts for 22% of the variance in counting skills. These results indicate the need for the development of number sense skills through the use of flexible strategies.

Development of Number Sense

Landmarks such as the Ten Frame (see Figure 2.1) are important tools in the development of a child's early number sense (Fosnot, 2001). Without these landmarks children rely on strategies such as "counting on" that can become problematic as the child needs to rework the strategy of counting from the beginning. For example, a student using a rudimentary "counting on" strategy when adding $3+5$ would have to count three times. First, they count three objects, then five objects and finally start from the beginning to re-count all eight items. In order for the child to discontinue the use of this

strategy, he/she needs to have developed a sense of cardinality and a hierarchical understanding of part-to-whole relationships. This struggle can be the root of many children's number sense difficulties (Fosnot, 2001). The use of a Ten Frame helps students develop a visual model which can assist them in organized and predictable ways, by providing a clear and concise context of the number ten as a reference. The Ten Frame allows students to develop an understanding of number, place value, and an anchor for computation, thereby enabling students to visually recognize ten as a tens unit and a group of singles. When students see six on a ten frame, they are able to easily visualize that four more will make ten, which is especially valuable since the human eye cannot easily subitize past three or four when the objects are not organized (Losq, 2005).

Table 2.1
Ten-Frame from Expeditions to Numeracy

Fosnot et al. (2001) reiterate the importance of number sense to the development of young mathematicians when she states that:

They make meaning in their world by setting up quantifiable and spatial relationships, by noticing patterns and transformations, by proving them as generalizations, and searching for elegant solutions. (p. 4)

To develop these ideas, students must develop a sense of numbers that Fosnot (2001) calls strategies of schemes. In order for students to be able to add numbers, they typically show perceptual correspondence by holding up fingers. Only when this is mastered can students begin to line up objects and count them, eventually using one-to-one correspondence. The development of these skills then allows students to develop strategies for addition and subtraction. In the absence of these skills, the order of mathematics and reasoning necessary for success is not developed in students.

Similarly, Fuson, Grandau, and Sugiyamam (2001) synthesize their previous research in number sense and identify core mastery goals for children at various ages. Figure 2.2 compares these mastery goals.

Table 2.2
Core Mastery Goals

Goals	Three-Year-Olds	Four-Year-Olds	Kindergartners	First Graders
Using disorganized counting				
Counting on fingers				
Recognizing patterns				
Relating words, numerals, and physical referents	three 3	eight 8	thirteen 13 one ten three ones $10 + 3$	thirty eight 38 three tens eight ones $30 + 8$

The goals emphasize which connections children should build in oral number words, written numbers, and numerical quantities. The authors believe that if students are having difficulty mastering these goals, they need to be given more opportunities to experience rich numerical lessons that can help them reach these goals. These experiences will form the student's "learning zone" that allows him or her to be informed mathematical citizens and build more complex mathematical ideas.

Wright, Stanger, Stafford, and Martland (2006) have a similar framework to situate a child's learning in early numeracy. Their Learning Framework in Number contains eleven different aspects of learning. One important aspect regarding number sense utilized in this study is called Strategies of Early Arithmetic Learning (SEAL). The authors identify the SEAL as the most important component of the Learning Framework,

as it is the foundation of number. Its seven stages start with Stage 0, where students cannot count and do not recognize one-to-one correspondence. Stage 1, called Perceptual Counting, is reached when children are able to count concrete objects, but not hidden objects as the child is relying on their senses to count. Figurative counting is obtained in Stage 2 when children are able to count concealed items, most likely inefficiently. However, if she is told how many in each group is to be counted, the child will start counting from the beginning. Stage 3, also called Initial Number Sequence, begins when a child is able to “count on” when solving problems such as $6+5$. A child at this stage will start at six and count up five to arrive at eleven. It is also at this stage in development when students can begin to develop concepts of ten. Intermediate Number Sequence is the fourth stage in SEAL. At this level children are able to count down to solve missing subtrahends. They are able to choose from a number of efficient strategies such as count down from and count down to. The final stage, called Facile Number Sequence, is achieved when a child no longer counts by one to solve basic addition and subtraction problems. She is able to use strategies such as compensation, commutative property, addition/subtractive inverse efficiently, and has an awareness of 10 as an anchor for operations. Identifying a child’s level using this model allows the educator to tailor a program to meet the student at the appropriate level for her development.

Another facet of the Learning Framework gives a sequential view of a student’s development in Base 10 strategies. As a child begins to develop her number skills, she does not see ten as a unit and focuses on the individual objects that make up the tens unit, counting up by ones. When a child reaches the intermediate level she is able to visualize ten as a unit of ten ones. She can use hidden or visual groups of ten to count. However,

she is not able to add or subtract without physical materials. Once the child reaches the highest level, called the Facile Concept of Ten, she can add or subtract with groups of ten without supplementary materials. Assessing the child's mathematical understanding in this framework can guide the development of instruction and an initial location in the curriculum to begin planning instruction.

Additionally, John Van De Walle (2004) describes four different types of number relationships students should master in order to use their number sense to increase their computational fluency. The first type of relationship is spatial, where students learn to recognize the patterns of arrangements to help them determine the total without counting. This is also referred to as 'subitizing'. Recognizing the number rolled on a die is an example of this relationship. The second relationship is one/two more and one/two less which involves counting on or back one or two. This enables students to think about their counting and see the relationship of one number to another. For example, students should understand that seven is one more than six and two less than nine. The third important relationship is the use of the benchmarks of five and ten. Ten is the foundation of our number system and is integral to the student's use of the distributive property to break apart numbers in order to solving problems mentally. David Wertz developed the Ten Frame in 1974 specifically for this purpose. Per Figure 2.1, the Ten Frame is a 2 by 5 array that allows users to illustrate numbers and their relationship to ten. The final and most important relationship identified by Van de Walle is called part-part whole. He asserts the importance of students being able to visualize numbers made up of many parts as the foundation of number sense.

Computation

The NCTM (2001) identifies three characteristics of computational fluency. The first, efficiency of recall, allows the students to achieve rapid recall of number facts which increases a student's ability to monitor sub-problems, as well as confidently implement all steps of a problem. The second characteristic, accuracy of fact recall, allows a child to carefully record results and double check the accuracy of the problem. The third characteristic, flexibility, helps the student see multiple solutions for an individual problem and choose the most efficient method. If a student is able to identify two strategies and solutions, she can use one to solve the problem and one to check the problem.

The ability to integrate these characteristics together to form a foundation for computational fluency is an important goal of mathematics instruction in the early grades (Fosnot et al., 2001; Ramos-Christian, Schleser, Varn, 2008; Russell, 2000). Students with greater fluency have been found to stay on task longer and are better able to resist distractions (Binder, 1996; Hassellbring, Goin, & Bransford, 1988; Lindsley, 1996). Understanding numerical magnitude relationships is positively correlated with acquiring new mathematical knowledge. Acquiring computational fluency is not a rote activity, rather a developed concept (Booth & Siegler, 2008). Students struggling with computational fluency spend more time using cognitive resources on fact recall, which leaves little working memory left to tackle the problem which has the computation embedded (Van De Walle, 2004). In fact developing routines for numbers without the semantic analysis is detrimental to students' ability to conceptualize mathematics,

indicating the need for strong understanding of concepts for success in mathematics (Wearne & Hiebert, 1988).

Henry and Brown (2008) studied first-graders' mastery of basic computation facts and the impact of the teacher's instruction on the computational fluency of the students in the classroom. Their results indicated that 86% of the first-graders used counting strategies at the end of first-grade, even if they were a high functioning mathematics student. When looking at fact retrieval, their research indicated that there is no statistically significant correlation between the use of textbook or supplemental materials and improved computational fluency. There was, however, a negative correlation between fluency and the use of timed tests. Teaching for memorization did not help students achieve fluency. However, the combination of memorization and the development of strategies were predictive of an increase in students' increased recall via fact-derived strategies. Interestingly, the use of double facts, usually the dominant strategy used in the United States, was not the primary predictor of fact fluency with sums over ten. In fact, the use of sums to ten facts had a larger contribution to the mastery of facts. When evaluating the importance of number sense to fact fluency, it was determined that only 27% of the first-graders had a strong understanding of place value.

A negative correlation was found between textbook reliance by the teacher and number sense. Additionally, teachers whose instruction emphasized correct answers reduced students' opportunities to develop proficiency with numbers. The correlation between number sense and fact derived addition strategies was strong. It appears there is a synergetic relationship between memorization and fact derived strategies (Henry et al., 2008).

In an effort to develop effective addition strategies, Fosnot (2001) stresses the importance of strategies as schemes or organized patterns of behavior. Children who are able to quickly master their facts do so because of their understanding of the relationships between numbers and facts. This development allows children to more readily comprehend higher level mathematics skills. For example, when learning double facts, students are able to recognize that counting by twos works when adding doubles. Additionally they are able to discuss odd and even numbers and the commutative property. When studying combinations to ten, students are able to see part-whole relationships and concepts such as compensation.

Kamii and Dominick (1997) assert that instruction that focuses solely on algorithms actually harms the mathematical development of students. One way to avoid this problem is the use of manipulatives (Fosnot, 2001; Reys, 1971). Long term use of manipulatives has resulted in an increase in mathematics achievement (Parhem, 1983; Sowell, 1989; Suydam & Higgins, 1977). However, it is crucial that teachers help students make the connection between the manipulative and the actions and symbols they represent. When students do not make this connection, the cognitive load is large (Ball, 1992; Kaput, 1989). In fact, Ball (1992) asserts that manipulatives can easily mask a lack of understanding. She cites the example of a student able to subtract using base ten blocks, but not able to transfer this skill to the algorithm because the use of blocks became a series of steps, not conceptual understanding. Manipulatives play a crucial role in enhancing student learning and communication only if used properly. The purpose of the manipulatives must be to construct knowledge.

Reys (1971), a seminal researcher in the use of manipulatives, asserted that the use of manipulatives is based on experiential learning theory. This includes the understanding that learning is based on experiences, and all experiences involve one's senses. He asserts that all students learn through a process that progresses from the concrete to the abstract, and involves active participation (Reys, 1971). Clements and McMillan (1996) posit two levels of knowledge from the use of manipulatives. Manipulatives develop concrete knowledge, also known as sensory concrete. This type of knowledge allows students to use sensory materials to make sense of a concept. Integrated concrete knowledge is an interconnected knowledge that involves the use of physical objects and abstractions to help form a mental picture. This is the type of knowledge necessary to build conceptual knowledge. An example of this is the use of Digi-Blocks to teach place value. Digi-Blocks are individual blocks that are packed into holders to create larger blocks of 10, 100, and 1000. Through use of Digi-Blocks or other manipulatives, student's conceptual knowledge is enhanced, thus student learning can increase. Clements and McMillan (1996) favor the development of number sense through the use of manipulatives, believing it to be crucial for the development of computational fluency, a necessary skill for the development of more advanced mathematical skills.

Number Sense Assessment

A large amount of research is available regarding reading and early intervention. This is not the case for mathematics. For example, research on mathematics assessment of number sense, especially for young children in grades kindergarten and one, is still in its infancy (Gersten et al., 2005; Jordan et al., 2007; Lembke et al., 2005). Counting and

quantity discrimination, the ability to identify the larger number when presented with two or more numbers written as digits, are considered by Okamoto (1996) to be two of the hallmarks of number sense through their six year study of instructional research. Okamoto determined that many young children can count to ten, but have no understanding of the magnitude of the number, meaning they are unable to identify whether four is bigger than six. Without this link, students are not able to connect the necessary components of number sense, such as estimation, to form more complex mathematical ideas (Gersten et al., 2005). Evidence that supports the importance of quantity discrimination can also be found in the work of Griffin, Case, and Siegler (1994). They posit that students who cannot identify larger quantities in kindergarten have difficulty in mathematical reasoning. This is especially true of low income students who have different experiences with informal instruction outside of school. However, there is evidence which suggests when students are given instruction in school, they are quickly able to catch up to their peers, indicating that number sense is a learned concept which needs to be accessed via a child's prior knowledge.

Early screening is of critical importance for students with potential mathematical disabilities in order to help facilitate number sense experiences early in their educational experience. Focus on the direct assessment of numerical skills is important for the identification of mathematical difficulties. In early education, the assessment of number sense is a better predictor of mathematical outcomes than non-mathematical assessments such as working memory (Fletcher, 2005). In order to assess the effectiveness of screening tools in the area of number sense, Clark and Shinn (2004) evaluated curriculum-based measurements in mathematics (CBM-M), which are short fluency

measures given to students to determine their growth in the curriculum. These measures are designed to assess first-grade student ability and identify students at-risk in the area of number sense. Most CBM-M probes have a ceiling for students in first-grade, and can only be used as a measurement once formal computational learning has begun to take place, which for many students is not until the middle of first-grade. This could lead to a critical failure to provide early interventions for struggling learners. This failure can influence a child's later ability to gain mathematical concepts, which in turn alters a child's educational trajectory.

In order to fill this gap, Clark and Shinn (2004), in their study of first-graders (n=52), assessed the effectiveness of using oral counting, number identification, quantity discrimination, and missing number measures as potential early mathematics assessments. Oral counting was identified as a student's ability to count starting at one for one minute. The number identification assessment asked students to identify numbers through twenty when presented with a set of number cards. The quantity discrimination assessment asked students to determine which of two visually represented numerals were larger. The missing number probe asked students to identify the number missing in a string of numbers between one and twenty. All probes were given for one minute. The results indicate that these measures were each reliable and valid measures to identify first-grade students with early mathematics difficulty when compared to standardized mathematics achievements tests such as the Test of Early Mathematics Abilities (TEMA). Additionally, the measures were sensitive to student growth. The oral counting and number identification seemed most sensitive to student development.

Lembke and Foegen (2005) built upon this research in an effort to add to the scant research on early measures of mathematics skills. They increased the number of students studied, introduced new measures, and investigated different criterion variables. The students' results were compared to Stanford Early Achievement Test and the Woodcock Johnson Mini Battery of Achievement, along with a teacher rating scale. Their results indicated that the Quantity Discrimination, Number Identification, and Missing Number probes all had test-retest reliability, with Quantity Discrimination and Number Identification being the strongest. The criterion validity of all the measures ranged from .57 to .63. When compared with standardized tests, the strongest correlations were with Quantity Discrimination (.45), Number Identification (.45), and Missing Number (.50).

Jordan, Kaplan, Olah, and Locuniak (2006), while working with the Children's Math Project, conducted a longitudinal study of the mathematical skills of students at-risk for math failure. Their number sense battery included five different assessments. The first was counting skills, where students used sequences, enumerated sets, recognized numbers, and counted. The second, a number knowledge assessment, asked students to compare quantities. The non-verbal calculations, the third assessment, required students to complete simple addition and subtraction. In the fourth assessment story problems were solved without the use of concrete objects. The final assessment, number combinations, was verbally presented with no concrete objects available for the students to use. Jordan et al determined that early number sense was a reliable and strong predictor of mathematics achievement at the end of first-grade. Additionally, they discovered a strong correlation between number combinations and story problem solving, even in kindergarten.

Mathematics Difficulties

Early screening of students in mathematics is crucial to the identification of students with potential mathematics difficulties. Math Disabilities (MD) have many definitions in the literature, with little or no standard criteria. For example, Fuchs, Fuchs, and Prentice (2004) have identified four different methods of labeling students with MD. In some schools, students identified with a MD are functioning two grade levels below their expected grade. Other schools use one grade below as their cut-off, while others use a score below the 35th percentile on a standardized assessment or the students receiving Chapter 1 services as their identification procedure. Regardless of the identification, students with MD need instructional methods that will help them achieve success.

Fletcher (2005) found statistical differences between children with mathematics difficulties (MD), reading difficulties (RD), and a co-morbidity of mathematics and reading difficulties (MD/RD), indicating differences between students who struggle in mathematics. Fletcher's research focused on the importance of direct assessment of numerical skills, which revealed that students with co-morbid associations are more likely to have a language-based difficulty than students with only mathematics difficulties. When looking at profiles found on the Woodcock-Johnson Psycho-Educational Test Battery-Revised, Fletcher found statistically significant differing profiles in sustained attention, procedural learning, concept formation, phonological awareness, rapid naming, vocabulary, paired associative learning, and visual motor sub-tests, thus indicating that MD, RD, and MD/RD students learn differently.

Regardless of the identification, Jordan, Hanich, and Kaplan (2003) identify common characteristics of students with math difficulties, whose population is estimated

to be between 6% and 14% (Barbarese, Katusic, Colligan, Weaver, & Jacobson, 2005).

However, many more students are struggling because of inadequate instruction or cognitive deficits (Geary, 2004). Students with only a MD have more success in mathematics than students with MD and RD. The MD students demonstrate different patterns of cognitive deficits. Their achievement is significantly higher, and they have an advantage over students with MD/RD in the areas of arithmetic combinations and story problem solving. They were also able to use finger counting strategies more accurately than MD/RD students, indicating that students with a MD have better counting procedures and problem solving because of the reading deficits in MD/RD students. Students with spatial representation issues, related to numerical magnitude, struggle with rapid retrieval and have trouble moving visual representations on a number line, which is critical for addition and subtraction conceptual development. This finding underscores the idea that computational fluency is a hallmark of MD. The two groups do not show differences in the areas of timed number combinations recall, estimation, place value and written computation at the end of third grade (Jordan et al., 2003).

Students with MD often struggle with multi-step problems (Bryant, Bryant, & Hammill, 2000). Fuchs, Fuchs, and Prentice (2004) determined that there was a general improvement when problem solving instruction was combined with explicit instruction to transfer broad schemas for identification of familiar problem types and the encouragement of meta-cognition in students. The authors define transfer as teaching rules for problem solving, along with building cumulative knowledge. It involves three components: explicitly transfer, immediate transfer, and near transfer. Explicit transfer involves looking for superficial changes in a problem, and finding novel changes in

regular problems. When teaching problem solving skills for immediate transfer, problems similar to those taught in class, it is reported that MD/RD students improved less than MD, RD, and non-disabled students. MD/RD, MD only, and RD only improved less than non-disabled students in computation. In the area of labeling all students improved at the same rate. When evaluating near transfer, solving problems with superficial changes to create novel problems, it was determined that MD/RD students improved less than non-disabled peers on conceptual underpinnings. Students with MD/RD, MD only, and RD only improved less than their non-disabled peers in computation. Labeling had similar improvement across all students. Overall, students with MD/RD in this study showed less growth than MD only and non-disabled peers. The MD students showed less growth than non-disabled peers. Therefore, it is important to provide students who have MD and/or RD with explicit instruction that includes supplemental, intensive tutoring in computation.

Aster and Shalev (2007) researched the neuropsychological underpinnings of developmental dyscalculia (DD) as a genetic disorder of number sense. He found that two-thirds of the students with DD had co-morbid conditions and under-activated regions of the brain, including the parietal area, which is important for number function, and the frontal region, which controls executive functions. The mental number line is important in the development of a child's spatial image and ability to construct, automatize, and enlarge ordinal numbers. A student needs to interlink the understanding of magnitude with symbolic and spatial properties of numbers. This requires cognitive functions, including language skills and working memory, which should develop during preschool and early primary grades.

Aster et al. (2007) developed a core system of representation organized in a hierarchy, which can predict the pathways of development. This system has four core steps. The first step begins in infancy and involves the bi-parietal area of the brain. It allows children to develop subitizing, approximation, comparison and gain a basic understanding of number, which enables a child to associate a perceived number of objects with spoken and written numbers later in development. If this step is not developed appropriately, names of numbers can be learned by rote but may be devoid of the meaning of numerical magnitude leaving children at risk for mathematical difficulties. The second step begins in preschool and involves verbal counting, counting strategies, and fact retrieval, which involves the left prefrontal lobe of the brain. However, if language development and the association between linguistic symbolization and non-verbal numerical properties are not established students are at risk for developing math difficulties, including the retrieval of mathematics facts. Step three involves the Arabic number system, including place value and the digits that represent numbers. This development occurs in the bi-occipital area of the brain that involves written calculations and concepts such as odd and even (Aster et al., 2007). Failure to develop at this step leads to difficulties in the construction of a mental number line, causing the child to be unable to place 52 on the number line between 50 and 60. The final step of number acquisition takes place in the bi-parietal area of the brain that helps one develop a mental number line via a mental image. It allows children to approximate calculation and identify numerical neighbors on the number line. The development of this mental number, a mental anchor, is strongly related to the student's experiences and learning environment.

Instruction needs to be provided that allows students to compensate for the skills which are lacking in their development. Students with MD have weak retrieval of mathematics facts and rely on counting long after their peers have developed more efficient strategies (Dowker, 2005). Baker, Gersten, and Lee (2002) determined that an explicit, teacher led, contextualized approach to instruction is paramount for students struggling in mathematics. Explicit instruction in the application of conceptual understanding and real-world applications led to statistically significant results on a mathematics achievement test. Explicit teaching of rules, concepts, principles, and problem solving had a moderate effect on the achievement tests of students identified as at-risk for MD, indicating the importance of such instruction for struggling mathematics students.

Addition Intervention

Carpenter and Moser (1984) identify five levels of basic fact development for students in grades one to three. Students at level 0 have no ability to solve any problem. At level 1, students need direct models to manipulate in their attempt to solve basic addition problems. Level 2 involves modeling with verbal and mental counting strategies. Students at level 3 can use verbal and mental strategies to solve problems. At the fifth and final level, students are able to use basic fact knowledge to solve many different types of problems. Countries which have high mathematics achievement scores on standardized international assessments, such as China, Taiwan, and Japan, teach strategies of ten to assist their students in their quest for Carpenter and Moser's fifth level (Henry et al., 2008).

Three strategies have emerged to assist students with anchoring their mathematics facts into their long term memory for easy recall (Fuson & Kwon, 1992; Fuson, Stigler, & Bartsch, 1988). The Up and Over Ten strategy teaches children to decompose addends into facts that have a sum of ten. The Down and Over Ten strategy is used with subtraction to decompose numbers down to make ten. A final strategy called Take from Ten teaches students to decompose the minuend into ten and a remainder and subtract multiples of ten using the minuend. Allowing students to learn such strategies via repeated practice, forms bonds for long-term memory retention (Ashcroft, 1995; Baroody, 2003; Fox, 1995; Geary, 1994; Siegler & Jenkins, 1989). Above average mathematics students use short cuts and deductive reasoning to solve basic fact problems, while students with below average achievement use derived facts, slowing down their computational fluency development (Gray, 1991). Research indicates a strong correlation between basic fact fluency and mathematics achievement on standardized assessments (Kilpatrick, Swafford, & Findell, 2001).

Henry and Brown's (2008) study of nine elementary schools in southern California provides further evidence to support this strong correlation. Data were collected on first-grade fluency facts, instructional strategies related to student achievement, and the interaction of instructional decisions and their relationship to number sense. Regular education teachers were given surveys to uncover information such as years of experience, level of textbook implementation, the use of supplemental activities, and the number of instructional events per week that explicitly helped students learn their basic facts. Students were given a basic fact pre-test which included 36 untimed items. Additionally, Mathematics Intervention Assessments were administered to

204 students that required them to solve problems and explain how they solved each problem. The results indicated that two-thirds of the first-grade students used counting as their primary solution method. Students who used this strategy had lower scores on number sense assessments. Only 6.9% of the students had their basic facts mastered 80% of the way through the school year, with subtraction scores being weaker than addition facts. Frequent use of timed tests actually worked against student memorization and successful mastery. The teacher reports indicate that textbook use had a negative correlation with student achievement, indicating the need for more implementation of supplemental conceptual activities.

Only if facts are mastered, and therefore stored in long term memory, can students generalize and use procedural knowledge for abstract mathematics principles (Gersten et al., 2005). A child's mathematics trajectory is set early in his or her academic career. Measures on mathematics and reading assessments between kindergarten and second grade are statistically related to mathematics and reading achievement in grades three through six (Jordan et al., 2002). A child whose learning growth curve indicated low levels of learning fell increasingly behind children with steeper growth curves (McCelland, Acock, & Morrison, 2006). Therefore early intervention is crucial to allow all students to reach their mathematics potential (Gersten et al., 2005; Van Luit & Schopman, 2000). Early intervention is beneficial, but immediate transfer must be taught explicitly through a structured model, with number sense being stimulated first (VanLuit et al., 2000). It should include three goals: fluency and accuracy with number combinations, mature efficient counting strategies, and number sense development

including magnitude comparisons along with the ability to use number lines. Gersten and Baker (1998) recommend a blend of a conceptual model with explicit instruction.

Concrete Representational Abstract

One model that has shown success when working with struggling students is the Concrete-Representational-Abstract (CRA) model (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Cass et al., 2003; Hudson et al., 2006; Miller & Mercer, 1993; Witzel et al., 2003). According to the Access Center (n.d.), a website supported by the United States Department of Education, CRA is an explicit instruction strategy that blends conceptual and procedural learning in a structured way that can be used with individuals, small groups, and whole classes. This graduated framework uses explicit instruction that helps all students make meaningful connections to mathematical concepts, especially passive learners. It also aligns with the recommendation of many researchers who stress the importance of learning disabled students receiving explicit instruction (Hudson et al., 2006).

This method of instruction is based on Jerome Bruner's ideas in *Toward a Theory of Instruction* (1966) which supports progress through the enactive, iconic, and symbolic stages. Students must manipulate concrete items throughout the enactive phase. The children are then ready to learn through the use of pictures and representations of objects in an iconic way. Once the enactive phase is mastered the child is ready to move to the abstract level of knowledge. In order for this level to be successful, students must have the prerequisite knowledge in place. The use of the concrete pictures and manipulatives anchor the abstract knowledge. However, it is important that the concrete work does not distort the abstract learning and that multiple solutions are provided to represent the same

concept (Devlin, 2000; Hudson et al., 2006). The use of this method is also supported in the NCTM Process Standards where it is recommended that students make connections between concrete and abstract ideas. NCTM Process Standards support the use of reasoning and proof that students demonstrate when they decide how to represent a problem and justify their solution.

The first level of CRA instruction, called the Concrete level, focuses on the use of concrete models to develop a concept. Students use manipulatives, such as blocks, chips or fraction pieces to learn the most basic ideas of a mathematical concept. This is the most critical phase of learning, thus making it crucial that students are provided with the appropriate concrete items to develop the concept. For example, when students are learning to add single digits, the students would count blocks for $5+4$ and put those chips on a Ten Frame. At the Representational level, students move to a semi-concrete level, drawing pictures on a Tens Frame to show $5+4$ and writing the equation $5+4=9$ underneath. In the Abstract level of instruction, students use only numbers and symbols to represent the equation $5+4=9$. It is essential that the teacher helps students to develop appropriate strategies to make the connections between the multiple levels. Furthermore, it is paramount that students be given independent practice after each stage of the model. Examples of this include peer tutoring, board games, self-correcting materials, homework, or computer-assisted instruction (Allsopp, 1999).

There are many research studies that have proven the effectiveness of the CRA method of instruction. Maccini and Hughes's (2000) meta-analysis of literature on learning disabled students found that there were few studies focusing on the development of conceptual knowledge. Most studies focus on teaching instructional rules. Their

analysis indicates that the use of explicit models to teach conceptual understanding, such as CRA, has proven successful for students with learning challenges. Kroesbergen and VanLuit (2003) also studied the effectiveness of mathematics intervention for students with special needs. Methods that used direct instruction and explicit instruction were determined to be more effective for teaching math facts and problem solving to children with learning differences than reform based mathematics programs. Indications are that students with special learning needs require a blend of strategies to be successful in the classroom.

Hudson et al. (2006) identifies explicit instruction as the design of lessons and arrangement of instruction to promote optimal student learning. The big ideas of the concept need to be developed with measurable outcomes where the teacher delivers instruction in four phases. In the first phase the teacher activates prior knowledge with advanced organizers, while explicitly stated objectives are provided for the students. The second phase involves demonstration or modeling of overt actions, along with modeling of meta-cognition and cognitive thinking. Questions and prompts are provided and adjusted to clarify misunderstandings. Guided practice is the hallmark of phase three. This is an opportunity for students to practice the new concept. High levels of support should be offered in the beginning and gradually decreased as the student gains confidence. The final phase is independent practice to allow students to practice and reinforce the skill on their own.

Researchers have incorporated CRA methods into a variety of mathematics instruction with success. Witzel et al. (2003) applied the CRA model to an algebra inclusion classroom. In this randomly assigned pre-post follow up study students were

identified by three criteria: performance below the classroom average, scores below the 50th percentile on recent statewide achievement tests, or low socio-economic status.

Results indicated that students who received CRA instruction over a four-week intervention period out-performed the abstract only similarly matched group. Their data showed significant improvement in the students' ability to solve single variable, multi-step problems. Both groups showed significant increases in their post-test scores, with the CRA students being significantly higher and maintaining those gains on a follow-up exam. These results also indicate that CRA can be used successfully as a whole-class intervention.

Butler et al. (2003) investigated the effects of CRA and Representational-Abstract on the teaching of fraction concepts to middle school students with mathematics disabilities. A four-phase model of instruction was used to deliver the CRA instruction. Their results revealed a statistically significant difference on a quantity fraction sub-test. Although the CRA group outperformed the control group on every subtest, lending evidence that the use of concrete manipulatives could increase a child's fraction understandings.

Miller and Mercer's (1993) study compared how long it takes students to "crossover" when they received a CRA approach to the instruction of basic addition facts. They defined "crossover" as the point where students were computing more basic facts correctly than incorrectly in one minute. The purpose of the study was twofold: to confirm that students were able to gain basic fact acquisition through the use of CRA and to determine how long this process can take. Their research indicated that students were able to master their basic computation facts through the use of the CRA method and it

took between three and seven lessons for the crossover effect to take hold. They also discovered that when the crossover was achieved, the students' rate of correct responses continued to rise, while their rate of incorrect responses continued to decrease. Additionally, it was shown that once students mastered the concrete learning they were able to successfully apply this knowledge to abstract learning.

There are many studies on the effective use of CRA in elementary school. Peterson, Mercer, and O'Shea (1988) studied the use of CRA to teach place value skills. They used place value cards, cubes, teacher-made cards, and picture representations of cubes. The results indicate immediate and delayed mastery of place value at the 80% mastery level. Funkhouser (1995) used manipulatives to assist kindergarten and first-grade students in the acquisition of number identification through five and sums up to five. After a four-week intervention, students had 90% mastery. Miller, Harris Strawser, Jones, and Mercer (1998) employed a CRA model, along with modeling, demonstration, feedback, and cognitive training, to a group of second graders' learning of multiplication. The students included learning disabled and underachieving students. The results indicate that all students benefited from the instruction provided in an inclusion setting with 25 to 27 other students.

Response to Intervention

This study is based on the identification of students through a Response to Intervention (RtI) model. RtI is a general education model that is meant to screen all students and identify those students who are most at risk for failure in the general education curriculum. It also allows students to be diagnosed with a specific learning disability without using the Discrepancy Model, which identified children solely on the

difference between their IQ and achievement. The Discrepancy Model often leaves children to flounder in the general education curriculum without support until the gap becomes wide, typically around third grade (National Research Center on Learning Disabilities). RtI seeks to determine if the student's struggles are due to a true disability or merely poor instruction. Providing access to quality instruction is important in first-grade, as prior approaches to learning predict future mathematics achievement (Byrnes & Wasik, 2009). A student's success in a research-based program, implemented with fidelity, eliminates the possibility of poor instruction as a reason for lack of growth (Fuchs et al., 2007).

According to the Pennsylvania Training and Technical Assistance Network (PaTTAN), an initiative of the Pennsylvania Department of Education, there are six core characteristics of RtI. The first characteristic is a standards-aligned curriculum. All students must receive a high-quality, research-based, general education curriculum that is standards aligned. This should include the use of formative and summative assessments.

The second characteristic promotes the use of universal screening. All students must be screened at least three times per year to determine their progress in the regular education curriculum, both academically and behaviorally. Data gained from these screenings should be organized and maintained in a friendly format with easy to read summaries and graphs. Teams should meet at least three times per year to review the data and make instructional decisions. The universal screening instrument should be research-based and predictive of future student growth. It should have norms and be sensitive to student improvement.

Shared decision making is the third hallmark of RtI. All educators in the building should take ownership of student learning. All teachers, including special educators, general educators, Title 1 teachers, and ESL teachers are responsible for assessment and implementing effective programs. Once data are collected, curriculum decision-making must be based upon the results. The data team will review all summative, formative, and normed data to determine those students who are at risk for failure in the regular curriculum. Progress monitoring should be used to determine instructional goals, intervention effectiveness, and drive any instructional changes to meet the child's learning needs. The data team should also choose the research-based program and plan for its implementation, determining staffing and time issues, along with fidelity checks.

According to PaTTAN (Pennsylvania Training and Technical Assistance Network), the fourth characteristic of RtI is the use of a tiered system. The Commonwealth of Pennsylvania uses a three-tier system. All students begin in the first tier where all children are provided with a quality regular education and are universally screened at least three times per year. The regular curriculum should have high expectations for all students and provide for high student engagement. Approximately 80% of the population should stay in this tier and meet with academic success. When more than 20% of the students are not meeting success in the regular curriculum, both the curriculum and instruction need to be re-evaluated. There are two possible student outcomes in Tier 1: responders and non-responders. Students who are responding to the regular curriculum continue to receive the instruction at this level.

The second tier is for those students called non-responders in the regular curriculum, those students whose assessments indicate they are not making adequate progress in the regular curriculum. Approximately 15% of the population is anticipated to fall in this tier. Students at this tier must receive research-based interventions that have been proven successful with students with similar learning profiles. Each intervention should last approximately 10-12 weeks. It is crucial that teachers implement the program with fidelity, as it was designed. Interventions need to include increased time for mastery of the learning objectives in small group instruction. Progress monitoring should be implemented in order to incorporate any necessary instructional changes. There are two potential outcomes at this tier: responders and non-responders. The responders either continue in the Tier 2 intervention or move back into Tier 1.

Non-responders, expected to be approximately 5% of the population, need to have either their Tier 2 intervention adjusted to better meet their instructional needs or be moved to Tier 3. Tier 3 involves more intensive instruction that includes weekly progress monitoring for students who are showing significant difficulties. Supplemental materials are used to develop specific skills. Additional tutoring, along with flexible grouping, may be added to give Tier 3 students more intensive instruction. Fidelity of intervention is also crucial at this tier. Those students who are responding either move to Tier 2 or are referred for a special education evaluation.

The final characteristic of the Pennsylvania model of RtI is parental engagement. It is imperative that parents receive information regarding their students' progress in the regular curriculum. If a student is receiving intervention services, parents should receive information about the process, intervention implemented, and regular progress reports.

This Pennsylvania model is meant to ensure that all students are receiving an appropriate education that meets their learning needs and includes parental support.

Little research has been conducted which investigates the implementation of Tier 2 interventions in mathematics. Gersten, Beckmann, Clarke, Foegen, Marsh, and Star (2009), in their analysis of effective RtI models, point to strong evidence supporting the use of explicit instruction which provides guided practice, frequent cumulative reviews, and verbalization of thought processes. They found moderate evidence supporting the use visual representations of mathematical concepts and the building of computational fluency. Evidence also supports the use of instruction models which develop properties of whole numbers and operations. This recommendation is supported by the NCTM's Focal Points (2006) and the National Mathematics Advisory Panel's 2008 report.

Most of the research into Tier 2 interventions studied the mastery of facts using drill and practice as the intervention (Fuchs et al., 2007). Fuchs et al. (2007) advocate for research that incorporates curriculum trends in mathematics. They designed a research study that targeted first-grade and had three purposes. They sought to assess the effectiveness of preventative small-group tutoring, investigate the usefulness of screening and progress monitoring tools, and to examine the prevalence, severity and sensitivity of RtI methods for determining lack of response to treatments. The researchers incorporated forty-one classrooms and identified students at risk for mathematics difficulties based on low performance. Their control group continued to receive instruction in the district's basal program. The experimental group received tutoring, using a CRA approach, and computer practice three times a week for 16 weeks. All students were assessed weekly with curriculum-based measures to monitor their progress. Study results indicate that the

experimental group's growth outperformed the control group, and in some cases surpassed the Tier 1 students. These results indicate that RtI has promise to reduce the number of students with math difficulties. In fact, the results indicate that RtI has the ability to reduce the number of students identified as having a mathematics disability. The Tier 2 intervention reduced the identification of a math disability by 35.6%. This statistic remained stable at the end of second grade indicating that RtI may be more sensitive to the identification of learning disabilities than commercial IQ tests.

Fuchs et al. (2007) also studied a group of third-grade students in need of a math intervention. They used a problem-solving schema broadening intervention with self-regulating strategies. Students in the third-grade cohort were evaluated to identify those at risk for mathematical difficulties and were then randomly assigned to four different groups: no primary or secondary intervention, primary intervention, only secondary intervention, and both primary and secondary interventions. Students were assessed in immediate, near, and far transfer of problem-solving skills. A lack of responsiveness was defined as being one or more standard deviation below the growth of the normative group. When evaluating immediate and near transfer, 86% and 100% respectively of the at-risk students who received no intervention were unresponsive. On contrary, students who received the primary intervention had their non-responsive rate drop to 29% on immediate transfer, and 62% on near transfer. However, when students received both primary and secondary interventions, the non-responsive rate dropped to 12% on immediate transfer and 26% on near transfer. These data also indicate that the use of a tiered intervention can assist students in overcoming mathematical difficulties.

Bryant, Bryant, Gersten, Scammacca, and Chavez (2008) studied the effects of a 10- to 12-week multi-tiered model of intervention on first- and second-grade students. Concepts were taken from Number and Operation and Quantitative Reasoning Skills assessment, as well as concepts in the NCTM standards for grades kindergarten through second grade. Magnitude comparisons, number sequences, place value, and addition/subtraction combinations were used as measures of student progress. Their results posit a significant main effect for the spring assessments given to second graders, indicating a positive effect for the intervention. The first-grade intervention did not have statistically positive effects. However, the authors believe this may indicate that the intervention should include a minimum of 20-minute sessions due to first-grade students' developmental level. The authors conclude that Tier 2 interventions for first- and second-grade students should focus on number sense development due to its predictive ability to identify future mathematics difficulties. They further recommend that future studies provide more intervention time, along with interventions that include practice with different representations, such as pictorial and abstract.

Summary

Clearly the groundwork has been laid to show that number sense is the foundation of students' ability to be successful in mathematics. Educators must provide interventions to determine if a student is struggling due to inadequate instruction or cognitive difficulties (Geary, 2004). Jordan and her colleagues (2007) recommend that future studies look at explicit instruction for Tier 2 students that develops number sense in order

to help students compensate for mathematical difficulties. If students are to be successful in mathematics, they need to have mental pictures of quantities and use visualization or counting to add numbers fluently (Jordan et al., 2007). The systematic model of CRA instruction will assist students in developing conceptual knowledge in both place value and addition combinations.

The need for more research in the area of early intervention in mathematics is evident. The identification of young students struggling in mathematics is in its infancy. Systematic and effective reading interventions have been thoroughly researched so that teachers are able to identify predictors and strategies that have been proven effective with struggling readers. This study will provide research into the use of early interventions using strategies that have proven successful in special education. Much of the literature focuses on the development of skills in mathematics. Through this lens, the researcher will add to the literature, providing guidance for educators in the field of Response to Intervention, and emphasizing the importance of including conceptual development in instructional practices.

CHAPTER 3 METHODOLOGY

Introduction

Using the procedures outlined below, the effects of a Concrete-Representation-Abstract (CRA) model of instruction was studied to determine its effectiveness with first-grade Tier 2 students in a Response to Intervention (RtI) model. These students were identified as at-risk on at least two of the following methods of evaluation: Lembke-Foegen Early Numeracy probes, Fuchs and Fuchs Curriculum based assessments, and teacher recommendation. Additionally, the CBM provided data to evaluate via a single subject design model. The TEMA-3 and Lembke-Foegen instrument were administered both pre-and post-intervention to determine student growth. In an effort to evaluate the effectiveness of this model, the following research questions were examined:

1. What are the effects of a concrete-representational-abstract instructional model on students' achievement as measured on a mathematical achievement test?
2. What are the effects of a concrete-representational-abstract instructional model on Tier 2 students' development of computational fluency?
3. What are the differences in student growth, measured via a progress monitoring tool, when students receiving concrete-representational-abstract instruction are compared to a control group?

A single subject design, as described by Rogers (2007), was used due to its ability to provide systematic data collection in the natural environment of the classroom. This allows the researcher to study and modify the design of the instruction to meet the learning needs of the learner on a continuous basis and serves to evaluate the process.

Comparisons are made within the subject, instead of between subjects making it ideal for an RtI model, as it provides a direct measure of the treatment effect which has strict controls (Tankersley, Harjusola, & Landrum, 2008, Zhan & Ottenbacher, 2001). The model consists of two phases of implementation: the baseline and intervention phases. The target behavior is repeatedly measured during both phases and plotted on a graph. This ensures a true representation of the student's performance due to the control of the random factors of the environment (Tankersley et al., 2008). In the baseline phase, data are collected over time, looking for consistency in performance. Once the baseline is established, any changes in the performance indicate a trend of the intervention. When the independent variable, Expedition to Numeracy program, is implemented, outcomes can be measured by the Math-Curriculum Based Measures, the dependent variable (Rogers, 2007).

Although statistical tests were performed on the data collected, visual inspection is one of the primary evaluations of Single Subject Design models. This is a process of systematic rules to be followed when evaluating the graphs for each student. The researcher examines the data noting changes in the dependent variable from the baseline to intervention time periods (Tankersley et al., 2008). The first examination requires an analysis of the mean using a progress monitoring tool, applied both pre- and post-intervention, to determine the magnitude of change (Alberto & Troutman, 2006). It is also important to look at the growth in performance immediately following the intervention, referred to as the 'level'. Researchers should look at both the magnitude and immediacy of changes in level; the larger the change in level, the greater the strength of the intervention. In single-subject design, as implemented in special education, a trend

in data points either systematically ascending or descending also provides solid evidence that the intervention is effective. The latency of change indicates the strength of the intervention. The more dramatic the growth in the desired direction, the stronger the intervention effects (Tankersley et al., 2008). However, there is little consensus as to the desired slope which would indicate significant progress in student learning (Zhan & Ottenbacher, 2001).

Growth Curve modeling, a type of Hierarchical Linear Modeling, was also used to add to the power of the data. This model allows the researcher to compare scores across time and look for changes in the slope and intercepts (starting scores) of the linear regression. This change may be either positive or negative in nature and allows the researcher to determine what variables account to the growth and rate of change.

Participants

Participants in this study represent a sample of convenience. The researcher is a Mathematics Specialist at the schools in which the study will occur. She has been trained in the implementation of the Expeditions to Numeracy program by the lead author. The school district is located outside of a major United States city in the northeastern part of the country. This suburban district has two high schools, one alternative school, three middle schools, and ten elementary schools. The total enrollment is 12,395. The researcher works in two of the elementary schools.

The researcher has obtained approval from the Internal Review Board at Temple University and obtained parent permission, as well as student assent. The participants were first-grade students attending three elementary schools in the district. All students are instructed using the Scott-Foresman Addison Wesley 2001 Grade 1 textbook, along

with a reform-based supplemental curriculum entitled *TERC Investigations*. The classrooms are heterogeneously grouped with approximately 25 students per classroom. Due to the nature of RtI, no students with IEP goals in mathematics were included in the study.

Students were chosen for inclusion in the study based on being labeled at-risk in a minimum of two of the following: teacher recommendation, performance on the Fuchs and Fuchs computation probes and performance on the Lembke-Foegen Early Numeracy probes. The control group for this study included first-grade students from another school in the same district. The students chosen as the control group were identified solely on the median of their first three computation probes. They were not screened with the Lembke-Foegen instrument or referred by teacher recommendation. These control students, along with all students at the control school, continued to take the computation probes throughout the study.

Instrumentation

Lembke-Foegen Early Numeracy Indicators

A universal screening tool designed specifically for first-grade students was used to identify those students who are experiencing difficulty in mathematics. (See Appendix A) This instrument, designed by Lembke and Foegen (2005), is based upon research that indicates that identifying numbers, quantity discrimination, and missing numbers prompts have strong criterion validity and reliability (Clarke & Shinn, 2004; Chard, Clark, Baker, Otterstedt, Braun, & Katz, 2005). Lembke and Foegen developed the universal screening during six years of study in an urban fringe school district. They demonstrated strong technical adequacy in the areas of number identification, quantity discrimination, and

missing number. These sub-tests had positive correlations with the TEMA-3, Woodcock Mini-Battery of Achievement, and the Stanford Early Achievement Test. The alternate form reliability had an r value between .79 and .89 and the test-retest coefficient was between .73 and .91, indicating that the two forms of the assessment are positively correlated. The criterion validity was between .36 and .71, with most of the r values between .5 and .6. These results reveal a significant correlation between the Lembke-Foegen instrument and other measures of early mathematics readiness.

The Lembke and Foegen Early Numeracy Indicators consist of three one-minute probes given to students in kindergarten and first-grade. The first sub-test, Number Identification, asks students to read a list of numbers presented in random order from 0-99. The Quantity Discrimination probes require students to indicate the larger number when presented with two numerals from 0-19, inclusively. The final sub-test, Missing Number, provides a series of four numbers which has one of the numerals missing, requiring students to indicate the missing number. The series does not usually start with one and can count by ones, fives, or tens. All sub-tests are administered in a similar fashion, the student page placed in front of the child. A scripted set of directions are provided for each sub-test that leads the student through three sample items. If the child's answer is correct the assessor responds, "Good", and repeats the answer in a complete sentence. If the response is incorrect, the assessor indicates the correct answer. For example, "The number that is bigger is 7. You should have said 7 is bigger." At the beginning of the assessment, the assessor then repeats the directions and informs the student to go across the page and try each one. The student starts with her finger on the first problem, and works for one minute giving answers verbally while the assessor marks

each student response on the examiner copy. When scoring each probe, protocol indicates if a student fails to answer a question in three seconds, she is instructed to try the next problem. If the student correctly answers the probe, one point is given. Conversely, if she states any other number, no points are awarded. A skipped probe is marked incorrect, as are all probes in an entire row that is skipped.

Curriculum-Based Measurement-Mathematics (CBM-M)

Fuchs and Fuchs have designed progress monitoring probes to be used weekly to assess student's growth in mathematics. Progress monitoring is a tool used to make educational decisions, specific to a skill. Three of the goals of progress monitoring are: 1.) estimating rates of improvement, 2.) identifying students not making adequate progress, and 3.) comparing the efficacy of different types of instruction (Fuchs et al., 2005). In the study the Fuchs and Fuchs probes were used to assess students weekly. These probes have been proven valid and reliable measures for each of these three goals (Deno, 1985; Marston, 1988; Shinn 1989).

The probes are designed to be used weekly and to include uniform instructions each time they are administered. Each assessment contains 25 computation problems which represent skills covered throughout first-grade, and include multiple forms that assess the same skills throughout the school year.

Test of Early Mathematics Ability (TEMA-3)

Students who are identified on the universal screening as at-risk were given the Test of Early Mathematics Ability-Third Edition (TEMA-3). This test is norm-referenced and appropriate for students between the ages of three and eight years, eleven months. It has parallel forms and provides a raw score, age equivalence, grade

equivalence, percentile rank, and Math Ability Score, also known as the standard score. Ginsburg and Baroody (2003) cite five purposes for using the TEMA-3:

1. Identify those students significantly ahead or behind their peers in mathematical thinking
2. Identify strengths and weaknesses in mathematical thinking
3. Provide instructional strategies appropriate for each child
4. Document a child's progress in learning mathematics
5. Serve as a measure for scholarly research

The TEMA-3 is a statistically valid and reliable assessment. It was normed in the fall of 2000 and the spring of 2001 on a sample of 1,228 students from 15 states. This sample selection is a representative sample of students from across the United States with regard to geography, region, gender, ethnicity, family income, educational level of parents, and disabling conditions. The percentages for each characteristic were compared to 1999 data obtain from the United States Census Bureau. The reliability of the test, as reported by the authors, has also been shown to be high. Table 3.2 lists the reliability.

Table 3.1
TEMA-3 Reliability

Form	Cronbach Alpha	Test-Retest
A	.94	.82
B	.96	.93

(TEMA-3 Examiner's Manual, 2003)

The content validity of the TEMA-3 was established through the assessment of informal and formal mathematics knowledge. Informal knowledge is measured through the use of items that assess numbering skills, number comparisons, calculation skills, and understanding of mathematical concepts. The items assessing formal mathematics

knowledge include: numeral literacy, number facts, calculation, and concepts such as base 10.

Procedures

Lembke-Foegen

All first-grade students in this study were assessed using the Lembke-Foegen instrument. Those students that scored at the 35th percentile on two of the three sub-tests participated were labeled at-risk on this instrument. The cut scores for each sub-test are listed in Table 3.2. The Winter and Spring scores, respectively, were used pre- and post-intervention.

Table 3.2
Lembke-Foegen Cut Scores

Assessment	Fall	Winter	Spring
Number Identification	30	38	41
Quantity Discrimination	26	28	31
Missing Number	13	14	17

Curriculum Based Assessment

The Fuchs and Fuchs probes were given once a week to all students in first-grade, at both study sites. Additionally, a control site at a third elementary school in the district was utilized. All classroom teachers received training in the implementation and received a written set of directions. The researcher and a cooperating Mathematics Specialist scored each CBM. These assessments are scored by counting the number of

digits correct for each problem. A correct two-digit answer received two points. If one digit is correct and the other incorrect, the student received one point. The points are totaled and recorded on a spreadsheet to track student progress. As recommended by the authors, three baseline data points were obtained. The median score was utilized when evaluating eligibility for the study. A score of twelve indicated a student was not on track to reach the first-grade benchmark score of 20 by the end of the year, and thus identified a student as at-risk for difficulties in mathematics.

Once the study began, the researcher continued to collect CBM data every week from all first-grade students, at all three schools. An increase of 0.35 points per week is considered average growth for a first-grade student. The benchmark for the end of first-grade was 20. Progress toward this goal was monitored for all students in both the study schools and the control school.

Teacher Evaluation

All first grade teachers at the two study schools were asked to submit a class rosters identifying students they believed were currently struggling in the first grade curriculum and were at-risk for future difficulties in mathematics. The researcher was careful not to discuss any assessments results with the teachers before the rosters were submitted.

TEMA

Each student identified as at-risk for mathematics difficulties on the Lembke-Foegen assessment and the computation probes were given the Test of Early Mathematics Abilities (TEMA). This test is an individually administered, un-timed test. In an effort to determine the entry point for each student and shorten the length of the test, the TEMA-3

manual provides suggested entry points. Students who are six years old were given on question 22 and students who are seven were given on question 32. The lower limit of the test, known as the basal, is achieved when the student gets five consecutive questions correct. If the student is not able to answer five consecutive questions correctly, the examiner continues the test until the student misses five questions in a row, and then work backwards from the entry point until a basal is reached. If no basal is attained, as may happen with students with severe mathematics difficulties, the test can still be scored. A student earns one point for each correct answer, and the test is concluded when a child hits the ceiling, meaning she answers five consecutive questions incorrectly. It is then assumed that all questions prior to the basal are correct. If there is more than one basal, the basal closest to the ceiling is used for scoring purposes.

Expeditions to Numeracy

Students were included in the study if they were labeled at-risk in a minimum of two out the three following measurements: teacher recommendation, Fuchs and Fuchs computation probes, and the Lembke-Foegen Early Numeracy probes. Once parent permission and student assent was obtained the students received instruction using the Expeditions to Numeracy program. This is an intervention program designed to help students develop conceptual knowledge of the base-ten number system (see Appendix B). It is divided into five mathematics regions: place value, addition and subtraction, multiplication and division, money, and proficiency practice. The most crucial aspect of the program is place value, as it contains the underlying principles of our number system and is the basis of the balance of the program. This region is divided into five different number sections: 0-9, 0-19, 0-99, 0-999, 0-1000+. The focus of this study was on the 0-9

place value due to the age and ability level of the students in the study. Additionally, addition and subtraction were covered from 0-9; as this region uses place value concepts to assist students in mastering their addition and subtraction facts.

At the core of the program is the use of Digi-Blocks, a mathematics manipulative system. These blocks are a proportional nesting system that gives a physical representation of the base-ten number system. The basis of the system is the one's block in the form of a small rectangular prism. Ten of these one's blocks are placed in a block to form a ten. Ten, ten's blocks can be placed in a hundred block. Similarly, ten of the hundred's blocks nest into a thousand's block. Through packing and unpacking different blocks, students are able to develop connections such as ten ones are equal to one-ten and ten-tens are equal to one-hundred or one-hundred ones. (See Appendix C)

It is crucial that students who qualify for the Tier 2 intervention be properly placed in the Expeditions to Numeracy program. In order to facilitate this, each student was given a placement test written by the authors of Expeditions to Numeracy (Akin & Rimbey, 2008). Proper placement ensures that students are unimpeded by gaps in their conceptual knowledge. Due to the young age of the students, the authors recommend that the users start at the beginning of the program and administer the first assessment in the Place Value 0-9 program. If students show mastery, the examiner will continue to move through each lesson's assessment until gaps in understanding are uncovered. Instruction is then offered and the pre-test for the next lesson is subsequently given to the student.

Formative and summative assessments are included in the program. Evidence-of-learning statements at the end of each lesson inform the teacher of which skill needs to be demonstrated in order to show evidence of mastery. The instructor recorded mastery on

the Student Progress Record sheet (Appendix D). When students have completed an exploration or lesson, a summative assessment was administered. These results and notes were also included on the Student Progress Record sheet. The following rubric was created to score student work:

Performance

- 4:** The student performs the task easily and with no errors.
- 3:** The student performs the task with a few minor errors.
- 2:** The student has difficulty performing the task and/or makes frequent and/or significant errors.
- 1:** The student is unable to perform the task

Criteria for Mastery

Pre-test: The student receives a score of 4.

Post-test: The student receives a score of 3 or 4.

The students received thirty half-hour instructional periods using the Program three times per week. According to the Leader's Guide, the teacher is the decision-maker in the classroom. The teacher must determine the proper instruction to meet each student's needs, and how long to spend on each concept, while using assessment to drive the instruction. Teacher monitoring of student progress is crucial. She must determine when to progress to additional instruction on the same concept, and when it is appropriate to spend multiple lessons on the same concept. Fidelity checks, conducted by independent members of the school, were completed using the attached Fidelity Checklist (Appendix E).

Curriculum Based Assessments

The Curriculum Based Measurements (CBM) was administered weekly, in a large group setting, to all students in grade on Fridays. If school is not in session, the probes were given on Thursday. Two minutes is allotted for the probe (Fuchs et al., 2005). When the Fuchs and Fuchs CBMs are distributed, students put their first and last name and date on the paper to ensure accurate record keeping. Students were then be instructed to: 1) complete as many problems as possible in two minutes, 2) remember that they will receive credit for each correct digit in the problem and 3) know to skip problems they find too challenging and can come back to them after they have completed the easier ones (see Appendix F). The probes are scored by counting the number of digits correct on each question, as the digit count is more sensitive to student growth than counting only completely correct answers (Fuchs & Fuchs, 2005). Outside assessors spot checked the scoring to ensure the accuracy of the data.

Data Analysis

TEMA

Once the TEMA-3 was administered to all qualifying students the results were analyzed for use as both a pre- and post-assessment. When this test was administered, the examiner recorded the student's birth date and calculated a precise age, including the years, months, and days. It was important that the examinee's age not be rounded when using the normative tables provided in the Examiner's manual. When the test was scored, it provided the examiner with raw scores, percentile ranks, and age and grade equivalents, and standard scores. The raw score has little significance in evaluating a student's mathematics knowledge and cannot be interpreted or compared with other

scores. The percentile score gives the researcher a measure of a student's score on a scale of 100. A percentile rank indicates the percentage of the students that score below the given student. The age and grade equivalents are "math ages" which are reported in terms of years and months. The standard score, called the Math Ability Score, provides a global measure of achievement. It has a mean of 100 and a standard deviation of 15. The table below provides the appropriate interpretation of the standard score. A *t-test* was used to compare the pre- and post-TEMA-3 scores for each student.

Table 3.3
Guide to Interpreting TEMA-3 Math Ability Scores (TEMA-3 Examiner's Manual, 2003)

Standard Score	Description	Percentage included in the Bell-Shaped Distribution
≥131	Very Superior	2.34
121-130	Superior	6.87
111-120	Above Average	16.12
90-110	Average	49.51
80-89	Below Average	16.1
70-79	Poor	6.87
≤69	Very Poor	2.34

Curriculum Based Measurement

Once the data were collected each score was graphed to provide a visual representation of student growth to assist the researcher in setting and modifying goals

and instruction. Zhan and Ottenbacher (2001) identify three advantages of visual analysis. It allows for continuous monitoring of student progress, is well suited for individualized treatment and outcomes, and it can be widely understood when analyzing slope, variability, and trend. Visual analysis also allows for additional statistical analysis, including correlating the data with other variables and assessing Expeditions to Numeracy's impact on achievement (Parker & Tindal, 1992).

The end of the year goal for first-grade students is 20 out of 30 digits correct in two minutes. A point is placed on the student's graph to represent the end of year benchmark and a line is then drawn from the median of the first three scores to the goal. This allowed the researcher to track trending toward the goal. After data are collected for approximately eight weeks, the researcher was able to evaluate a student's actual progress toward the desired goal set at the beginning of instruction. Since visual inspection is subjective, other statistical methods will also be implemented.

Fuchs and her colleagues (2005) provide a nationally normed guideline for student improvement. The weekly rate of growth for a typical first-grade student is .35, indicating that students should add approximately .35 points to their score every week. In order to evaluate this, the researcher used the Tukey Method to give an accurate picture of how the student's actual growth is trending (Parker & Tindal, 1992). The researcher collected at least seven data points. The data was divided into three equal groups, each containing approximately the same number of assessments in each section. Vertical lines were then drawn on the graph to show the division of the three groups. The median score of the first and last group was calculated, along with median week number. An X was

then placed on the graph at the appropriate score and week in the first and last group. The line created shows the trend of growth and line of improvement.

There are other techniques that can be utilized to evaluate the data collected. Olive and Smith (2005) describe an analysis technique found in the literature which assess change in the baseline and intervention data and appear to provide consistent measures across data. Percentage of Over-lapping Data is utilized to judge the effectiveness of the intervention. Since the current study is focusing on increasing CBM scores, the highest baseline data point was identified. Next the total number of data points falling below this point was calculated and divided by the total number of data points. If the score is above 90%, the treatment is considered very effective. Likewise, percentages between 70 and 90% are considered effective, scores between 50 and 70% are considered questionable. Any percentage below 50 is considered ineffective (Scruggs & Mastropieri, 1998).

Zhan and Ottenbacher (2001) recommend the use of Split Middle Trend Line, also called the celeration line approach. In this quasi-experimental technique, the baseline data were divided in half. The median of the data of both halves was plotted. The two points are then connected indicating the trend of the data. If the slope of the celeration line is similar in the baseline and intervention data the program would be proven ineffective. If the proportion of data points above the celeration line in the treatment phase is greater than the baseline data one can conclude that the treatment was effective (Zhan et al., 2001).

Hierarchical Linear Modeling

Hierarchical Linear Modeling (HLM) was also be used to offer validity to the single subject design and evaluate the change in the Fuchs and Fuchs computation probes. Growth Curve modeling, a specific kind of HLM, was used to study the many layers of units of the computation scores. A study eligible for this method of analysis must have at least three waves of data, have outcomes that change systematically over time, and have a sensible metric for clocking time. In this study, thirteen waves of CBM data was collected, scores changed throughout the study, and the probes were given on a weekly basis, thus meeting the criteria for inclusion in this model. The assessments being observed are nested with-in people and observations over time are independent of one another. The first layer measures with-in subject changes from the baseline data and characterizes the individual student's growth over time. The second level of analysis accounts for the observed variance among students. It allows the researcher to predict those differences among the subjects that could account for the differences found in level one. In this statistical model the change in scores was analyzed, not the individual scores, along with the steepness of the slope and the beginning scores on the CBMs (Singer & Willett, 2003).

This hierarchical structure has many benefits in research studies with a small number of participants (Singer & Willett, 2003). The structure is able to control for Type 1 errors regardless of the sample size or serial dependence and provides adequate power, being able to detect moderate and large effects (Jenson, Clark, Kircher, & Kristjansson, 2007; Singer & Willett, 2003). Additionally, it can accommodate varying numbers of phases and number of data points in each phase, including data with missing data points.

This allows for fewer participants (Singer & Willett, 2003). It can be used when the researcher is “reasonably certain” of the interventions’ effects noted in visual analysis (Godbold, 2005), and allows for data to be linear, curvilinear, or exponential.

CHAPTER 4 RESULTS

Introduction

Twelve first-grade students were chosen to participate in the Concrete Representational Abstract (CRA) study based on identified needs as determined equally by teacher recommendation, Lembke-Foegen Early Numeracy screening probes, and/or the Fuchs and Fuchs Computation Curriculum-Based Measures (CBM). Students who failed to meet the criteria in at least two of the three screening tools were eligible to participate in the intervention. This CRA instructional model intervention was implemented three times a week beginning in March and ending in June, with each instructional session lasting thirty minutes. One group of six received instruction immediately following lunch and recess, and six students were instructed during the last thirty minutes of the day. The Expeditions to Numeracy program was used with fidelity, following the prescribed lessons as designed by the authors. The twelve students involved in the study were students that did not qualify for special education, but were having difficulty in the district's prescribed curriculum. There were six boys and six girls. The group which met after lunch had four girls and 2 boys, while the end of the day group had two girls and four boys. Students ranged in age from six years seven months to seven years two months. No student had repeated first grade.

At the beginning of the intervention, students were given a placement assessment and then progressed through the sessions, completing one lesson at each meeting. This assessment demonstrated that the students were proficient in counting objects to ten,

reading and writing numerals to ten, counting backwards from ten, and matching numerals and sets. Therefore, the researcher began with ten-frame formation since the students had little experience with the manipulative. There were occasions when it was necessary to extend a lesson into two sessions. Occasionally, it was possible to combine two lessons into one day, especially at the beginning of the program when students were being introduced to the materials and routines. A total of 30 sessions occurred. Anecdotal notes were taken daily as an auxiliary record of student progress. At the conclusion of the program, the students were given the Test of Early Mathematics Ability (TEMA-3), the Lembke –Foegen Early Numeracy probes, and a final Fuchs and Fuchs CBM.

During the initial sessions, students were instructed on place value of numbers 0-10 using ten-frames and Digi-blocks. The introductory ten-frame lessons focused on assisting students in the development of subitizing the Digi-blocks on the ten-frame. This instruction exposed students to ten-frame formations and enabled them to recognize the number of blocks on the frame without hesitation. Additionally, they were instructed in modeling numbers ten and below on the frame. One lesson in place value had students identifying quantities on the ten-frame without counting, thus creating mental pictures of numbers for use in addition and subtraction. For example, when a student identified a ten-frame with seven blocks, discourse in class centered around seven being two more than five and three less than ten, using the anchors of five and ten. The visual of the ten-frame assisted the students in subitizing and creating visual images to be used in subsequent lessons. These skills provided the necessary background knowledge to begin instruction in addition and subtraction.

Once place value instruction was completed, students moved to the addition and subtraction with numbers from 0-9. This exploration led students through addition on a number line and the use of a ten-frame as a learning tool. Students initially manipulated the Digi-blocks on the ten-frame to concretely add numbers. During the initial phases of instruction, only the top row of the ten-frame was utilized. Students added numbers such as two and one by combining the correct number of blocks on the first row of the ten-frame. After practice and mastery was achieved, the students began using addends that required the use of both rows of the ten-frame for representation. In this case the students solved expressions such as $2+7$ by placing two blocks on one ten-frame and seven on another. The students then combined the blocks together to determine the sum of nine. The discussions in class centered around determining if the sum would be more or less than five and identifying the sum by subitizing. Nine of the twelve students became confident in making pairs of ten within two lessons. The remaining three students continued to use the ten-frame to make ten and showed signs of frustration. They required more one-on-one modeling by the instructor. The next step had students creating number sentences to represent their concrete manipulations. Once the students had mastered the representations, instruction moved to visualization. The students were given an addition problem and asked to visualize the first addend on the ten-frame by mentally putting an X on the correct box, or even their finger if necessary. The second addend was then added mentally and read from the ten-frame. Some students initially began by putting their finger on the starting number, but were able to move beyond with practice. Rich discussions occurred as students shared their strategies and

solutions. Activities such as Addition Solitaire and Go Fish for Ten, allowed students to practice these skills over the course of the lessons.

Interestingly, all students were incorrectly solving problems involving zero at the beginning of the intervention. Some students even changed the number of blocks on the ten-frame to fit their answer. For example, when solving the equation $2+0$ on the CBMs, all students incorrectly answered zero. When presented with a ten-frame to solve this problem, some students cleared the frame to represent the answer they believed to be correct. As students were led through a series of questions similar to those used in the ten-frame instruction, they changed their perceptions of adding and subtracting with zero. Some students were able to shift their thinking after one lesson; others required up to five lessons. However, all were able to master the concept. The use of the ten-frame enabled the student to see that adding and subtracting zero did not change the number of blocks on the ten-frame.

Mastery of addition allowed the students to move toward an abstract model of subtraction, where the students were eventually asked to visualize the addition and subtract problems on the ten-frame. Activities such as Go Fish, solitaire, and determining addition and subtraction partners were used at the end of the exploration to help students make the abstract connections necessary to store the number facts into their long-term memory. Finally, at the end of the program, the students began to formally identify the connections between addition and subtraction.

Two students, Matt and Emily, were chosen to demonstrate the typical development of students in the study because their growth on the computation probes represented characteristic progress for those students involved in the intervention.

Emily's growth was linear in its shape, and demonstrated regression in two of the weeks. Matt, on the other hand, started very low, made substantial gains followed by small regressions. Unfortunately, he ended the program with regression, but this may be attributed to the timing of the end of the intervention. The program and assessments continued into the last week of school in an effort to provide 12 weeks of intervention. This time frame was problematic for the researcher and subjects, as student motivation decreases and activity level rises as the school year draws to a close.

Test of Early Mathematics Ability-3rd Edition

In an effort to measure goal one of the study, the effects of a CRA instructional model on students' achievement, twelve children were given the TEMA-3 Form A both pre- and post-intervention. All students, except one, demonstrated growth from pre- to post-assessment. One student remained at the 42nd percentile both pre and post-intervention. This score does indicate growth for a typical seven year old student. However, the other students' scores indicated growth exceeding expected growth on this measurement. The range of improvement on the TEMA-3 was 0 to 30 percentile points. A paired t-test was conducted to compare the pre- and post- assessment scores. There was a significant increase in the scores pre- ($M=32.33$, $SD=18.13$) and post- ($M=52.92$, $SD=18.54$) assessment; $t(11)=5.86$, $p<.001$. The students' scores on the TEMA-3 increased dramatically after the ten week intervention. Table 4.1 reports the students' pre- and post-intervention TEMA scores, along with the increase in percentile.

Table 4.1
Test of Early Mathematics Ability-3 (TEMA) scores expressed in percentiles

Student	TEMA-A March	TEMA-A June	Gain Score
1	12	21	+9
2	19	61	+42
3	68	79	+11
4	42	68	+26
5	21	45	+24
6	21	47	+26
7	42	42	0
8	19	25	+6
9	25	55	+30
10	35	63	+28
11	63	79	+16
12	21	50	+29

When analyzing the target students, it was found that Emily was at the 21st percentile prior to the intervention. After the intervention, her score rose to the 45th percentile, an increase of 24 percentile points. Her standard score rose from an 88, a below average score, to a 98, an average achievement score. Emily was able to determine number magnitude for numbers through one hundred, read and identify three digit numbers, and verbally count by tens through 190. Matt began the study at the 25th percentile and improved to the 55th percentile, an increase of 30 percentile points. Matt's standard score rose from 90 to 102, both of which are considered average achievement scores. He was able to read two digit numbers write three digit numbers, count by tens through 190, and determine number magnitude through three digit numbers. Table 4.2 specifically reports Emily and Matt's percentile scores.

Table 4.2
Selected Students' TEMA-3 Scores

Student	Pre-Intervention	Post-Intervention
Emily	21	45
Matt	25	55

Curriculum Based Assessments

The Curriculum Based Measurements (CBM) were used to measure the development of computational fluency in the first-grade Tier 2 mathematics students, as defined by the second research question. This measurement was also used as one of three criteria for inclusion in the intervention. If a student's median score on the first three CBMs was below 13, he/she was considered eligible for participation in the intervention. Two of the twelve intervention students had median scores above the established criteria. However, these students fell below the benchmark on both the Lembke-Foegen screening and teacher evaluations. The CBMs were given to all students in first-grade at three elementary schools in the district. Two of the schools had students which participated in the study. The third school took weekly CBMs which were used as a control group.

At the conclusion of the study, all students demonstrated adequate progress, as defined by Fuchs and Fuchs. Ten students of the students surpassed the minimum growth

of three points. Two students met the minimum growth requirements. Three students met or surpassed the end of first-grade benchmark score of 20. Given the low beginning scores, it was not realistic to expect all students to reach the benchmark. Table 4.3 provides the baseline and post intervention data for all students.

Table 4.3
Computation Probes Median Scores

Student	Pre	Post	Increase
1	*9	*15	6
2	*9	*18	9
3	*10	*13	3
4	*7	24	17
5	*12	25	13
6	*8	*11	3
7	13	20	7
8	*10	*15	5
9	*4	*16	12
10	*10	*15	5
11	14	21	7
12	*6	*16	10

*score below benchmark

Three data analysis techniques were used to evaluate the effectiveness of the intervention. The first technique was the use of a Tukey, which provides a visual analysis of student's growth by comparing the growth of a student against the expected growth, and allows the researcher to compare the expected and actual student growth. This analysis demonstrated positive growth for all students. The second technique, the Percentage of Non-Overlapping Data, which calculates the percent of data which is above the median baseline score, determined that only three students had more than 10% of their data points below their highest baseline data point. In all three of these cases, the scores which fell below the highest baseline data point occurred at the beginning of the study indicating the program proved successful for all students after about three to four

weeks of instruction. The final method of analysis was the Split Middle Trend Line.

In this analysis, the trend line of the baseline and intervention data is calculated and plotted on a graph, providing evidence that students made greater than expected growth when instructed using the CRA instructional model. All students exceeded their projected growth from their baseline data.

The target students demonstrated significant gains. Emily's median score pre-intervention was 9 digits correct out of 30. By the end of the intervention, Emily's median score from the last three weeks of instruction was 18 digits correct. This represents an increase of 9 points, 5.5 points above the weekly acceptable growth Fuchs and Fuchs established to assist educators in measuring adequate weekly gains. At the end of the third week of instruction, Emily's weekly CBM score increased 5 points and began to demonstrate a steady increase, except during weeks seven and ten. She surpassed the end of the first-grade benchmark three times throughout the intervention, specifically during the last two weeks of the intervention. Figure 4.1 illustrates Emily's weekly scores.

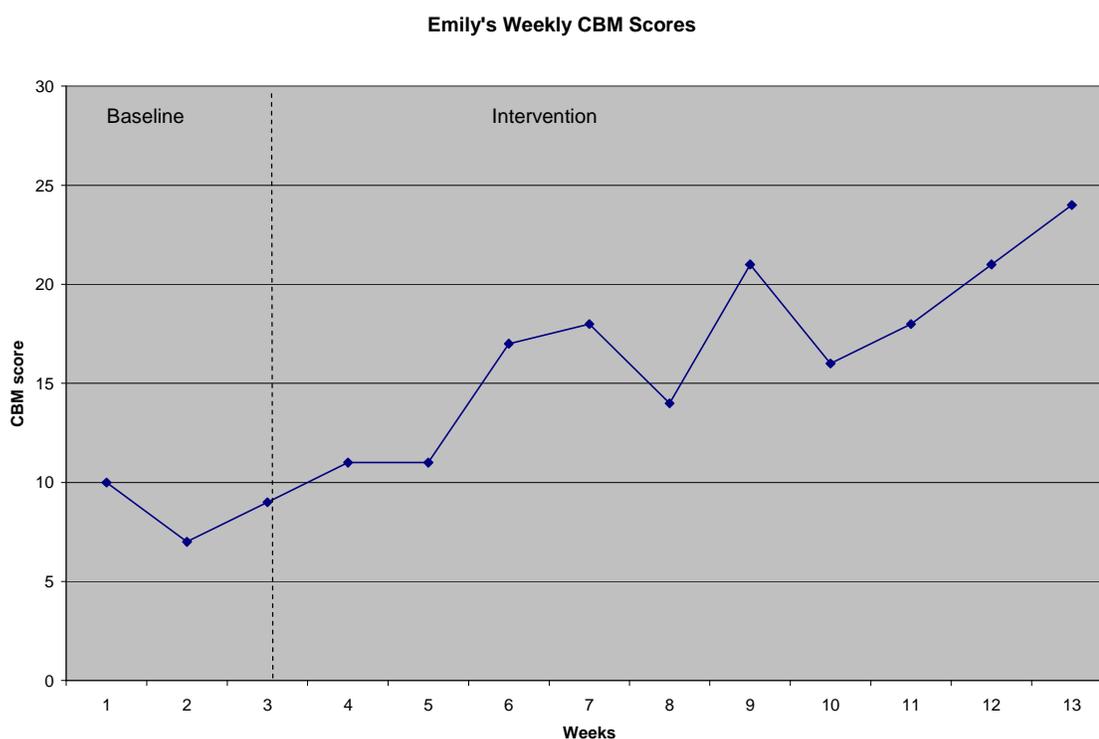


Figure 4.1 Emily's Weekly CBM

Matt began the intervention with a median score of four. During last three weeks of instruction Matt had a median score of 16, an increase of 12 points during the intervention. He showed an immediate increase in scores of eight points during the first two weeks. However, he had a temporary setback in week six, but showed steady upward growth after this decline in score. His scores did decline in the last two weeks of the intervention. This could be attributed to the end of the school year, as the intervention was completed during the last week of school, when students become less enthusiastic about learning and more interested in summer activities. Matt's CBM scores are represented in Figure 4.2.

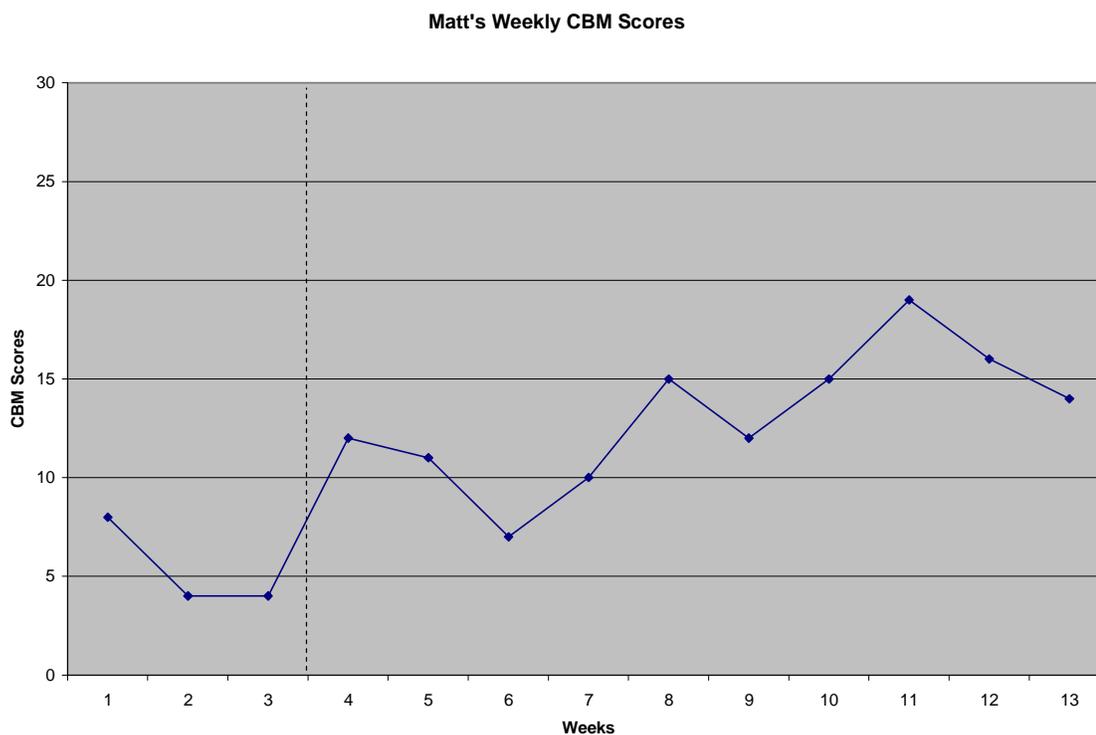


Figure 4.2 Matt's Weekly CBM

Both of the CBM graphs indicate a steady increase in the students' score throughout the intervention, especially since the final data were collected the last week of school. Emily was able to achieve the end of year benchmark of 20. Matt's scores steadily increased throughout the study. In week eleven he did approach the benchmark scoring a 19. The latency of change on both graphs indicates about two weeks of instruction was required to facilitate a large increase in the CBM score, suggesting the impact of the program requires instruction in number sense before a change in score can be realized. All twelve students exceeded the recommended rate of growth of 0.35 points per week. Sixty-six percent of the students surpassed the acceptable growth level by two or more points. This positive impact of the instruction is evident in continued student growth.

Tukey Method

Another data analysis technique in Single Subject design is the Tukey method, which provides a visual analysis of student's growth. By comparing the growth of a student against the expected growth, the researcher is able to compare expected and actual student growth. Figures 4.3 and 4.4 illustrate the expected growth of Emily and Matt respectively. The graph with Emily's data indicates a two-point difference between actual and expected growth in the first two weeks of instruction. After week two, the growth accelerated well above the expected outcome. By the end of the study, Emily's score was 12 points above the expected growth rate, indicating an accelerated rate of growth. Matt's actual growth also exceeded the expected growth on the CBM. Week 11 had the largest difference between expected and actual growth, with a positive difference of 12 points. Even at the end of the study, when Matt's score declined, he remained seven points above the expected growth. These data give further evidence that each student's overall growth exceeded the expected growth rate when instructed using a CRA instructional model.

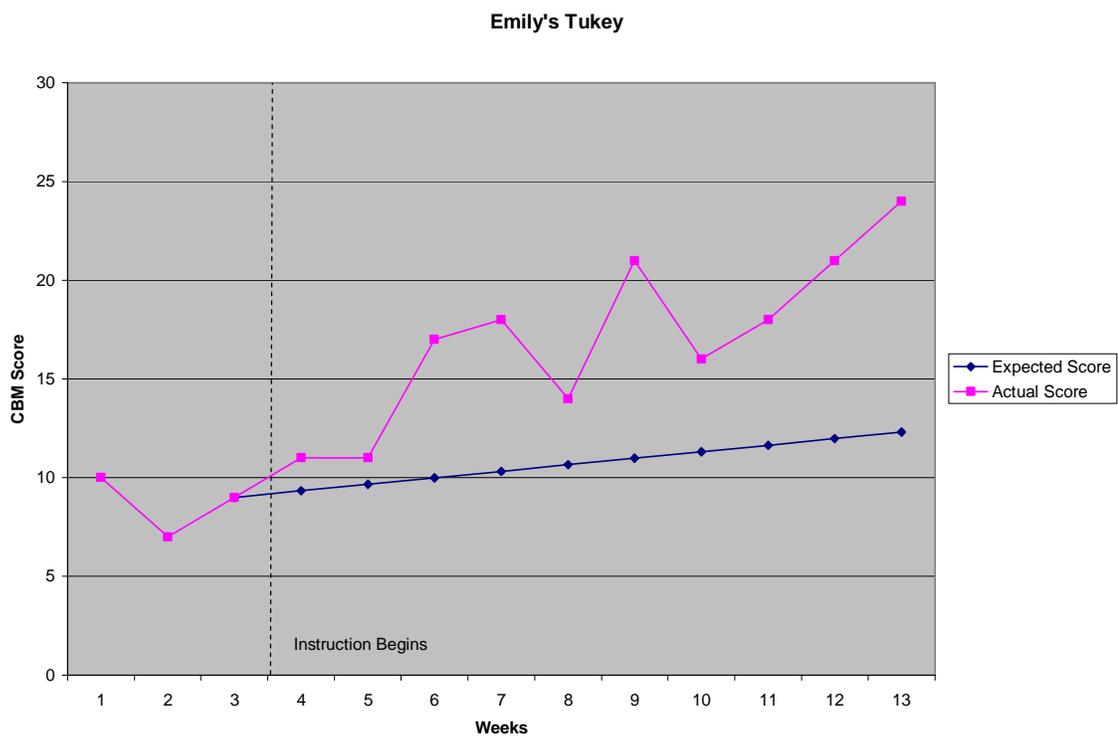


Figure 4.3 Emily's Tukey Analysis

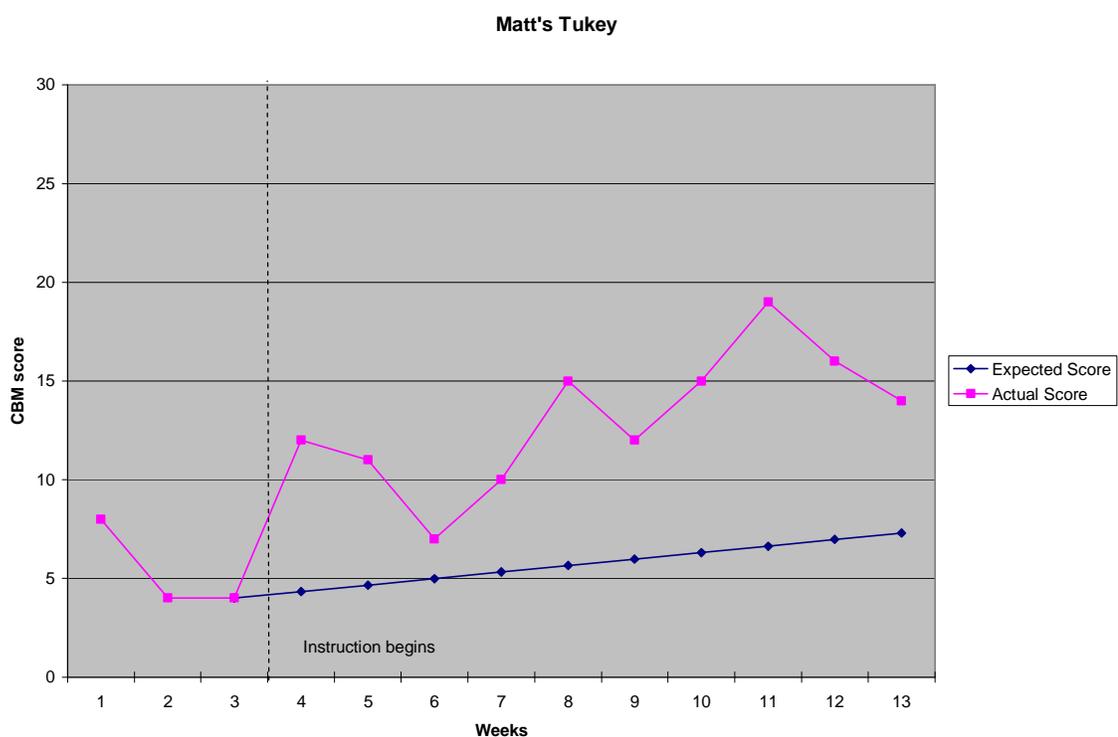


Figure 4.4 Matt's Tukey Analysis

Percent of Non-Overlapping Data

The percentage of Non-Overlapping data (PND) (Scruggs et al., 1998) provides another meaningful analysis of the effect of the intervention. Scruggs and Mastopieri (1998) identify PNDs greater or equal to 0.80 as highly effective. All of Emily's data points during the intervention were above highest baseline data points, as demonstrated in Figure 4.5. Her PND score was 1.0 indicating a strong effect from the intervention program. As stated earlier, her CBM score exceeded the first-grade benchmark. Matt's PND, as represented in Figure 4.6, was 0.9. Only one data point in the third week of instruction fell below the highest baseline data point, which indicated the intervention was highly effective as measured with this statistic.

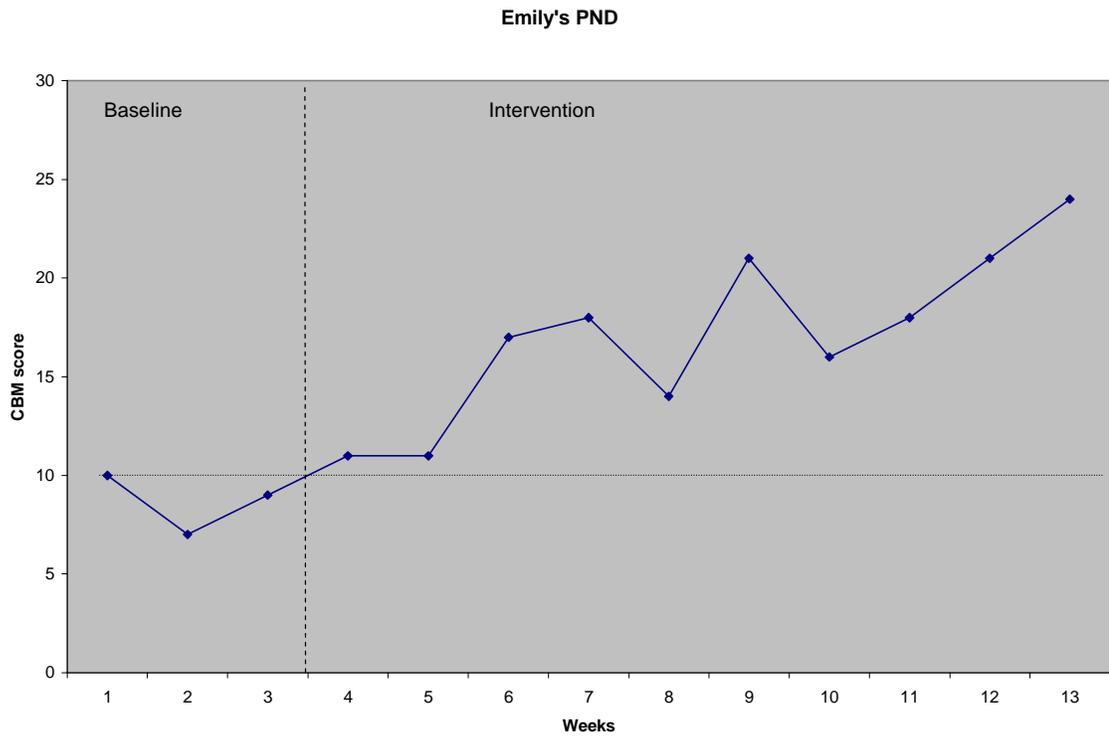


Figure 4.5 Emily's PND Analysis

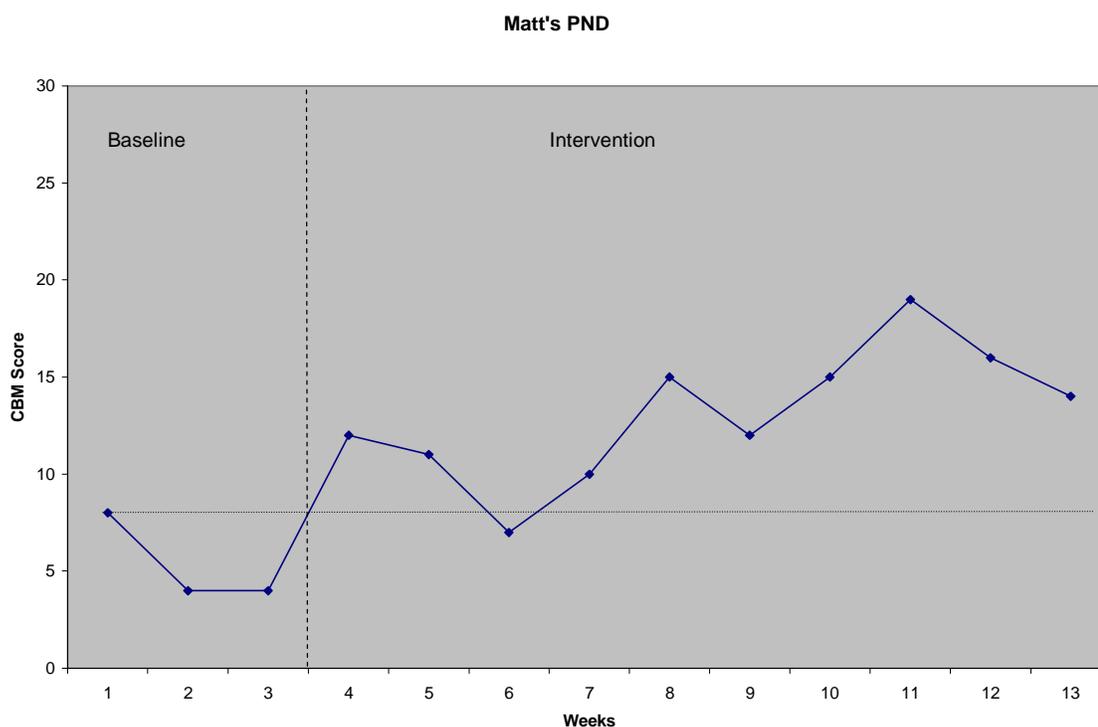


Figure 4.6 Matt's PND Analysis

Split Middle Trend Line

The Split Middle Trend Line provides further evidence that the intervention was successful. In this analysis, the trend line of the baseline and intervention data are calculated and plotted on a graph, providing evidence that students made greater than expected growth when instructed using the CRA instructional model. The baseline data for both Emily and Matt indicate the scores are low and seem to be trending downward. Although it should be noted that the students' scores most likely would not have continued downward, rather they would have shown slower growth than was evident on the graph post intervention. After the intervention began, the scores climbed at a steady rate, although scores can rise or fall from week to week. As indicated by the slope of the regression line, Emily showed an average of 1.17 weekly growth. Matt's weekly growth

on the CBMs was .75. Figures 4.7 and 4.8 show the expected growth based on baseline data, along with the actual growth, adding further to the strength of the intervention for first-grade students.

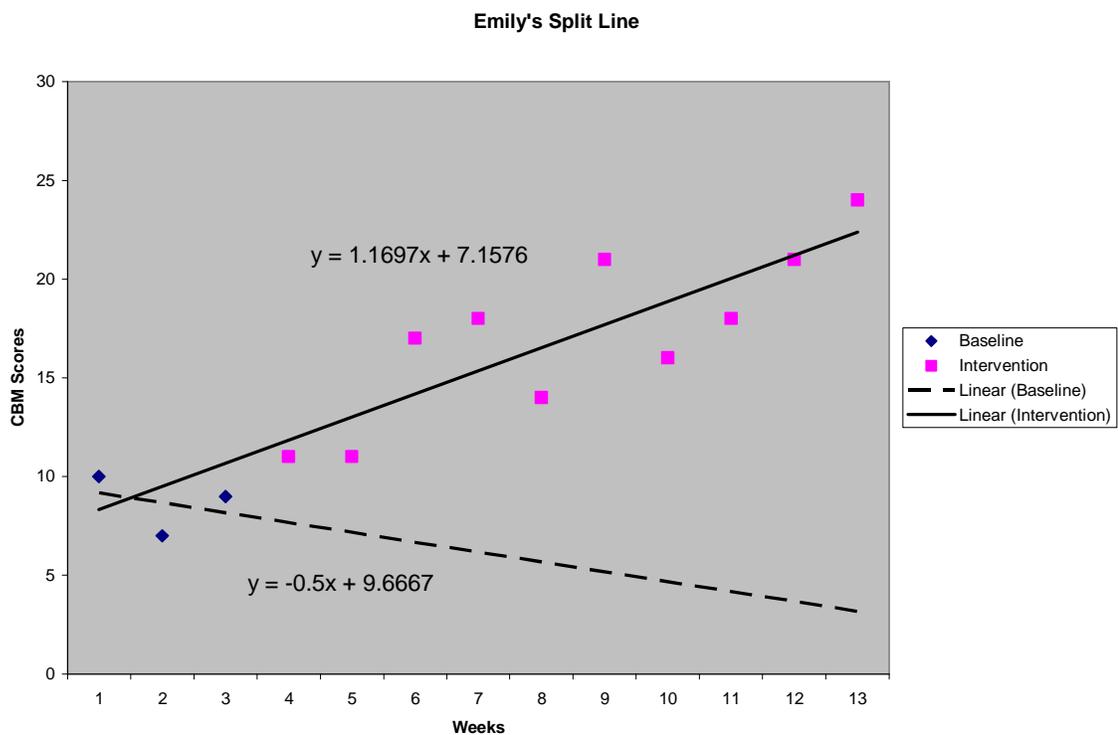


Figure 4.7 Emily's Split Trend Line Analysis

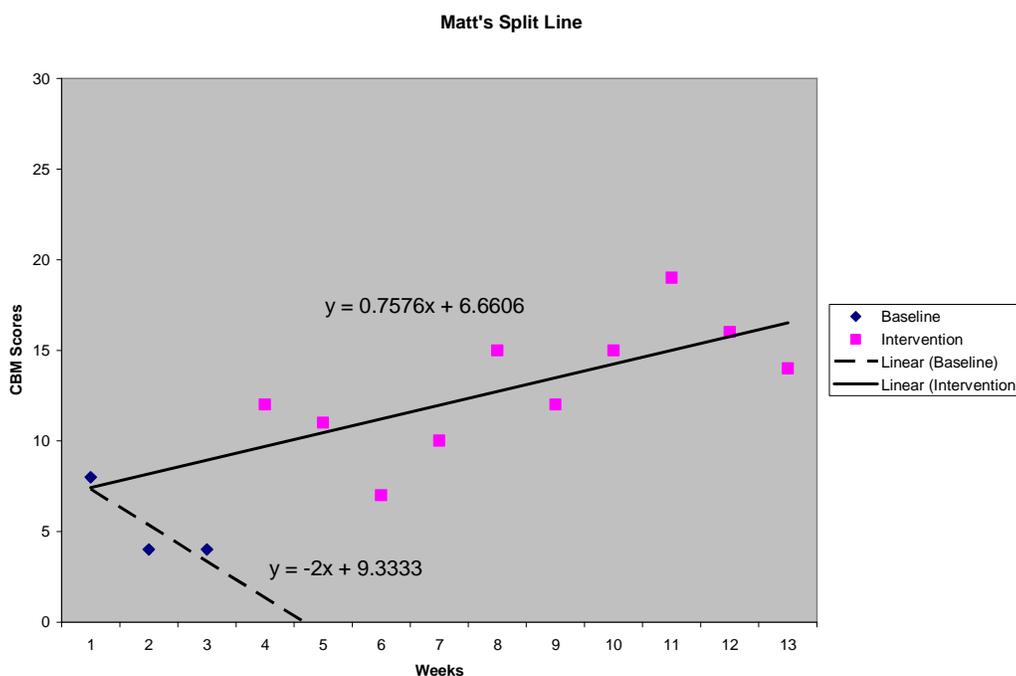


Figure 4.8 Matt's Split Trend Line Analysis

Lembke-Foegen Early Numeracy Indicators

All students participating in the study were given the Lembke-Foegen Early Numeracy indicators, both pre- and post-intervention. This assessment was utilized as one of three screening tools to identify those students struggling with numeracy in first-grade. The probes were utilized to screen students for inclusion in the study. A paired t-test was conducted to compare the pre- and post-intervention Early Numeracy scores for students who received the intervention. There was a significant increase in the Number Identification probe between the pre- ($M=28.92$, $SD=11.39$) and the post- ($M=48.08$, $SD=10.54$) assessment; $t(11)=5.68$, $p<.001$. The Quantity Discrimination probe also had a significant increase in the pre- ($M=23.92$, $SD=8.73$) and post- ($M=29.33$, $SD=6.58$)

assessment; $t(11)=2.29$, $p=.04$. The Missing Number probes did not have a significant increase pre- ($M=12.08$, $SD=5.07$) and post- ($M=14.33$, $SD=3.65$) assessment, $t(11)=1.10$, $p=.29$. Table 4.3 shows student scores both pre- and post- intervention raw scores.

These scores indicate the number of correct answers in one minute.

Table 4.4
Lembke-Foegen Early Numeracy Scores

Student	Winter Scores			Spring Scores		
	NI	QD	MN	NI	QD	MN
1	8*	6*	23	51	28*	6*
2	27*	29	16	43	35	15*
3	32*	15*	13*	67	31	16*
4	33*	26*	13*	63	37	19
5	40	37	17	47	37	19
6	34*	30	9*	46	28*	13*
7	51	25*	8*	59	25*	17
8	15*	17*	4*	31*	13*	13*
9	24*	32	11*	38*	29*	12*
10	29*	30	8*	39*	31	13*
11	34*	23*	9*	46	33	17
12	20*	17*	14	47	25*	12*

*indicates score below 25%ile

Emily's scores are found in Table 4.4. These scores demonstrate that Emily made progress in the area of number identification and quantity discrimination, but not in missing number. The missing number probe required students to fill in the blank in a pattern that was counting by ones, twos, fives, or tens. This is not a skill covered by the intervention program, and therefore may not accurately reflect the student's growth in mathematical achievement and computational fluency. Matt's scores, as shown in Table 4.5, indicate he made improvement in the area of number identification. His quantity discrimination and missing number assessments show little (if any) growth. The quantity discrimination score does not align with his improvement in computational fluency, as

measured on the CBMs or his overall mathematics achievement as measured on the TEMA. It is plausible that the lack of performance may be due to the administration of the assessments during the last week of the school year when motivation, attention and persistence may have been lower.

Table 4.5
Emily's Lembke-Foegen Scores

Assessment	Pre-Intervention	Post-Intervention
Number Identification (NI)	27*	43
Quantity Discrimination (QD)	29	35
Missing Number (MN)	16	15

*indicates scores at or below the 25%ile

Table 4.6
Matt's Lembke-Foegen Scores

Assessment	Pre-Intervention	Post-Intervention
Number Identification (NI)	24*	38*
Quantity Discrimination (QD)	32	29*
Missing Number (MN)	11*	12*

*indicates scores at or below the 25%ile

Growth Curve Modeling

The third research aim of the study was to measure the differences in student growth on the computational probes. In an effort to add more power to the data analysis and measure these differences in students' computational fluency over time Growth Curve modeling was used. This model of Hierarchical Linear modeling evaluates the heterogeneity of change in scores across individuals. This study was especially conducive to this type of evaluation due to its multiple waves of data, whose values change over time, and are measured at regular intervals (Singer, et al., 2003). Growth Curve modeling determined the effectiveness of the CRA instructional model on the intervention group. The model analyzed the shape and rate of change in students' growth. For this analysis students, who had met or exceeded the first-grade benchmark of 20 in March were removed from the data in an effort to remove outlier that may skew the results of the study.

In many phases of early learning, growth follows a curve, where students may begin at different points, and some grow faster than others. In the present analyses, the

treatment group or students who received the CRA instruction was used as a predictor of students' beginning scores (intercept), and also as a predictor of students' rate of growth in CBM fluency. In an effort to arrive at a model which best predicted student growth on the Fuchs and Fuchs CBM, four models were examined. Figure 4.10 compares the data in all of the models.

The first model, an unconditional means model, examined each individual's mean CBM score without consideration of any predictors. Each student's mean score was compared to the mean for all participants. This first model indicates that the average score on the computation assessments was statistically significantly greater than zero. When scores were collapsed across time, the average score for all students on the CBMs across time was 15.68 with a standard deviation of 0.39. A statistically significant variance in the intercept was found, indicating that students' mean scores varied from the grand mean at the beginning of the study. A significant variance in the Level 1 (student) residuals was found, indicating that each student had scores that varied around his/her own mean. The Intra-class Correlation Coefficient, which describes how related the scores are to each other, for Model 1 was 0.52, suggesting that analyzing an alternative model that accounted for time was warranted.

As described above, results of the unconditional means model warranted exploration of a second model, an unconditional growth model, which examined student scores over time to determine if there was a difference in scores as the study progressed. Visual inspection of the data suggested a linear growth pattern, rather than a curvilinear growth pattern. Model 2 fit significantly better than Model 1 (difference in deviance = 607.6, change in $df = 1$, $p < .001$, Pseudo $R^2 = 36.71$). Model 2 indicated that students'

scores did grow over time, with a mean starting score of 12.36 and a mean growth of 0.58 per week. This model indicated that students had varying intercepts and varying slopes in their rates of linear growth. There is a statistical difference in growth over time between students, and there are differences between students when their beginning scores are considered. No significant relationship was found in the covariance of the slope and starting score. This finding indicates the need to investigate further the predictors that account for the differences in starting scores and growth in students' computational fluency.

The third model examined the differences in intercept (starting score) between students who received the intervention and those students who were eligible but did not participate in the intervention, also known as the control group. Model 3 fit significantly better than Model 2 (difference in deviance = 54.9, change in $df = 2$, $p < .001$, Pseudo $R^2_{\text{intercept}} = 38.91\%$, $R^2_{\text{slope}} = 0.5\%$). Those students who were not eligible for the intervention (due to median scores above twelve points, but less than twenty points at the beginning of the study) had a significantly higher mean intercept of 13.58. The control group had a starting score of 8.7, while the treatment group had an starting score of 8.39. The differences between starting scores of the treatment and control group were found to be statistically insignificant. In the third model, the slopes of the CBM scores are not modeled as different for control and treatment students; the common slope of growth was 0.57. The variance in the slopes was found to be significant; therefore, further inquiry was warranted to determine if the slopes for students in the treatment group varied from the control group. Variance in Level 1 residuals and intercepts remained significant,

suggesting there are some predictors that are not accounted for in the model. The intercept-slope covariance remained non-significant, however.

The final model investigates whether the slopes of growth for students in the treatment group were steeper than the slopes for students in the control group. Model 4 fit significantly better than Model 3 (difference in deviance = 60.06, change in $df = 2$, $p = .001$, Pseudo $R^2_{\text{intercept}} = 39.03\%$, $R^2_{\text{slope}} = 5.9\%$). With regard to intercepts (starting scores), non-eligible students had a mean intercept of 13.87, which was significantly higher than eligible participants (8.76) and eligible non-participants (8.67, nsd). The results indicate the non-eligible students had a slope of .56. The control group's slope was .49, which was found to be statistically non-significantly different than the non-eligible students. However, the treatment group's slope (.38) was higher than the control's (.93), statistically significantly greater than the other members of the study. This finding indicates a positive effect for the use of a CRA instructional model. Table 4.6 demonstrates questions answered in each model and how the slopes and intercepts were calculated. Figure 4.9 shows fitted growth trajectories for ineligible students, eligible participants, and control participants in the final model, indicating that students in the treatment group began to close the achievement gap among first-grade mathematics students in the study.

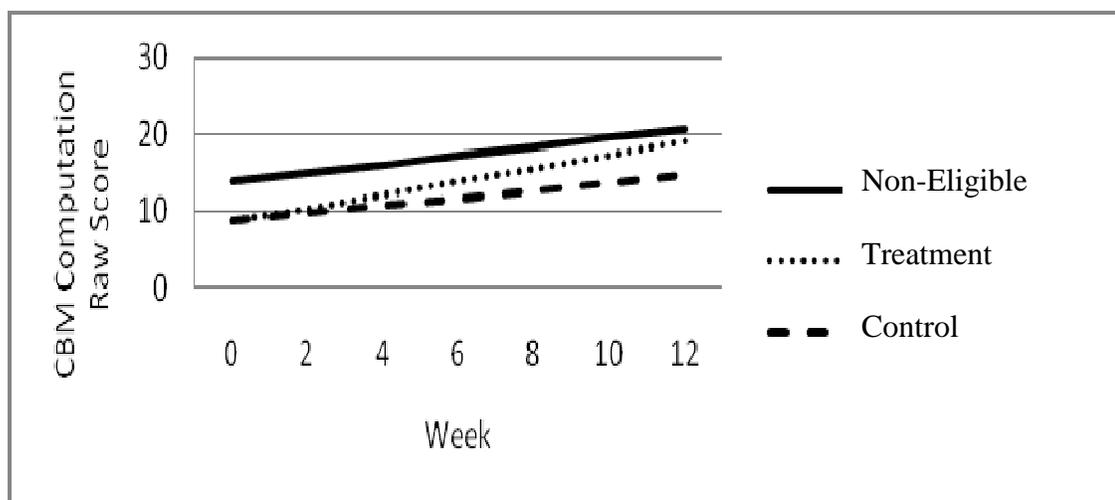


Figure 4.9 Fitted Growth Trajectories

Table 4.7
Individual Growth Model Statistics

		Parameter Estimates (SE)			
		Model 1: Uncond. Means Model	Model 2: Uncond. Growth Model	Model 3: Intervention on Intercepts	Model 4: Intervention on Intercepts and slopes
Fixed effects					
	Intercept	15.68* (.39)	12.36* (.35)	13.86* (.34)	13.87* (.34)
	Control on intercept			.31 (1.17)	-.09 (1.18)
	Treatment on intercept			-5.19* (.71)	-5.10* (.71)
	Slope: Time (weeks)		.58* (.04)	.58* (.04)	.56* (.04)
	Slope: Treatment x time				.38* (.17)
	Slope: Control x time				-.07 (.10)
Random effects					
	Residual L1, estimate	19.20* (.67)	12.22* (.45)	12.22* (.45)	12.22* (.45)
	Intercept, estimate	20.81* (2.62)	14.26* (2.08)	8.71* (1.45)	8.69* (1.45)
	Slope, estimate	NA	.18* (.03)	.17* (.03)	.16* (.02)
	Intercept-Slope covariance		.13 (.18)	.16 (.15)	.17 (.15)
Proportional reduction in variance from baseline model					
	Residual, L1		.37		
	Intercept		.59	.39	.39
	Slope			<.01	.06
	Deviance	10650.65	10043.04	9988.19	9982.98
	Akaike's information criteria	10656.66	10055.04	10005.18	10002.97

*p<.01

Table 4.8
Explanation of Growth Curve Data

	Model 1: Uncond. Means Model	Model 2: Uncond. Growth Model	Model 3: Intervention on Intercepts	Model 4: Intervention on Intercepts and slopes
Question answered by model:	What is the median score for all students collapsed across time?	Is there a difference between students and their intercepts?	Is there a difference between the treatment and control starting scores?	Is there a difference in slopes between the treatment and control group?
Intercept-start score (SD)	15.68* (.39)	12.36* (.35)	13.86* (.34)	13.87* (.34)
Treatment on intercept			-5.19* (.71)	-5.10* (.71)
Control on intercept			13.86-5.19=8.7 .31 (1.17)	13.87-5.1=8.76 -.09 (1.18)
			8.7-.31=8.39	8.76-.09=8.67
Slope: Time (growth per week)		.58* (.04)	.58* (.04)	.56* (.04)
Slope: Treatment x time				.38* (.17)
				.56+.38=.94
Slope: Control x time				-.07 (.10)
				.56-.07=.49

Summary

The 12 students who participated in the intervention made significant gains on their weekly curriculum based assessments and the TEMA-3, a math achievement measure, indicating improvement in student number sense and place value understanding. The two target students' growth was indicative of the growth seen across the majority of the intervention participants. Their CBM scores rose significantly by the end of the intervention, far above the expected growth. The Tukey, Percent of Non-Overlapping Data, and the Split Trend Line data all indicate the intervention had a significant impact on student computational fluency. Growth Curve modeling adds to the abundance of data for student improvement. The results of the final model, which indicate the growth in intervention students' CBM scores, were statistically greater than the control students' growth on the same measure.

CHAPTER 5 DISCUSSION, IMPLICATIONS, AND RECOMMENDATIONS

Introduction

This chapter presents an overview of the major findings of a study focused on the effects of a Concrete, Representational, Abstract (CRA) instructional model on Tier 2 first-grade mathematics students in a Response to Intervention (RtI) model. The findings were discussed and linked to prior research as well as the theoretical framework. Educational implications and recommendations for future studies is offered at the conclusion of this chapter.

Summary of Major Findings

The purpose of this study was to determine whether a Concrete, Representational, Abstract (CRA) instructional model was an effective model to assist first-grade students who were eligible for remedial services under a Response to Intervention (RtI) model. All first-grade students in two elementary schools were universally screened as prescribed by the Pennsylvania Training and Technical Assistance Network (PaTTAN), an initiative of the Pennsylvania Department of Education. Each child was screened using three tools. First, students were given Fuchs and Fuchs computation probes once a week for three weeks. Those students whose median score was below 12 out of 30 were labeled at-risk by this indicator. Next, the students were individually screened by the researcher using the Lembke-Foegen Early Numeracy Indicators. Students below the 35th percentile were identified as being at-risk by this indicator. Finally, teachers were asked to provide a list of students whom they believed were struggling in the

mathematics curriculum. Only the twelve students who were identified as at-risk by a minimum of two out of three indicators were included in the research study. These students were given the Test of Early Mathematics Ability-3 both pre- and post-intervention to measure their mathematical achievement.

During the intervention, 10 weeks of instruction was provided to the students three times a week for 30 minutes. A CRA method of instruction was implemented through a program entitled Expeditions to Numeracy. Students were instructed on place value and addition and subtraction of numbers 0-9 through the use of Digi-blocks and a ten-frame. Once the concept was mastered, the abstract representation of number sentences was introduced. Throughout the intervention, all students in first-grade at the study sites (regardless of inclusion in the study) were given the Fuchs and Fuchs first-grade computation probes weekly as a progress monitoring tool to assess computational fluency. Additionally, a control group from a third school in the same district was given identical first-grade computational probes. At the conclusion of the study, all students in the intervention were given the TEMA-3 a second time to measure growth in mathematical achievement. Additionally, the Lembke-Foegen Early Numeracy probes were re-administered.

The results of the study indicated dramatic student growth on the Fuchs and Fuchs computation probes and the TEMA-3, with significant growth on the Lembke –Foegen Early Numeracy probes in the area of Number Identification and Quantity Discrimination. No significant growth was found in the Missing Number probes. Two students were chosen as a representative sample of the twelve involved in the study for the single subject design. Both target students far exceeded the anticipated growth on the

computation probes, regardless of the method of analysis used. A visual analysis of the CBM growth model showed steady progress above the expected growth level. Tukey and Percentage of Non-Overlapping Data analyses also added to the strength of the achievement claim. Although the students did show regression in scores during a few weeks, the intervention students' CBM growth far exceeded the control group, which was comprised of students from the control school whose median score was below 13. In fact, the achievement of the intervention students began to approach the achievement level of students who demonstrated typical achievement in first-grade mathematics. The use of linear growth modeling revealed a statistically significant difference between those students who received the intervention and eligible students who did not receive an intervention.

The TEMA-3 also indicated significant mathematical achievement growth as measured by the instrument. This assessment was meant to measure students' formal and informal mathematics skills, including numeral literacy, number facts, and understanding of the base 10 number system. Participating students showed statistically significant gains in achievement ranging from 0-42 percentile points, with the average gain of 20.6 percentile points. Clearly the CRA method of instruction, as measured by the TEMA-3, was successful for the students who participated in the intervention.

The Lemke-Foegen Early Numeracy Indicators were administered both pre- and post-intervention. Initially, the probes were used as part of the screening process as an indicator of number sense in young children. Jordan et al. (2007) have identified number sense as a predictor of mathematics achievement at the conclusion of first-grade. The probes evaluated students' quantity discrimination, number identification, and ability to

identify numbers missing in a pattern. Emily, one of the target students, demonstrated consistent growth throughout the probes. Her pre-intervention missing number score was below the 25th percentile, after the intervention the score rose above the bottom quartile. Matt's score remained in the bottom quartile but increased fourteen points. Both students showed growth on the number identification probe. Emily and Matt's scores on the missing number probe remained constant.

Lembke and Foegen (2009) identify number identification and missing number probes as having the highest predictive validity of mathematics achievement, when compared to teacher ratings of student achievement. The correlation was smaller when compared to the TEMA-3, especially in the spring of first grade. These data support the rise in TEMA-3 scores for Matt and Emily, but lack of growth on some Lembke-Foegen probes. The probes were designed for kindergarten and first grade students; however, Matt and Emily were at the end of first-grade, when growth on the probes begins to slow (Lembke & Foegen, 2009). In fact due to this slower growth rate, Lembke and Foegen question the use of these probes as a progress monitoring tool at the end of first grade.

Links to Prior Research

The framework for this study originates in Bruner's theory of Cognitive Development. This framework asserts that knowledge is developed through three modes of representation that allow the student to construct an understanding of his/her world: 1) enactive, 2) iconic, and 3) representational models (Bruner, 1966). The Concrete, Representational, and Abstract (CRA) instructional model is based on this framework and provided the foundation for the instruction offered in the study intervention. In this model, students started with concrete objects to represent numbers on a ten-frame. They

first progressed to writing numbers which describe the formations on the ten-frame and finally to writing addition and subtraction sentences. In addition, students worked to solve activities that presented only abstract number sentences. This model offers explicit instruction for students in mathematics and assists the learner in constructing meaningful knowledge of numbers, thus developing the students' number sense. The Expeditions to Numeracy program provided students with the necessary instructional activities to increase both their number sense and computational fluency achievement.

Maccini and Hughes's (2000) meta-analysis of literature of students with learning challenges found few studies focused on the development of conceptual knowledge. Their analysis, along with that of Kroesbergen and VanLuit (2003), indicated that the use of explicit models to teach conceptual understanding, such as the CRA instructional model used in this study, delivered successful outcomes for students with learning challenges. Using the State of Pennsylvania's model for Response to Intervention, results from this study reinforce these research findings, and further underscores the importance of explicit instruction in place value and computational fluency for struggling learners.

The instructional model, which utilized direct and explicit instruction to develop conceptual understanding, was determined to be more effective for teaching math facts. Clement and McMillian (1996) favor the development of number sense through the use of manipulatives, believing it to be crucial for the development of computational fluency. Combined use the ten-frame, along with the Digi-blocks, facilitated students' progress from the concrete to the abstract level of understanding. The students pictured the ten-frame in their mind as they completed basic addition and subtraction problems. These findings support earlier research in the use of manipulatives, which indicated students

with conceptual understanding of the physical objects to effectively lighten the cognitive load of the student when mastering a new concept (Ball, 1992; Kaput, 1989). Students' evolving use of their fingers throughout the study further supports the value of the ten-frame in relieving the cognitive load for students. Students began to utilize their fingers as a ten-frame and utilized them more effectively, with many eventually dropping their use altogether. Subtraction proved to be more challenging for the students. Many students were only beginning to master the abstraction of subtraction by the end of the study. All students were able to successfully use the ten-frame for both addition and subtraction.

Henry and Brown (2008) cite the importance of assisting children in the development of automaticity in recall of basic facts. They determined that 86% of first graders continued to use counting strategies at the end of first grade, indicating that students need to develop efficient strategies and conceptual understanding to become computationally fluent. This study furthers this research and provides strategies which have proven effective in the development of computational fluency. This study includes the use of the ten-frame and explicit instruction first in the development of place value understanding, and then in its application to sums of zero to nine. Furthermore, this study adds to research by Henry et al. (2008), suggesting that there is a synergetic relationship between fact-derived strategies and memorization of facts.

Miller and Mercer's (1993) study compared the amount of time necessary for students to "crossover" to recall basic addition facts when they received a CRA approach to the instruction. This study's results concur with their findings that three to four weeks is the average time necessary for students to begin mastering basic facts. Curriculum-

based measurement scores in this study rose after approximately three weeks of intervention, and continued to rise at a statistically significant pace throughout the intervention, thus indicating an increase in the students' computational fluency. The use of the ten-frame was an integral piece of the intervention and provided the foundation for the CRA instructional model.

Although there have been no studies using the CRA instructional model in a first-grade Response to Intervention model, the data cited in this study support the use of the model for young students. Many studies have cited successful use of a CRA model with early elementary students who have mathematical learning difficulties (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Flores, 2009; Miller, Harris, Strawser, Jones, & Mercer, 1998; Miller & Mercer, 1993). This study adds to this literature and begins to investigate the use of the model for students identified as at-risk in mathematics.

The results of the TEMA-3 confirm research indicating a strong correlation between basic fact fluency and mathematics achievement on standardized assessments (Kilpatrick, Swafford, & Findell, 2001). Early intervention for students struggling is crucial. Assisting low-achieving first-grade mathematics students allows the students to begin to close the gap with their typically developing peers. Without such intervention, the gap between the high achievers and at-risk students continues to widen (Rasanen, Salminen, Wilson, Aunio, Dehaene, 2009). Clearly, the improvement on the TEMA-3, an achievement assessment, indicates that instruction using the CRA instructional model via the Expeditions to Numeracy program with the use of ten-frames proves to be a valuable tool for students in a Tier 2 Response to Intervention model.

Limitations of the Study

There are limitations due to the nature of the study. The first limitation is the use of students who were not randomly selected for this intervention. The sample was a sample of convenience. The 12 students in the study attended first grade at the two elementary schools where the researcher is employed. Students were chosen from a sample of approximately 150. However, only those students who were having difficulty in the regular mathematics curriculum were included in the study. This number was limited by the scope of the RtI model. By design only 10-15% of the students should be eligible for a Tier 2 intervention. If the number of eligible students were larger, it would indicate a flaw in either the teacher's instruction or the district's curriculum. In this study approximately 10% of the population was eligible for and received services.

A second limitation is the sample size of 12 students. However, the nature of an RtI model by definition requires a small number of students. The single subject design and the use of linear growth modeling were designed to account for this limitation. Students' scores were analyzed individually in the single subject analysis in an effort to effectively evaluate individual growth. Linear growth modeling added to the power of the research model (Singer & Willett, 2003). Thus the model for this study is specifically designed to compensate for a small sample size. Additionally, the researcher was able to compare a significantly larger control group's CBM scores to the subjects in the current study, even when a few data points were missing in the control group.

A final limitation is the selection process for the control group. Students in the control group were identified as at-risk based solely upon their median score on the first three computational probes. Students in the invention group were subjected to further

screening via the Early Numeracy probes and teacher recommendations. Control group students did not receive this extra level of screening. It is possible that one or more students could have been eliminated from the control group (reclassified as typical and not at-risk) had they received these two additional screenings. Expedition to Numeracy intervention results indicated that computational fluency scores for the treatment group grew at a significantly higher rate than did fluency scores for the control group. If, in actuality, the control group included students who were incorrectly included in the control group of at-risk students, then the study results for the success of Expedition to Numeracy are even more substantial

Summary of Conclusions

This study was designed to evaluate the effects of a CRA instructional model on Tier 2 first-grade mathematics students in an RtI model. *No Child Left Behind* (NCLB) and the *Individuals with Disabilities Act* (IDEA) have heightened the awareness of the need for reform in the methods used to identify for special education. The RtI model was developed in response to this legislation. This model is divided into three tiers, each with screenings for learning challenges. All students begin in Tier 1 with a standards-based education. Students labeled as non-responders, due to lack of adequate progress in Tier 1, are then placed in a Tier 2 intervention where they receive intensive, research-based instruction in a small group. Weekly or bi-weekly progress monitoring is used to evaluate student progress throughout the intervention. Those students who are non-responders in Tier 2 move to Tier 3 where they receive more intensive interventions, which may include special education placement.

The TEMA-3 is part of Tier 2 monitoring, and is designed to assess formal mathematics knowledge, including number literacy and the concept of place value. Results from this study indicate dramatic student growth. This assessment was used to evaluate the students' development in number sense and place value. The TEMA-3 provides the assessor with a standard score, called the Math Ability Score, which provides an overall assessment of a students' mathematical achievement. At the end of the intervention, 9 of the 12 students participating in the study scored in the average range on the TEMA-3, indicating a gain in their mathematics achievement throughout the study. In fact, two of the students scored in the above average range at the end of the study. All but one student in the study demonstrated growth on the TEMA-3, as measured by an increase in standard score.

The effects of a CRA instructional model on students' computational fluency were measured through the use of weekly CBMs administered to all first-grade students. Only those students who had not yet met the first-grade benchmark were included in the analysis of growth over time. If a child had reached the benchmark at the beginning of the study, he/she continued to take the weekly CBM, but his/her scores were not excluded in this study. All intervention students included in the study demonstrated significant growth in CBM scores throughout the study, meeting or exceeding the recommended weekly growth of 0.35. Two-thirds of the students exceed the recommended growth by two or more points.

Analysis of the CBM data indicates the strength of the study intervention. Three methods of Single Subject Design analysis were used to evaluate these data. The first, the Tukey method, demonstrated positive growth for all students. The second analysis

method, the Percentage of Non-Overlapping Data, revealed that scores recorded below the highest baseline data point occurred at the beginning of the study and accounted for 10% or less of all CBM scores. Thus, the program proved successful for all students after about three to four weeks of instruction. The final method of analysis was the Split Middle Trend Line, which revealed that all students (but one) exceeded their projected growth from their baseline data. The lone student followed the expected growth and grew by 3 points on the computation probes by the end of the study.

Analysis of student growth on the CBMs, as well as the variables that effect this change, was also analyzed using Growth Curve Modeling. The slope of the linear regression line demonstrated the growth in CBM scores over the course of the study. This final method of analysis indicated that intervention group's slope was statistically significantly greater than the slope for other members of the study. In fact, the intervention group's slope also exceeded the slope for students progressing at a typical first-grade level who were therefore not eligible for services. Study results clearly indicate a statistically significant effect when using a CRA instructional model with Tier 2 students in a Response to Intervention Model.

Educational Implications

The results of this study indicate a Tier 2 intervention using a CRA instructional model is able to improve student achievement in computational fluency. These findings have several educational implications. First, this study provides a model for RtI that can prove successful in an elementary school. The Lembke-Foegen Early Numeracy indicators, CBM computation probes, and teacher recommendation identified the appropriate students in need of a Tier 2 RtI intervention. Additionally, the CRA

instructional model, via the Expeditions to Numeracy program, provided instruction that assisted in the improvement of students' over-all mathematical achievement and computational fluency. It is crucial that policy makers at the school district level develop creative scheduling that allows teachers to provide additional instruction in mathematics to struggling students. In this study, intervention students continued to attend their regular instruction in mathematics and received their tiered instruction.

A second educational implication of the study underscores the need for early intervention for first-grade students struggling with mathematics. Many in the educational community hope that struggling students will eventually catch up to their peers as they mature and develop. In some instances this may be the case; however it is in the best interest of a child to provide specific educational opportunities to facilitate this growth. There is compelling evidence that this is especially true for minority students, as the gap often widens for these students if specialized instruction is not offered year round (Alexander, Entwisle, & Olson, 2007). Identifying students before the discrepancy between their mathematical achievement and their peers becomes significant is crucial. However, many schools are reluctant to conduct formal educational evaluations on students in first-grade due to staffing and monetary concerns, as well as flaws in IQ and achievement tests for young children.

A final implication involves the use of a CRA instructional model in first-grade classrooms. There are many studies which validate the effectiveness of a CRA model in the classroom (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Flores, 2009; Miller, Harris, Strawser, Jones, & Mercer, 1998; Miller & Mercer, 1993). Studies have also shown the power of manipulatives in the classroom (Ball, 1992; Clement and McMillan,

1996; Kaput, 1989; Reys, 1971). This study has shown the appropriateness of this model in an RtI intervention using Digi-blocks and ten-frames. The Expedition to Numeracy program provided systematic instruction through the use of manipulatives and the ten-frame. Ten-frames are scattered throughout many first-grade mathematics textbooks. However, few programs offer on-going instruction in its use, or development of place value skills beyond ten, using this valuable tool. The foundation of the CRA instruction in this program was the use of the ten-frame. Remedial mathematics programs should incorporate the development of place value and number sense understanding through the use of this tool. In fact, every primary classroom should have frequent instruction involving the use of a ten-frame in place value development and computational fluency skills.

Recommendations

This study has provided valuable insights into the use of an RtI model with first-grade mathematics students. The CRA instructional model proved valuable to students with needs in the area of computational fluency and over-all mathematics achievement. The following recommendations are offered to further the study of RtI and the use of a CRA instructional model:

1. Future studies should investigate the long-term effects of this intervention on students' mathematical achievement. This study followed students for approximately 12 weeks. Following students throughout the early elementary years could uncover the learning patterns of students who would later be identified as having a mathematical learning disability.

2. Future studies should determine if the use of this model was able to decrease the number of students placed in special education. Future research is necessary in order to further investigate the value of early intervention in computation and global mathematics achievement, thereby allowing students to develop strategies to assist in overcoming learning challenges earlier in their academic experience.
3. It would be helpful to replicate this study in a larger setting that involves students from different socio-economic backgrounds to confirm these findings. This study was conducted in a suburban school district with 12 students from higher socio-economic families than is the norm in most public schools.
4. While the CRA instructional model has been used with many different populations, the Expeditions to Numeracy program has not been formally assessed with diverse populations. Future studies should specifically investigate program use with a more ethnically diverse student body.
5. A future study finding relationships between the CRA instructional model and the development of problem solving proficiency in first-grade students should be investigated, as students' skills in problem solving were not investigated in this study.

Clearly, there has been sufficient evidence presented to validate the use of the RtI and CRA instructional model to increase first-grade student achievement in mathematics. The results are promising, as they identify an instructional model which can increase first-grade student achievement in the areas of computational fluency and

global mathematical achievement. More research in the area of early intervention in mathematics is critical for mathematics reform in the United States.

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APPENDICES

APPENDIX A
LEMBKE-FOEGEN INSTRUMENT

Number Identification, Fall—1

Number Identification

Example

6	15	1	44
---	----	---	----

Missing Number, Fall—1

Missing Number

Example I

0 1 2 ___	1 ___ 3 4	5 10 15 ___
-----------	-----------	-------------

Quantity Discrimination, Individual—1

Examples

1	7	6	2	8	0
---	---	---	---	---	---

**APPENDIX B
EXPEDITIONS TO NUMERACY
CURRICULUM**

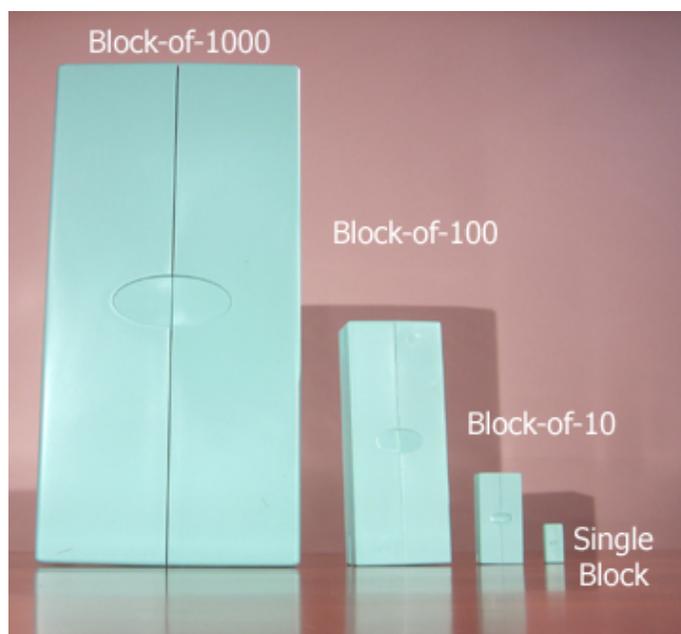
**EXPEDITIONS TO NUMERACY
A Place-Value-Based Intervention for K-4**

Math Region	Number Set Section	Exploration	Number of Activities	Proficiency Park Games and Exercises	
Place Value City	0-10	1: One, Two, Three	5	<ul style="list-style-type: none"> •Subitizing •Numeral Writing •More-Less-Equal •Teen Games •Two-Digit Place Value •Big-Number Place Value •Equality 	
		2: Counting Objects to 10	11		
		3: Reading and Writing Numerals 0 to 10	7		
		4: Matching Numerals and Sets	10		
		5: More, Less, Equal	10		
		6: Ten-Frame Formations	7		
		7: Ordinal Words and Numbers	4		
		8: Ten Ones Equal One Ten	9		
		9: Benchmarks within 0-10	5		
		10: Counting Backward from 10	6		
			1: Counting Past 10		5
			2: Place Value Meaning for Teen Numbers		10
			3: Ordering and Comparing Numbers 0-19		3
					1: Counting by Tens
	2: Place Value Meaning for Two-Digit Numbers				7
3: Comparing and Ordering Numbers 0-99	7				
4: Equivalent Block Representations	3				
0-999		1: Place Value Meaning for Three-Digit Numbers	6		
		2: Comparing and Ordering Numbers 0-999	4		
		3: Equivalent Representations	3		
		1: Place Value Meaning for Large Numbers	6		
		2: Ordering and Comparing Large Numbers	2		
Add'n/Subtract Town	0-10	1: Adding on the Ten-Frame	11	<ul style="list-style-type: none"> •Number Combinations •Addition/Subtraction Concepts •Hidden Numbers •Addition/Subtraction Fact Practice •Connecting Addition/Subtraction •Regrouping •Large Number Addition •Large Number Subtraction •Equality •Word Problems •Sample Word Problems 	
		2: Subtracting on the Ten-Frame	13		
		3: Expressions and Equations	3		
		4: Getting Ready to Regroup	5		
			1: Place Value Addition (No Regrouping)		7
			2: Place Value Subtraction (No Regrouping)		8
			3: Expressions and Equations		3
			4: Place Value Addition (Regrouping)		6
			5: Place Value Subtraction (Regrouping)		8
	0-99		1: Place Value Two-Digit Addition		6
			2: Place Value Two-Digit Subtraction		7
	0-999		1: Using Models to Add and Subtract Three-Digit Numbers		7
			2: Using Numerals to Add and Subtract Three-Digit Numbers		8
		1: Adding and Subtracting Large Numbers	3		
		2: Subtracting Large Numbers	2		

EXPEDITIONS TO NUMERACY
A Place-Value-Based Intervention for K-4

Math Region	Number Set Section	Exploration	Number of Activities	Proficiency Park Games and Exercises	
Place Value City	0-10	1: One, Two, Three	5	<ul style="list-style-type: none"> •Subitizing •Numeral Writing •More-Less-Equal •Teen Games •Two-Digit Place Value •Big-Number Place Value •Equality 	
		2: Counting Objects to 10	11		
		3: Reading and Writing Numerals 0 to 10	7		
		4: Matching Numerals and Sets	10		
		5: More, Less, Equal	10		
		6: Ten-Frame Formations	7		
		7: Ordinal Words and Numbers	4		
		8: Ten Ones Equal One Ten	9		
		9: Benchmarks within 0-10	5		
		10: Counting Backward from 10	6		
			1: Counting Past 10		5
			2: Place Value Meaning for Teen Numbers		10
			3: Ordering and Comparing Numbers 0-19		3
			1: Counting by Tens		5
2: Place Value Meaning for Two-Digit Numbers			7		
3: Comparing and Ordering Numbers 0-99			7		
4: Equivalent Block Representations			3		
0-999		1: Place Value Meaning for Three-Digit Numbers	6		
		2: Comparing and Ordering Numbers 0-999	4		
		3: Equivalent Representations	3		
		1: Place Value Meaning for Large Numbers	6		
		2: Ordering and Comparing Large Numbers	2		
Add'n'Subtract Town	0-10	1: Adding on the Ten-Frame	11	<ul style="list-style-type: none"> •Number Combinations •Addition/Subtraction Concepts •Hidden Numbers •Addition/Subtraction Fact Practice •Connecting Addition/Subtraction •Regrouping •Large Number Addition •Large Number Subtraction •Equality •Word Problems •Sample Word Problems 	
		2: Subtracting on the Ten-Frame	13		
		3: Expressions and Equations	3		
		4: Getting Ready to Regroup	5		
			1: Place Value Addition (No Regrouping)		7
			2: Place Value Subtraction (No Regrouping)		8
			3: Expressions and Equations		3
			4: Place Value Addition (Regrouping)		6
			5: Place Value Subtraction (Regrouping)		8
	0-99		1: Place Value Two-Digit Addition		6
			2: Place Value Two-Digit Subtraction		7
	0-999		1: Using Models to Add and Subtract Three-Digit Numbers		7
			2: Using Numerals to Add and Subtract Three-Digit Numbers		8
			1: Adding and Subtracting Large Numbers		3
2: Subtracting Large Numbers			2		

APPENDIX C DIGI-BLOCKS



**APPENDIX D
STUDENT PROGRESS RECORD SHEET
(Sample pg. 1 of 5 Place Value City)**

Name _____

Student Progress Record for Place Value City 0-10

Performance	Criteria for Mastery
4: The student performs the task easily and with no errors. 3: The student performs the task with a few minor errors. 2: The student has difficulty performing the task and makes frequent and/or significant errors. 1: The student is unable to perform the task.	Pre-test: The student receives a score of 4. Post-test: The student receives a score of 3 or 4.

Exploration	Learning Objective	Assmt Task or Activity	Date	Comments	Score 4, 3, 2, 1
1: One, Two, Three	Demonstrate the concepts 1, 2, and 3.	Task 1 as Pre-test			
		Activity 1-1			
		Activity 1-2			
		Activity 1-3			
		Activity 1-4			
		Activity 1-5			
		Task 1 as Post-test			
2: Counting Objects to 10	Count objects to 10.	Task 2 as Pre-test			
		Activity 2-1			
		Activity 2-2			
		Activity 2-3			
		Activity 2-4			
		Activity 2-5			

APPENDIX E
EXPEDITIONS TO NUMERACY
FIDELITY SELF-CHECK

I . . .	Date:
1. Pre-and post-test according to instructions.	5: Always 4: Usually 3: Sometimes 2: Seldom 1: Never
Comments:	
2. Have each student working at his or her level, based on pre-test results.	5: Always 4: Usually 3: Sometimes 2: Seldom 1: Never
Comments:	
3. Prepare each lesson before teaching it.	5: Always 4: Usually 3: Sometimes 2: Seldom 1: Never
Comments:	
4. Teach to the lesson objective.	5: Always 4: Usually 3: Sometimes 2: Seldom 1: Never
Comments:	
5. Follow the lesson procedure.	5: Always 4: Usually 3: Sometimes 2: Seldom 1: Never
Comments:	
6. Engage every student in learning.	5: Always 4: Usually 3: Sometimes 2: Seldom 1: Never
Comments:	
7. Challenge each student to his or her potential.	5: Always 4: Usually 3: Sometimes 2: Seldom 1: Never
Comments:	
8. Check for evidence of learning	5: Always 4: Usually 3: Sometimes 2: Seldom

before advancing to the next lesson.	1: Never
Comments:	
9. Maintain instructional materials for every student in an individualized binder or folder.	5: Always 4: Usually 3: Sometimes 2: Seldom 1: Never
Comments:	
10. Keep student progress sheets up-to-date.	5: Always 4: Usually 3: Sometimes 2: Seldom 1: Never
Comments:	

**APPENDIX F
FUCHS AND FUCHS
CURRICULUM BASED ASSESSMENT**

Sheet #1

Computation 1

Password: ACT

Name: _____ Date: _____

A $\begin{array}{r} 0 \\ + 3 \\ \hline \end{array}$	B $\begin{array}{r} 7 \\ + 3 \\ \hline \end{array}$	C $\begin{array}{r} 0 \\ + 7 \\ \hline \end{array}$	D $\begin{array}{r} 54 \\ + 33 \\ \hline \end{array}$	E $\begin{array}{r} 7 \\ + 2 \\ \hline \end{array}$
F $\begin{array}{r} 10 \\ - 0 \\ \hline \end{array}$	G $\begin{array}{r} 9 \\ + 0 \\ \hline \end{array}$	H $\begin{array}{r} 0 \\ + 9 \\ \hline \end{array}$	I $\begin{array}{r} 6 \\ - 0 \\ \hline \end{array}$	J $\begin{array}{r} 8 \\ - 5 \\ \hline \end{array}$
K $\begin{array}{r} 10 \\ - 1 \\ \hline \end{array}$	L $\begin{array}{r} 8 \\ - 1 \\ \hline \end{array}$	M $\begin{array}{r} 10 \\ - 7 \\ \hline \end{array}$	N $\begin{array}{r} 1 \\ 7 \\ + 1 \\ \hline \end{array}$	O $\begin{array}{r} 6 \\ - 2 \\ \hline \end{array}$
P $\begin{array}{r} 65 \\ + 23 \\ \hline \end{array}$	Q $\begin{array}{r} 45 \\ - 4 \\ \hline \end{array}$	R $\begin{array}{r} 5 \\ + 1 \\ \hline \end{array}$	S $\begin{array}{r} 8 \\ 1 \\ + 0 \\ \hline \end{array}$	T $\begin{array}{r} 7 \\ - 5 \\ \hline \end{array}$
U $\begin{array}{r} 8 \\ + 1 \\ \hline \end{array}$	V $\begin{array}{r} 99 \\ - 8 \\ \hline \end{array}$	W $\begin{array}{r} 10 \\ - 3 \\ \hline \end{array}$	X $\begin{array}{r} 9 \\ - 7 \\ \hline \end{array}$	Y $\begin{array}{r} 9 \\ + 1 \\ \hline \end{array}$

APPENDIX G IRB CONSENT



**Curriculum, Instruction and
Technology in Education - CITE**
1301 Cecil B. Moore Avenue
Ritter Hall 3rd Floor (003-00)
Philadelphia, PA 19122

phone 215-204-2117/8377
fax 215-204-1414

Participant's Name:

February 23, 2010

Project Title: The Effects of a Concrete, Representational, Abstract (CRA) Instructional Model on Tier 2 First Grade Mathematics Students in a Response to Intervention Model: Educational Implications for Number Sense and Computational Fluency

Investigator(s) Name(s), Department, Phone Number:

Student Investigator: Julie Eastburn, Doctoral Candidate, CITE Department

Principal Investigator: Jacqueline Leonard Ph.D., CITE Department (215) 204-8042

My name is Julie Eastburn and I am completing my dissertation in Mathematics Education at Temple University. I am conducting a research study to determine if a CRA instructional model can help students develop number sense and computational fluency (rapid math fact recall). Your child has been identified as a student who may benefit from additional instruction in mathematics. As in previous years, a mathematical support program is being offered to students who have shown areas of need in regular classroom performance and assessment. The support program I am offering works hand-in-hand with the regular classroom curriculum. Your child will have greater opportunities, in a small group setting, to get specific help in areas that he or she may be struggling with in the classroom. Thirty minutes of instruction will be offered three times a week for 12 weeks. I will be using a program entitled Expeditions to Numeracy, a base-10 intervention program that targets number sense and operations. It operates under the idea that numeracy, the ability to understand and work with numbers, underlies success in mathematics. Its goal is to identify gaps in student knowledge and provide systematic instruction to fill those gaps while developing a strong base for continued mathematical growth. I will be assessing your child's mathematics achievement and monitoring his/her weekly progress. These results will be available at the end of the study.

The information I collect on your child's progress will be kept confidential. Furthermore, your child's name will not be used. An ID number will be assigned to refer to him/her during the study. There are no risks to participating in this study. However, your child will benefit from participating in small group activities to improve his/her skills in math.

Your questions about the study are welcome at any time. Please call me at 215-944-1541 or email me at julieast@temple.edu. Your child's participation in this study is voluntary. There is no penalty for not participating. It will not affect your relationship with your child's school or teacher.

Questions about your rights as a research subject may be directed to Mr. Richard Throm, Institutional Review Board, Temple University, 3400 N. Broad St., Philadelphia, PA 19140, phone (215) 707-8757.

Signing your name below indicates that you have read and understand the contents of this Consent Form and that you agree to allow your child to take part in this study. You may withdraw consent at any time.

Parent Consent

Date

Principal Investigator's Signature

Date

APPENDIX H STUDENT ASSENT



Curriculum, Instruction and
Technology in Education - CITE
1501 Cecil B. Moore Avenue
Ritter Hall 3rd Floor (003-00)
Philadelphia, PA 19122

phone 215-204-2117/8377
fax 215-204-1414

Minor Assent Form

TITLE: The Effects of a Concrete, Representational, Abstract (CRA) Instructional Model
on Tier 2 First Grade Mathematics Students in a Response to Intervention Model:
Educational Implications for Number Sense and Computational Fluency

Mrs. Julie Eastburn, Math Education, Temple University

**I am planning to see how first grade students learn about
numbers and learn their addition facts.**

**If you want to be involved, I will meet with you three times per
week to practice your math skills.**

You can ask me questions any time about what I am studying.

Questions about your rights as a research subject may be directed to Mr. Richard Throm,
Office of the Vice President for Research, Institutional Review Board, Temple
University, 3400 N. Broad Street, Philadelphia, PA, 19140, (215) 707-8757.

**Signing your name below means you understand what we will be doing
and you agree to take part in this study.**

Participant's Signature _____ Date _____

Investigator's Signature _____ Date _____