RISK, RETURN AND CREDIT

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ABSTRACT

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This dissertation investigates the role of credit in the evaluation of risk and return. The research comprises three essays, which analyze the use of credit from different perspectives.

In the first essay, we propose a comprehensive theory for the assessment and implementation of "acceptable" underwriting and rating variables. Because of disparities in the way underwriting and rating classifications affect members of different social classes and ethnic/racial groups, government policymakers and consumers often question the appropriateness of certain controversial variables – for example, credit scores. We argue that a rating classification may be appropriate when the cost to society is relatively small. The use of personal credit in the automobile-insurance industry is addressed as a potential application of the proposed models, and other considerations are explored.

In the second essay, we investigate the use of credit by speculators (i.e., gamblers or investors) as a means of increasing expected survival time and/or the probability of

achieving gains. We propose a strategy in which a speculator engages in bet doubling and uses credit to maximize the probability of winning a specified amount. It is shown that with sufficient credit, it is possible to win with an arbitrarily high degree of certainty over the long run, even when facing random trials that are individually unfavorable. However, adopting such a strategy would eventually lead to large losses and negative expected profits. By limiting liability, total losses are restricted and the speculator's chances of obtaining positive gains increase. It is also shown that the cost of obtaining credit is an important consideration, and that it is disadvantageous for the speculator to engage in bet-doubling strategies when the cost of obtaining credit is high relative to the probability of winning.

In the third essay, we investigate "hazardously immoral" contracts that force external parties to bear significant losses without their consent. By concentrating risks and reducing visibility, such contracts obscure the expected cost of failure by not taking routine charges for predictable, albeit uncertain, future losses. Abuses are particularly likely to occur when the threat of system-wide disruption is sufficient to make governments and/or international agencies bail out the offending organizations in order to limit total damages. The models provided in the second essay are presented as possible strategies for concentrating risks, and several results are derived. In particular, it is shown that credit is extremely valuable, and that if credit can be obtained at sufficiently low rates of interest, then it is a simple matter to develop strategies in which the probability of achieving gains is practically guaranteed, despite large negative expected profits.

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"For the LORD gives wisdom; From His mouth come knowledge and understanding" (Proverbs 2:6, New King James Version).

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To Walt, Troy and Zanét

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EQUAL VERSUS FAIR: A PLACE FOR CONTROVERSIAL UNDERWRITING AND RATING CLASSIFICATIONS?

1.1 Introduction

Risk classification is an important means by which insurers compete to reduce the cost of providing insurance (Crocker and Snow, 2000). However, disparities in rates among social classes continue to be a public concern. Public policymakers (i.e., regulators and elected officials), as well as the public at large, often differ with insurers regarding what are appropriate rating classifications. Actuaries consider rating systems to be fair if the difference in rates reflects the material difference in expected costs. Individuals purchasing insurance have vastly different rating characteristics and offer numerous of risk levels, and premiums (net of expenses and profits) should be equal to expected losses. On the other hand, correlation with expected losses may not always justify the use of a rating characteristic to policymakers and the general public. Causation, controllability, and disproportionate premiums are all concerns that may prevent policymakers from accepting the use of a proposed rating characteristic in writing insurance.

One of the main reasons for the disagreement in rating variables between the insurance industry and policymakers is the lack of information regarding risk characteristics. While insurers have the statistical knowledge to justify the use of rating variables, policymakers are not privy to the same information. This is especially true when controllability is not met and causality cannot be determined. Policymakers' concern may be increased further if the rating characteristic is related to personal characteristics of the insured and unrelated to the insured property. Under many circumstances, society at large may be willing to share the costs of insurance when socio-political classes are disproportionately affected by the inclusion of controversial rating characteristics. In practice, insurers have to maintain a balance between using all available information when pricing policies and foregoing rating characteristics that are considered as controversial, consequently sharing the risk among all policyholders.

The Problem

According to the American Academy of Actuaries, a risk classification should serve three primary purposes (American Academy of Actuaries, ND). It should protect the insurance system's financial soundness, permit economic incentives to operate and thus encourage widespread availability of coverage, and finally, be fair. Although the latter of these may be the most intuitive, it can also be the most difficult to implement in practice. What the insurer determines as actuarially fair may not be equitable across all rating classes. As a result, policymakers may feel that the insurer is exemplifying discriminatory practices, especially if the rating characteristic has a disparate impact on certain groups.

There are several fundamental reasons for skepticism regarding certain classification variables. These include:

(1) *Doubts about actual correlation with risk* – For example, are younger drivers really more likely to have automobile accidents than older drivers? After all, one often hears about aged drivers with poor eyesight and/or slow reflexes causing accidents. If that is so, then why do "senior citizen" discounts exist?

(2) *Doubts about actual causality of risk* – Granting that younger drivers actually are more likely to have accidents than older drivers, is age the true causal factor? Another obvious possibility is that driver inexperience provides the fundamental causal link. Should experience be used instead of age to avoid treating young but experienced drivers unfairly?

(3) *Concerns about individual fairness* – Assuming that age is a true causal factor because younger people are intrinsically less careful (and therefore more likely to drive fast, drive while intoxicated, etc.), should this factor be applied to everyone equally? For example, there may be young drivers who are teetotalers, and therefore never will drive while intoxicated. Is it fair for them to bear the full impact of the age variable?

(4) *Concerns about group fairness* – Finally, even if one accepts that age is a true causal factor that applies to everyone equally (i.e., without need for modification for such things as alcohol avoidance), does society really want to penalize young people for their ages? After all, a young person cannot take any action to change his or her age, and younger people tend to be the least able to afford higher insurance premiums. Why not simply remove age as an underwriting and/or rating classification and thereby "spread the risk?"

When setting new rating classifications, insurers have to obtain the approval of insurance regulators in regulated markets and of consumers in competitive markets (i.e., markets in which the purchase of insurance is completely voluntary). In doing so, they must not only show a correlation between the proposed rating characteristic and losses, but also provide a theoretical explanation of why the rating variable affects losses as it does. Even if these two conditions are met, insurers may have a difficulty setting rates that are deemed as controversial. In this study, controversial rating characteristics are assumed to be characteristics that are not related to the insured property itself but related to a personal characteristic of the insured. For example, the age and make of an automobile will not be considered as controversial, whereas the age and sex of the driver will be. Adding to the controversy is the fact that many personal characteristics are not within a policyholder's control. It is often easier to justify the increase in insurance premium if the risk level can be changed by the insured. However, in many cases a policyholder's risk level is linked to variables in which he or she has no control over; i.e., gender, age, race, and genetics.

Purpose of Study

The purpose of this study is to develop parameters under which the use of controversial rating classifications in the underwriting and ratemaking process would be appropriate. Although this paper specifically addresses the property-liability insurance industry, some of the theory and models can be extended to health and life insurance as well. Four models will be presented that will assist insurers in deciding when to include controversial rating characteristics in the process of writing insurance and calculating premiums.

The remainder of this chapter is structured as follows. Section two describes two opposing principles under which insurance is written. Section three identifies constraints and lists a spectrum of variables that should be considered before accepting a proposed rating characteristic. Section four provides theory and a simple statistical model is presented. In section five, a look at the use of personal credit history by property line insurers is provided as an application, followed by further considerations and conclusions.

1.2 Equivalence versus Solidarity

In the insurance industry, it is not fair to rate everyone equally. Insurers often have to find a compromise between producing rates that accurately portray the risk characteristics of the insured and maintaining their approval by policymakers. The two main principles that are developed in the insurance literature are the *principle of premium equivalence* (or *equivalence principle*) and *solidarity*.

The equivalence principle as defined by De Wit (1986) states that “for each single risk there should be equivalence between the risk premium (or net premium) and the mathematical expectation of the claims” (p. 645). Similarly, “for each homogeneous group, the amount of premium accessed to the group should reflect the level of risk within the group.” While the equivalence principle makes sense, pure premium equivalence is difficult to obtain. Each individual insured possesses a unique set of risk characteristics, causing all policyholders to be heterogeneous. As a consequence, there will not be any two premiums that are identical. This will lead to a wide range of rates, making it difficult for the insurer to write business. Simply stated, it is impossible for insurers to charge insureds premiums (net of expenses and fees) that are exactly equal to the mathematical expectation of all future claims.

The next best alternative is to create rating classes in which the policyholders in each cohort have *similar* risk profiles. This is what is usually done in practice. Underwriters assign prospective insureds to groups in which the risk characteristics are approximately the same and assign rates accordingly. However, within each individual group, the equivalence principle is lost and solidarity ensues in which fellow policyholders pay for each other.

Different from the equivalence principle, solidarity suggests that individuals should not be obligated to pay their specific portion of the risk pool but instead, everyone equally contributes to the overall cost of the group. The basis for this theory is that people in some way feel responsible for others and are willing to share their burdens when necessary. Some believe that this principle is what drives insurance. In insurance, the "lucky" ones (those who do not file a claim) pay for the "unlucky" ones and the "good" risks pay for the "bad" risks. In the case of pure solidarity the insurer (total risk pool of insureds) takes on the entire risk of all expected losses and individuals are not responsible for their particular risk profiles, which is a direct contradiction of the equivalence premium.

De Wit and Van Eeghen (1984) expand these two principles and offer the following mathematical interpretations. If no insurance is purchased, the insured on average will pay the expected cost of the loss, $E[X]$ (where X is the random loss variable). The insured will also incur some risk since losses can have a high or unbounded limit. This risk is defined as the variance of the loss, $Var[X]$. See Table 1.

Table 1. Obtaining Insurance without Risk Classification

	Before Insurance		*After Insurance*	
	Insured	Insurer	Insured	Insurer
Incurred Loss	X	0	$E[X]$	$X - E[X]$
Expected Loss	$E[X]$	0	$E[X]$	0
Variance of Loss	$Var[X]$	0	0	$Var[X]$

De Wit and Van Eeghen, 1984

Once an insurance policy is purchased, the policyholder pays the expected cost of the loss to the insurer in the form of premiums (not including profit and expenses) in exchange for the transfer of risk to the insurer. Under this rating structure, pure solidarity is achieved when the total risk is paid by the insurer. Total solidarity, S is defined as $Var[X]$.

This primitive form of insurance only works if insureds are not aware of their own risk levels. As we move into a more informed society, policyholders will gain knowledge of their risk characteristics and loss potential. Eventually, the "good risks" will realize that they are subsidizing the policyholders with greater loss potential and will ultimately leave the insurance pool. The total premium collected from the "bad risks" will be lower than the expected costs of their losses. This will cause insurers to incur a positive total expected loss equal to the difference between the premium they collect and the actual expected loss of the "bad" risks. To decrease expected losses to zero, the insurer would have to include risk classifications, and charge premiums equitable to expected loss. The challenge then is for the insurer to identify the available risk characteristics, R in order to rate policies accordingly, and consequently share some of the risk with the insured.

Table 2 shows the differences in expectation and variance of losses for the insured and the insurer, with and without risk classification. The premium paid by the insured is no longer constant but varies with the insured's risk characteristics. For example, a policyholder with automobile insurance may face the risk of changes in premium due to accidents, address changes and/or traffic violations.

Table 2. The Importance of Risk Classification

		After Insurance Without Risk Classification		*After Insurance With Risk Classification*	
		Insured	Insurer	Insured	Insurer
Good Risks	Incurred Loss	-	-	$E[X_G]$	$X_G - E[X_G]$
	Expected Loss	-	-	$E[X_G]$	0
Bad Risks	Incurred Loss	$E[X]$	$X_B - E[X]$	$E[X_B]$	$X_B - E[X_B]$
	Expected Loss	$E[X]$	$E[X_B] - E[X]$	$E[X_B]$	0
	Variance of Loss	0	$Var[X']$	$Var_R[E(X_R)]$	$E_R[Var(X_R)]$

De Wit and Van Eeghen, 1984
Note: R denotes Risk classification; $Var[X] = Var_R[E(X_R)] + E_R[Var(X_R)]$

With risk classification, premiums more accurately reflect potential future losses. As a result, expected losses for the insurer are reduced to zero. Total solidarity $S = Var[X]$, can now be separated into risk solidarity $S_R = Var_R[E(X_R)]$ and probabilistic solidarity $S_P = E_R[Var(X_R)]$ (De Wit and Van Eeghen, 1984). If insurers can rigorously identify each risk classification and rate policies on the basis of individual risk profiles, risk solidarity will ultimately remain with the insureds and the risk level incurred by the insurer (group of insureds) will be reduced to probabilistic solidarity, i.e., the lucky ones paying for the unlucky ones. The structure created under the Equivalence Principle becomes the optimal rating structure and it then becomes necessary for insurers to use all available information to accurately price insurance.

1.3 Evaluating Controversial Rating Characteristics

Although the equivalence principle is seemingly the optimal approach, it is important for insurers to consider the socio-political factors impacting the acceptability of potential rating classifications. Insurers may be pressured by the public and regulators to ensure that insurance is both available and affordable to all economic classes regardless of their risk profiles. Insurers in particular may have difficulty in assigning rate classes that are seemingly related to controversial classification variables – such as age, gender, genetics, income, and ethnicity/race.

When evaluating controversial rating characteristics, it is important to determine the level of controversy that the variable creates. Although policymakers typically frown on the use of any rating characteristic that is seemingly related to socio-political variables, some characteristics may be looked upon as being more prejudicial than others. Under each of the following headings: Causality, Controllability and Social Sensitivity, a series of spectra is given which shows the importance of the related variable to each line of business. For example, under Social Sensitivity; variables related to age and gender, are at the lower ends because they in general cause the least amount of concern. In fact, in some lines of business, rates differ predominantly on the basis of these two characteristics alone. However, the use of variables related to income and/or race are more inflammatory putting them at the higher ends of our spectra. It is suggested that insurers keep in mind where the proposed rating characteristic fits on each scale and make their decisions accordingly.

Causality

One major level of controversy is a lack of evidence supporting causality. It is not enough to show that a variable is correlated with losses to justify its use as a rating classification. Although the existence of a causal relationship is not a formal requirement for rating characteristics (American Academy of Actuaries, ND), insurers should be able to provide some theory to explain how the proposed variable would cause a change to the policyholder's loss potential. It is important that the public has an understanding of why personal characteristics can affect a policyholder's potential to generate losses. Refer to the following table.

Table 3: Level of Controversy Relating to Causality

Level of Controversy	***Life Insurance***	***Health Insurance***	***Auto Insurance***
High	Income	Income	Race
↑	Race	Race	Current Health[1]
	Gender	Gender	Income
	Health Behavior[1]	Age	Health Behavior[1]
	Current Health [1]	Health Behavior[1]	Gender
Low	Age	Current Health[1]	Age

In the Life Insurance industry, age and health variables are at the lower end of the spectrum since they are more widely accepted as causes of future claims. However, studies have shown a significant relationship between income (source and amount) and

[1] Here we make the distinction between current health conditions (including genetics) and health conditions that are related to behaviors (i.e. smoking).

mortality (Snyder and Evans, 2006). It is worth noting that this table does not show actual levels of causality amongst the given variables but rather what society perceives as causal factors. Similarly, in the Health Industry, health related variables are at the lower end of the spectrum since they are perceived as being directly related to loss potential and in the Auto Industry, age and gender are more accepted as causes of risk than race and income. It is important for insurers to know where the potential rating classification fits in the spectra for the given line of business. If insurers cannot make a strong case for the use of new rating variables and can successfully show causality, policymakers may have a negative response to its use.

Controllability

Here controllability is defined as the ability a policyholder has to control the rating characteristic(s) under which he/she is written. If the insured can control his/her level of risk, they can ultimately decide which rating class they are assigned. If a socio-economic class is disproportionately affected by a risk characteristic that is within their control, they can decrease or even eliminate that risk. This will reduce the level of controversy associated with the potential rating classification since the policyholder can change which rating class they are assigned to and the insurer can remain impartial. Controllability decreases the likelihood of unfair premiums and eliminates discriminatory practices. In addition, policyholders will have the economic incentive to reduce their exposure to risk and ultimately reduce claims.

Table 4. Level of Controversy Relating to Causality

Low End	⟶		***High End***
Health Behavior	Income	Gender, Health Genetics	Age, Race

Table 4 measures the level of controversy regarding the policyholder's ability to control the proposed rating variable. Health behavior is listed at the lower end since it is the only variable (in the spectrum) in which the policyholder has complete control, with income being a close second. Health genetics, gender, age and race are at the high end since these variables cannot be manipulated.

Social Sensitivity

Perhaps the greatest level of concern is general social welfare. The problem arises when a potential rating classification is highly correlated with potential loss, the causality constraint is met yet the classification is related to variables in which the policyholder cannot control. The equivalence principle tells us to use the rating class in the process of writing insurance, decreasing solidarity to S_P. However, some may feel that individuals should not have to pay higher rates for characteristics that are beyond their control, especially if those characteristics are seen as proxies for variables at the high end of the spectrum. One major concern is that there may be a disparate impact on groups already faced with disproportionate burdens. Table 5 lists a hierarchy of controversial variables each line of business as they relate to social welfare. Although doubts have been raised (at one time or another) about most demographic variables that cannot be controlled by insureds, the use of age and gender generally have been found more acceptable than the use of income and race/ethnicity. Such determinations typically arise from either (or both) of two considerations:

- the *likelihood of implicit cost averaging* (e.g., age may be more acceptable because most individuals eventually pay rates as both young and old insureds during their lifetimes, and gender may be more acceptable because higher insurance rates for a given sex in one line of business can be offset by lower rates in another line of business); and

- *support for social mobility/equity* (e.g., income and race/ethnicity may be less acceptable because they are viewed as oppressing certain vulnerable classes of society).

Table 5. Level of Controversy Relating to Social Welfare

Level of Controversy	***Life Insurance***	***Health Insurance***	***Auto Insurance***
High ↑	Race	Race	Race
	Income	Age, Income	Income
	Gender		Health Genetics
	Health Genetics	Gender	Health Behavior
	Age	Health Genetics	Gender
Low	Health Behavior	Health Behavior	Age

Once an underwriting and/or rating variable has been deemed sufficiently unacceptable, such as race/ethnicity for all lines of business in the United States, policymakers invariably must address the potential use of improper proxy variables as substitutes for the excluded variable. For example, some American lawmakers and regulators have objected to the use of credit scores in automobile-insurance underwriting and rating out of a concern that this classification merely provides a less conspicuous stand-in for race/ethnicity and/or income.

1.4 Theory and Methodology

When proposing the use of a potential controversial rating classification, the insurer should first determine if the benefits of using the variable outweigh the social costs. Figure 1 lists a series of recommended tests that the proposed rating classification should pass before being considered in the underwriting/ratemaking process. The first is the test of causality following by a test to determine the level of control the policyholder has over the proposed rating variable. If both the causality and controllability tests have been passed, then the insurer could use the rating variable without prejudice. However, if the variable cannot easily be manipulated, then the insurance should test to determine if the proposed rating classification is a proxy for one (or more) of the socio-political variables listed in the above tables. The proxy test is important to ensure that the use of the proposed rating classification will not have a disparate impact on any of the social classes. Failing the proxy test does not necessarily mean that the rating variable cannot be used, but rather it should be adjusted to remove the controversial component.

Theoretical Model

To evaluate the effect of using such potential proxy variables, we propose a simple statistical model to determine whether or not the explanatory value of the subject variable (e.g., credit score) is diminished by including the related socio-political variable (i.e., race/ethnicity or income), after controlling for all other classification effects.

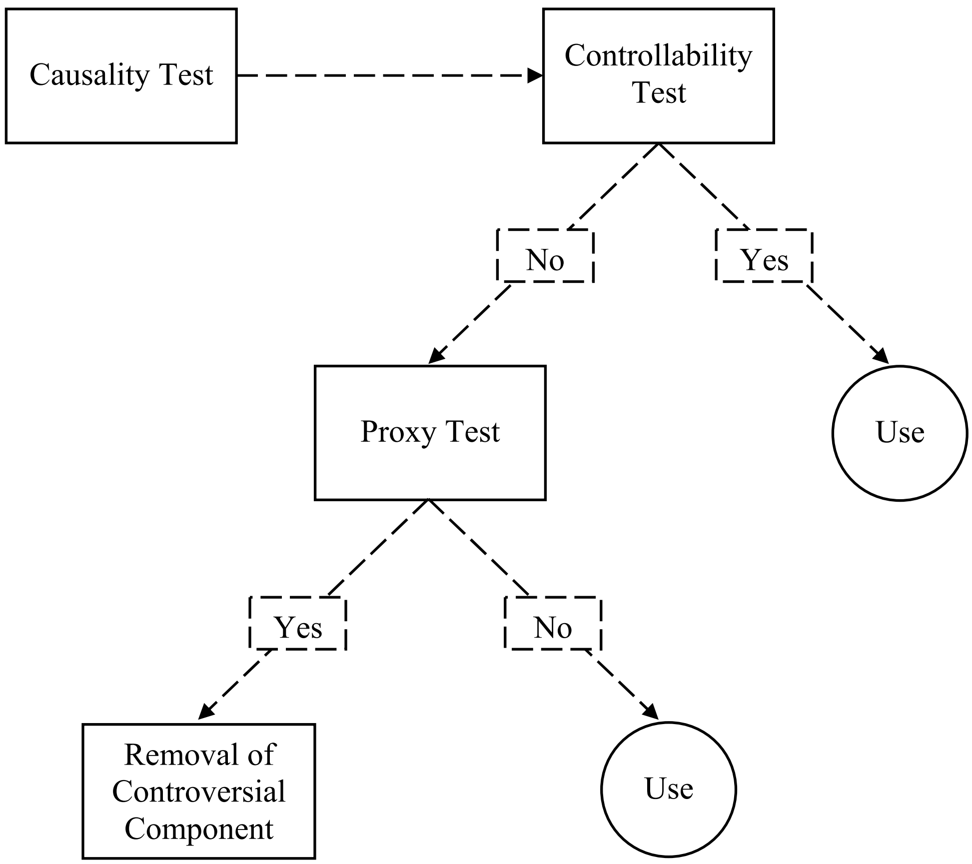

Figure 1. Test to Determine the Use of Proposed Rating Classification

Specifically, consider the following set of regression models for insured i 's historical average loss as a function of k classification variables, $X_{1i}, X_{2i}, \ldots, X_{ki}$, along with zero, one, or two members of the pair (C_i, R_i), where C_i (for credit score) denotes the potential proxy, and R_i (for race/ethnicity) denotes the prohibited indicator:

$$L_i = \alpha_0 + \alpha_1 X_{1i} + \alpha_2 X_{2i} + \cdots + \alpha_k X_{ki} + \varepsilon_i \qquad (1)$$

$$L_i = \alpha_0 + \alpha_1 X_{1i} + \alpha_2 X_{2i} + \cdots + \alpha_k X_{ki} + \beta_0 C_i + \varepsilon_i \qquad (2)$$

$$L_i = \alpha_0 + \alpha_1 X_{1i} + \alpha_2 X_{2i} + \cdots + \alpha_k X_{ki} + \gamma_0 R_i + \varepsilon_i \qquad (3)$$

$$L_i = \alpha_0 + \alpha_1 X_{1i} + \alpha_2 X_{2i} + \cdots + \alpha_k X_{ki} + \beta_1 C_i + \gamma_1 R_i + \varepsilon_i \qquad (4)$$

Case 1: Inclusion of controversial variable C_i (i.e. credit), in model (2), provides a significantly better fit to the loss data than the first model. However, the inclusion of the related social class R_i (i.e. race), in model (3), does not provide a better fit than model (1). We can conclude that C_i (credit variable) is a legitimate predictor of losses and is not a proxy for R_i (race variable).

Case 2: Both models (2) and (3) provide significantly better fits to the loss data than model (1). In addition, model (4) provides a significantly better fit than model (3). That is the marginal effects of including the controversial variable C_i are statistically significant. We can conclude that C_i s a legitimate predictor of losses and is not a proxy for R_i.

Case 3: Both models (2) and (3) provide significantly better fits than model (1), and model (4) does not provide a significantly better fit than model (3). We may conclude that C_i is indeed a proxy an illegitimate proxy for R_i. This does not mean that C_i cannot be used as a classification variable, but rather that if it is to be used, then it first must be adjusted to remove the effect of R_i. For example, the insurer simply may replace C_i with $C_i - \left(\hat{\gamma}_1 / \hat{\beta}_1\right)\hat{R}_i$ in model (2), where $\hat{\beta}_1$ and $\hat{\gamma}_1$ are estimated from model (4).

1.5 Application: Use of Credit Scores in Writing Automobile Insurance

One of the more recent controversial developments in the United States automobile insurance industry has been the use of personal credit to price and underwrite insurance. Personal-line insurers contend that the use of personal credit, in particular credit-based insurance scores, will enable them to rate and underwrite prospective policyholders more accurately. Using the equivalence principle as their basis, they argue that individuals who poorly manage their personal finances are higher risks and should therefore pay higher premiums. In addition, they insist that the use of credit information has, on average, decreased premiums due to the higher accuracy of pricing insurance.

Although credit-based insurance scores have proven to be an invaluable tool to insurers, its use has generated a number of concerns. Critics not only question the significance of using personal credit as a rating variable but are also concerned with the opportunity it creates to indirectly discriminate against groups otherwise protected under the Fair Housing Act. Controversies surrounding the use of credit-based insurance scores include:

- Whether there is a correlation between personal credit and loss potential? If so, does theory support a causal relationship?
- Is credit data related to other risk characteristics already used in the underwriting / ratemaking process?

- Does the use of personal credit information have a disparate impact on minority and low-income populations?
- Are credit-based insurance scores a proxy for ethnicity and/or income level?

Applying the Model

Causality/Controllability Test

The first two concerns are related to society's perception of causality. The general public does not understand why there is a relationship between personal credit and loss potential. In addition, they argue that credit information is most likely linked to other variables such as age and geographical location, already used to price automobile insurance.

There are a number of published studies which support the use of personal credit in the Automobile Insurance industry. Federal Trade Commission (2007), Hartwig and Wilkinson (2003), Monaghan (2000), and Wu and Guszcza (2003) were all able to show that an insured's credit history provides valuable information regarding their potential to generate losses. In addition, Monaghan (2000) and the Federal Trade Commission (2007) have both shown, through multivariate analysis, that the inclusion of other underwriting variables such as age and territory did not cause the relationship between credit and losses to disappear. Although the study performed by the Federal Trade Commission (2007) demonstrates that credit-based insurance scores are "effective predictors of risk," the FTC also acknowledged that at the time of their study, research was not available to support causality.

In 2007, Brockett and Golden conducted a study to determine a causal link between poor credit history and increased loss potential. In their analysis, they found that the biological, psychological, and behavioral attributes of financial risk takers were similar to the biological, psychological, and behavioral attributes of risky automobile drivers and consequently incurred losses. They further concluded that credit history can give valuable information regarding an individual's bio-psychological makeup apart from that used in standard underwriting, which will be useful in writing insurance. Since poor credit history is linked to biological, psychological and behavioral attributes, it is easy to argue that credit-based insurance scores cannot easily be manipulated.

Proxy Test

As stated previously, one of the major controversies with using credit-based insurance scores is its perceived correlation with race and income. Several studies have shown significant relationships between personal credit history and ethnicity and income levels (Kabler, 2004; Federal Trade Commission, 2007). In particular, the FTC study found that the inclusion of the credit variable has caused African-Americans and Hispanics to experience premium increases of approximately ten percent and four percent respectively while Asians and Non-Hispanic Whites experienced decreases in premiums.

To determine if the credit variable is a proxy for race or income, the FTC tested the marginal effects of using while controlling for race and income. The FTC concluded:

> Credit-based insurance scores predict risk within racial, ethnic, and income groups. Scores have only a small effect as a "proxy" for membership in racial and ethnic groups in estimating of insurance risk, remaining strong predictors of risk when controls or race, ethnicity and income are included in risk models. (Federal Trade Commission, 2007)

The results from the FTC study showed that while minorities and low-income populations bear a disproportionate burden of increased costs, controlling for these variables did not cause for the relationship between credit and risk to disappear. From their analysis, the FTC found credit-based insurance scores to be the strongest single predictor of loss potential and concluded that credit scores were not a proxy for race or income.

Removal of Controversial Component

Although, credit-based insurance scores have not been found to be illegitimate proxies of race or income, the disproportionate burden on minorities and low income insureds continue to be a public concern. The FTC suggests that an alternate credit-based scoring model that continues to predict risks effectively, but also decreases the disparities among racial groups, is ultimately desired. The obvious solution is the removal of the controversial components; however, it cannot easily be determined which of the credit variables are related to race and/ or income.

1.6 Other Considerations

Removing Controversial Factors

One of the steps in the above model is the removal of controversial components. In theory, one can make a proposed variable less controversial if they can properly identify and remove the attributes associated with the variables listed in Tables 3 through 5. In the case of credit-based insurance scores, the level of controversy is mainly attributed to its correlation with race and income. However, there are many credit variables that are included in the calculation of the score. If those variables (or parts of) related to race and or income can be identified and removed, then the remainder may only be that portion solely related to risk. This will make for a more accurate and more preferred rating characteristic.

Technological Solutions

As the insurance industry becomes more informed and technologically advanced, actuaries will be able to identify risk characteristics more accurately without the controversial components. Insurers will successfully be able to transfer total risk solidarity to the insured without disproportionately affecting socio-political groups. Policymakers' fears of disparate impacts will be eliminated and their role in the insurance industry will be minimized.

1.7 Conclusions

In this research, we have proposed a comprehensive theory for the assessment and implementation of "acceptable" underwriting and rating variables. While there are several fundamental reasons for skepticism regarding certain classification variables, extensive research and careful analysis can help alleviate the concern. The major contribution of this study to the insurance industry is that it gives insurers a bargaining chip when proposing the use of controversial rating classifications. By conducting the series of suggested tests, insurers can diminish the skepticism regarding the use of controversial rating classifications. The best-case scenario would be to remove the controversial component from the potential rating classification. However, without the technological advances to do so; insurers will ultimately have to weigh the benefits of using complete information against the social impact.

CHAPTER 2

BETTING AGAINST THE ODDS:

ANOTHER LOOK AT BET-DOUBLING STRATEGIES

2.1 Introduction

A challenge that gamblers face is to develop strategies to reach monetary and/or survival goals. Objectives can range from maximizing the probability of winning a specified amount, to maximizing the expected time in the game before ruin. Gamblers often have to manipulate the size of the bet at each trial to achieve one or both of these goals. In the coin tossing example provided by Thorp (2006), the author looks at two extremes in which a gambler first plays to maximize expected gains and then to maximize survival time. Thorp (2006) defines the expected funds after n trials as

$$E[X_n] = X_0 + \sum_{k=1}^{n}(p-q)E[B_k],$$

where

p = probability of winning on a given trial

$q = 1 - p$ = probability of losing on a given trial

B_k = size of bet on kth trial

X_0 = initial capital

Thorp shows that the gambler must bet all resources at each trial in order to maximize $E[B_k]$ and thus maximize $E[X_k]$. However, the probability of ruin, given by $1-p^n$ is almost certain for large n, Thorp (2006).

The opposite approach would be to minimize the probability of ruin (maximize survival time) by placing minimal bets at each trial as given by Feller (1966). However, as shown above, this strategy also minimizes $E[X_k]$ and may not be an attractive strategy for gamblers with a profit maximizing goal. An optimal strategy would be to find the appropriate balance between monetary and survival objectives by choosing a betting amount that best matches the player's goal.

2.2 The Kelly Criterion and Other Strategies

One of the more well known gambling strategies was developed by J.L. Kelly in 1956. The Kelly Criterion has been a significant development in the gambling literature but has also been a dominant strategy increasingly used by investors. The Kelly Criterion also referred to as the geometric mean maximizing portfolio strategy, is a betting rule which maximizes the logarithm of expected gains. According to "Kelly's Rule", the gambler should only bet/invest a fraction of his wealth at each instance in order to maximize the growth rate of expected gains in the long run.

Case 1: Gambler is faced with 1-1 odds

$$f = p - q = 2p - 1$$

where

f = fraction of wealth

p = probability of winning on a given trial.

$q = 1 - p$ = probability of losing on a given trial

For example, if $p = 2/3$, gambler should bet $1/3$ of his bankroll and if $p = 3/4$, gambler should bet $1/2$ of his bankroll.

Case 2: Gambler is faced with betting odds, b

$$f = \frac{bp - q}{b}$$

where

f = size of bet

p = probability of winning on a given trial.

$q = 1 - p$ = probability of losing on a given trial

For example, if the gambler is faced with a proposition paying 2 to 1, $(i.e.\ b = 2)$ and $p = 2/3$, gambler should bet $1/2$ of his bankroll and if $p = 3/4$, gambler should bet $5/8$ of his bankroll.

Kelly's Criterion has been a significant development in the gambling and finance literature. However, the strategy has a few limitations. While this strategy maximizes expected gains, it is a long run strategy in which the game has to be repeated over an infinite horizon. Adopting such a strategy also does not increase the probability of winning a fixed amount before ruin. Another disadvantage to adopting such a strategy is the inability to place positive bets when faced with unfavorable odds (i.e. $bp < q$). Even in the event where odds are evenly matched $(bp = q)$, the Kelly Criterion says that the bet should not be placed.

Another common strategy used by gamblers is bet doubling. The gambler starts with a betting size equal to his initial objective and doubles the bet until a win occurs. By consistently doubling the size of bets, the gambler will be able to recover all previous losses and also obtain his initial objective, increasing the probability of walking home a winner. Turner (1998) found that the main reason why gamblers adopt such a strategy is due to the faulty assumption that successive trials are not independent. That is, gamblers tend to believe that a string of losing trials will inevitably lead to a win. Turner found that a doubling strategy was more profitable in the short run. However, in the long run, bet doublers suffered losses much greater than constant betters with the same probabilities of winning.

In this research, we propose a strategy in which a gambler/investor engages in bet doubling and uses credit to maximize the probability of winning a specified amount. As shown by Turner (1998) a doubling strategy could lead to high expected losses which we will attempt to diminish by including limited liability in our model. With limited liability, the gambler will be able to maintain positive expected losses while facing a losing proposition.

The remainder of this paper is structured as follows. In Section III, we present a simplified bet-doubling strategy followed by a more formal and general model which provides some results regarding the size of bet, probability of loss, and expectations of profit. Sections IV and V will analyze expected gains in the cases of limited liability and cost of obtaining credit respectively. Finally, Section VI concludes.

2.3 Simple Discrete Model

Consider a simple discrete model in which a speculator tries to raise a sum of money by betting on a series of Bernoulli trials and doubling the bet on each successive trial until a win occurs. The speculator obtains credit, possibly from several lenders, to increase his probability of winning. A full discussion of the models presented in this paper and real life applications will be provided in Chapman and Getzen (2009).

Credit Requirements for Speculating on Bernoulli Trials Using Bet Doubling

Model:

A speculator with initial wealth $W > 0$ and initial credit line of $C > 0$ seeks a return of $b \in (0,1]$ units by betting on a series of i.i.d. Bernoulli trials each with probability of success, $p = 1 - q \in (0,1)$. The speculator places an initial bet b on the first trial, and then doubles the size of the bet on each successive trial. The speculator discontinues the series of trials at the earlier of two events (see Figure 2):

(1) he is successful on a given trial, thereby winning b units (net); or

(2) his current credit level, including initial wealth, is inadequate (i.e. falls below the amount needed to make the next bet).

If the speculator selects a value $b = 1/m$ for $m \in \Im^{+}$, and seeks an ultimate return equal to 1, then he must repeat the above procedure m times, but discontinue immediately if his current credit level is inadequate during any sub-series of trials.

Definitions:

Let $X \sim f_G(x) = \Pr\left(1^{\text{st}} \text{ success occurs on trial } x\right) = pq^{x-1}$, for $x = 1,2,3,\ldots$

Let $T = \begin{cases} X \text{ if } X = 1,2,\ldots,\left\lfloor \log_2\left(\frac{W+C}{b}+1\right)\right\rfloor \\ \left\lfloor \log_2\left(\frac{W+C}{b}+1\right)\right\rfloor \text{ if } X > \left\lfloor \log_2\left(\frac{W+C}{b}+1\right)\right\rfloor \end{cases}$ denote the trial on which the game ends.

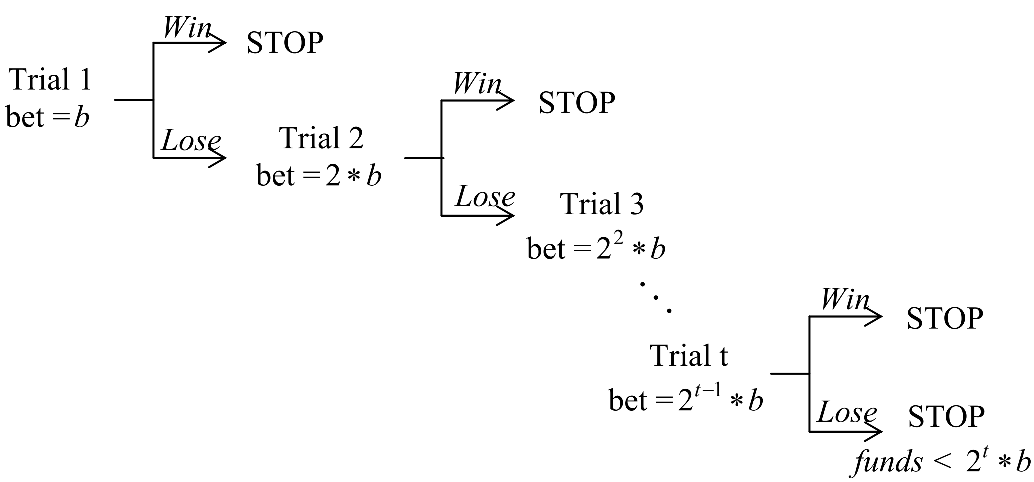

Figure 2: Possible Outcomes and Stopping Times; Simplified Model

Let $\omega(W,C,b) = 1 - \lambda(W,C,b)$

$$= \Pr\left(\text{net gain} = b \text{ before funds are inadequate} \,\middle|\, \begin{array}{l}\text{initial wealth} = \text{W, credit line} = C, \\ \text{initial bet} = b\end{array}\right)$$

$$= \Pr\left(1^{\text{st}} \text{ success occurs on or before trial} \left\lfloor \log_2\left(\frac{W+C}{b}+1\right)\right\rfloor\right)$$

$$= \sum_{x=1}^{\left\lfloor \log_2\left(\frac{W+C}{b}+1\right)\right\rfloor} f_G(x) = 1 - q^{\left\lfloor \log_2\left(\frac{W+C}{b}+1\right)\right\rfloor}.$$

Case I (Initial Bet = 1, Objective = 1):

$\tau = Max\{T\} = \lfloor \log_2 (W + C + 1) \rfloor$

$\omega(W, C, 1) = 1 - \lambda(W, C, 1)$

$= \Pr(\text{net gain} = 1 \text{ before funds are inadequate} \mid \text{initial funds} = W + C, \text{initial bet} = 1)$

$= 1 - q^{\lfloor \log_2 (W+C+1) \rfloor} = 1 - q^{\tau}$.

$$E[T] = E[T|\text{win}](1 - q^{\tau}) + E[T|\text{loss}]q^{\tau},$$

where

$$E[T|\text{win}] = \frac{1}{1-q^{\tau}}\left(\sum_{x=1}^{\tau} xpq^{x-1}\right) = \frac{p}{1-q^{\tau}}\left[\sum_{x=1}^{\tau}\frac{d}{dq}(q^x)\right] = \left(\frac{p}{1-q^{\tau}}\right)\frac{d}{dq}\left(\sum_{x=1}^{\tau} q^x\right)$$

$$= \frac{p}{1-q^{\tau}}\left[\frac{1-q^{\tau}-(1-q)\tau q^{\tau}}{(1-q)^2}\right] = \left(\frac{1}{1-q}\right) - \left(\frac{\tau q^{\tau}}{1-q^{\tau}}\right) = \left(\frac{p}{1-q^{\tau}}\right)\frac{d}{dq}\left(\frac{q-q^{\tau+1}}{1-q}\right)$$

and

$$E[T|\text{loss}] = \tau .$$

Thus,

$$E[T] = \left[\left(\frac{1}{1-q}\right) - \left(\frac{\tau q^{\tau}}{1-q^{\tau}}\right)\right](1-q^{\tau}) + \tau q^{\tau} = \frac{1-q^{\tau}}{1-q} .$$

$$E[Gain] = E[Gain|\text{win}](1 - q^{\tau}) + E[Gain|\text{loss}]q^{\tau},$$

where

$$E[Gain|\text{win}] = 1$$

and

$$E[Gain|\text{loss}] = -(2^{\tau} - 1).$$

Thus,

$$E[Gain] = (1)(1-q^{\tau}) - (2^{\tau}-1)q^{\tau} = 1-(2q)^{\tau}.$$

Table 6. Expected Gains as a Function of q

$Max\{T\}$	$q = 0.25$	$q = 0.49$	$q = 0.50$	$q = 0.51$	$q = 0.75$
1	0.5000	0.0200	-	-0.0200	-0.5000
2	0.7500	0.0396	-	-0.0404	-1.2500
5	0.9688	0.0961	-	-0.1041	-6.5937
10	0.9990	0.1829	-	-0.2190	-56.6650
25	1.0000	0.3965	-	-0.6406	-25250.1682

The asymmetry in expected gains, as shown in the above table, is due to the polynomial relationship between expected gains and q $\left(i.e.\ E[Gains] = 1-(2q)^{\tau}\right)$. When $q < 0.50$ $(p > 0.50$ and $2q < 1)$, $E[Gains]$ are positive and increase with τ. However, when $q > 0.50$ $(p < 0.50$ and $2q > 1)$, $E[Gains]$ are negative and decrease more rapidly as τ increases.

Case II (Initial Bet = 1/m, Objective = 1):

$$\tau = Max\{T\} = \sum_{i=1}^{m} \left\lfloor \log_2 \left(m(W+C) + (i-1) + 1 \right) \right\rfloor = \sum_{i=1}^{m} \tau_i$$

$$\omega\left(W, C, \frac{1}{m}\right) = 1 - \lambda\left(W, C, \frac{1}{m}\right)$$

$$= \Pr\left(\text{net gain} = 1 \text{ before funds are inadequate} \,\middle|\, \text{initial funds} = W + C, \text{initial bet} = \frac{1}{m} \right)$$

$$= \prod_{i=1}^{m} \left[1 - q^{\lfloor \log_2 (m(W+C)+(i-1)+1) \rfloor} \right] = \prod_{i=1}^{m} \left(1 - q^{\tau_i} \right).$$

$$E[T] = E[T|\text{win}] \cdot \prod_{i=1}^{m} \left(1 - q^{\tau_i} \right) + \sum_{j=1}^{m} \left\{ E[T|\text{loss during sub-series } j] \cdot q^{\tau_j} \prod_{i=1}^{j-1} \left(1 - q^{\tau_i} \right) \right\},$$

where

$$E[T|\text{win}] = \sum_{i=1}^{m} \left[\left(\frac{1}{1-q} \right) - \left(\frac{\tau_i q^{\tau_i}}{1-q^{\tau_i}} \right) \right]$$

and

$$E[T|\text{loss during sub-series } j] = \sum_{i=1}^{j-1} \left[\left(\frac{1}{1-q} \right) - \left(\frac{\tau_i q^{\tau_i}}{1-q^{\tau_i}} \right) \right] + \tau_j .$$

Thus,

$$E[T] = \sum_{i=1}^{m} \left[\left(\frac{1}{1-q} \right) - \left(\frac{\tau_i q^{\tau_i}}{1-q^{\tau_i}} \right) \right] \cdot \prod_{i=1}^{m} \left(1 - q^{\tau_i} \right)$$

$$+ \sum_{j=1}^{m} \left(\left\{ \sum_{i=1}^{j-1} \left[\left(\frac{1}{1-q} \right) - \left(\frac{\tau_i q^{\tau_i}}{1-q^{\tau_i}} \right) \right] + \tau_j \right\} \cdot q^{\tau_j} \prod_{i=1}^{j-1} \left(1 - q^{\tau_i} \right) \right)$$

$$E[Gain] = E[Gain|\text{win}] \cdot \prod_{i=1}^{m} \left(1 - q^{\tau_i} \right)$$

$$+ \sum_{j=1}^{m} \left\{ E[Gain|\text{loss during sub-series } j] \cdot q^{\tau_j} \prod_{i=1}^{j-1} \left(1 - q^{\tau_i} \right) \right\}$$

where

$$E[Gain|\text{win}] = 1$$

and

$$E[Gain|\text{loss during sub-series } j] = -\left(\frac{2^{\tau_j} - 1}{m} \right) + \left(\frac{j-1}{m} \right) = -\left(\frac{2^{\tau_j} - j}{m} \right).$$

Thus,

$$E[Gain] = (1)\cdot\prod_{i=1}^{m}\left(1-q^{\tau_i}\right) - \sum_{j=1}^{m}\left[\left(\frac{2^{\tau_j}-j}{m}\right)\cdot q^{\tau_j}\prod_{i=1}^{j-1}\left(1-q^{\tau_i}\right)\right].$$

Behavior of Tail Probabilities

Theorem 1: For any initial funds, $K_j = 2^j; j \in \{0,1,2,3,\ldots\}$, (where $K = W + C$), initial bet $b_n = \frac{1}{2^n}; n \in \{0,1,2,3,\ldots\}$, and Bernoulli probability $p = 1-q \in (0,1)$, it follows that for fixed b_n, as $k_j \to \infty$,

$$\lambda(K_j, b_n) \sim {b_n}^{\gamma-1}{K_j}^{-\gamma} \to 0,$$

where $\gamma = \log_{1/2} q > 0$.

Proof: From Case II above, we know that

$$\omega(K_j, b_n) = 1 - \lambda(K_j, b_n)$$

$$= \prod_{i=1}^{2^n}\left[1 - q^{\left\lfloor \log_2\left(2^n \cdot 2^j + (i-1) + 1\right)\right\rfloor}\right].$$

Since

$\left\lfloor \log_2\left(2^n \cdot 2^j + (i-1)+1\right)\right\rfloor = n + j$ for all $i \in \{1,2,3,\ldots,2^n\}$, it follows that

$$\omega(K_j, b_n) = \left(1 - q^{n+j}\right)^{2^n}$$

$$= \left[1 - q^{\log_2\left(\frac{1}{b_n}\right) + \log_2\left(K_j\right)}\right]^{\frac{1}{b_n}}$$

$$=\left[1-q^{\frac{\log_q\left(\frac{1}{b_n}\right)}{\log_q(2)}+\frac{\log_q(K_j)}{\log_q(2)}}\right]^{\frac{1}{b_n}}$$

$$=\left[1-\left(\frac{1}{b_n}\right)^{-\gamma} K_j^{-\gamma}\right]^{\frac{1}{b_n}}. \qquad (1)$$

Thus, for fixed b_n, as $K_j \to \infty$,

$$\omega(K_j, b_n) = 1-\left(\frac{1}{b_n}\right)^{1-\gamma} K_j^{-\gamma} + O\left(K_j^{-2\gamma}\right)$$

$$= 1 - b_n^{\gamma-1} K_j^{-\gamma} + O\left(K_j^{-2\gamma}\right),$$

and so

$$\lambda(K_j, b_n) = b_n^{\gamma-1} K_j^{-\gamma} + O\left(K_j^{-2\gamma}\right) \sim b_n^{\gamma-1} K_j^{-\gamma}.$$

Theorem 2: For any initial funds, $K_j = 2^j; j \in \{0,1,2,3,\ldots\}$, initial bet $b_n = \frac{1}{2^n}; n \in \{0,1,2,3,\ldots\}$, and Bernoulli probability $p = 1-q \in (0,1)$, it follows that for fixed K_j, as $b_n \to 0$,

(i) $\lambda(K_j, b_n) \sim b_n^{\gamma-1} K_j^{-\gamma} \to 0$ for $\gamma = \log_{1/2} q > 1$,

(ii) $\lambda(K_j, b_n) \to 1-\exp\left(-K_j^{-\gamma}\right)$ for $\gamma = \log_{1/2} q = 1$, and

(iii) $\omega(K_j, b_n) \sim \exp\left(-b_n^{\gamma-1} K_j^{-\gamma}\right) \to 0$ for $\gamma = \log_{1/2} q < 1$.

Proof:

(i) For $\gamma = \log_{\frac{1}{2}} q > 1$, consider equation (1),

$$\omega(K_j, b_n) = \left[1 - \left(\frac{1}{b_n}\right)^{-\gamma} K_j^{-\gamma}\right]^{\frac{1}{b_n}}.$$

From Lemma 1, we know that for fixed K_j, as $b_n \to 0$,

$$\omega(K_j, b_n) = \exp\left(-b_n^{\gamma-1} K_j^{-\gamma}\right) + O\left(b_n^{2\gamma-1} \exp\left(-b_n^{\gamma-1} K_j^{-\gamma}\right)\right),$$

and so

$$\lambda(K_j, b_n) = 1 - \exp\left(-b_n^{\gamma-1} K_j^{-\gamma}\right) + O\left(b_n^{2\gamma-1} \exp\left(-b_n^{\gamma-1} K_j^{-\gamma}\right)\right).$$

Since $\lim_{b_n \to 0} \frac{\lambda(K_j, b_n)}{1 - \exp\left(-b_n^{\gamma-1} K_j^{-\gamma}\right)} = 1$ (by l'Hôpital's rule), it follows that

$$\lambda(K_j, b_n) \sim 1 - \exp\left(-b_n^{\gamma-1} K_j^{-\gamma}\right) \sim b_n^{\gamma-1} K_j^{-\gamma} \to 0.$$

(ii) For $\gamma = \log_{1/2} q = 1$, rewrite equation (1) as

$$\omega(K_j, b_n) = \left[1 - \left(\frac{1}{b_n}\right)^{-1} K_j^{-1}\right]^{\frac{1}{b_n}}.$$

For fixed K_j, as $b_n \to 0$,

$$\omega(K_j, b_n) \to \exp\left(-K_j^{-1}\right),$$

and so

$$\lambda(K_j, b_n) \to 1 - \exp\left(-K_j^{-1}\right).$$

(iii) For $\gamma = \log_{\frac{1}{2}} q < 1$, equation (1) implies that for fixed K_j,

$$\lim_{b_n \to 0} \frac{\omega(K_j, b_n)}{\exp\left(-\left(\frac{1}{b_n}\right)^{1-\gamma} K_j^{-\gamma}\right)} = 1.$$

Equivalently,

$$\omega(K_j, b_n) \sim \exp\left(-b_n^{\gamma-1} K_j^{-\gamma}\right) \to 0.$$

Discussion

The above theorems show that the parameter $\gamma = \log_{1/2} q > 0$ is critical in determining the behavior of the tail probabilities, $\lambda(K_j, b_n)$ or $\omega(K_j, b_n)$. For large values of K_j, Theorem 1 shows that $\lambda(K_j, b_n) \to 0$. In the "favorable" case of $\gamma > 1$ $(i.e.\ q < 1/2)$, $\lambda(K_j, b_n)$ decreases rapidly over K_j, and increases over b_n. On the other hand, in the "unfavorable" case of $\gamma < 1$ $(i.e.\ q > 1/2)$, $\lambda(K_j, b_n)$ decreases less rapidly over K_j, and decreases over b_n. In the special case of $\gamma = 1$ $(i.e.\ q = 1/2)$, we see that $\lambda(K_j, b_n) \sim \frac{1}{2K_j}$, which is unaffected by b_n.

For small values of b_n, Theorem 2 shows that $\lambda(K_j, b_n) \to 0$ only in the favorable case of $\gamma > 1$ $(i.e.\ q < 1/2)$, whereas $\lambda(K_j, b_n) \to 1$ $(\text{or } \omega(K_j, b_n) \to 0)$ in the unfavorable case of $\gamma < 1$ $(i.e.\ q > 1/2)$. In the special case of $\gamma = 1$ $(i.e.\ q = 1/2)$, $\lambda(K_j, b_n) \to 1 - \exp\left(-K_j^{-\gamma}\right)$. See Table 7.

Table 7. Winning Certainty as a Function of Betting Size

m	$p = 0.75$	$p = 0.51$	$p = 0.50$	$p = 0.49$	$p = 0.25$
1	0.9961	0.9424	0.9375	0.9323	0.6836
2	0.9980	0.9443	0.9385	0.9322	0.5817
4	0.9990	0.9458	0.9389	0.9315	0.4566
8	0.9995	0.9470	0.9392	0.9304	0.3178
16	0.9998	0.9481	0.9393	0.9292	0.1849
32	0.9999	0.9492	0.9394	0.9279	0.0823

Size of bet = $1/m$; $K = 16$

Ceteris paribus, one can say that for favorable Bernoulli trials, the speculator should select as small a value of b_n as possible, whereas for unfavorable Bernoulli trials, the speculator should select as large a value of b_n as possible.

2.4 Expected Profits with Limited Liability

The above equations assume that the speculator will be financially able to pay total losses in the event a win does not occur. Let's now suppose that the speculator is unable to pay losses in excess of total net worth. In the event that losses exceed initial wealth, the speculator will either seek help from an outside organization (i.e. parent company, government agency) or declare bankruptcy. The speculator will not have the incentive to stop the game once losses exceed assets, so the game will end at the earlier of two events, a win occurs or credit is insufficient to make another bet.

Case I (Initial Bet = 1, Objective = 1):

Since the rules of the game are unchanged, the expected time in which the game will end will also remain unchanged. However expected profits may differ substantially dependent upon the cap on total losses.

$\tau = Max\{T\} = \lfloor \log_2 (W + C + 1) \rfloor$

$$E[T] = \frac{1 - q^{\tau}}{1 - q} \text{ from above.}$$

$$E[Gain] = E[Gain|\text{win}](1 - q^{\tau}) + E[Gain|\text{loss}]q^{\tau},$$

where

$$E[Gain|\text{win}] = 1$$

and

$$E[Gain|\text{loss}] = -(2^{\tau} - 1 - C).$$

Thus,

$$E[Gain] = 1 - q^{\tau} - (2^{\tau} - 1 - C)q^{\tau}.$$

In the special case where, $\log_2 (W + C + 1) \in Z$,

$$E[Gain] = 1 - q^{\tau}(1 + W) > 0$$

when

$$C > (1 + W)^{1/\gamma} - W - 1 \qquad (2)$$

where

$$\gamma = \log_{1/2} q > 0.$$

Case II (Initial Bet = 1/m, Objective = 1):

$\tau = Max\{T\} = \sum_{i=1}^{m} \lfloor \log_2 (m(W+C) + (i-1) + 1) \rfloor = \sum_{i=1}^{m} \tau_i$ and

$$E[T] = \sum_{i=1}^{m} \left[\left(\frac{1}{1-q} \right) - \left(\frac{\tau_i q^{\tau_i}}{1-q^{\tau_i}} \right) \right] \cdot \prod_{i=1}^{m} \left(1 - q^{\tau_i}\right)$$

$$+ \sum_{j=1}^{m} \left(\left\{ \sum_{i=1}^{j-1} \left[\left(\frac{1}{1-q} \right) - \left(\frac{\tau_i q^{\tau_i}}{1-q^{\tau_i}} \right) \right] + \tau_j \right\} \cdot q^{\tau_j} \prod_{i=1}^{j-1} \left(1 - q^{\tau_i}\right) \right)$$

from above.

$$E[Gain] = E[Gain|\text{win}] \cdot \prod_{i=1}^{m} \left(1 - q^{\tau_i}\right)$$

$$+ \sum_{j=1}^{m} \left\{ E[Gain|\text{loss during sub-series } j] \cdot q^{\tau_j} \prod_{i=1}^{j-1} \left(1 - q^{\tau_i}\right) \right\}$$

where

$$E[Gain|\text{win}] = 1$$

and

$$E[Gain|\text{loss during sub-series } j] = -\left(\frac{2^{\tau_j} - 1}{m} \right) + \left(\frac{j-1}{m} \right) + C = -\left(\frac{2^{\tau_j} - j - mC}{m} \right).$$

Thus,

$$E[Gain] = (1) \cdot \prod_{i=1}^{m} \left(1 - q^{\tau_i}\right) - \sum_{j=1}^{m} \left[\left(\frac{2^{\tau_j} - j - mC}{m} \right) q^{\tau_j} \prod_{\mathrm{i}=1}^{j-1} \left(1 - q^{\tau_i}\right) \right].$$

In the special case where, $\log_2 [m(W+C) + (j-1) + 1] \in \mathrm{Z}$,

$$E[Gain] = \prod_{i=1}^{m} \left(1 - q^{\tau_i}\right) - \sum_{j=1}^{m} \left[W q^{\tau_j} \prod_{i=1}^{j-1} \left(1 - q^{\tau_i}\right) \right] > 0$$

when

$$W < \frac{\prod_{i=1}^{m}\left(1-q^{\tau_i}\right)}{\sum_{j=1}^{m} q^{\tau_j}\prod_{i=1}^{j-1}\left(1-q^{\tau_i}\right)}.$$

Expected Gains and Size of Bet

For any initial funds, $K_s = 2^s;\ s \in \{0,1,2,3,\ldots\}$ where $K = W + C$, initial bet

$b_n = \frac{1}{2^n};\ n \in \{0,1,2,3,\ldots\}$, and Bernoulli probability $p = 1-q \in (0,1)$, it follows that for

fixed K_s, as $b_n \to \infty$, $E[Gain] \to -(K_s - C) < 0$

Proof:

$$E[Gain] = \prod_{i=1}^{2^n}\left(1-q^{n+s}\right) - \sum_{j=1}^{m}\left[\left(\frac{2^{n+s}-j-mC}{m}\right) q^{n+s}\prod_{i=1}^{j-1}\left(1-q^{n+s}\right)\right]$$

$$= (1-q^{n+s})^{2^n} - \sum_{j=1}^{2^n}\left[\left(\frac{2^{n+s}-j-2^n C}{2^n}\right) q^{n+s}(1-q^{n+s})^{j-1}\right]$$

$$= (1-q^{n+s})^{2^n} - \left(2^s - C\right) q^{n+s}\sum_{j=1}^{2^n}\left[\left(1-q^{n+s}\right)^{j-1}\right] + \frac{q^{n+s}}{2^n}\sum_{j=1}^{2^n} j\left(1-q^{n+s}\right)^{j-1}$$

$$= \left(1-q^{n+s}\right)^{2^n} - \left(2^s - C\right) q^{n+s}\left[\frac{1-\left(1-q^{n+s}\right)^{2^n}}{1-\left(1-q^{n+s}\right)}\right]$$

$$+\left(\frac{q^{n+s}}{2^n}\right)\left\{\frac{1-\left(1-q^{n+s}\right)^{2^n+1}}{\left[1-\left(1-q^{n+s}\right)\right]^2} - \frac{\left(2^{\mathrm{n}}+1\right)\left(1-q^{n+s}\right)^{2^n}}{1-\left(1-q^{n+s}\right)}\right\}$$

$$= \left(1 - q^{n+s}\right)^{2^n} - \left(2^s - C\right)\left[1 - \left(1 - q^{n+s}\right)^{2^n}\right]$$

$$+ \left\{ \frac{1 - \left[\left(1 - q^{n+s}\right)^{2^n}\left(1 - q^{n+s}\right)\right]}{2^n q^{n+s}} - \frac{\left(1 - q^{n+s}\right)^{2^n}}{2^n} - \left(1 - q^{n+s}\right)^{2^n} \right\}$$

$$= -\left(2^s - C\right)\left[1 - \left(1 - q^{n+s}\right)^{2^n}\right] + \left[\frac{1 - \left(1 - q^{n+s}\right)^{2^n}}{2^n q^{n+s}} + \frac{\left(1 - q^{n+s}\right)^{2^n} q^{n+s}}{2^n q^{n+s}} - \frac{\left(1 - q^{n+s}\right)^{2^n}}{2^n}\right]$$

$$= -\left(2^s - C\right)\left[1 - \left(1 - q^{n+s}\right)^{2^n}\right] + \left[\frac{1 - \left(1 - q^{n+s}\right)^{2^n}}{2^n q^{n+s}}\right]$$

$$= \left[-\left(K_s - C\right) + b_n^{1-\gamma} K_s^{-\gamma}\right]\left\{1 - \left[1 - \left(\frac{1}{b_n}\right)^{-\gamma} K_s^{-\gamma}\right]^{\frac{1}{b_n}}\right\}.$$

Thus, for fixed K_s, as $b_n \to 0$,

$$E\left[Gain\right] \to -\left(K_s - C\right) < 0$$

Discussion:

Section III above shows the behavior of the tail probabilities $\lambda\left(K_j, b_n\right)$ and $\omega\left(K_j, b_n\right)$ and the effects of the parameter: $\gamma = \log_{1/2} q > 0$. While these tail probabilities can be manipulated by the size of the bet, expected gains remain negative when the speculator is faced with unfavorable odds $\left(i.e.\ q > 1/2\right)$. By capping potential loss and thus limiting liability, the speculator increases his chances of obtaining positive profits regardless of p.

Refer to equation (2) above. The right hand side of the equation, $(1+W)^{1/\gamma} - W - 1$ is negative when $\gamma \geq 1$ (i.e. $q \leq 1/2$), and expected gains are positive for any positive credit line, C. In the event that $0 < \gamma < 1$ $(q > 1/2)$, positive profits can be obtained when the credit line, C, is large with respect to initial wealth.

By decreasing the size of the bet, positive expected gains become less probable. In the limiting case where $2^n \to \infty$ $(b_n \to 0)$, expected gains decrease to the loss of initial wealth, $W = K_s - C$.

2.5 Cost of Obtaining Credit

Revised Model

A speculator with initial wealth, $W > 0$, obtains an initial credit line of $C > 0$ and agrees to pay to the creditor an amount of $C(1+r)^t$ at the end of t periods, where r is equal to the required rate of interest on the loan. The speculator seeks a return $b \in (0,1]$ units (net of interest) by betting on a series of i.i.d. Bernoulli trials each with probability of success, $p = 1 - q \in (0,1)$. The speculator places an initial bet b on the first trial, and then doubles the size of the bet on each successive trial. The speculator discontinues the series of trials at the earlier of three events (see Figure 3):

(1) he is successful on a given trial, thereby winning b units (net);

(2) his current credit level, including initial wealth, is inadequate; or

(3) amount of interest owed on the loan exceeds 1. The amount of interest, $C\left[(1+r)^t - 1\right]$ paid on the loan varies directly with the number of trials.

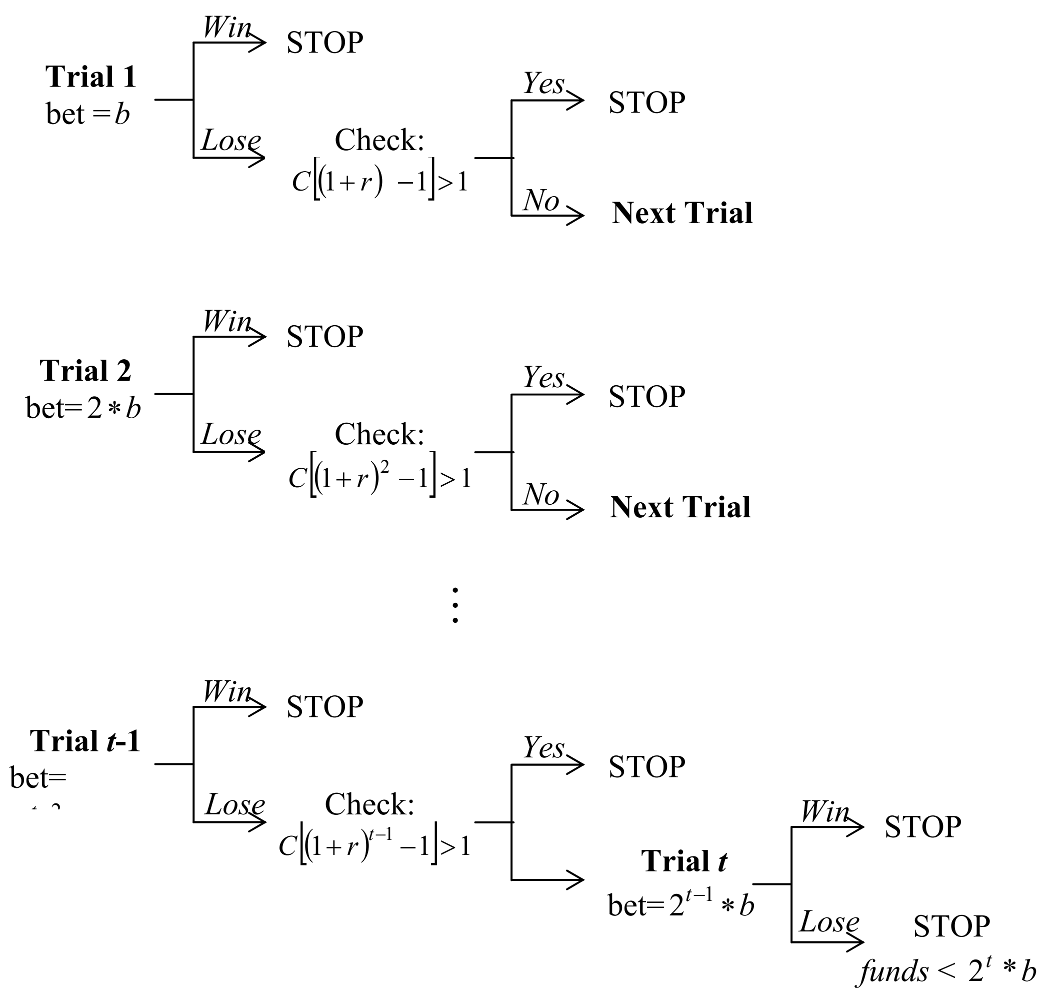

Figure 3: Possible Outcomes and Stopping Times; Including Cost of Credit

Definitions:

Let $X \sim f_G(x) = \Pr\left(1^{\text{st}} \text{ success occurs on trial } x\right) = pq^{x-1}$, for $x = 1, 2, 3, \ldots$

$$\text{Let } T = \begin{cases} X \text{ if } X = 1,2,\ldots,\min\left\{\left\lfloor \log_2\left(\dfrac{W+C}{b}+1\right)\right\rfloor, \left\lfloor \dfrac{\log_2(1+C)-\log_2 C}{\log_2(1+r)} \right\rfloor\right\} \\ \\ \min\left\{\left\lfloor \log_2\left(\dfrac{W+C}{b}+1\right)\right\rfloor, \left\lfloor \dfrac{\log_2(1+C)-\log_2 C}{\log_2(1+r)} \right\rfloor\right\} \\ \qquad \text{if } X > \min\left\{\left\lfloor \log_2\left(\dfrac{W+C}{b}+1\right)\right\rfloor, \left\lfloor \dfrac{\log_2(1+C)-\log_2 C}{\log_2(1+r)} \right\rfloor\right\} \end{cases}$$

denote the trial on which the game ends.

$$\tau = Max\{T\} = \min\left\{\left\lfloor \log_2\left(\frac{W+C}{b}+1\right)\right\rfloor, \left\lfloor \frac{\log_2(1+C)-\log_2 C}{\log_2(1+r)} \right\rfloor\right\}$$

$$\omega(W,C,b) = 1 - \lambda(W,C,b)$$

$$= \Pr\left(\text{net gain} = b \text{ before funds are inadequate} \;\middle|\; \begin{array}{l}\text{initial wealth} = \text{W},\ \text{credit line} = C, \\ \text{initial bet} = b\end{array}\right)$$

$$= \Pr\left(1^{\text{st}} \text{ success occurs on or before trial } \tau\right)$$

$$= \sum_{x=1}^{\tau} f_G(x) = 1 - q^{\tau}.$$

$$E[Gain] = E[Gain|\text{win}](1-q^{\tau}) + E[Gain|\text{loss}]q^{\tau},$$

where

$$E[Gain|\text{win}] = 1 - C\left[(1+r)^{\tau} - 1\right]$$

and

$$E\left[Gain \middle| \text{loss}\right] = -\left\{2^{\tau} - 1 + C\left[(1+r)^{\tau} - 1\right]\right\}.$$

Thus,

$$E\left[Gain\right] = 1 - (2q)^{\tau} - C\left[(1+r)^{\tau} - 1\right].$$

$$E[Gain] > 0$$

when

$$q < \frac{\left[1 - C(1+r)^{\tau} + C\right]^{1/\tau}}{2}$$

for $r < \left(\frac{1+C}{C}\right)^{1/\tau} - 1$.

Cost of Credit with Limited Liability

$$E\left[Gain \middle| \text{win}\right] = 1 - C\left[(1+r)^{\tau} - 1\right]$$

and

$$E\left[Gain \middle| \text{loss}\right] = -\left(2^{\tau} - 1 - C\right).$$

Thus,

$$E\left[Gain\right] = \left\{1 - C\left[(1+r)^{\tau} - 1\right]\right\}\left(1 - q^{\tau}\right) - \left(2^{\tau} - 1 - C\right)q^{\tau}.$$

$$E\left[Gain\right] = 1 - C(1+r)^{\tau} + C - q^{\tau} + C(1+r)^{\tau} q^{\tau} - Cq^{\tau} - 2^{\tau} q^{\tau} + q^{\tau} + Cq^{\tau}$$

$$E\left[Gain\right] = 1 - C(1+r)^{\tau} + C + C(1+r)^{\tau} q^{\tau} - 2^{\tau} q^{\tau}.$$

$$E[Gain] > 0$$

when

$$q < \left[\frac{1 - C(1+r)^{\tau} + C}{2^{\tau} - C(1+r)^{\tau}} \right]^{1/\tau}$$

for $r < \left(\frac{1+C}{C} \right)^{1/\tau}$.

Discussion

With bet doubling, the probability of winning increases with the amount of available funds the player has at the beginning of the game. This is due to the fact that ample amounts of credit enables the spectator to stay the in the game longer which increases their chances of winning. When there is a cost of obtaining credit, the game changes. The number of trials starts to work against the speculator (i.e. the longer the speculator stays in the game the more interest is owed on the loan). Due to the amount of interest, positive expected gains are less probable and can only be obtained when the probability of failure on any one trial is minimal. $\left(i.e.\ q < \frac{1}{2}\left[1 - C(1+r)^{\tau} + C\right]^{\frac{1}{\tau}} \right)$.

With limited liability, $E[Gain \mid loss]$ is limited to the loss of initial wealth making positive expected gains more probable. In section IV, we saw that positive expected gains are possible when C is large with respect to initial wealth, W, regardless of p. However, when we add in the cost of credit, $E[Gain \mid win]$ decreases by the amount of interest owed on the loan. Expected gains are dependent upon the relationship between,

p the probability of winning, and r, the required rate of interest on the loan. Positive profits can be obtained for $q < \left\{\left[1 - C(1+r)^{\tau} + C\right] / \left[2^{\tau} - C(1+r)^{\tau}\right]\right\}^{1/\tau}$.

2.6 Conclusions

Gamblers adopt strategies to increase their chances of coming home a winner. In this research, a bet doubling strategy was used to increase a speculator's chances of achieving a specified monetary goal. While bet doubling maximizes the probability of winning, it could also lead to obsessive losses and negative expected gains when the speculator is faced with less than favorable odds $(p < q)$. By limiting liability, total losses will be restricted and the speculator can expect positive gains even while facing a losing proposition. In this research it was also shown that the size of the bet is dependent upon the speculator's chance of winning on a given trial, and splitting the bet should only be considered when $p > 1/2$. The cost of obtaining credit should also be considered when analyzing expected gains and it is suggested that the gambler should not engage in such a strategy when the cost of obtaining credit is high relative to the probability of winning.

APPENDIX

Lemma 1: If $\gamma > 1$ and $K_j^{-\gamma} \in \Re^+$ are fixed as $b_n \to 0$, then

$$\left[1-\left(\frac{1}{b_n}\right)^{-\gamma} K_j^{-\gamma}\right]^{\frac{1}{b_n}} = \exp\left(-b_n^{\gamma-1} K_j^{-\gamma}\right) + O\left(b_n^{2\gamma-1} \exp\left(-b_n^{\gamma-1} K_j^{-\gamma}\right)\right).$$

Proof: Let η be an odd positive integer and let $\alpha \in \Re$. Then

$$\left(1-\frac{\alpha}{\eta}\right)^{\eta} = \sum_{i=0}^{\eta} (-1)^i \binom{\eta}{i} \left(\frac{\alpha}{\eta}\right)^i = \sum_{i=0}^{\frac{\eta-1}{2}} (-1)^i \binom{\eta}{i} \left(\frac{\alpha}{\eta}\right)^i + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\eta\left(\frac{\eta-1}{2}\right)!}\right)$$

and

$$\exp(-\alpha) = \sum_{i=0}^{\infty} (-1)^i \left(\frac{\alpha^i}{i!}\right) = \sum_{i=0}^{\frac{\eta-1}{2}} (-1)^i \left(\frac{\alpha^i}{i!}\right) + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\left(\frac{\eta+1}{2}\right)!}\right),$$

from which it follows that

$$\left(1-\frac{\alpha}{\eta}\right)^{\eta} - \exp(-\alpha) = \sum_{i=0}^{\frac{\eta-1}{2}} (-1)^i \left[\binom{\eta}{i}\left(\frac{\alpha}{\eta}\right)^i - \frac{\alpha^i}{i!}\right] + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\eta\left(\frac{\eta-1}{2}\right)!}\right) + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\left(\frac{\eta+1}{2}\right)!}\right)$$

$$= \sum_{i=0}^{\frac{\eta-1}{2}} (-1)^i \frac{\alpha^i}{i!}\left[\frac{\eta!}{(\eta-i)!\eta^i} - 1\right] + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\left(\frac{\eta+1}{2}\right)!}\right)$$

$$= \alpha^2 \sum_{i=2}^{\frac{\eta-1}{2}} (-1)^{i-2} \frac{\alpha^{i-2}}{i!} \left[\frac{\eta!}{(\eta-i)!\eta^i} - 1 \right] + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\left(\frac{\eta+1}{2}\right)!} \right)$$

$$= \alpha^2 \sum_{i=2}^{\frac{\eta-1}{2}} (-1)^{i-2} \frac{\alpha^{i-2}}{i!} \left[-\frac{i(i-1)}{\eta} + O\left(\frac{1}{\eta^2}\right) \right] + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\left(\frac{\eta+1}{2}\right)!} \right)$$

$$= \frac{\alpha^2}{\eta} \sum_{i=2}^{\frac{\eta-1}{2}} (-1)^{i-2} \frac{\alpha^{i-2}}{(i-2)!} \left[-1 + O\left(\frac{1}{\eta}\right) \right] + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\left(\frac{\eta+1}{2}\right)!} \right)$$

$$= \frac{\alpha^2}{\eta} \left[-1 + O\left(\frac{1}{\eta}\right) \right] \left[\exp(-\alpha) + O\left(\frac{\alpha^{\frac{\eta-3}{2}}}{\left(\frac{\eta-3}{2}\right)!} \right) \right] + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\left(\frac{\eta+1}{2}\right)!} \right)$$

$$= -\frac{\alpha^2}{\eta} \left[\exp(-\alpha) + O\left(\frac{\alpha^{\frac{\eta-3}{2}}}{\left(\frac{\eta-3}{2}\right)!} \right) \right] + \frac{\alpha^2}{\eta} O\left(\frac{1}{\eta}\right) \left[\exp(-\alpha) + O\left(\frac{\alpha^{\frac{\eta-3}{2}}}{\left(\frac{\eta-3}{2}\right)!} \right) \right] + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\left(\frac{\eta+1}{2}\right)!} \right)$$

$$= -\frac{\alpha^2}{\eta} \exp(-\alpha) + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\eta\left(\frac{\eta-3}{2}\right)!} \right) + O\left(\frac{\alpha^2}{\eta^2} \exp(-\alpha) \right) + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\eta^2\left(\frac{\eta-3}{2}\right)!} \right) + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\left(\frac{\eta+1}{2}\right)!} \right)$$

$$= O\left(\frac{\alpha^2}{\eta} \exp(-\alpha) \right) + O\left(\frac{\alpha^{\frac{\eta+1}{2}}}{\eta\left(\frac{\eta-3}{2}\right)!} \right)$$

$$= O\left(\frac{\alpha^2}{\eta}\exp(-\alpha)\right) + O\left(\frac{\alpha^{\frac{\eta+1}{2}} e^{\frac{\eta-3}{2}}}{\eta\sqrt{2\pi}\left(\frac{\eta-3}{2}\right)^{\frac{\eta-2}{2}}}\right)$$

$$= O\left(\frac{\alpha^2}{\eta}\exp(-\alpha)\right) + O\left(\alpha^2 \frac{(2e\alpha)^{\frac{\eta-3}{2}}}{\eta^\eta}\right),$$

where the penultimate step is given by Stirling's approximation.

In short,

$$\left(1-\frac{\alpha}{\eta}\right)^\eta = \exp(-\alpha) + O\left(\frac{\alpha^2}{\eta}\exp(-\alpha)\right) + O\left(\alpha^2 \frac{(2e\alpha)^{\frac{\eta-3}{2}}}{\eta^\eta}\right).$$

Now, by setting $\eta = \frac{1}{b_n}$ and $\alpha = \left(\frac{1}{b_n}\right)^{1-\gamma} K_j^{-\gamma}$, we find that

$$\left[1-\left(\frac{1}{b_n}\right)^{-\gamma} K_j^{-\gamma}\right]^{\frac{1}{b_n}} = \exp\left(-b_n^{\gamma-1}K_j^{-\gamma}\right) + O\left(\left(K_j^{-\gamma}\right)^2 b_n^{2\gamma-1}\exp\left(-b_n^{\gamma-1}K_j^{-\gamma}\right)\right)$$

$$+ O\left(\left(K_j^{-\gamma}\right)^2 \left(2eK_j^{-\gamma}\right)^{\frac{1}{2}\left(b_n^{-1}-3\right)} b_n^{\frac{1}{2}\left[b_n^{-1}(1+\gamma)+\gamma-1\right]}\right)$$

$$= \exp\left(-b_n^{\gamma-1}K_j^{-\gamma}\right) + O\left(b_n^{2\gamma-1}\exp\left(-b_n^{\gamma-1}K_j^{-\gamma}\right)\right) + O\left(\left(2eK_j^{-\gamma}\right)^{\frac{1}{2}\left(b_n^{-1}\right)} b_n^{\left(\frac{1+\gamma}{2}\right)\left(b_n^{-1}\right)} b_n^{\frac{\gamma-1}{2}}\right)$$

$$= \exp\left(-b_n^{\gamma-1}K_j^{-\gamma}\right) + O\left(b_n^{2\gamma-1}\exp\left(-b_n^{\gamma-1}K_j^{-\gamma}\right)\right).$$

Chapter 3

HAZARDOUS IMMORALITY:

STRATEGIC EXTERNALIZATION OF RISK AND CREDIT PRICING

3.1 Introduction

When an organization undertakes a venture relying on an incomplete contract which forces other parties to bear significant losses without their consent, that part of the risk is said to be externalized. Insurance based on the presumption that not all losses would be covered (i.e., payment of only “ordinary” and not “extreme” events) is underpriced relative to an actuarially fair premium and thus would routinely generate underwriting profits even though its expected long-run financial position was neutral or even negative. Since the term “moral hazard” most specifically refers to incentive distortions created within a contract, a slightly different term, “hazardously immoral,” is here used to refer to similar distortions created externally by contractual incompleteness that force extreme losses to be borne by unwitting and unpaid outsiders.

This chapter proceeds as follows. In Section II, hazardous contracts are briefly reviewed, and the use of contracts to concentrate risks is described. In section III, a strategy to concentrate risk is presented and a more formal and general model is analyzed to provide some results regarding the size of contracts, probability of loss, and expectations of profit. In Section IV, the model is extended to include limiting liability and the associated benefits. Section V discusses the cost of obtaining credit, and Section VI concludes.

3.2 Hazardous Contracts and Concentration of Risks

Traditional insurance contracts pool risks, to obtain gains from trade. Uncertain and potentially disruptive large losses are exchanged in the market for more certain losses with smaller bounds. Such contracts achieve two ends: 1) they manage risk, and 2) they make the price of uncertain and infrequent occurrences more visible, thus making current financial statements more clearly reflective of expected future income. For example, the purchase of flood insurance for construction in a flood plain makes property owners set aside funds for damage repair and engage in risk reduction, and it makes their current income statement reflect the true cost of property in this area. Without flood insurance, the implied rental rate is too cheap, and appears to provide excess profits that cannot realistically be expected over the long run.

Hazardous contracts do the opposite, concentrating risks and reducing visibility. Myriad small losses are contractually linked to create a bundle where all are more likely to fail simultaneously. Such contracts obscure the expected cost of failure by not taking routine charges for predictable, albeit uncertain, future losses. Just as moral hazard provides incentives for insured firms to take too many risks, the false promise of safe profits arising from hazardously immoral contracts provides a zone of ignorance that lures investors into taking too many risks. The promise of guaranteed profits is false. In the end, failure is much worse. Indeed, the strategy relies on making it worse, so much worse that none of the principals can be held fully responsible.

There are, of course, a number of legitimate reasons for crafting contracts that concentrate risk. Venture capital financing, by its use of preferred debt, typically increases the leverage and risk of start-up firms. Such intensification of risk is intentional and efficient. Without it, the funds provided by the venture firm would create moral hazard on the part of owners, who would use the venture funds for salaries, office amenities and retirement packages rather than product development. Similarly, large pharmaceutical firms spin off research biotech entities to usefully concentrate risks and create high-powered incentives for the principals to "succeed or die trying." The capital markets provide a cost-effective method to diversify the risks to investors, and stock in the large firm is not sufficiently responsive to the efforts of the researchers to reward successful innovation.

A contract may be said to have created a "toxic" risk when it enables a concentration of losses sufficient to destroy the organization that entered into the contract and to significantly harm other organizations. As risk becomes more concentrated, informational asymmetry between management and stockholders, and between management and creditors, becomes worse. If a risk becomes bad enough, it is worth hiding. With heavy losses in prospect, the transparency required for accurate assessment of credit risk and pricing is likely to be compromised. Sometimes, what appears superficially to be a risk management strategy that breaks a large risk into many parts, may actually work to hide, rather than spread, the risk. For example, a firm obtaining loans from many different creditors who are not fully aware of the extent of aggregate leverage has the effect of hiding the systematic risks that links all of the loans together--and not incidentally allows the firm to access a larger amount of credit at lower prices. The examination of the credit worthiness of a thousand mortgage applicants may focus on the details, rather than the systemic risk that links them together (earthquake, interest rates, the use of a single construction firm). In section III, a model of a strategy to concentrate risks is presented and credit requirements for obtaining positive profits are discussed.

3.3 Model of a Strategy to Concentrate Risk

Consider a highly simplified example, in which a firm wishes to obtain \$1m by investing \$1m in a business venture with a probability of positive profits equal to p. In this example the business venture is represented by a bernoulli trial with probability of success, p equal to 51%. Since gains are expected, it is profitable to make such an investment, and to do so repeatedly. Yet it may sometimes be necessary to withstand a string of losses in order to reap the expected benefits. Thus some form of insurance, equity risk capital, or credit, will be required.

By obtaining ample amounts of credit, and playing the game long enough, an investor can turn a risky venture into one with a high probability of winning. One strategy to provide certainty with regard to profit would be to double the investment on each successive trial until a win occurred. With two runs, the probability of winning rises to .7599, with four runs to .9424, and with ten runs to .9992--less than 1/1000 chance that the firm will not reach its profit goal. Obtaining greater certainty in this manner requires credit, potentially quite large amounts of credit. In order to have the capacity to withstand losses and still make four runs requires almost 2^4 = \$16m, for ten runs \$1024m. Yet profits in the venture become a “near certainty” if credit can be obtained. If obtaining \$1 billion in credit is prohibitively difficult or costly, the strategy could be modified slightly by making smaller bets, perhaps of \$1,000 or even \$10, but carrying out the doubling-down of bets for a sufficient number of winning cycles so that successive small wins add up to \$1m. Consider the following:

Let

$W =$ initial wealth

$C =$ credit line

$K = W + C$ = amount of available funds

b = size of bet

t = number of trials

p = probability of winning on each trial

$\omega(W, C, b)$ = certainty of winning sequence.

$\lambda(W, C, b) = 1 - \omega(W, C, b)$ = probability of not reaching initial objective

The total amount of funds available to play the game is equal to the sum of initial wealth and available credit. The amount of funds required is independent of the probability of winning and rises exponentially with the number of tosses.

K (avalable funds) $= b * (2^t - 1)$

Note that in this formulation, the potential loss is slightly less $(K - b)$ than total funds available, K. That is, for 4 turns, the firm will potentially lose \$1m on the first trial, \$2m on the 2nd, \$4m on the third, and \$8m on the fourth, for a total loss of \$15m $(2^4 - 1)$.

The probability of winning is the product of not losing $(1 - p)$ on each turn, and rapidly approaches certainty as *t* increases (see Table 8).

Table 8. Winning Certainty and Expected Gains as a Function of p

# of trials	$p = 0.51$		$p = 0.50$		$p = 0.49$	
	Probability of winning	Expected Gains	Probability of winning	Expected Gains	Probability of winning	Expected Gains
1	0.5100	20,000	0.5000	-	0.4900	-20,000
2	0.7599	39,600	0.7500	-	0.7399	-40,400
5	0.9718	96,079	0.9688	-	0.9655	-104,081
10	0.9992	182,927	0.9990	-	0.9988	-218,994
20	0.9999	332,392	0.9999	-	0.9999	-485,947

A few points are worth noting in this table. First, while there is a high degree of certainty (.9992 for $t = 10$) with such a strategy, the expected value is actually rather small ($180 thousand) relative to the amount that must be put at risk ($1 billion). Second, expected gains are not symmetric about $p = 0.50$. This is due to the relationship between expected gains and q, that is $E[Gains] = 1 - (2q)^{\tau}$, as shown in chapter 2. The final point worth noting from this table is that it is quite possible to create a "winner" (likelihood greater than 50%) even with a losing proposition.[2] Indeed, for a firm that can access $1 billion in available funds and pursues this strategy, but with a business venture that is just as unfavorable as the initial proposition was favorable (i.e. $p = 49\%$ rather than 51%), the difference in the likelihood of winning is almost vanishingly small, (.9988 versus .9992).

[2] Unlike Parrondo's Paradox, where a random alternation of losing games aggregates to a winning strategy (see Paulos, 2003, pp.52-54), "winning" here does not imply positive expected value. However, the strategy outlined above does have the advantage of being easy to implement, and indeed has been profitably practiced by many entrepreneurs throughout history.

That is, if type I firms had 51% businesses with expected profits, and type II firms had 49% businesses with expected profits, but both followed the same risk concentration strategy, then financial analysts reviewing the results of a thousand of each would have only a 0.4 percent likelihood of finding that the II's performed worse than the I's. Even for a major difference in expected outcomes: firms that win $2/3$ of the time versus firms that lose $2/3$ of the time, the difference is just .9999 versus .9827 -- so such big losers could look like winners 98% of the time (well above the usual confidence limit).

Discussion of Credit Requirements and Size of Bet

With ample amounts of credit, the firm is able to stay in the game longer, increasing their chances of winning (see Table 8). However, when faced with unfavorable odds, $(i.e.\ p < 0.50)$, the firm will incur negative expected profits which decrease with the number of trials. This is due to the firm making and possibly losing larger and larger bets. The large amounts of available funds will allow the firm to win with a great deal of certainty. However, in the unlikely event that the firm loses, the loss could be catastrophic.

Consider a slightly modified strategy in which the investor places an initial bet, $b = 1/m$ and doubles the bet until a win of $1/m$ occurs. The investor repeats the series of bets m times, and if successful, reaches his initial objective of $1m (note this strategy can be modified for objectives different than $1m). By placing smaller bets, the investor can stretch his available funds over a longer period of time, thereby increasing the number of trials. However, the probability of winning in the long run, with such a strategy, can differ substantially dependent upon the value of p. The following diagram shows changes in winning certainty as m increases for each of the given levels of p, where the size of the initial bet, b, is equal to $1/m$. The amount of available funds in the given diagram is equal to 16 million and the number of runs is inversely related to the size of the bet.

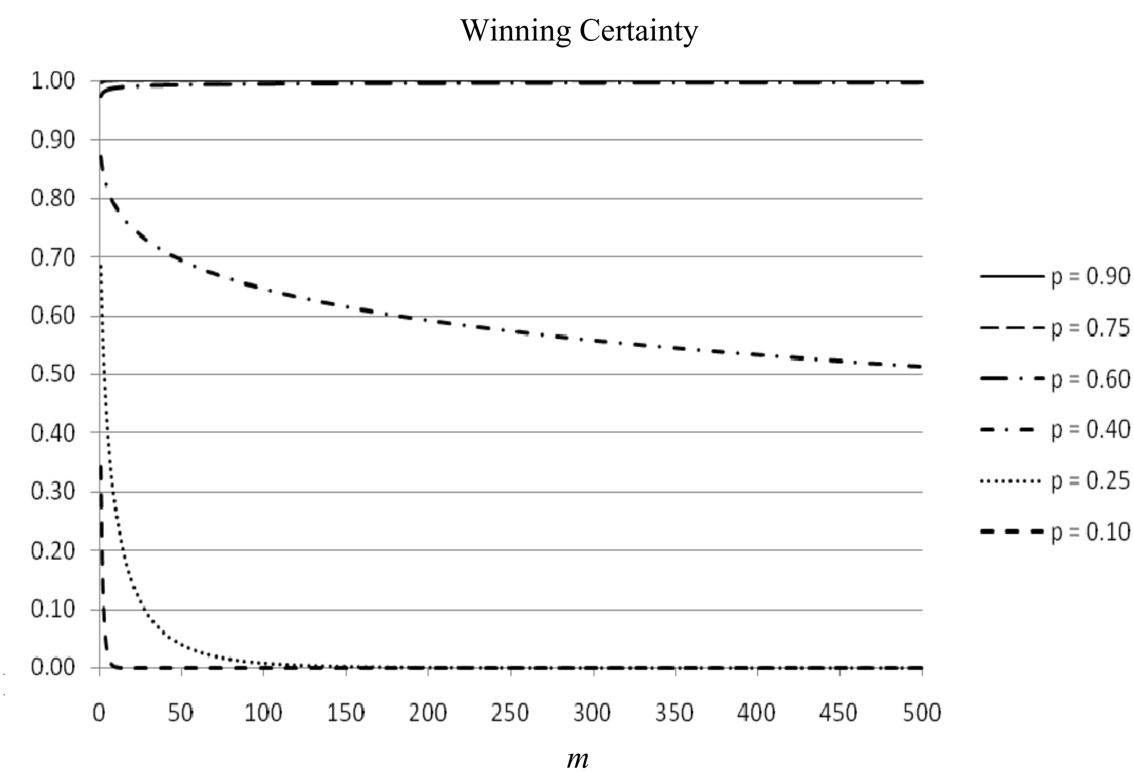

Figure 4. Tail Probabilities as a Function of Betting Size

Figure 4 shows convergence of $\omega(W, C, b)$ in which the limiting value is dependent upon the value of p. In the favorable cases $(p > 0.50)$, for fixed K, as m gets larger, the probability of winning approaches one. That is, when the investor is faced with favorable odds, it is beneficial for him to make bets as small as possible. In the unfavorable cases $(p < 0.50)$, reducing the initial bet size incurs a penalty; and the joint likelihood of success falls as the number of runs increases. With a greater chance of failing on a given trial, it is beneficial for the investor to place bets as large as possible, limiting the number of betting series required to obtain the initial objective. A mathematical explanation of the divergent behavior is outlined in the following theorems. The details are omitted here, but the proofs can be found in chapter 2.

Theorem 1: For any initial funds, $K_j = 2^j$; $j \in \{0,1,2,3,\ldots\}$, (where $K = W + C$), initial bet $b_n = \frac{1}{2^n}$; $n \in \{0,1,2,3,\ldots\}$, and Bernoulli probability $p = 1 - q \in (0,1)$, it follows that for fixed b_n, as $K_j \to \infty$, the probability of not reaching the initial objective, $\lambda(K_j, b_n) \sim b_n^{\gamma-1} K_j^{-\gamma} \to 0$, where $\gamma = \log_{1/2} q > 0$.

Theorem 2: For any initial funds, $K_j = 2^j$; $j \in \{0,1,2,3,\ldots\}$, initial bet $b_n = \frac{1}{2^n}$; $n \in \{0,1,2,3,\ldots\}$, and Bernoulli probability $p = 1 - q \in (0,1)$, it follows that for fixed K_j, as $b_n \to 0$,

(i) $\lambda(K_j, b_n) \sim b_n^{\gamma-1} K_j^{-\gamma} \to 0$ for $\gamma = \log_{1/2} q > 1$,

(ii) $\lambda(K_j, b_n) \to 1 - \exp(-K_j^{-\gamma})$ for $\gamma = \log_{1/2} q = 1$, and

(iii) $\omega(K_j, b_n) \sim \exp(-b_n^{\gamma-1} K_j^{-\gamma}) \to 0$ for $\gamma = \log_{1/2} q < 1$.

As outlined in the above theorems, the behavior of the tail probabilities, $\lambda(K_j, b_n)$ and $\omega(K_j, b_n)$ is dependent on the parameter $\gamma = \log_{1/2} q$, where $q = 1 - p$. In all cases, the probability of winning increases with K_j while holding the size of bet constant. However, when the investor is faced with unfavorable odds, $\gamma = \log_{1/2} q < 1$ and $p < 0.50$, $\omega(K_j, b_n)$ increases less rapidly and more runs are required to obtain a high degree of certainty. Also, in the unfavorable case, $\omega(K_j, b_n)$ rapidly decreases with the size of bet but decreases less rapidly for smaller values of q. In the special case where $\gamma = \log_{1/2} q = 1$ $(i.e.\ p = 0.5)$, the tail probabilities are not affected by the size of the bet.

While splitting the bet increases the ability to take advantage of favorable odds, the magnitude of the effects is not large. For example, with initial available funds of $16 million allowing for a maximum of 5 runs, and the initial bet set at $1, the likelihood of winning for an investment with favorable odds of 0.60 is .9744. Slitting the initial bet to $\$1/2$ to allow for a maximum of 6 tosses raises the joint probability of winning $\$1/2$ twice to .9796. Further reductions raise the probability, but again only modestly. A smaller bet of $\$1/50$ raises the probability to .9870 and a further reduction to one-hundredth, increases likelihood to .9896.

The effects of splitting the bet, are also sensitive to other parameters (see Tables 9 and 10). When the investor is faced with approximately fair odds, the effect of the betting size is minimal, however, when $p < 0.50$, the investor incurs a penalty for decreasing the size of the bet, which is reflected in expected gains.

Table 9. Winning Certainty as a Function of Bet Size; Favorable Odds

$b = 1/m$	$p = 0.51$		$p = 0.60$		$p = 0.75$	
	Winning Certainty	Expected Gains	Winning Certainty	Expected Gains	Winning Certainty	Expected Gains
1	0.9424	77,632	0.9744	590,400	0.9961	937,500
2	0.9443	94,722	0.9796	668,878	0.9980	968,277
5	0.9327	111,041	0.9797	731,836	0.9988	983,894
10	0.9342	127,922	0.9837	784,484	0.9994	991,915
50	0.9217	159,789	0.9870	860,245	0.9998	997,994
100	0.9233	175,887	0.9896	888,009	0.9999	998,976

Available funds, $K = 16$

Table 10. Winning Certainty as a Function of Bet Size; Unfavorable Odds

$b = 1/m$	p = 0.49		p = 0.40		p = 0.25	
	Winning Certainty	Expected Gains	Winning Certainty	Expected Gains	Winning Certainty	Expected Gains
1	0.9323	(82,432)	0.8704	(1,073,600)	0.6836	(4,062,500)
2	0.9322	(102,285)	0.8505	(1,430,454)	0.5817	(5,811,386)
5	0.9151	(121,800)	0.7875	(1,809,114)	0.3753	(7,293,789)
10	0.9138	(142,823)	0.7528	(2,280,915)	0.2387	(9,174,878)
50	0.8897	(184,341)	0.6026	(3,280,402)	0.0202	(9,772,284)
100	0.8877	(206,578)	0.5453	(3,904,503)	0.0030	(10,031,840)

Available funds, $K = 16$

3.4 Limited Liability and the Externalization of Risk

The models presented in section III assumes that the firm will be willing and financially able to pay total losses in the event a win does not occur. However, losses may be so significant that the firm may deny liability or seek help from an outside organization. In the event of a catastrophe, it is not unusual for the organization which was supposed to have borne the risk to claim that such a loss was too large to have ever been anticipated, and hence they should be excused from financial responsibility. The intuition behind the analysis of this paper is that excess profits accrue to an organization to the extent that it is paid for catastrophic risks that it would not, in the event, actually be able to cover. Indeed, it may even become a business strategy for a firm to concentrate risks rather than diversify in order to go bankrupt in the case of the "unanticipated" and “overwhelming” loss.

Revised Model: Limited Liability

Limited liability is an essential aspect of such a strategy. The firm can only lose, at most, the amount invested. All additional losses imposed by the firm are borne by someone else (disappointed customers, creditors, employees, government).

Let’s return to the previous model now supposing that the firm is unwilling or unable to pay losses in excess of total net worth. In the event that losses exceed initial wealth, the speculator will either seek help from an outside organization (i.e. parent company, government agency) or declare bankruptcy. Since the firm can depend on being “bailed out” once losses exceed total net worth, the firm will not have the incentive to end the game and cut their losses. Consider the following example, where the firm has initial wealth, $W = 500\, million$ and an initial credit line, $K = 524\, million$.

Table 11. Changes in Expected Profits with Limited Liability

p	Prob of Winning	Expected Profits Unlimited liability	Expected Profits Limited liability
75%	99.99%	999,023	999,522
51%	99.92%	182,927	600,241
49%	99.88%	(218,994)	403,597
33%	98.18%	(17,665,859)	(8,132,417)
25%	94.37%	(56,665,039)	(27,213,071)

In the case of unlimited liability, the firm can expect negative profits once the probability of winning, *p*, on a single trial decreases to less than fifty percent. However, by limiting liability, it is possible for the firm to experience positive profits even while facing unfavorable odds $(i.e.\ p < 0.50)$. In chapter 2 we found that positive profits can be obtained when the amount of available credit is large with respect to initial wealth; $K > (1+W)^{1/\gamma} - W - 1$ where $\gamma = \log_{1/2} q > 0$. This is illustrated in table 12.

As the ratio of initial wealth to credit line decreases, expected gains rise for all values of p . At some point, the firm can expect positive gains, regardless of p as long as initial wealth is small with respect to initial available funds. By concentrating risks and obtaining enough credit the firm can guarantee positive profits regardless of how risky the business venture.

Table 12. Changes in Expected Profits Due to Changes in Wealth

p	Prob of Winning	Expected Profits $C = 774$ m; W = 250 m	Expected Profits $C = 924$ m; W = 100 m
75%	99.99%	999,761	999,904
51%	99.92%	799,721	919,410
49%	99.88%	701,204	879,767
33%	98.18%	(3,575,323)	(841,066)
25%	94.37%	(13,134,692)	(4,687,665)

Available funds, $K = 1.024$ billion

3.5 Limitations of Bet Doubling

Cost of Obtaining Credit

As shown above, by using a bet-doubling strategy, an investor can obtain positive profits with a high degree of certainty even while facing less than favorable odds. However, the ability to withstand a string of losses is necessary and a sufficient amount of credit is required. When credit is costly, the number of trials starts to work against the investor and the strategy changes. While staying in the game as long as possible increases the investor's overall chances of winning, the longer the game is played, the more interest is owed on the loan. After some point (max t) the interest due will exceed the initial winning objective and the investor will stop the game. By limiting the number of trials the probability of obtaining the initial objective decreases substantially dependent upon r (here r is defined as the required rate of interest on the loan), see Table 13.

Table 13. Effects of Cost of Credit

Interest rate	Credit Line = 250		Credit Line = 500		Credit Line = 750	
	Max t	$\omega(W,C,b)$	Max t	$\omega(W,C,b)$	Max t	$\omega(W,C,b)$
0.005%	10	.9992	10	.9992	10	.9992
0.01	10	.9992	10	.9992	10	.9992
0.02	10	.9992	9	.9984	6	.9862
0.04	9	.9984	4	.9424	3	.8824
0.06	6	.9862	3	.8824	2	.7599
0.08	4	.9424	2	.7599	1	.5100
0.10	3	.8824	1	.5100	1	.5100
0.15	2	.7599	1	.5100	-	-
0.20	1	.5100	1	.5100	-	-

Assumptions: $p = 0.51$; $K = W + C = 1024$

With available funds equal to \$1.024 *billion*, the investor will be able to engage in a bet-doubling strategy and stay in the game for a maximum of 10 trials, giving them a 99.92% chance of reaching their initial objective of \$1 *million* (when $p = 0.51$). However, when the initial credit line is large, the amount of interest accrued for the ten rounds will exceed \$1 *million* and the investor will limit the number of runs. For example, when the initial credit line is 750 million, the investor will continue his strategy for a maximum of 6 runs when $r = 0.02\%$, and a maximum of 1 run, when $r = 0.10\%$. By limiting the number of runs, the probability of winning decreases and when $C\left[(1+r)^t - 1\right]$ is large, the investor will not invest in the business venture, see Table 14.

Recall from section III, that when the investor has 1.024 *billion* in available funds, the expected value of profits is approximately 180 *thousand* when $p = .51$ and zero, when $p = .50$. However, expected profits are decreased substantially once the cost of credit is considered. Due to the amount of interest, positive expected gains are less probable and can only be obtained when the probability of failure on any one trial is minimal with respect to $r \left(i.e.\ q < \frac{1}{2}\left[1 - C(1+r)^{\tau} + C\right]^{1/\tau} \right)$. See the following table.

Table 14. Expected Profits Including Cost of Credit

Interest rate (Max t)	*Expected Profits* $p = 0.51$	$p = 0.50$	$p = 0.49$
0.005% (10 trials)	(67,129.06)	(250,056.26)	(469,050.68)
0.01 (10 trials)	(317,297.87)	(500,225.06)	(719,219.48)
0.02 (9 trials)	(734,468.10)	(900,720.34)	(1,095,812.90)
0.04 (4 trials)	(722,848.29)	(800,480.13)	(882,912.29)
0.06 (3 trials)	(841,732.11)	(900,540.11)	(961,748.11)
0.08 (2 trials)	(760,720.00)	(800,320.00)	(840,720.00)
0.10 (1 trials)	(480,000.00)	(500,000.00)	(520,000.00)
0.15 (1 trials)	(730,000.00)	(750,000.00)	(770,000.00)
0.20 (1 trials)	(980,000.00)	(1,000,000.00)	(1,020,000.00)

Assumptions: $C = 500$
$K = W + C = 1024$

3.6 Conclusions and Implications

The main contribution of this paper is to demonstrate clearly the following proposition: showing that a prospect is profitable to a high degree of certainty--even 99.9%, does not mean that it has positive expected profits. The corollary lesson is that credit is valuable, and lots of credit obtainable at low rates is so valuable that anyone can come up with a strategy to use such cheap credit to make millions of dollars, even if their plan has negative expected profits.

Like many risky businesses, hazardously immoral contracts are about distribution rather than average returns -- who wins, and not whether the winners could even in principle compensate the losers. There are two complementary steps in the process:

(1) concentrating risk, so that a rare small probability event has high enough value to be worth hiding and

(2) obscuring the actual expectation of returns, so that credit can be obtained cheaply.

Often the second step requires partitioning a giant risk (catastrophe) into many little pieces, each of which seems innocuous enough, through contracts with multiple counter-parties, or a chain of transactions which would each be considered low-risk if considered independently. This makes it difficult for any one creditor to see the whole picture and assess the magnitude of possible total systemic losses -- an obfuscation that is vital in order to get them to provide low-cost credit for borrowing which exceeds the available equity used as collateral by an order of magnitude. The stage is then set for a collapse that is so large that one can call on the government to come in and pick up the pieces.

Concentration of risk is not pernicious in itself, and is clearly welfare enhancing when done correctly. As pointed out by Gron (1999) a major purpose of reinsurance is not the underwriting gains (substantial as they may be) or simple protection from insolvency, but rather to provide liquidity and capture profits in the chaotic conditions which exist post-disaster. Having credible insurance in times of need makes it possible for business to continue. Having credible regulation of genetically modified organisms makes it possible to capture the gains from biotech innovation. Having flood insurance makes it possible to build profitable yet risky housing in low-lying areas. However, concentration of serial (and thus inherently correlated) risks into toxic securities obfuscates real expected values leading to under-pricing of insurance and credit instruments. In this way, even losing business proposition can be made to seem safe and able to generate net positive returns to any required degree of "safety" (e.g., "will earn at least $1 million more than 99.4% of the time").

When extreme risks are externalized through limited corporate liability and (potential) government rescue of firms considered "too big to fail" the ordinarily beneficial pursuit of profit will inevitably create contracts and financial instruments that magnify the losses of extreme events rather diversify them, leading to chaos and destruction of value rather than risk management. When entered strategically, in order to deceptively obtain credit under-pricing, such contracts are hazardously immoral.

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