EXECUTIVE SKILLS AND PROCEDURAL FLEXIBILITY IN MIDDLE SCHOOL MATHEMATICS

A Dissertation
Submitted to
The Temple University Graduate Board

In Partial Fulfillment
of the Requirements of the Degree
DOCTOR OF PHILOSOPHY
in School Psychology

---

by
Tera Gibbs
Diploma Date: May 2022

Examining Committee Members:

Dr. Renée M. Tobin, Advisory Co-Chair, Department of Psychological Studies in Education
Dr. Julie L. Booth, Advisory Co-Chair, Department of Teaching & Learning
Dr. W. Joel Schneider, Department of Policy, Organizational, & Leadership Studies
Dr. Frank Farley, Department of Psychological Studies in Education
Dr. Kristie Newton, External Member, Department of Teaching & Learning

ABSTRACT

As procedural flexibility, previously understood as adaptive reasoning, emerges as an important consideration in math skill development, it is important to account for executive functioning in that process as well, as executive functioning a well-researched factor in math performance. The current study, a secondary data analysis, explores how students rate themselves on the Executive Skills Questionnaire – Revised (ESQ-R), an informal executive skills measure, and how those scores relate to procedural flexibility scores, which accounts for students' efficiency in math problem solving.

Using the factor structure relevant to the current sample, which varies significantly from the current ESQ-R, findings indicate that procedural flexibility is lower in seventh grade when compared to sixth and eighth grades. Perceived executive skills vary positively across sixth, seventh, and eighth grades, indicating more perceived difficulties with executive skills as students move up in grade. Additional analyses explored the relationships between procedural flexibility and ESQ-R scores. Although there was no evidence of a significant relationship between procedural flexibility and ESQ-R scores, the relationship varied across grade level, yielding a negative relationship for sixth grade, a neutral relationship fore seventh grade, and a positive relationship for eighth grade. This pattern indicates that procedural flexibility may become more readily demonstrated, and possibly more valuable, as students gain mastery of skills and procedures and students may become more critical of their executive skills. Procedural flexibility is also highly sensitive to context and curriculum, based on the Common Core State Standards for Mathematics.

ACKNOWLEDGEMENTS

Thank you to my committee, comprised of Dr. Julie Booth who made me a part of her research team and provided access to this data as well as invaluable feedback; Dr. Renée Tobin who provided unconditional encouragement and helped me realize that finishing this was possible; Dr. Joel Schneider who guided me through these analyses with patience and precision; Dr. Frank Farley who consistently checked in during lulls in productivity and supported my goals with such enthusiasm; and Dr. Kristie Newton who introduced me to procedural flexibility and quickly agreed to serve as external reader. To the entire committee, I will take your lessons with me moving forward.

To the faculty of the Temple University School Psychology Program, thank you for the solid foundation on which I will keep building a skill set as a practitioner. Thank you to my classmates for your friendship and professional support that extends beyond our education. Many thanks to my supervisors and colleagues at CORA Services and my assigned schools who have been immensely supportive as I chipped away at this project. You made this logistically possible. To Dr. Christine Espin, thank you for encouraging me to finish this and for sending me the book, *How to Write a Lot* (Silva, 2018) so I could learn how to practically move forward while maintaining my other roles.

To my parents, my siblings, their wonderful partners, and my dear friends, thank you for your constant love, support, and encouragement. To my amazing partner, Eric, thank you for nudging me down the more challenging path, helping me realize I was worthy of the time, and keeping things moving at home when I was distracted. To Cici and Dante, I am grateful to be your parent. Thank you for your patience and forgiveness and for helping me find a way to pursue this process with less fear and more joy.

TABLE OF CONTENTS

APPENDICES

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

In 2001, the National Research Council emphatically promoted the importance of high-quality math education when they posited the reciprocal relationship between academic success and math learning, stating that "All young Americans must learn to think mathematically, and they must think mathematically to learn" (p. 1). In recent history, rapid developments in science, technology, engineering, business, and politics have increasingly made basic mathematical skills a prerequisite for full participation in society, and a lack of mathematical skills deprives individuals of both competence in basic daily activities and opportunities for academic and/or professional advancement (National Research Council, 2001). According to the National Center for Education Statistics (2016), students who do not advance past Algebra I score lower on standardized tests, which impacts their likelihood of taking more advanced science and math courses. Therefore, success in science, technology, engineering, and mathematics (STEM) courses throughout a student's education can inform one's trajectory for opportunities long term. Postsecondary STEM graduates earn higher pay, experience higher job security, and boost innovation in ways that sustain economic growth locally and globally (National Governors Association, 2011). Additionally, students who study STEM are equipped to enter a variety of occupational fields and tend to earn competitive salaries in non-STEM positions (National Governors Association, 2011).

Although mathematics is just one component of STEM, it uniquely relates to future success. For example, students who took advanced math classes in high school (e.g., trigonometry, pre-calculus, and calculus) earned higher overall grade point averages, scored higher on college entrance exams, and expected to finish their degrees at a higher rate than peers who did not enroll in advanced math courses (Chen, 2009). In addition, Knuth and colleagues (2016) suggest that math participation relates to students' quality of life. More specifically, math proficiency positively relates to a person's ability to navigate health insurance choices, medication management, and overall societal self-advocacy; therefore, math proficiency is connected to one's life skills, not just their likelihood to access and maintain careers in STEM (Knuth et al., 2016). Success in math, therefore, relates to overall success, whether a student pursues postsecondary STEM or not, which may be attributable to the complexity of the learning process and the transferable skills that result.

Math learning is a complex developmental process that involves both procedural and conceptual understanding (Hiebert & Lefevre, 1986) and imitative and creative processes (Lithner, 2008) that, in many ways, can inform critical thinking skills and systematic problem solving across many areas of study (Chen, 2009; National Governors Association, 2011). Within the STEM field, fundamental math skills contribute to the foundation on which innovation is built, as the top ten fields that generated the highest number of patents in the United States between 2006-2010 were all STEM fields (Rothwell et al., 2013). As such, STEM continues to dominate in terms of innovation, while requiring fundamental math skills to do so. Math learning, therefore, cannot be overlooked whether considering micro-level individual student success or macro-level innovation and economic growth.

Procedural Flexibility

There is a specific component of mathematical reasoning that is particularly relevant in terms of math reasoning and its role in critical thinking and innovation, and that is procedural flexibility. Procedural flexibility, which Star and colleagues (2015) define as the ability to learn multiple procedures and use them appropriately to solve various problems, is a valued element of mathematics learning (National Council of Teacher of Mathematics, 2006; National Mathematics Advisory Panel, 2008). Within the Common Core State Standards (CCSS, 2019), mathematical understanding is viewed as having a “flexible base” that allows students to engage in mathematical practices and adjust problem-solving strategies, accordingly, allowing students to “deviate from a known procedure to find a shortcut.” Students, therefore, learn multiple methods to solve problems so that they can then choose an efficient, effective procedural solution. For example, according to CCSS (2019), students learn to compare fractions using both equivalent and benchmark fractions, which bolsters a well-rounded understanding of the content so that they do not become completely dependent on one procedure to solve fraction problems.

Previously understood as adaptive reasoning, procedural flexibility includes the extent to which a student can think logically, reflect on, explain, and justify their problem-solving decisions to reach an outcome efficiently (Kilpatrick et al., 2001). In that way, it is a unique combination of procedural understanding and practical skill that contributes to successful math learning and elegant demonstration of expertise (Newton et al., 2020; Star & Newton, 2009). Schneider and colleagues (2011) determined that there is a stable bidirectional relationship between conceptual and procedural knowledge that was not moderated by students’ prior knowledge of equation solving, and although both conceptual and procedural knowledge contribute to procedural flexibility during equation solving, it is still unclear how exactly procedural flexibility fits into the framework of procedural and conceptual knowledge. To understand how math learners get to a point of expertise, creativity, and innovation, a deeper understanding of the development of procedural flexibility is vital.

Understanding procedural flexibility can affect the development of effective instructional strategies as well. For example, Star and colleagues (2015) found that algebra teachers whose students had the most flexibility gains frequently asked students to verbalize what they had learned from comparing two strategies. Additionally, Newton and colleagues (2010) used carefully chosen side-by-side examples of different strategies and found that algebra students who found mathematics difficult felt reassured when they knew multiple methods to solve problems. Much research on procedural flexibility to date pertains to algebra learning, but students are capable of learning multiple methods for solving mathematics problems as early as second grade (Blöte et al., 2001).

Though the CCSS encourages educators to support the development of their students' flexible math base for efficient math computation and problem solving, students' math performance may be judged by their ability to demonstrate each step of their math problem solving process as it relates directly to a specific strategy taught in class. Students who do use efficient, flexible computation strategies, therefore, may not be rewarded for doing so. Alternative computation options are worthy of attention both in instruction and research. In this study, I will explore the use of efficient, flexible procedural strategies and how those align with students' perceived executive skills, as executive skills have been shown to impact math achievement (Cragg et al, 2017).

Executive Skills

Similarly, executive functions are vital for students' academic success in mathematics and in life. Executive functions are the neurological processes that occur mostly in the prefrontal cortex of the brain, although additional cortical and subcortical circuits also impact efficiency of these complex brain activities (Hale & Fiorello, 2004). Executive functions of the brain are those that are required for humans to execute or perform certain tasks needed for general functioning (Dawson & Guare, 2009). While cognitive executive functions support one's ability to execute tasks, executive skills refer to the actual abilities carried out as a result of those brain functions, such as planning, organizing, initiating tasks, attending to necessary content, inhibiting emotional and behavioral responses to an appropriate extent, recalling and manipulating information, and shifting fluently from task to task (Dawson & Guare, 2009). Referring to these skills as such emphasizes the malleability inherent in the idea of "skills" rather than "abilities," leaving room for a growth mindset in this work (Strait et al., 2020). The distinction between executive functions and executive skills is an important one for this project, as most math performance research in this area has focused on the relationship with executive functions of the brain, and no formal research has operationalized executive skills as defined by Dawson and Guare (2009), as this project will attempt to do. In other words, focusing on executive skills will lend itself to better understanding the practical, informal, developmental angle currently missing from this literature.

Math education literature shows that executive functioning is a vital component and predictor of math learning (St. Clair-Thompson & Gathercole, 2006). Executive functions, including working memory, attention, shift, and inhibitory control, are vital for students' academic success in mathematics. Specifically, working memory is positively related to overall math outcomes (Destefano & Lefevre, 2004; Raghubar et al., 2010; Frisco-Van Den Bos et al., 2013; Bull & Lee, 2014; Dekker et al., 2016), as are inhibition, attention, and shift (Hecht et al., 2003; Friso-van den Bos et al., 2013; Jordan et al., 2013). Interestingly, little research has explored how executive skills contribute to the fundamental processes upon which mathematical achievement is built, such as retrieval of math facts and math fact fluency, understanding numerical and operational relationships, and choosing and performing arithmetic methods (Cragg et al., 2017). Additionally, students often show various patterns of strengths and challenges across these complex and unique processes (Dowker, 2005; Gilmore & Papadatou-Pastou, 2009; Gilmore et al., 2017). Another key limitation of the literature available in this area is that these studies have explored these topics solely by way of teacher questionnaires, which measure only one point of view, or neuropsychological subtests, which measure very specific task demands to understand students' cognitive tendencies based on the tasks at hand, in testing environments that are not always reminiscent of students' actual learning environments.

Executive Skills and the Student's Voice

In this way, most studies that connect executive functions and math achievement are missing the student's voice, as they focus solely on measuring the executive brain functions employed during mathematical problem solving rather than the perceived executive skills students understand themselves to have (Bull & Lee, 2014; Raghubar et al., 2010). These studies use standardized, norm-referenced tests that are highly structured, closely facilitated by a skilled adult, and as free from distractions as possible, which automatically reduces the executive skills demands placed on the student examinees as they perform the standardized tasks (Dawson & Guare, 2018). Executive skills are most in demand when solving complex problems that require unique, creative solutions in real-life contexts, including classroom settings, and norm-referenced tests do not always yield a complete understanding of a student's abilities (Dawson & Guare, 2018).

Although formal, norm-referenced executive functioning assessments can give important information about students' cognitive functioning, there are limits to the data that formal assessments produce as they relate to the executive skills required in daily life, including students' learning environments, making multiple sources of data imperative to yield a full representation of a student's abilities. Moreover, the National Association of School Psychologists (NASP, 2009) encourages practitioners to use a range of assessment tools to bolster sensitivity to contextual influences, and student-report data is included as a possible assessment method to ensure a comprehensive understanding of students and their performance.

Executive Skills Questionnaire, Revised (ESQ-R)

Because most studies measuring executive skills and math performance use neuropsychological subtests or teacher rating scales, there is room in the literature to employ different measures of executive skills, including self-report questionnaires, especially one that is administered in the students' actual learning environment. Informal questionnaires can serve as helpful student-level self-report measures regarding how students perceive their own executive skills in their learning environment. One example of this type of measure is the Executive Skills Questionnaire, Revised (ESQ-R; Dawson & Guare, 2018; Strait et al., 2020), which is the latest version of the Executive Skills Questionnaire (Dawson & Guare, 2010). Originally, the ESQ was designed for use with middle and high school students (Dawson & Guare, 2010), although researchers have since developed a new five-factor structure for the ESQ-R based on a sample of undergraduate students with internal consistency estimates at .70 or more for all five factors (Strait et al., 2020).

Additionally, the ESQ-R demonstrated higher test-retest reliability (r=.70 for the ESQ-R) when compared to other executive functioning scales for adults, such as the Adult Executive Functioning Inventory (ADEXI) and the Current Behavior Scale (CBS), which both had test-retest correlations of .52 (Strait et al., 2020). Criterion validity assessments of the ESQ-R indicated negative correlations with grade point average (GPA), although there is still need for exploration of how the ESQ-R correlates with lower GPA scores, as GPA's in studies to date have skewed toward better performance overall (Strait et al., 2020). There are no similar comparisons of the ESQ-R with school-age rating scales. Strait and colleagues (2020) have called for additional psychometrics based on samples of students at various developmental levels, including middle school students.

As middle school students shift into adolescence and gain autonomy in their education, understanding the students' perspective of their own executive skills is vital to fully understand how their math performance and executive skills are related. There is indirect evidence suggesting the importance of self-perception in this relationship, including work on the use of self-regulation strategies, which are a particular component of learners' executive skills. More specifically, Pintrich and Zusho (2002) posit that students who perceive mathematics skills as useful are more likely to apply self-regulation skills to bolster their learning. In a recent study exploring the relationship between confidence and procedural flexibility in calculus students, Maciejewski (2020) found no significant interactions between the two variables, suggesting that there are "confident students who are not flexible and flexible students who lack confidence" (p. 13). Extending this concept into further exploration of students' perceptions of themselves and their executive skills, therefore, is warranted, especially in the context of math learning in middle school.

Rationale For This Project

The data gathered for this project are part of a larger study examining the specific variables that contribute to algebra readiness. Some of the considered variables include fraction computation, magnitude understanding, procedural flexibility, and executive skills. This project is a secondary data analysis examining the relationship between executive skills—such as planning, time management, organization, emotional regulation, and behavioral regulation—and procedural flexibility, or the ability to learn multiple procedures and use them appropriately to solve various problems (Star et al., 2015) in students' solving of fraction operations problems over the course of the school year for middle school mathematics students. It also explores the factor structure of the ESQ-R for this sample of middle school students. An increased understanding of this specific information will help educators understand the relationship between executive skills and procedural flexibility in fraction computation during a particularly formative stage of middle school students' math learning. It will also better inform how educators gather data on their students' perceived executive skills so that appropriate executive skills instruction can be implemented. This information could better inform academic interventions and consultation strategies to better support educators and their students.

Middle school is an especially important time to be evaluating these connections, as it is a vital time in students' math learning. As noted above, passing Algebra I is crucial for students' long-term success in secondary and postsecondary education, and ultimately in life. Before students can become proficient in algebra, however, they need to be proficient in their understanding of fractions, which is one major focus of middle school mathematics instruction. Siegler and colleagues (2012) determined that students' fraction knowledge at age 10 was a stronger predictor of algebra knowledge at age 16 than students' ability to perform arithmetic with whole numbers. Fraction knowledge, therefore, is a unique component to long-term success in math that has the potential to set the trajectory toward secondary and post-secondary education opportunities. This project is uniquely situated within the context of fraction computation but will focus on the broader concept of procedural flexibility. Most research on procedural flexibility to date has been in algebraic contexts; therefore, a stronger understanding of procedural flexibility in fraction computation would provide necessary information about how to situate procedural flexibility in another mathematics context specific to middle school students.

Middle school is also a critical time for executive skill development. Lee and colleagues (2013) found a period of executive functioning development in the study's 11 to 14-year-old age group that differed significantly from their younger age group quantifying the increasing intricacies of executive functions through middle adolescence. More specifically, neurological pruning, or the reduction in neurons and synapses to facilitate efficient neurological functioning, occurs increasingly in adolescence (Dawson, 2014), which influences the emergence of more complex executive skills such as planning and goal setting (De Luca & Leventer, 2008) and metacognition (Chapman et al., 2012). The current sample of middle school students, therefore, is a particularly interesting representation of a formative time in executive skill development.

It is well documented that executive functioning informs math achievement (Bull & Lee, 2014; Raghubar et al., 2010), and because of the urgency of math learning in middle school, executive skills are an important component of that. Also, executive skill demands increase dramatically during early adolescence, requiring students to navigate complex school schedules and daily class and assignment changes, plan and carry out long-term projects, organize systems for assignment completion and organization of materials, and inhibit behavioral responses, especially as independence increases (Dawson & Guare, 2018). Determining how perceived executive skills relate to procedural flexibility could yield insight into how skills like planning, organization, inhibitory control impact the way in which students demonstrate solutions to fraction computation problems, potentially making space for more creative ways to learn and demonstrate procedural and conceptual knowledge in fraction computation. To date, there is no literature linking procedural flexibility to executive skills, and there is a call for research on the factor structure of the ESQ-R on various age ranges, including middle school students.

In summary, understanding students' perceived executive skills in the context of middle school mathematics will help educators make informed recommendations for integrating executive skills training into academic contexts for middle school mathematics students. By exploring the interaction between specific executive skills domains and a specific component of math problems solving demonstrated by experts—procedural flexibility—this research can inform additional evidence-based interventions for students studying, or preparing to study, algebra, which opens the door to additional math study and postsecondary opportunities. Previous research on procedural flexibility has already informed effective instructional strategies and questioning techniques

(Newton et al., 2010; Star et al., 2015). Building upon this work is vital and will benefit educators as they provide recommendations for appropriate instructional and intervention strategies for students in middle school mathematics classes.

In the present study, I aim to determine how these two predictors, procedural flexibility and perceived executive skills, are related to one another in middle school students. Specifically, I investigate the connections between middle school students demonstrated procedural flexibility and their perceived executive skills, including time management, organization, emotional regulation, behavioral regulation, and overall executive skills, when solving fraction problems. I, therefore, propose the following research questions (RQ):

RQ1: Do procedural flexibility and perceived executive skills vary across sixth, seventh, and eighth grades?

RQ2: How do perceived executive skills relate to procedural flexibility scores and does this relationship vary across sixth, seventh, and eighth grades?

CHAPTER 2: LITERATURE REVIEW

This chapter will review the literature on math procedural knowledge and, more specifically, procedural flexibility and its developmental trajectory. The literature linking executive functioning, math achievement, and fraction knowledge will also be reviewed below. Finally, the ways in which executive functioning is typically measured in the context of math will also be reviewed.

Theoretical Basis

The theory around procedural knowledge in mathematics has evolved over time. Österman and Bråting (2019) elaborate on this evolution by synthesizing the ideas of Dewey (1997,1938), Skemp (1976), Hiebert and Lefevre (1986), Lithner (2008), and Star (2005). Österman and Bråting (2019) suggest that the understanding of procedural and conceptual knowledge has, perhaps unintentionally, become dichotomous over time, which is likely unnecessary and inaccurate, as the two are both necessary and bidirectional in nature (Schneider et al., 2011). This idea is supported by the foundation of learning Dewey (1997, 1938) set regarding expertise and application of content psychologically and socially. Because students are active, unique individuals with various backgrounds, strengths, challenges, and interests, academic content and its delivery needs to channel these unique qualities into progress (Dewey, 1997, 1938). Instruction on algorithms alone does not always lend itself well to upholding this instructional ideal. Instead, educators must master abstract elements of mathematics in order to make it interesting and intelligible for students (Österman & Bråting, 2019).

Skemp (1976) expands on the distinction between the algorithmic and applied elements of mathematics in the operationalization of two discrete types of understanding, instrumental and relational. Instrumental understanding is what develops when a student learns a series of fixed plans to solve a particular type of task (Skemp, 1976). This type of understanding is more algorithmic in nature and Skemp is critical of this instrumental understanding, which lacks a foundation of connection between fixed plans used and the result of the task or problem. Skemp (1976) instead prioritizes relational understanding, which is the gradual development of a conceptual structure that can lead to unlimited, unique plans that students can use to solve novel problems successfully. In this way, plans are based on connections and are no longer prescribed or fixed. Students with relational understanding, therefore, have freedom to solve problems more creatively.

Hiebert and Lefevre (1986) further develop Skemp's ideas about understanding when they make the distinction between conceptual and procedural knowledge in mathematics education. Procedural knowledge consists of the ability to use procedures as prescribed chains for manipulating symbols (Hiebert & Lefevre, 1986), which is similar to instrumental understanding. By contrast, conceptual knowledge is largely based on relationships and connections between a network of separate pieces of information (Hiebert & Lefevre, 1986), which is reminiscent of Skemp's idea of relational understanding previously described. In both cases, the student's goal is a system of concepts and understanding of the connected information within them.

Rather than using understanding or knowledge, Lithner (2008) uses reasoning to define the basis of mathematical ability development. Lithner posits two categories of mathematical reasoning, imitative and creative. Imitative reasoning is reflective of rote learning, or the process of repeating something demonstrated until a student memorizes it (Lithner, 2008). Similar to procedural knowledge or instrumental understanding, with only imitative reasoning a student can recall an algorithm but may lack the ability to explain each step's meaning and/or purpose. A student demonstrates creative reasoning when generating a novel reasoning sequence with arguments supporting each strategic choice, much like one might when demonstrating conceptual knowledge or relational understanding.

Star (2005) argues for a new characterization of procedural knowledge that addresses the assumptions that limit a comprehensive understanding of procedural knowledge as it relates to mathematical understanding and performance. He notes that research on procedural knowledge has evolved over the years from a cognitive lens in the 1980's to a developmental lens through the 1990s and into the 2000s. Kieran (2013) also emphasizes the need for continued evolution of procedural and conceptual knowledge, since work in this area to date has fabricated a dichotomous relationship between the two types of mathematical knowledge. To address this Österman and Bråting (2019) propose the idea of operational skill in which students learn by engaging in mathematical tasks and activities and engage in both procedural and conceptual skill sets simultaneously. This perspective reduces the oppositional relationship of procedural and conceptual knowledge and captures numerical and computation skills in mathematics understanding as students apply them thoughtfully to yield a correct mathematical solution, much like the complementary relationship of structural and operational processes suggested by Sfard (1991). In general, parsing out a dichotomous relationship of mathematical concepts and processes has been difficult and somewhat muddled due to the cognitive complexities involved in mathematics problem solving.

Procedural Knowledge

Star (2005) suggests a new perspective on procedural knowledge in mathematics learning that reduces the limiting dichotomous emphasis between conceptual and procedural knowledge and instead deepens the perspective of procedural knowledge to include procedures associated with comprehension, flexibility, and critical judgement. Initial definitions of conceptual and procedural knowledge developed opposing views of the two knowledge types. Hiebert and Lefevre (1986) defined conceptual knowledge as

> knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network, in which the linking relationships are as prominent as the

> discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network (pp. 3-4).

They define procedural knowledge as

> a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols…[or] rules of procedures for solving mathematical problems. Many of the procedures that students possess probably are chains of prescriptions for manipulating symbols (pp. 7-8).

The preceding definitions limit procedural knowledge to superficial understanding, lacking connection between sequential problem-solving steps, and Star (2005) suggests expanding the definition of procedural knowledge to include justification of and the goals for each step, the specific context of strategy use, including the inherent common-sense knowledge applicable to given situations. By including these complex elements of procedural knowledge, rich in relational understanding, the definition deepens.

Procedural Flexibility

A student's ability to demonstrate procedural flexibility involves a complex combination of factors including prior knowledge (Star & Rittle-Johnson, 2008; Fazio et al., 2016), exposure to multiple methods (Rittle-Johnson et al., 2012), and other contextual factors (Vershaffel et al., 2009). To better understand the gap between prior knowledge and flexible strategy use, Newton and colleagues (2020) measure six evidence-based aspects of procedural flexibility to develop a flexibility development continuum, which starts with a preference for efficiency, then moves to the evaluation of efficiency, recognition of multiple methods, occasional use of efficient methods, generation of multiple methods, and finally consistent use of efficient methods. Their findings highlight the foundation of a preference for efficiency, which develops before the use of efficient methods and knowledge of multiple solutions (Newton et al., 2020).

If students are to have procedural flexibility, they need to be familiar with multiple strategies. Some research has helped inform instructional practices to teach multiple strategies. Newton and colleagues (2010) suggest that using worked examples to promote multiple strategies in algebra problem solving increase students' knowledge of multiple strategies and increase their appreciation of them; however, students do not always use alternative strategies, which mostly depend on familiarity, efficiency, and understandability of the strategies, as well as type of algebra problem. In a study with eighth grade students, Rittle-Johnson and colleagues (2012) posited that novice students who compared different procedures immediately during algebra instruction solved problems more flexibly than those who did not. Additionally, greater flexibility was associated with greater procedural and conceptual knowledge (Rittle-Johnson et al., 2012). During interviews with middle and high school mathematics teachers, Lynch and Star (2014a) found that teachers at this developmental level viewed teaching multiple strategies as one way to address students' unique learning needs and increase their likelihood of success and motivation as they develop mathematical understanding; however, teachers also perceived teaching multiple strategies to be laden with barriers, including time constraints, student resistance, and the possibility of confusing the students. Additionally, these results differ from those that inform elementary math instruction, which recommend making instruction on multiple strategies a priority for preparing students to transition from intuitive to more formal problem-solving strategies (Lynch & Star, 2014a).

Lynch and Star (2014b) note that there is some confusion in the field about how best to conduct instruction of multiple strategies. These efforts sometimes include solving non-routine problems that do not employ a rote algorithm, students inventing their own strategies, students comparing multiple methods for problem-solving, and engaging in whole-class discussion about multiple strategies. After conducting interviews with students about instruction on multiple strategies, Lynch and Star (2014b) report students cited disadvantages of multiple strategy instruction much less than they cited advantages. Advantages of instruction on multiple strategies include having an increased awareness of differing methods, increasing the possibility of finding a preferred and/or "easier" method, reducing anxiety around mathematics tasks and assignments, improving accuracy, and having a "back-up" strategy. Students see the advantages of teaching multiple strategies and the freedom it could give them to feel more comfortable solving mathematics problems, and these findings counter teachers' perspectives on the possibility of confusing students with too many options for problem-solving. Although teachers' concerns about teaching multiple strategies are valid and likely based on their own prior experience, Lynch and Star (2014b) indicate that students could benefit from employing this instructional approach. More specifically, Star and colleagues (2015) found that teachers who asked more open-ended questions during instruction yielded higher flexibility gains from their students. New developments in curricular design have started to guide teachers through instructional strategies that build procedural flexibility in their students, including the Comparing and Discussing Multiple Strategies approach (Durkin et al., 2021).

To better understand students' justification of procedural strategy use, Maciejewski and Star (2019) conducted interviews with university students as they completed a matrix task. Participants' justifications were broadly categorized as algorithmic or anticipatory, which similarly reflects the creative and imitative reasoning framework (Lithner, 2008), which was initially less procedural in nature. Lithner (2008) suggests creative and imitative reasoning, which involves sensible reasons backing the process of task-solving, even if the conclusion is incorrect. Maciejewski and Star (2019) propose that the algorithmic and anticipatory procedural justifications can effectively contribute to deeper, more flexible knowledge in mathematics.

Executive Functioning and Mathematics

Connections Between EF and Math Achievement

Raghubar and colleagues (2010) completed a systematic review of this content revealing that most have used norm-referenced or other task-based tests to show that working memory is positively related to overall math outcomes. To extend that study, Bull and Lee (2014) conducted a meta-analysis of studies linking executive functions with math achievement to find that updating, or working memory, is the most conclusive area of executive functioning that predicts math performance. As such, there is a great deal of literature linking the executive function of working memory, or the ability to recall and manipulate information (Hale & Fiorello, 2004), to general math achievement; some of this work also examines developmental change in this relation and/or drills down to demonstrate relations to specific domains of mathematical competence, such as number sense (Frisco-Van Den Bos et al., 2013).

Beyond working memory, other studies inspect areas of executive functioning more broadly to include working memory, attention, and inhibition and explore their connection to specific areas of math achievement, such as fraction conceptual and procedural understanding. These studies have also used task-based neuropsychological subtests and teacher-report questionnaires as measures of student executive skills (Jordan et al., 2013; Hansen et al., 2015).

In a review of literature on executive functions and mathematics achievement, Bull and Lee (2014) conclude that students with executive functioning difficulties also have difficulty acquiring mathematics skills through development and over the course of classroom instruction. Specifically, Bull and Lee (2014) base these difficulties on executive functioning challenges pertaining to memory, completing instructions, filtering unnecessary information, focusing on specific tasks, self-monitoring progress, and shifting to appropriate strategies.

Historically, executive functions in mathematics education literature have been divided into three specific facets. These include working memory or updating, which is the ability to attend to and manipulate information encoded in the mind; inhibition, which is the ability to suppress extraneous information and unsuitable responses; and shifting, or the ability to think flexibly and switch attention between varying tasks (Miyake et al., 2000). Executive functioning extends far beyond these three facets, however. Additional areas of executive functioning that are less often included in the math education literature include plan management, time management, organization, emotional regulation, and behavioral regulation. These areas are also included on the ESQ-R used in this study. I will, therefore, explore the traditional areas of executive functioning reported in the math education literature and also attempt to parse out the specific areas of executive functioning included in the ESQ-R, as they relate to mathematics achievement in the literature and, specifically, fraction computation, as much as possible.

Working Memory, Inhibition, and Shift

Cragg and colleagues (2017) studied 293 participants' executive functions and overall mathematics achievement abilities and how the relationship between both change throughout development. Participants' ages ranged from 8 to 25 years, and findings revealed that working memory indirectly contributed to mathematics abilities via procedural skill and factual knowledge (Cragg et al., 2017). Working memory also contributed to mathematics achievement through conceptual knowledge, though to a lesser extent than procedural skill and factual knowledge, and interestingly, the relationship between working memory and mathematics achievement was stable throughout development (Cragg et al., 2017). Relationships between inhibition and mathematics achievement and, similarly, shifting and mathematics achievement seem to be less stable. A meta-analysis by Friso-van den Bos and colleagues (2013) found that shifting and inhibition are less important for mathematics achievement than working memory.

More specifically, Friso-van den Bos and colleagues (2013) determined that shifting contributed less to mathematics procedural skills as age increased, whereas visuospatial working memory's contribution increased with age, and verbal short-term working memory and inhibition's contributions to procedural skills remained stable. Conversely, Jansen and colleagues (2013) examined whether math practice influenced executive functions for children with mild intellectual disability and found that there were no transfer effects between mathematics training and executive functions, though visuo-spatial memory skills were positively related to addition and subtraction skills.

Executive functions' contributions to conceptual understanding appear to be slightly different than to procedural skills. For example, Robinson and Dube (2013) concluded that children, aged eight to ten years, who had difficulties with inhibition used conceptually based shortcuts less than children with stronger inhibition, which may indicate that children find inhibiting well-learned algorithms to be difficult. Although working memory still seems to contribute to mathematics performance, it does not seem to impact conceptual understanding to the same degree as inhibition, especially in fraction problem solving (Hecht et al., 2003; Jordan et al., 2013). Instead, mathematical knowledge (Hecht et al., 2003) and, specifically, fraction magnitude understanding seem to make significant contributions to students' demonstration of fraction conceptual and procedural understanding (Jordan et al., 2013). Interestingly, Hansen and colleagues (2015) report that working memory and attentive behavior, as well as other elements of general mathematics knowledge, predict fraction conceptual and procedural understanding for sixth grade students, although the combined predictability of all variables is higher for fraction conceptual understanding than it is for fraction procedures.

Plan Management, Time Management, and Organization

There is little to no research on the relationship between plan management, time management, and organizational skills and how they specifically relate to math achievement. Instead, these areas have traditionally been conceptualized in the literature as elements of inhibition. Specifically, if inhibition implies the ability to attend to the necessary information for problem solving (Miyake et al., 2000), then that indirectly applies to how one plans, manages time, and organizes thoughts and information to reach a conclusion. Plan management, time management, and organization, therefore, could be understood as functions of what is traditionally framed in the literature as inhibition.

Emotional and Behavioral Regulation

Wang and colleagues (2019) incorporated a self-regulation intervention into third-grade fraction instruction to foster a growth mindset and help manage motivation, cognition, and behavior throughout their specific learning process. They found that the embedded self-regulation intervention, which included goal setting, self-directed learning, and ongoing progress monitoring contributed to stronger fraction learning outcomes (Wang et al., 2019). The self-regulation intervention employed executive skills, such as planning, organization, attention, and inhibitory control, which highlighted the importance of executive skills, specifically for fraction learning in third grade.

More specifically, Wang and colleagues (2019) examined how regulatory focus, a motivational factor, moderates the relationship between potential and practical flexibility. Potential flexibility is the knowledge of multiple strategies and how to use them efficiently, whereas practical flexibility involves actually using the most efficient strategy to solve mathematics problems (Wang et al., 2019). Using the two distinct types of motivational orientations present in regulatory focus theory, promotion-focused (i.e., those striving toward gains and positive outcomes and who are more likely to take risks) and prevention-focused (i.e., those striving to avoid mistakes or negative outcomes by employed more cautious, conservative methods), results indicated that a predictive relationship between potential and practical flexibility was related to whether a person was promotion- or prevention-focused (Wang et al., 2019).

It is clear that executive functions and skills are important for mathematics learning and demonstrating understanding of various mathematics skills; however, the extant research does not address fully how executive skills and functions contribute to the overlap in procedural and conceptual understanding of fractions. It is for this reason that applied executive skills be explored in relation to fraction-based procedural flexibility, which is a possible representation of procedural and conceptual overlap.

Measuring Executive Functioning

Up to this point, most studies on mathematics understanding and executive functions have used task-based measures in isolated testing environments (Cragg et al., 2017; Hansen et al., 2015; Robinson & Dubé, 2013; Hecht et al., 2003) or teacher report questionnaires (Hansen et al., 2015). Research indicates that behavior rating scales are of particular value in terms of ecological validity, when compared to direct performance tests (Barkley, 2012; Toplak et al., 2013). Although student-report and adult-report (e.g., parent, teacher) do not typically align, especially on internalized factors, such as inattention, anxiety, withdrawal, and excess motor tension (Peach & Cobb, 1990), student-report measures of their perceived executive skills administered in their authentic learning environments could yield important information about how students understand themselves and how that relates to their procedural flexibility when solving fraction problems.

The Executive Skills Questionnaire, Revised (ESQ-R; Strait et al., 2018) is a useful questionnaire to assess how students perceive their own executive skills. The ESQ-R is the updated version of the Executive Skills Questionnaire (ESQ; Dawson & Guare, 2010), which was not initially designed as a norm-referenced measure. Instead, it was designed to be an informal, brief assessment tool to help educators inform classroom executive skills interventions to meet their students' needs. Additionally, as a self-report measure, the ESQ and ESQ-R guide students to identify their own perceived challenges and strengths regarding executive skills. It currently measures the following areas:

Plan management refers to a student's perceived ability to create a route to a goal or to complete a task, while employing decision-making skills around important and unimportant components of that process and how to focus one's attention accordingly, which is measured by eleven items on the ESQ-R (Dawson & Guare, 2018).

Time management refers to a student's perceived ability to navigate the amount of time available and how to use it to complete tasks within time limits and deadlines, which is measured by four items on the ESQ-R (Dawson & Guare, 2018).

Organization refers to a student's perceived ability to arrange and maintain systems for classifying and structuring information and materials, which is measured by three items on the ESQ-R (Dawson & Guare, 2018).

Emotional regulation refers to a student's perceived ability to manage emotional responses in order to complete tasks, achieve goals, and control or direct their behavior, which is measured by three items on the ESQ-R (Dawson & Guare, 2018).

Behavioral regulation refers to a student's perceived ability to think before acting, which also involves the capacity to resist an urge or impulse to do or say something, allowing for time to evaluate a given scenario to determine the potential consequences, which is measured by four items on the ESQ-R (Dawson & Guare, 2018).

Overall executive skills total refers to a students' total score on the ESQ-R, which is a sum of their scores on all 25 items (Dawson & Guare, 2018).

In summary, research exploring the links between procedural flexibility and executive skills in the context of fraction computation could yield specific insight into how to approach instruction and learning for students with perceived executive skill strengths and challenges. It will be important to address the developmental components of perceived executive skills and procedural flexibility by measuring how procedural flexibility and perceived executive skills vary across sixth, seventh, and eighth grades. Further insight into the relationship between perceived executive skills and procedural flexibility scores and whether this relationship varies across middle school grades will also be important. I hypothesize there will be a positive relationship between perceived executive skills and procedural flexibility. In particular, I expect the strongest relationships to be found with the perceived plan management and behavioral regulation subscales, as these areas of the ESQ-R include items that address how students implement procedures and conceptualize their approach to problem solving. The links between math performance and executive functioning reported in the literature have traditionally emphasized working memory, which is not included on the ESQ-R, as well as inhibition and shifting. Though the evidence on inhibition and shifting is less extensive and conclusive, there is some research that indicates that inhibition, which includes elements of plan management and behavioral regulation, impacts student decision making as they solve fraction problems. Finally, examination of the factor structure of the ESQ-R for middle school students will contribute to the research base on this measure and may yield a more accurate understanding of how students perceive themselves in this area.

Hypotheses

I predict the current factor structure will look different for middle school students, as changes in the differentiation and stability of various domains of executive functioning seem to occur between the ages of 11 and 15 years (Lee et al., 2013). Specifically, the areas of shift, which includes plan management on the ESQ-R, and inhibitory control, which includes emotional regulation and behavioral regulation on the ESQ-R may be less pronounced (Lee et al., 2013). I also predict that ESQ-R scores and procedural flexibility scores will vary across grade level, improving linearly as grade level increases. Regarding the relationship between procedural flexibility and ESQ-R scores, I predict there will be a positive correlation between the two variables and that the correlation will vary across grade level.

CHAPTER 3: METHOD

The current study is a secondary analysis of data from a larger research project funded by the Institute of Education Sciences (IES) that examines how specific components of fraction knowledge are related to middle school students' and preservice teachers' algebra readiness, overall performance, and learning. It also examines when certain components of fraction knowledge are the most meaningful to students' algebra preparation. Specifically, the larger study looks at the relationship between areas of fraction knowledge and algebra readiness and performance, the relationship between the type of targeted fraction knowledge and improvements in algebra readiness, and what mechanisms connect fraction knowledge and algebra. The current study is one small part of the IES study and looks specifically at procedural flexibility in the context of fraction computation and its relationship to perceived executive skills.

Participants

Participants are 499 sixth, seventh, and eighth grade students drawn from four public and charter schools in suburban areas in the northeastern United States. Participants were evenly distributed across grades (38.89% sixth grade, 29.66% seventh grade, and 31.46% eighth grade) and gender identification (48.09% male, 51.10% female). Ethnicity distribution of participants was 69.93% White/Caucasian, 14.60% Black/African-American, 7.21% Latinx, 3.01% Asian/Pacific Islander, 1.60% Multiracial, 0.40% American Indian/Alaskan Native/Native Hawaiian, and 1.60% Other. Assent and parental consent were obtained for all participants. Participants completed all procedural flexibility measures in the fall at the start of year (SOY). Each pretest included twelve items measuring fraction computation, six of which also measured procedural flexibility. The ESQ-R was completed once by each student either during the EOY of Year 2 of the study (eighth grade) or SOY of Year 3 of the study (sixth and seventh grades).

Measures

Fraction Computation

Pre- and posttests included twelve items measuring fraction computation, which were presented on one sheet of paper. See Appendix A for this particular problem set. The problems included all operations (i.e., two addition problems, three subtraction problems, four multiplication problems, and three division problems). Each problem included two rational numbers. Seven of the problems involved two fractions, two involved a mixed number and a whole number, one involved a fraction and a whole number, and two involved both mixed numbers. The directions stated, "We are interested in what processes you use to solve these problems, so please show what you did or tell what you thought to get your answer." Correct points were awarded to students who demonstrated correct procedures. Participants were not penalized for whole number arithmetic mistakes. The percentage of problems correctly solved was computed for each student.

Procedural Flexibility

Of the twelve fraction computation items presented to participating students, six were eligible for flexibility coding, meaning there were multiple methods participants could have used to correctly complete the problem. See Appendix A where the problems coded for flexibility are circled. The problems coded for flexibility included three subtraction problems, two multiplication problems, and one division problem. Of those problems, two included only fractions, three included mixed numbers, and one included a whole number and a fraction. Participants' solutions to the fraction-based computations problems were coded for efficiency. Participants who used what were deemed efficient or unconventional methods received points for procedural flexibility. In other words, the standard, conventional procedure was determined by math education experts (Newton,

2008) and any other, more efficient methods used to solve the problem correctly were coded as procedurally flexible using a binary coding scheme. Specifically, the problems included on the current study's materials were developed and tested by a group of math experts comprised of math education professors and doctoral students, who collaborated to ensure adequate sampling of flexibility opportunities within the included problems (Newton, 2008). Another group of experts, which included mathematicians, engineers, math educators, post-secondary math students, and a secondary math teacher, provided further validation of the flexibility measure, which ultimately indicated eleven possible items for demonstrating flexibility (Newton, 2008). Six of the eleven items are included in this study. Some examples of procedurally flexible methods included cancellation, distributive property, reducing, avoiding conversion to improper fractions, and showing no work, which implies mental calculation. The flexibility scores used for this study were obtained at SOY.

Executive Skills

Sixth-, seventh-, and eighth-grade participants completed the Executive Skills Questionnaire - Revised (ESQ-R; Strait et al., 2018), an informal, self-report measure of executive skills. The ESQ-R includes 25 items that reflect an ecologically valid understanding of executive skills as they apply directly to academic functioning and interventions. On the ESQ-R, participants rate each item on a frequency Likert-type scale related to how often they experience an executive skills issue (0=Never or Rarely, 1=Sometimes, 2=Often, 3=Very Often). Based on factor analysis, subscales on the original ESQ-R include plan management (11 items), time management (4 items), organization (3 items), emotional regulation (3 items), and behavioral regulation (4 items). Because this factor structure has been validated for undergraduate students, this factor structure was reassessed for this sample of middle school students, as called for in current literature (Strait et al., 2020). Exploratory factor analysis was used. To score the questionnaires, the examiner added the numerical value of the student's responses stated above. Missing data on the ESQ-R was prorated by subscale as follows: two missing items for plan management (18.2% missing), one missing item for time management (25% missing), organization (33.3% missing), emotional regulation (33.3% missing), and behavioral regulation (25% missing), and three missing items for the ESQ-R total score (12% missing). R code for subscale creation is included for review in Appendix B.

The readability of the ESQ-R was determined using a free Text Readability Consensus Calculator (www.readabilityforms.com) to informally determine the readability of the ESQ-R's directions and items. Readability formulas used on the Text Readability Consensus Calculators included the Flesch Reading Ease Formula (Flesch, 1948), the Flesch-Kincaid Grade Level (Kincaid et al., 1975), the Gunning FOG Formula (Gunning, 1952), the SMOG Index (Hedman, 2008), the Automated Readability Index (Senter & Smith, 1967), and the Linsear Write Formula (Klare, 1974). For the ESQ-R as it was presented in this study, The Flesch Reading Ease Formula score was 89.3, which is described as "easy to read." The Gunning Fog score was 5.4, which is described as "easy to read." The Flesch-Kincaid Grade Level formula determined the text to be a third grade reading level (3.2), and SMOG Index score was a 4.4, which indicated fourth grade reading level. The Automated Readability Index yielded a score of 1.9, which indicated a reading level for ages six to eight years, or first and second grade students. The Linsear Write Formula indicated a fourth grade reading level (4.4). Based on the readability formulas included here, the Readability Consensus was that the text in the ESQ-R was appropriate for students in fourth and fifth grades or students eight to nine years old. Though there was some variability across the formulas' results, all of them fell below a sixth grade reading level, which is important to consider, given that the sixth-grade students who participated in this study did so at the beginning of their sixth-grade year, just after completing fifth grade.

Some additional details, based on word statistics, are also important to consider when gauging the readability of the ESQ-R. The average number of words per sentence in the ESQ-R was 10 words per sentence, and there was an average of 3.9 characters per word and an average of one syllable per word. Within the text, 80% of all included words were single-syllable words, 15% of the words included had double syllables, and 5% of the words had three or more syllables, making them “hard” words. The “hard” words included in the ESQ-R were “executive,” “questionnaire,” “directions,” “impulse,” “finishing,” “priorities,” “estimating,” “appointments,” “solution,” “different,” “activities,” “decisions,” and “interrupted.” When strictly considering age and grade, the ESQ-R's readability was appropriate for this sample.

Procedure

Students completed the start-of-year (SOY) pre-test, intervention packets, immediate post-test, and end-of-year (EOY) post-test during class time. Researchers administered the SOY pre-test and EOY post-test during a full class period for each administration. Students could work through the rest of the SOY and EOY packets at their own pace throughout the rest of the class period. Researchers circulated the classrooms during SOY and EOY pre-/post-test administration and were available to answer questions, though researchers gave no overt feedback or problem-solving guidance on any SOY or EOY task. The ESQ-R was included at EOY for 8th grade participants during Year 2 of the study and at SOY for 6th and 7th grade participants during Year 3 of the study.

Variables

This study specifically measures the relationship between procedural flexibility and specific domains of executive skills across grade levels. Operational definitions for each variable are included below:

Procedural flexibility refers to the student's ability to solve fraction problems correctly and efficiently, according to a predetermined coding scheme based on the traditional method of solving the given math problem.

Grade level refers to the student's academic year as set by the United States Department of Education. In this study, participants are in either sixth, seventh, or eighth grade.

Classroom refers to the student's assigned learning environment in which they received math instruction.

ESQ-R total score refers to a students' total score on the ESQ-R, which is a sum of their scores on the items identified as necessary for middle school students for this sample, based on preliminary analyses.

Analysis

Preliminary analyses were conducted to examine the factor structure of the ESQ-R. Using Classical Test Theory (DeVellis, 2006), we conducted a parallel analysis based on factor analysis with a principal-axis extraction. For the factor analysis, iterations were set at 1000 to confirm the parallel analysis and factor structure for this sample. The resulting factor structure was used for the remaining analyses. In determining the factor structure, we also ran item analyses to determine if all the ESQ-R items were necessary to support optimum reliability using both Cronbach's alpha and McDonald's omega.

The first research question asked whether procedural flexibility and/or perceived executive skills varied across grade level. To determine if procedural flexibility and perceived executive skills varied across grade level, we fitted constant (intercept-only) linear mixed models to create the null comparison for each. We then used linear mixed models to obtain standardized parameters, including students' classroom as a random effect. ANOVA’s then generated the necessary p values to determine significance. I hypothesized that each variable would correlate positively across grade.

The second research question asked about the relationship between procedural flexibility and perceived executive skills and whether that relationship varied across grade level. To determine the nature of the relationship between procedural flexibility and self-rated executive skills, we fitted a linear mixed model to predict procedural flexibility scores with ESQ-R total scores using students’ classroom as random effect. To get to the interaction effect and determine if the relationship varied by grade level, we first added grade level as a covariate to understand the null hypothesis. Then we determined the standardized parameters by fitting a linear mixed model to predict procedural flexibility points with ESQ-R total points and grade, using students’ classroom as a random effect. I predicted that ESQ-R scores would improve across grade level.

CHAPTER 4: RESULTS

Preliminary Analyses to Understand ESQ-R Factor Structure

Using Classical Test Theory (DeVellis, 2006), preliminary analyses indicated that, for the current sample of middle school students, the factor structure of the ESQ-R does not resemble the structure established using a sample of post-secondary students. A parallel analysis based on factor analysis with a principal-axis extraction suggested either four or five factors. Factor analysis (with maximum iterations set at 1000) confirmed this, and, ultimately, I decided to use four factors. Proposed titles for each factor included Planning/Future Orientation (12 items, factor loadings of .63--.29), Negative Emotionality (5 items, factor loadings of .62--.27), Disorganization/Messiness (3 items, factor loadings of .67--.49) and Impulsivity (5 items, factor loadings of .46--.29). After examining the factor loadings, two items, initially included in the Impulsivity factor, yielded especially low correlations across factors (e.g., "19. I go with my gut," and "25. I live for the moment."). Item analyses using both McDonald's omega and Cronbach's alpha indicated that reliability would increase if these items were removed. With the two items in question included, Cronbach's alpha was .88 and McDonald's omega total was .90. Without these items, reliability measures increased (Cronbach's alpha = .89, McDonald's omega total = .90). Given the increased reliability, these two items were removed when calculating the ESQ-R total score. Given the inconsistencies in the factor loadings between the original sample and the present sample, the ESQ-R total score was used as the executive skills measure. This score did not include the two items that reduced reliability (e.g., "19. I go with my gut," and "25. I live for the moment."). R code for preliminary factor analysis can be found in Appendix C. For the purposes of testing hypotheses, we conducted analyses using the ESQ-R total score as the executive skills measure because of the inconsistencies in the factor loadings between the original sample and the present sample. These results are reported below.

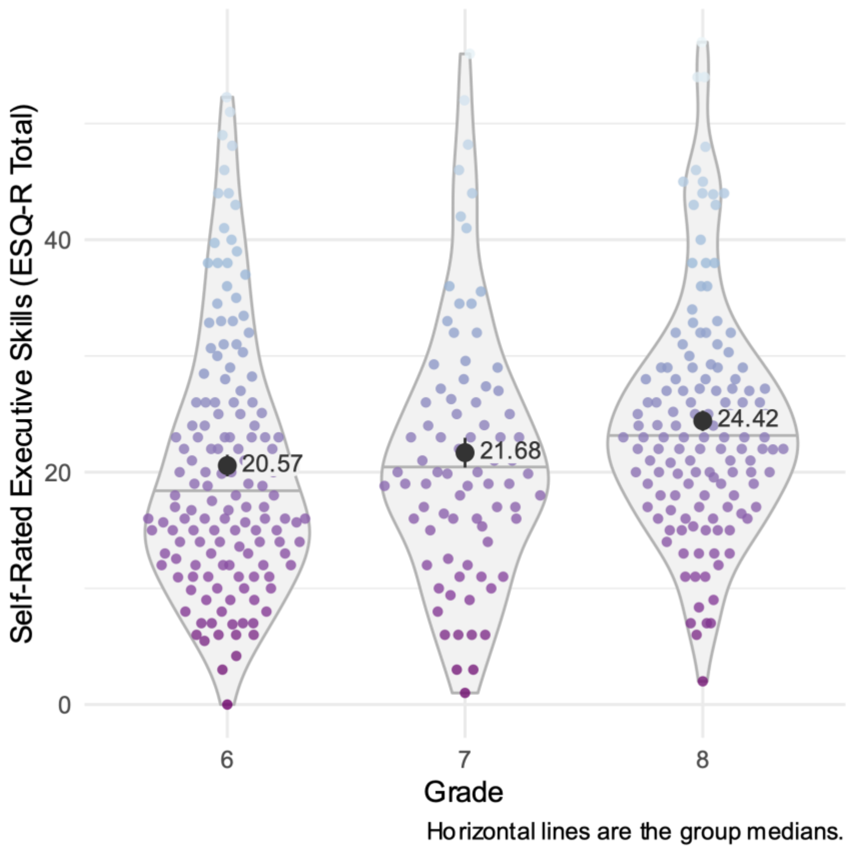
Procedural Flexibility and Grade Level

To test if procedural flexibility varied across grade level, we fitted a linear mixed model to predict procedural flexibility points with grade level. This model included students' classrooms as random effects. The effect of grade level was statistically non-significant ($p=.197$) and negative. Interestingly, procedural flexibility points were lower for the seventh-grade group (*M*=.94) than for the sixth-grade group (*M*=1.15) and eighth-grade group (*M*=1.31). R code for this analysis can be found in Appendix D.

Executive Skills and Grade Level

Next, to test if ESQ-R totals varied across grade level, we fitted a linear mixed model to predict the ESQ-R total with grade level. The effect of grade level was statistically significant ($p= 0.01$) and positive in this model. Means increased consistently across sixth grade (*M*=20.57), seventh grade (*M*=21.68), and eighth grade (*M*=24.42) scores, meaning students perceived more executive skills difficulties as grade level increased, as shown in Figure 1. R code for this analysis and for generating Figure 1 can be found in Appendix E.

Figure 1. ESQ-R Total Scores Across Grade



Relationship Between ESQ-R Total and Procedural Flexibility

To get to the interaction effect and determine if the relationship varied by grade level, we first added grade level as a covariate to understand the null hypothesis. More specifically, we fitted a linear mixed model to predict procedural flexibility scores with ESQ-R total scores using students' classroom as random effect. The model's total explanatory power was moderate (conditional $R^2 = 0.19$). The model's intercept, corresponding to ESQ-R total score and grade level, was at 0.54 ($p = 0.564$). Within this model, the effect of ESQ-R total score was statistically non-significant and negative ($p = 0.630$). The effect of grade level was statistically non-significant and positive ($p = 0.467$). R code for this analysis can be found in Appendix F.

Relationship Between ESQ-R Total and Procedural Flexibility Across Grade

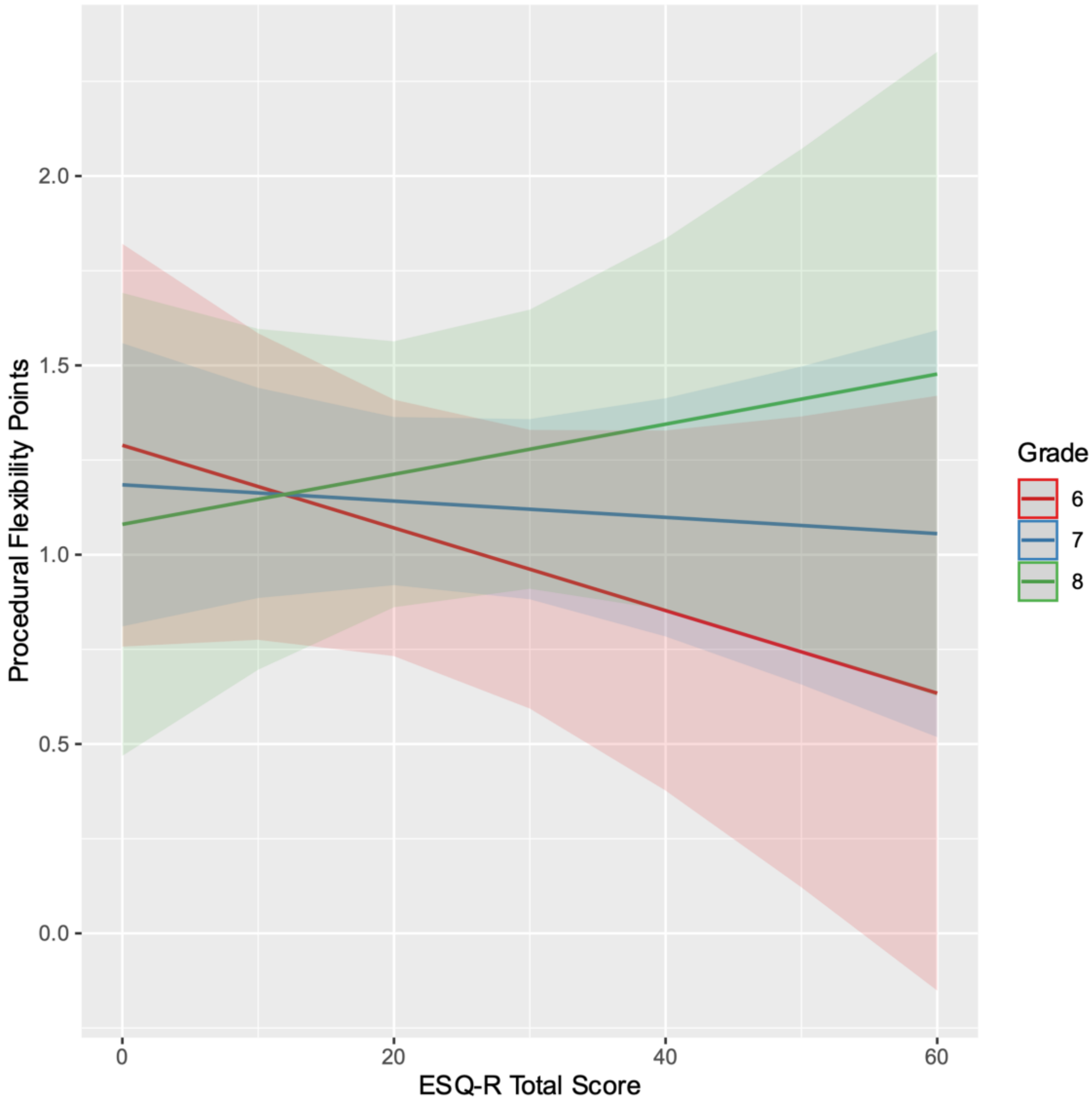
To test the null hypothesis and more fully understand if the relationship between procedural flexibility and ESQ-R total scores varied across grade level, we obtained standardized parameters by fitting a linear mixed model to predict procedural flexibility points with ESQ-R total points and grade, using students' classroom as a random effect. As such, we fitted a linear mixed model, estimated using the restricted maximum likelihood approach to predict procedural flexibility points with ESQ-R total scores and grade level with students' classroom as a random effect. The model's total explanatory power was moderate (conditional $R^2 = 0.19$). The model's intercept, corresponding to ESQ-R total score and grade level, was at 1.92 ($p = 0.21$). Within this model, the effect of ESQ-R total score was statistically non-significant and negative ($p = 0.24$). The effect of grade level was statistically non-significant and negative ($p = 0.64$). The interaction effect of grade level on ESQ-R total score was statistically non-significant and positive ($p = 0.26$), as shown in Table 1. R code for this analysis and for generating Table 1 can be found in Appendix G.

Table 1. The Relationship Between ESQ-R Total and Procedural Flexibility Across Grade

Predictors	*Estimates*	*CI*	*p*	*Estimates*	*CI*	*p*	*Estimates*	*CI*	*p*	*Estimates*	*CI*	*p*
(Intercept)	1.04	0.86 – 1.23	**<0.001**	1.21	0.84 – 1.58	**<0.001**	0.54	-1.30 – 2.38	0.564	1.92	-1.10 – 4.94	0.213
ESQ-R Total				-0.00	-0.02 – 0.01	0.642	-0.00	-0.02 – 0.01	0.630	-0.06	-0.17 – 0.04	0.237
Grade							0.10	-0.16 – 0.36	0.467	-0.10	-0.54 – 0.33	0.637
ESQ-R Total * Grade										0.01	-0.01 – 0.02	0.257
Random Effects												
σ^2	1.47			1.62			1.62			1.62		
τ_{00}	0.32 class_id			0.35 class_id			0.37 class_id			0.37 class_id		
ICC	0.18			0.18			0.19			0.19		
n	71 class_id			66 class_id			66 class_id			66 class_id		
Observations	465			337			337			337		
Marginal R^2/ Conditional R^2	0.000 / 0.179			0.001 / 0.179			0.004 / 0.189			0.007 / 0.193		

Interestingly, eighth grade students' correlation between ESQ-R total and procedural flexibility was positive (as ESQ-R score increases, so does flexibility points), seventh grade student's correlation was neutral, and sixth grade student's correlation was negative (as ESQ-R total increased, procedural flexibility decreased), as shown in Figure 2.

Figure 2. The Relationship of Procedural Flexibility and ESQ-R Scores Across Grade



I also tested the new factor structure in a similar way, fitting a linear mixed model with the updated subscales determined by the preliminary analyses. The new subscales included Planning/Future Orientation (12 items), Negative Emotionality (5 items), Disorganization/Messiness (3 items), and Impulsivity (3 items). To complete the analyses, I fitted a linear mixed model to predict procedural flexibility points with these subscales across grade using students' classroom as a random effect. The model's total explanatory power was moderate (conditional R^2=.21). The model's intercept was at 1.23 (p=.44). There was no evidence of significance across grade for any of the updated subscales across, including Planning/Future Orientation (p=.69), Negative emotionality (p=.62), Disorganization (p=.71), and Impulsivity (p=.65). There was also no evidence of an interaction effect across grade (p=.80). The code for these analyses is included in Appendix G.

For the sake of completeness, analyses using the original ESQ-R subscales were also conducted to examine my specific hypotheses. In particular, I examined whether the ESQ-R subscales including the plan management and behavioral regulation subscales predicted procedural flexibility. I found no evidence that these subscales predicted procedural flexibility (p=.62). Similarly, I tested the interaction of grade across all ESQ-R subscales and found no evidence that these interactions predicted procedural flexibility (p=.89). The R code for multilevel modeling with the original ESQ-R is included in Appendix H.

Post Hoc Analyses to Account for Prior Knowledge

In addition, I ran additional post hoc analyses to take into account students' prior knowledge. Specifically, I tested whether content moderated the relationship between procedural flexibility and ESQ-R total scores. First, I fitted a linear mixed model to predict procedural flexibility scores with ESQ-R total scores and their fraction computation scores. As before, the effect of ESQ-R total score was not statistically significant ($p = 0.56$). The effect of students' fraction computation score was statistically significant and positive ($p < .001$). When I added the interaction of ESQ-R total score and the fraction computation score, interaction effect was not statistically significant, $p = .14$.

Thus, there was no evidence that the effect of executive skills on flexibility differed depending on prior content knowledge. The R code for multilevel modeling with the original ESQ-R is included in Appendix I.

CHAPTER 5: DISCUSSION

Preliminary Analyses

Preliminary analyses supported a new factor structure for the ESQ-R for this sample of middle school students, showed increased reliability after the removal of two items, and indicated low factor loadings across items. As predicted, a factor analysis showed the ESQ-R's factor structure for this sample differs significantly from the one currently established from a sample of post-secondary students. Middle school is a particularly complex time for executive skill building, both practically and developmentally. Executive skill demands increase during middle school years, requiring students to navigate more complicated school schedules, plan and complete long-term projects, organize assignments and materials, and monitor their behavior with increasing autonomy (Dawson & Guare, 2018). Research has consistently shown the unique phase of executive skills development that occurs in middle school years (Lee et al., 2013), which involves neurological pruning (Dawson, 2014) and burgeoning complex skills like planning and goal setting (De Luca & Leventer, 2008), as well as metacognition (Chapman et al., 2012). Although there are plenty of executive skills demands present for post-secondary students, the overall experience and level of change is different than middle school students, so a new factor structure seems warranted here.

In addition, preliminary analyses supported the removal of two items from the current ESQ-R for this sample. The removed items were written in a way that emphasizes egosyntonic impulses or what comes naturally to the respondent, whereas the remaining items lend themselves more to an egodystonic perspective. There is some published literature in the measurement of psychological and personality disorders, that supports including both egosyntonic and egodistonic elements of the measured construct (Paris, 2015; Serpell et al., 2004); however, in this case, the balance of egosyntonic and egodystonic test items was not established. Therefore, removing these two egosyntonic items increased reliability of the ESQ-R.

The ESQ-R yielded relatively low factor loadings across items, which suggests a diverse selection of items, making high reliability a challenge. There is some value to this approach, as having diverse items is one way to avoid a bloated specific. Current research in the development of personality measures explores this in the context of measuring maladaptive traits and encourages balanced scale development (Oltmanns & Widiger, 2018). The ESQ-R's item diversity can, therefore, be an asset, especially when used as it is intended, as an informal measure of executive skills.

Research Questions

Procedural Flexibility Across Grade

Procedural Flexibility did not vary across grade level as expected. Seventh grade students' mean procedural flexibility score was lower than that of sixth grade students. In this way, the procedural flexibility variable was not behaving as expected if procedural flexibility were conceptualized as a linear skill that increases consistently over time. This obtained pattern may be attributable to valued classroom expectations of completing work thoroughly, as well as instructional strategies that emphasized the early stages of fraction procedures. It is important to note that all sixth- and seventh-grade measures were collected at the start of the school year, when students were reviewing course content, learning new material, and settling back into school. Perhaps at that time of the school year, less flexibility and greater emphasis on full procedures was more highly valued by students and/or instructors. Given the procedural flexibility measures were obtained early in the year, flexibility scores may have reflected students' performance based on their previous grades, as students were likely in the thick of reviewing materials from their previous year. However, each grade's curriculum must be considered when interpreting the data at these time points. Curricular design and standards could be informing the ways in which teachers review material and prepare their students for upcoming content in given grades during their reviews.

There is curricular evidence that seventh grade requires more focus on procedural operations, which could inform less efficient problem-solving. More specifically, Common Core State Standards for Mathematics (CCSS, 2019) emphasize math operations in seventh grade stating that instructional time should be dedicated in part to developing an understanding of operations with rational numbers. Additionally, CCSS (2019) recommend that seventh grade students extend addition, subtraction, multiplication, and division to all rational numbers while maintaining the specific properties of each. As such, seventh grade students are required to use arithmetic of rational numbers as they formulate math expressions and equations to solve problems (CCSS, 2019). One way to ensure students are understanding procedures is to have them demonstrate each step of a problem in their work. The expectation to carry out each step directly impacts the likelihood that a student would use an efficient problem-solving method.

In contrast, sixth grade standards encourage the use of reasoning about multiplication and division to solve ratio and rate problems, as well as using the meaning of fractions, multiplication, and division (and the relationship between the two), to explain why procedures for dividing fractions make sense (CCSS, 2019). Sixth grade curricula that align with CCSS, therefore, likely place less emphasis on carrying out each intricate step of a procedure, which could inform the higher procedural flexibility scores in sixth grade. For eighth grade students, CCSS (2019) recommend that students "strategically use and efficiently implement procedures" to solve linear equations. Linear equations involve fractions, and the emphasis on strategic and efficient use of procedures directly informs what constitutes procedural flexibility as it is operationalized and coded in this study.

To integrate the time of data collection in this study with curricular considerations, it is important to note that sixth-grade students who focused on developing fluency in addition and subtraction of fractions in fifth grade (CCSS, 2019) were likely reviewing that material in preparation for their sixth-grade curriculum. Seventh-grade students who focused on operational mastery of fraction multiplication and division with further emphasis on rationale in sixth grade (CCSS, 2019) were likely reviewing that material in preparation for the highly procedural curriculum that they would be learning in the months ahead. Finally, eighth-grade students, who had spent the previous year studying fraction addition, subtraction, multiplication, and division procedures, could have been reviewing that material in ways that inform future successful implementation of efficiency, which is a key component of eighth-grade curricular design (CCSS, 2019). In summary, the students in the current sample were in a phase of transition, and the standards being taught at each grade level could be informing the non-linear pattern of procedural flexibility seen across grades.

The emphasis on operations in seventh grade possibly aligns with part of the flexibility development continuum, posited by Newton and colleagues (2020). The continuum begins with a preference for efficiency, then shifts to the evaluation of efficiency, recognition of multiple methods, occasional use of efficient methods, initiation of multiple methods, and finally regular use of efficient methods. Based on the CCSS (2019), seventh grade is likely at the stage involving the recognition of multiple methods as students prepare for the complex standards to come. Seventh grade's emphasis on operations, therefore, may be an effort to teach multiple methods in preparation for eighth grade when students are taught to begin using efficient methods as part of their curriculum (CCSS, 2019).

ESQ-R Scores Across Grade

ESQ-R scores varied across grade level; however, they did not vary as expected. Specifically, ESQ-R scores increased across grade level. I predicted scores would improve, but instead scores increased consistently across grade level. Interestingly, higher scores on the ESQ-R indicate more perceived difficulties with executive skills. ESQ-R scores were lowest for sixth grade students and highest for eighth grade students, indicating that students report more difficulties with executive skills as grade level increases. These results may be an indication environmental and/or developmental contributors.

First, the increase in ESQ-R scores across grade level may be a result of a time point of data collection. Six and seventh grade students completed the ESQ-R at the beginning of the year when they were just starting to settle into their school routines, being taught how to function within their learning environments, and possibly were reviewing content from previous years. It is unlikely that any long-term projects or other assignments requiring a great deal of executive skills had begun yet either, making it likely that the executive skills demands were lower for students in these grades when the assessment was conducted.

Also, sixth grade students reported lower scores, meaning they perceive better executive skills. Developmentally, this finding may be due to experiencing fewer executive skill demands in a more supportive, scaffolded environment around skills, such organization, time management, and behavioral regulation, speaking to the increasing executive skills demands that gradually unfold throughout adolescence (Dawson, 2014). Perhaps most important is the idea of self-awareness. Sixth grade students may have approached their answers with less of a critical perspective than higher grade levels. Metacognition develops throughout adolescence (Chapman et al., 2012), and sixth graders may have approached the ESQ-R with less metacognition than the students in older grades.

ESQ-R scores for seventh grade students increased from that of sixth grade students. As middle school progresses, students may experience more demands as support gradually decreases. Additionally, as students grow and develop, they may be gaining more self-awareness and possibly more critical metacognition (Chapman et al., 2012) to inform their responses. Eighth-grade students' responses yielded results that support this explanation to a higher degree, as their scores increased in comparison to seventh grade. Considerations may be that eighth-grade students continue to have higher executive skill demands as they grow and develop with decreasing support (Dawson, 2014), as they report more executive skill difficulties than seventh and sixth grade students, respectively.

Relationship Between ESQ-R Scores and Procedural Flexibility Across Grade

There was no evidence that ESQ-R scores predicted procedural flexibility. There are several factors that may have contributed to this finding, including the informality of the ESQ-R; procedural flexibility score's non-linear behavior statistically, as represented by the decrease in procedural flexibility in seventh grade; and procedural flexibility's sensitivity to context. First, preliminary analyses indicated that the most appropriate factor structure of the ESQ-R for this sample varied significantly from the one validated for post-secondary students. Although the ESQ-R is an important resource for informal self-assessment of executive skills (Dawson & Guare, 2018; Strait et al., 2020), it is not yet the most accurate gauge of a middle school student's executive skills for use as a predictor. It instead needs to be refined for this population before being used in this capacity.

The second point is related to the outcome measure in this study, namely procedural flexibility. I found a non-linear pattern of procedural flexibility, such that there was a decrease in procedural flexibility in seventh grade relative to sixth and eighth grades. This pattern may be attributable to the increased emphasis placed on math procedural operations in seventh grade, when compared with sixth and eighth grade standards, which focus more on reasoning and conceptual knowledge (CCSS, 2019). Given that this study is one of the first studies to compare procedural flexibility at these grade levels, future research examining its presentation over time is warranted.

Additionally, there is a great deal of nuance inherent in procedural flexibility, which is an intentionally complex component of math learning that addresses the limitations of dichotomous models of math learning and moves toward a more comprehensive understanding of math learning and performance (Star, 2005; Kieran, 2013; Österman & Bråting, 2019). This recommendation goes back to Dewey (1997, 1938) and the need to channel students' unique qualities into instruction. Moving away from binary models makes way for complexity and messiness. In the literature, procedural flexibility shifts away from the previously dichotomous perspectives of math knowledge (Star, 2005; Kieran, 2013; Österman & Bråting, 2019), such as algorithmic and relational understanding (Skemp, 1976), procedural and conceptual knowledge (Hiebert & Lefvre, 1986) and imitative and create reasoning (Lithner, 2008), making it especially sensitive to contextual factors as a variable. For example, the relationship between ESQ-R and procedural flexibility did not significantly vary across grade level, although eighth grade's correlation was positive (i.e., as ESQ-R goes up, so do procedural flexibility points), seventh grade's correlation was neutral, and sixth grade's correlation was negative (i.e., as ESQ-R total increased, procedural flexibility decreased), providing further evidence that procedural flexibility is not linear. The meandering nature of procedural flexibility, as evidenced in the non-linear way it varies across grade level, as well as the way its relationship with ESQ-R scores changed across grades, indicates that it is likely impacted a great deal by context, including developmental level as well as instructional and curricular preferences and values (CCSS, 2019).

The shifts across grade levels in the relationship between procedural flexibility and the ESQ-R scores, could inform the argument that procedural flexibility may be helpful or useful for more experienced students and less so for students with fewer years of mathematics instruction, though seventh grade disrupts this notion. Research has shown that struggling students find flexible procedures comforting in a high support context when being overtly taught multiple ways to solve problems then giving each student a choice in their method (Newton et al., 2010). Perhaps seventh grade CCSS (2019) are a universal attempt at intentionally teaching multiple procedures at another phase of their education so that students can prepare for the efficiency required in eighth grade. This possibility seems to warrant further research examining which problem solving methods are most prevalent across grades, how curricular design impacts efficient math problem solving, and at which points in math learning the specific components of the procedural flexibility development continuum—including preference for efficiency, evaluation of efficiency, recognition of multiple methods, occasional use of efficient methods, generation of multiple methods, and consistent use of efficient methods—are most useful for direct instruction (Newton et al., 2020).

Post Hoc Analyses to Account for Prior Knowledge

After hypotheses were not upheld, I included post hoc analyses to account for students' prior knowledge, which is upheld in the literature as a predictor of procedural flexibility (Star & Newton, 2009) and a crucial component of procedural flexibility development (Newton et al., 2020). Results indicated that, although fraction computation scores were a significant predictor of procedural flexibility scores for this sample, there was no evidence of an interaction effect of fraction computation scores and ESQ-R scores on procedural flexibility.

Limitations

The current study has several limitations, including demographic distribution, students' anecdotal confusion on some ESQ-R items, inability to parse out specific executive skills using the ESQ-R for this sample, and the possibility of refining how to code the procedural flexibility variable. Addressing these limitations in future research could add to the current literature on executive skills and procedural flexibility and yield a great deal of possibilities for future research directions.

First, in terms of demographics, the sample is not evenly distributed across ethnic identities. This is an equity consideration and warrants attention as further research is conducted in this area. Research samples more accurately representative of the broader population will serve the field well as we develop additional understanding of executive skills and procedural flexibility. Specifically, according to the Kids Count Data Center (Annie E. Casey Foundation, 2020) the population of school age children by race includes 50% White/Caucasian, 14% Black/African American, 26% Latinx, 5% Asian, <.5% Native Hawaiian/Pacific Islander, 5% Multiracial, and 1% American Indian/Alaskan Native. The current sample includes 69.93% White/Caucasian, 14.60% Black/African-American, 7.21% Latinx, 3.01% Asian/Pacific Islander, 1.60% Multiracial, 0.40% American Indian/Alaskan Native/Native Hawaiian, and 1.60% Other. In this way, the current sample overrepresents White/Caucasian students and underrepresents Latinx, Asian/Pacific Islander, and Multiracial students. This difference in representation is an important consideration, given that previous research indicates achievement gaps in math between various ethnic groups in eighth grade. According to the National Center for Education Statistics (NCES; 2019) eighth grade math achievement scores between White/Caucasian and Black/African American students and

White/Caucasian and Latinx students differed by 32 points and 24 points, respectively, with White/Caucasian students earning higher scores. Additionally, Asian students out-performed White/Caucasian students by 18 points on eighth grade math achievement measures (NCES, 2019). Because the current sample overrepresents White/Caucasian students and underrepresents Latinx and Asian student populations, it is susceptible to exacerbated achievement score gaps between White/Caucasian and Black/African American and Latinx students and diminished representation of high scores among Asian students. Procedural flexibility scores could be sensitive to population representation in eighth grade, since flexible problem solving is an overt goal of eighth grade students, according to the CCSS (2019).

Secondly, during data collection students frequently asked the meaning of two items ("I act on impulse," and "I have a short fuse."). These were not the items with low correlations that were eventually removed. Anecdotally, these items were easily clarified by researchers with terminology already used on other items (e.g., "I act without thinking" or "I get easily upset."). Although the reading level of the ESQ-R was appropriate according to several readability formulas (Flesch, 1948; Kincaid et al., 1975; Gunning, 1952; Hedman, 2008; Senter & Smith, 1967; Klare, 1974), students still had difficulties interpreting unfamiliar idioms and phrases included on the ESQ-R. When refining the ESQ-R for use with middle school students, adjusting terminology, jargon, and slang, may be helpful for this population moving forward.

Another limitation is the undeniable result that there is something non-linear happening with procedural flexibility. If procedural flexibility were a linear math skill that builds upon itself, it would be expected that procedural flexibility would increase steadily across grade levels. Instead, it decreased between sixth grade and seventh grade, then increased between seventh grade and eighth grade. Although this pattern is likely attributable to curricular design based on the CCSS (2019), there is new evidence supporting the non-linear nature of procedural flexibility as a variable for calculus students (Maciejewski, 2020) and preliminary evidence of specific strategies used across grade level (Gibbs & Newton, 2020). These developments give reason to believe that the non-linearity of procedural flexibility may need to be accounted for in future coding schemes as measuring it becomes more refined. Taking a closer look at what methods were used to solve problems flexibly may lead to new conceptualizations of coding schemes to measure procedural flexibility at this developmental level.

Additionally, solving problems efficiently or creatively may not have been rewarded in students' learning environments, whereas demonstrating each step of the procedure may have been required or more highly valued by various instructors or curricula. Finally, over the course of this study, students were not aware that efficiency and creativity would be valued in their approach to solving problems. Directions stated, "We are interested in what processes you use to solve these problems, so please show what you did or tell what you thought to get your answer." Notably, these directions were intentionally vague to ensure authentic, unprompted responses. It is possible that, had they been prompted to solve the problem as efficiently as possible, the variable may have yielded different results, as students may have felt that they needed permission to show less work. During the current study, students were not made aware that flexible problem solving would be valued, so they may have been capable but did not engage in it based on previous experiences in which they were encouraged to show all work.

Conclusions

Although there was generally no evidence of statistically significant results in this study, there are some conclusions of note inherent in the data, including the factor structure of the ESQ-R for middle school students and the non-linear nature of procedural flexibility. First, to truly answer the call for research on the factor structure of the ESQ-R (Strait et al., 2020), more research on the ESQ-R factor structure is needed for middle school students using a sample more representative of the population. There is a need for validated informal measures that teachers can easily access for classroom use, and to be able to use a well-researched, equitable tool to gauge students' understanding of their executive skills could benefit instruction (Dawson, 2014) and possibly even relationship building in the classroom (Dawson & Guare, 2018).

Secondly, procedural flexibility is a valuable piece of math learning, although I did not find evidence that it was related to this measure of perceived executive skills, the ESQ-R. Procedural flexibility is, instead, a nuanced, complex variable that will require a great deal of attention in future research. Procedural flexibility was meant to encompass the nuances of math learning, reducing the limited dichotomous emphasis between conceptual and procedural knowledge and instead adding to the idea of procedural knowledge so that it includes comprehension, flexibility, and critical judgement (Star, 2005). As research continues, it will be important to find new ways to measure and code for procedural flexibility, especially for middle school students.

Procedural flexibility is a complex variable that accounts for students' unique perceptions, strengths, and preferences, as well as the instruction employed to build flexibility. A recent study exploring the relationship between confidence and procedural flexibility in calculus students indicated no significant interactions between the two variables, which suggests that some students with confidence lack flexibility, and some students who demonstrate flexibility lack confidence (Maciejewski, 2020). In this way, it is difficult to identify patterns between students' perceptions and procedural flexibility. Procedural flexibility's complexity is seen in both the student-centered and teacher-centered research on procedural flexibility to date. As noted previously, a preference for efficiency precedes efficient problem solving (Newton et al., 2020). Also, Newton and colleagues (2010) found that using worked examples to develop multiple strategies in algebra problem solving increased students' knowledge and appreciation of multiple strategies, although students do not always use them. The use of multiple strategies depends on several factors including familiarity, efficiency, and understandability of the strategies, in addition to the type of algebra problem assigned (Newton et al., 2010), making both internal and external factors relevant in the process of solving problems efficiently. During class, novice students who compared different procedures immediately during algebra instruction solved problems more flexibly than those who did not, and stronger flexibility was associated with stronger procedural and conceptual knowledge (Rittle-Johnson et al., 2012). There are also new curricula being developed to guide teachers through procedural flexibility instruction in Algebra I (Durkin et al., 2021). As we continue to learn more about how students balance their own learning with environmental factors, new ideas about the specifics of procedural flexibility will become clearer.

Teachers' perspectives also add to the complexity of procedural flexibility. Middle and high school teachers viewed teaching multiple strategies as one way to address students' unique learning needs and increase their likelihood of success and motivation in math education; however, teachers also perceived many barriers to teaching multiple strategies, including time constraints, student resistance, and student confusion (Star et al., 2015). These results conflict with those that inform elementary math instruction, which have determined multiple strategies to be imperative for student preparation for more formal problem-solving strategies (Lynch & Star, 2014a). To address this discrepancy, curricular developments, such as the Comparing and Discussing Multiple Strategies approach, are starting to gain traction with researchers, educators, and policy makers alike (Durkin, 2021). The results of the current study demonstrate the iterative process of math learning and the nuance in procedural flexibility as a variable.

Continued work on procedural flexibility will undoubtedly refine new ways of exploring this important, complex variable for both students and teachers.

APPENDIX A. FRACTION COMPUTATION PROBLEMS

We are interested in what processes you use to solve these problems, so please show what you did or tell what you thought to get your answer.

A) $\frac{2}{3} \times \frac{1}{5} =$

B) $2\frac{1}{2} \times 4 =$

C) $2\frac{1}{3} \div 9 =$

D) $\frac{3}{8} + \frac{2}{8} =$

E) $\frac{2}{15} \times \frac{7}{15} =$

F) $\frac{2}{3} + \frac{3}{8} =$

G) $5 - \frac{3}{8} =$

H) $\frac{9}{10} \div \frac{3}{10} =$

I) $6\frac{2}{5} - 2\frac{4}{5}$

J) $\frac{2}{4} - \frac{3}{6} =$

K) $1\frac{4}{5} \times 2\frac{1}{3} =$

L) $\frac{2}{9} \div \frac{3}{8} =$

APPENDIX B. R CODE FOR CREATING ESQ-R SUBSCALES AND TOTAL

```
```{r}

# function to prorate if number of missing items is less than a minimum
prorate_min_missing <- function(x, min_missing = 1) {
  n_missing <- sum(is.na(x))
  n <- length(x)
  if (n_missing > min_missing) {NA} else n * mean(x, na.rm = TRUE)}

# esq-r plan management
# We decided to prorate the scale, allowing for 2 missing items.
D_plan_management <- d %>%
  select(id, `ESQ-R_Q6`, `ESQ-R_Q7`, `ESQ-R_Q12`, `ESQ-R_Q13`, `ESQ-R_Q14`,
`ESQ-R_Q16`, `ESQ-R_Q17`, `ESQ-R_Q18`, `ESQ-R_Q22`, `ESQ-R_Q23`, `ESQ-
R_Q24`) %>%
  pivot_longer(-id) %>%
  group_by(id) %>%
  summarise(esq_r_plan_management = prorate_min_missing(value,
                                             min_missing = 2))

# esq-r time management
# We decided to prorate the scale, allowing for 1 missing item.
D_time_management <- d %>%
  select(id, `ESQ-R_Q10`, `ESQ-R_Q11`, `ESQ-R_Q15`, `ESQ-R_Q20`) %>%
  pivot_longer(-id) %>%
  group_by(id) %>%
  summarise(esq_r_time_management = prorate_min_missing(value,
                                             min_missing = 1))

# esq-r organization
# We decided to prorate the scale, allowing for 1 missing item.
D_organization <- d %>%
  select(id, `ESQ-R_Q3`, `ESQ-R_Q8`, `ESQ-R_Q9`) %>%
  pivot_longer(-id) %>%
  group_by(id) %>%
  summarise(esq_r_organization = prorate_min_missing(value,
                                             min_missing = 1))

# esq-r emotion regulation
# We decided to prorate the scale, allowing for 1 missing item.
D_emotion_regulation <- d %>%
  select(id, `ESQ-R_Q4`, `ESQ-R_Q5`, `ESQ-R_Q21`) %>%
  pivot_longer(-id) %>%
  group_by(id) %>%
  summarise(esq_r_emotion_regulation = prorate_min_missing(value,
                                             min_missing = 1))
```
```

```
# esq-r behavioral regulation
# We decided to prorate the scale, allowing for 1 missing item.
D_behavioral_regulation <- d %>%
  select(id, `ESQ-R_Q1`, `ESQ-R_Q2`, `ESQ-R_Q19`, `ESQ-R_Q25`) %>%
  pivot_longer(-id) %>%
  group_by(id) %>%
  summarise(esq_r_behavioral_regulation = prorate_min_missing(value,
                                          min_missing = 1))

#Updated MS ESQR variables

#MS ESQR future orientation
#prorated for 2 missing items
d_future_orientation <- d %>%
  select(id, `ESQ-R_Q10`, `ESQ-R_Q11`, `ESQ-R_Q12`,`ESQ-R_Q14`,`ESQ-
R_Q15`,`ESQ-R_Q16`,`ESQ-R_Q17`,`ESQ-R_Q18`, `ESQ-R_Q21`,`ESQ-
R_Q22`,`ESQ-R_Q23`,`ESQ-R_Q24`) %>%
  pivot_longer(-id) %>%
  group_by(id) %>%
  summarise(esq_r_future_orientation = prorate_min_missing(value,
                                          min_missing = 2))

#MS ESQR negative emotionality
#prorated for one missing item
d_negative_emotionality <- d %>%
  select(id, `ESQ-R_Q1`, `ESQ-R_Q4`, `ESQ-R_Q5`, `ESQ-R_Q6`,`ESQ-R_Q7`) %>%
  pivot_longer(-id) %>%
  group_by(id) %>%
  summarise(esq_r_negative_emotionality = prorate_min_missing(value,
                                          min_missing = 1))

#MS ESQR disorganization
#prorated for 1 missing item
d_disorganization <- d %>%
  select(id, `ESQ-R_Q3`, `ESQ-R_Q8`, `ESQ-R_Q9`) %>%
  pivot_longer(-id) %>%
  group_by(id) %>%
  summarise(esq_r_disorganization = prorate_min_missing(value,
                                          min_missing = 1))

#MS ESQR impulsivity
#prorated for one missing item
d_impulsivity <- d %>%
  select(id, `ESQ-R_Q2`, `ESQ-R_Q13`, `ESQ-R_Q20`) %>%
  pivot_longer(-id) %>%
```

```
  group_by(id) %>%
  summarise(esq_r_impulsivity = prorate_min_missing(value,
                                           min_missing = 1))


# esq-r total
# We decided to prorate the scale, allowing for 3 missing items.
D_total <- d %>%
  select(id, `ESQ-R_Q1`:`ESQ-R_Q25`) %>%
  select(-`ESQ-R_Q19`, -`ESQ-R_Q25`) %>%
  #item analysis suggested removing these items
  pivot_longer(-id) %>%
  group_by(id) %>%
  summarise(esq_r_total = prorate_min_missing(value,
                                       min_missing = 3))

d <- d %>%
  left_join(d_plan_management, by = 'id') %>%
  left_join(d_time_management, by = 'id')  %>%
  left_join(d_organization, by = 'id') %>%
  left_join(d_emotion_regulation, by = 'id') %>%
  left_join(d_behavioral_regulation, by = 'id') %>%
  left_join(d_future_orientation, by = 'id') %>%
  left_join(d_negative_emotionality, by = 'id') %>%
  left_join(d_disorganization, by = 'id') %>%
  left_join(d_impulsivity, by = 'id') %>%
  left_join(d_total, by = 'id')
```

APPENDIX C. R CODE FOR PRELIMINARY FACTOR ANALYSIS

````
## Parallel Analysis of ESQ-R

```{r}
d_esqr <- d %>%
  dplyr::select(`ESQ-R_Q1`:`ESQ-R_Q25`)
colnames(d_esqr) <- d_esqr_items$ESQR.Abbreviated
d_esqr %>%
  WJSmisc::parallel_analysis()
```{r}
d_esqr %>%
  psych::fa(fm = “pa”, nfactors = 4) %>%
  WJSmisc::plot_loading()

d_esqr %>%
  psych::fa(fm = “pa”, nfactors = 4,max.iter = 1000) %>%
  WJSmisc::plot_loading()
# item analysis
# calculate internal consistency measure using McDonald’s Omega
```{r}
print(psych::omega(d_esqr,nfactors = 5),sort=TRUE,cut=0)
```
#Item analysis using alpha
```{r}
#alpha with all items
psych::alpha(d_esqr)
#Removed 25LiveFormoment due to low item total correlation
d_esqr %>%
  select(-`25LiveForMoment`) %>%
  psych::alpha()
#Removed 19GoWithGut due to low item total correlation
d_esqr %>%
  select(-`25LiveForMoment`,-`19GoWithGut`) %>%
  psych::alpha()
d_esqr %>%
  select(-`25LiveForMoment`,-`19GoWithGut`) %>%
  psych::omega()
```
````

APPENDIX D. R CODE FOR PROCEDURAL FLEXIBILITY ACROSS GRADE

````
```{r}
fit_0 <- lmer(FlexPoints ~ 1 + (1 | class_id), data = d)
report(fit_0)
performance::check_model(fit_0)
fit_grade <- lmer(FlexPoints ~ Grade + (1 | class_id), data = d)
report(fit_grade)
tab_model(fit_0,fit_grade)
anova(fit_0,fit_grade)
compare_performance(fit_0,fit_grade)
```

```{r}
d %>%
  group_by(teacher_id,Section,Grade) %>%
  summarize(n=n()) %>%
  view()
```
````

APPENDIX E. R CODE FOR ESQ-R SCORES ACROSS GRADE LEVEL

````
```{r}
fit_esqr0 <- lmer(esq_r_total ~ 1 + (1 | class_id), data = d)
report(fit_esqr0)
performance::check_model(fit_esqr0)
fit_esqrgrade <- lmer(esq_r_total ~ Grade + (1 | class_id), data = d)
report(fit_esqrgrade)
tab_model(fit_esqr0,fit_esqrgrade)
anova(fit_esqr0,fit_esqrgrade)
compare_performance(fit_esqr0,fit_esqrgrade)
```

```{r}
d %>%
  ggplot(aes(factor(Grade),esq_r_total)) +
  geom_violin(width=.8,draw_quantiles = c(.5),color=”gray70”,fill=”gray95”)+
# geom_jitter(alpha=.8,pch=16,width=.07,height=.1,aes(color=esq_r_total))+

ggbeeswarm::geom_quasirandom(alpha=.8,pch=16,width=.35,height=.1,aes(color=esq_r
_total))+

stat_summary(color=”gray20”,geom=”richtext”,mapping=aes(label=round(..y..,2)),fun.y
=mean,hjust=-
0,label.color=NA,alpha=1,text.color=”gray20”,fill=”gray95”,label.margin=unit(2,”mm”),
size=3,label.padding=unit(0,”mm”))+
  stat_summary(color=”gray20”,size=.5)+
  scale_color_distiller(palette = “BuPu”)+
  theme_minimal()+
  theme(legend.position = “none”)+
  labs(x=”Grade”,y=”Self-Rated Executive Skills (ESQ-R Total)”,caption=”Horizontal
lines are the group medians.”)
````
````

APPENDIX F. R CODE RELATING PROCEDURAL FLEXIBILITY AND ESQ-R SCORES

```
``{r, warning=FALSE}

fit_0 <- lmer(FlexPoints ~ 1 + (1 | class_id), data = d)
report(fit_0)
performance::check_model(fit_0)
fit_total <- lmer(FlexPoints ~ esq_r_total + (1 | class_id), data = d)
report(fit_total)
d %>%
  select(FlexPoints, esq_r_total, class_id) %>%
  visdat::vis_dat()
d %>%
  select(FlexPoints, esq_r_total, class_id) %>%
  describe()
anova(fit_0, fit_total)
library(naniar)
ggplot(d,aes(FlexPoints, esq_r_total))+
  geom_point()+
  geom_smooth(method = "lm")+
  geom_miss_point()
sjPlot::plot_model(fit_total,type = "pred")$esq_r_total+
  scale_y_continuous("Flex Points",limits = c(0,6))
```
```
```

APPENDIX G. R CODE FOR RELATING PROCEDURAL FLEXIBILITY AND ESQ-R SCORES ACROSS GRADE LEVEL

````
```{r}
#adding grade as covariate (provides null for below, need this to get to the interaction effect)
fit_grade <- lmer(FlexPoints ~ esq_r_total + Grade + (1 | class_id), data = d)
report(fit_grade)
#does the effect of esqr vary by grade?
Fit_gradeinteraction <- lmer(FlexPoints ~ esq_r_total * Grade + (1 | class_id), data = d)
report(fit_gradeinteraction)
anova(fit_grade,fit_gradeinteraction)
sjPlot::plot_model(fit_gradeinteraction,type = “pred”,terms = c(“esq_r_total”,”Grade [6,7,8]”))
sjPlot::tab_model(fit_0,fit_total, fit_grade,fit_gradeinteraction)

```
 #Multilevel modeling with new subscales
```{r}
##complete data
d_subscale <- d[(d %>% select(FlexPoints,Grade,esq_r_future_orientation,esq_r_negative_emotionality,esq_r_disorganization,esq_r_impulsivity) %>% complete.cases()),]
#null model with complete data
fit_subscale0 <- lmer(FlexPoints ~ 1 + (1 | class_id), data = d_subscale)
summary(fit_subscale)
#predicting flex points with all 4 new subscales of MS esqr
fit_subscale <- lmer(FlexPoints ~ esq_r_future_orientation+esq_r_negative_emotionality+esq_r_disorganization+esq_r_impulsivity + (1 | class_id), data = d_subscale)
summary(fit_subscale)
#interaction of grade with all 4 new subscales of MS esqr
fitsubscale_bygrade <- lmer(FlexPoints ~ esq_r_future_orientation*Grade+esq_r_negative_emotionality*Grade+esq_r_disorganization*Grade+esq_r_impulsivity*Grade + (1 | class_id), data = d_subscale)
summary(fitsubscale_bygrade)
report(fitsubscale_bygrade)
anova(fit_subscale0,fit_subscale,fitsubscale_bygrade)
```
````

APPENDIX H. R CODE FOR MULTILEVEL MODELING WITH ORIGINAL ESQ-R

```
```{r}

##complete data

d_subscale <- d[(d %>%
select(FlexPoints,Grade,esq_r_plan_management,esq_r_time_management,esq_r_organization,esq_r_emotion_regulation,esq_r_behavioral_regulation) %>% complete.cases()),]

#null model with complete data

fit_subscale0 <- lmer(FlexPoints ~ 1 + (1 | class_id), data = d_subscale)

summary(fit_subscale)

#predicting flex points with all 5 subscales of esqr

fit_subscale <- lmer(FlexPoints ~
esq_r_plan_management+esq_r_time_management+esq_r_organization+esq_r_emotion_regulation+esq_r_behavioral_regulation + (1 | class_id), data = d_subscale)

summary(fit_subscale)

#interaction of grade with all 5 subscales of esqr

fitsubscale_bygrade <- lmer(FlexPoints ~
esq_r_plan_management*Grade+esq_r_time_management*Grade+esq_r_organization*Grade+esq_r_emotion_regulation*Grade+esq_r_behavioral_regulation*Grade + (1 |
class_id), data = d_subscale)

summary(fitsubscale_bygrade)

report(fitsubscale_bygrade)

anova(fit_subscale0,fit_subscale,fitsubscale_bygrade)
```
```

APPENDIX I. R CODE FOR POST HOC ANALYSES

```{r}

#adding FractionComputationTotal as covariate (provides null for below, need this to get to the interaction effect)

fit_fractioncomp <- lmer(FlexPoints ~ esq_r_total + FractionComputationTotal + (1 | class_id), data = d)

report(fit_fractioncomp)

#does the effect of esqr vary by grade?

Fit_fractioncompinteraction <- lmer(FlexPoints ~ esq_r_total * FractionComputationTotal + (1 | class_id), data = d)

report(fit_fractioncompinteraction)

anova(fit_fractioncomp,fit_fractioncompinteraction)

sjPlot::plot_model(fit_fractioncompinteraction,type=”pred”,terms = c(“esq_r_total”, “FractionComputationTotal”))

sjPlot::plot_model(fit_fractioncompinteraction,type=”pred”,terms = c(“FractionComputationTotal”, “esq_r_total”))


```

REFERENCES

Annie E. Casey Foundation (2020). Kids Count Data Center. Child population by race in the United States. Retrieved from: https://datacenter.kidscount.org/

Barkley, R. A. (2012). Executive functions: what they are, how they work, and why they evolved. *In Choice, 50*, p. 762.

Blöte, A. W., Van der Burg, E., & Klein, A. S. (2001). Students' flexibility in solving two-digit addition and subtraction problems: Instructional effects. Journal of Educational Psychology, 93, 627-638.

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives*, *8*(1), 36–41.

Chapman, S. B,m Gamino, J. F., & Mudar, R. A. (2012). Higher order strategic gist reasoning in adolescence. In V. F. Reyna, S. B. Chapman, M. R. Dougherty, & J. Confrey (Eds.), *The adolescent brain: Learning, reasoning, and decision making* (pp. 123-151). Washington, DC: American psychological Association.

Chen, X. July 2009. "Students Who Study Science, Technology, Engineering, and Mathematics (STEM) in Postsecondary Education." National Center for Education Statistics. NCES 2009–161.

Common Core State Standards Initiative (2019). Common Core State Standards for Mathematics. Retrieved from: http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf

Cragg, L., Keeble, S., Richardson, S. Roome, H. E., & Gilmore, C. (2017). Direct and indirect influences of executive functions on mathematics achievement. *Cognition*, 162, 12-26.

Dawson, P. (2014). Best practices in assessing and improving executive skills. In P. L. Harrison & A. Thomas (Eds.) *Best practices in school psychology: Student-level services* (pp. 269-285). Bethesda, MD: National Association of School Psychologists.

Dawson, P. & Guare, R. (2009). *Smart but scattered.* New York, NY: The Guilford Press.

Dawson, P. & Guare, R. (2010). *Executive skills in children and adolescents: A practical guide to assessment and intervention, 2nd Edition*. New York, NY: The Guilford Press.

Dawson, P. & Guare, R. (2018). *Executive skills in children and adolescents: A practical guide to assessment and intervention, 3rd Edition*. New York, NY: The Guilford Press.

Dekker, M. C., Ziermans, T. B., & Swaab, H. (2016). The impact of behavioural executive functioning and intelligence on math abilities in children with intellectual disabilities. Journal of Intellectual Disability Research, 60(11), 1086-1096.

De Luca, C. R. & Leventer, R. J. (2008). Developmental trajectories of executive functions across the lifespan. In V. A. Anderson, P. Jacobs, & P. Anderson (Eds.) *Executive functions and the frontal lobes: A lifespan perspective* (pp. 23-55). New York, NY: Taylor & Francis.

Destefano, D. & Lefevre, J. A. (2004). The role of working memory in mental arithmetic. European Journal of Cognitive Psychology, 16, 353-386.

DeVellis, R. F. (2006). Classical test theory. *Medical Care*, *44*(11), 50-59.

Dewey, J. (1997, 1938). Experience and education. New York, NY: Touchstone.

Dowker, A. (2005). *Individual differences in arithmetic implications for psychology, neuroscience and education.* New York: Psychology Press.

Durkin, K., Rittle-Johnson, B., Star, J. R., & Loehr, A. (2021): Comparing and Discussing Multiple Strategies: An approach to improving algebra instruction. *The Journal of Experimental Education*, 1-18.

Fazio, L. K., DeWolf, M., & Siegler, R. S. (2016). Strategy use and strategy choice in fraction magnitude comparison. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *42,* 1-16.

Flesch, R. (1948). A new readability yardstick. *Journal of Applied Psychology, 32*(3), 221-233.

Frisco-Van Den Bos, I., Van Der Ven, S. H. G., Kroesbergen, E. H., & Van Luit, J. E. H. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review, 10,* 29-44.

Gibbs, T. L. & Newton, K. J. (2020). Procedural flexibility in middle school mathematics: Exploring students' strategy use. Poster presented at the National Association of School Psychologists Convention, Baltimore, MD.

Gilmore, C. K., Keeble, S. Richardson, S., & Cragg, L. (2017). The interaction of procedural skill, conceptual understanding, and working memory in early mathematics achievement. *Journal of Numerical Cognition, 3*(2), 400-416.

Gilmore, C. K. & Papadatou-Pastou, M. (2009). Patterns of individual differences in conceptual understanding and arithmetical skill: A meta-analysis *Mathematical Thinking and Learning, 11*(1), 25-40.

Gunning, Robert (1952). *The Technique of Clear Writing*. McGraw-Hill. Pp. 36–37.

Hale, J. B. & Fiorello, C. A. (2004). *School neuropsychology: A practitioner's handbook.* New York, NY: The Guilford Press.

Hansen, N., Jordan, N. C., Fernandez, E., Siegler, R. S., Fuchs, L., Gersten, R., & Micklos, D. (2015). General and math-specific predictors of sixth-graders' knowledge of fractions. *Cognitive Development*, *35*, 34-49.

Hecht, S. A., Close, L., & Santisi, M. (2003). Sources of individual differences in fraction skills. *Journal of Experimental Child Psychology, 86*(4), 277–302.

Hedman, Amy S. (2008). Using the SMOG formula to revise a health-related document. *American Journal of Health Education*. *39* (1), 61–64.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), Conceptual and procedural knowledge (pp. 1-27). Hillsdale, NJ: Erlbaum.

Jansen, R.J., De Lange, E., & Van der Molen, M. J. (2013). Math practice and its influence on math skills and executive functions in adolescents with mild to borderline intellectual disability. *Research in Developmental Disabilities, 34,* 1815-1824.

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology, 116*(1), 45–58.

Kieran, C. (2013). The false dichotomy in mathematics education between conceptual understanding and procedural skills: An example from algebra. In K. R. Leatham (Ed.) Vital directions for mathematics education research (pp. 153-171).

Kincaid, J.P., Fishburne, R.P., Rogers, R.L., & Chissom, B.S. (1975). Derivation of new readability formulas (automated readability index, fog count, and flesch reading ease formula) for Navy enlisted personnel. Research Branch Report 8–75. Chief of Naval Technical Training: Naval Air Station Memphis.

Klare, George R. (1974). Assessing Readability. *Reading Research Quarterly, 10*(1): 74.

Knuth, E., Stephens, A., Blanton, M, & Gardiner, A. (2016). Build an early foundation for algebra success. *Phi Delta Kappan, 97*(6), 65-68.

Lee, K., Bull, R. and Ho, R. M. H. (2013) Developmental changes in executive functioning. *Child Development, 84*(6), 1993-1953.

Lithner, J. (2008). A research framework from creative and imitative reasoning. Educational Studies in Mathematics, 67(3), 255-276.

Lynch, K. & Star, J. (2014a). Teachers' views about multiple strategies in middle and high school mathematics. *Mathematical Thinking and Learning,16*(2), 85-108.

Lynch, K. & Star, J. (2014b). Views of struggling students on instruction incorporating multiple strategies in algebra I: An exploratory study. *Journal for Research in Mathematics Education, 45*(1), 6-18.

Maciejewski, W. (2020). Between confidence and procedural flexibility in calculus. *International Journal of Mathematical Education in Science and Technology*, 1-18.

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wagner, T. I. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology, 41*(1), 49-100.

National Association of School Psychologists. (2009). *School Psychologists' Involvement in Assessment* (Position Statement). Bethesda, MD: Author.

National Center for Education Statistics (2019). Status and trends in the education of racial and ethnic groups. Indicator 11: Mathematics achievement. Retrieved from: https://nces.ed.gov/programs/raceindicators/indicator_rcb.asp

National Council of Teachers of Mathematics (2006). Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence. Reston, VA: Author.

National Council of Teachers of Mathematics (2016). Mathematics Education in the United States: A capsule summary facto book. Reston, VA: Author.

National Governors Association (2011). Building a Science, Technology, Engineering and Math Agenda: An update of state actions. Washington, DC: Author.

National Mathematics Advisory Panel (2008). Foundations for success: The final report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education.

National Research Council (U.S.). (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Newton, K. J. (2008). An extensive analysis of preservice elementary teachers' knowledge of fractions. *American Educational Research Journal*, *45*(4), 1080 – 1110.

Newton, K. J., Lange, K., & Booth, J. L. (2020). Mathematical flexibility: aspects of a continuum and the role of prior knowledge. *The Journal of Experimental Education*, *88*(4), 503-515.

Newton, K. J., Star, J. R., & Lynch, K. (2010). Understanding the development of flexibility in struggling algebra students. *Mathematical Thinking and Learning, 12*(4), 282-305.

Oltmann, J. R. & Widiger, T. A. (2018) Maladaptive variants of adaptive traits and bloated specific factors. *Journal of Research in Personality, 76*, 177-185.

Österman, T., & Bråting, K. (2019). Dewey and mathematical practice: revisiting the distinction between procedural and conceptual knowledge. *Journal of Curriculum Studies, 51*(4), 457–470.

Pintrich, P. R. & Zusho, A. (2002). The development of academic self-regulation: The role of cognitive and motivational factors. In A. Wigfield and J. S. Eccles (Eds.) *Educational Psychology: The Development of Achievement Motivation* (pp. 249-284).

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Journal of Child Psychology and Psychiatry, 37*, 51-87.

Robinson, K. M., & Dubé, A. K. (2013). Children's additive concepts: Promoting understanding and the role of inhibition. *Learning and Individual Differences, 23*, 101–107.

Rothwell, J., Lobo, J., Strumsky, D., and Muro, M. (2013). Patenting Prosperity: Invention and economic performance in the United States and its metropolitan areas. Washington, DC: The Brookings Institute.

Rittle-Johnson, B., Star, J. R., & Durkin, K. (2012). Developing procedural flexibility: Are novices prepared to learn from comparing procedures? *British Journal of Educational Psychology, 82*, 436-455.

St. Clair-Thompson, H. L. & Gathercole, S. E. (2006). Executive function and achievement in school: Shifting, updating, inhibition, and working memory. *The Quarterly Journal of Experimental Psychology*, *59*, 745-759.

Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations among conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology, 47*(6), 1525-1538.

Senter, R.J.; Smith, E.A. (November 1967). Automated Readability Index. *Amrl-Tr. Aerospace Medical Research Laboratories (U.S.)*. Wright-Patterson Air Force Base: 1–14.

Serpell, L., Teasdale, J. D., Troop, N. A., & Treasure, J. (2004). The development of the P-CAN, a measure to operationalize the pros and cons of anorexia nervosa. *International Journal of Eating Disorders*, *36*(4), 416–433.

Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as two sides of the same coin. *Educational Studies in Mathematics, 22*, 1–36.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engle, M., …Chen, M. (2012). Early predictors of high school mathematics achievement. Psychological Science, 23(7), 691-697.

Silva, P. J. (2018). *How to Write a Lot: A practical guide to producing academic writing. (Second Edition).* Washington, D.C.: American Psychological Association.

Skemp, R. (1976). Relational understanding and instrumental understanding. Mathematics Teaching, 77, 20-26.

Star, J. R. (2005). Reconceptualizing procedural knowledge. Journal for Research in Mathematics Education, 36(5), 404-411.

Star, J. R., Caronongan, P., Foegen, A., Furgeson, J., Keating, B., Larson, M. R., Lyskawa, J., McCallum, W. G., Porath, J., & Zbiek, R. M. (2015). Teaching strategies for improving algebra knowledge in middle and high school students (NCEE 2014-4333). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. Retrieved from the NCEE.

Star, J. R., & Newton, K. J. (2009). The nature and development of experts' strategy flexibility for solving equations. ZDM - The International Journal on Mathematics Education, 41, 557–567

Star, J. R., Newton, K., Pollack, C., Kokka, K., Rittle-Johnson, B., & Durkin, K. (2015). Student, teacher, and instructional characteristics related to students' gains in flexibility. Contemporary Educational Psychology, 41, 198-208.

Star, J. R. & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction*, *18*(6), 565-579.

Strait, J. E., Dawson, P. Walther, C. A. P., Strait, G. G., Barton, A. K., McClain, M. B. (2020). Refinement and psychometric evaluation of the executive skills questionnaire-Revised. *Contemporary School Psychology*, *24*, 378-388.

Toplak, M. E., West, R. F., & Stanovich, K. E. (2013). Practitioner re- view: Do performance-based measures and ratings of executive function assess the same construct? Journal of Child Psychology and Psychiatry and Allied Disciplines, 54(2), 131–143.

U.S. Department of Education, National Center for Education and Statistics (2016). The Condition of Education 2016.

Verschaffel, L., Lowel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education*, *24*(3), 335-359.