TEACHERS' BELIEFS AND PRACTICES IN RELATION TO REFORM ORIENTED MATHEMATICS TEACHING

A Dissertation

Submitted to

The Temple University Graduate Board

In Partial Fulfillment

Of the Requirements for the Degree

Doctor of Philosophy

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by

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August 2009

ABSTRACT

Teachers' Beliefs and Practice in Relation to
Reform Oriented Mathematics Teaching

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Doctor of Philosophy

Temple University, August 2009

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The core purposes of this study were twofold: (1) to ascertain whether mathematics teachers support reform oriented teaching practices, and (2) to discover whether there is correspondence between what classroom mathematics teachers say they should do when they teach mathematics and what they really do in the classroom. To carry out this investigation, elementary, middle and high school mathematics teachers responded to survey questions about their beliefs and practices and were observed. There are two major research questions that underlie this research and several secondary questions. The primary questions are:

1. Do in-service mathematics teachers support the major principles of reform oriented mathematics instruction?
2. To what extent do in-service mathematics teachers exhibit reform-oriented teaching in their classrooms?

Among the secondary research questions are the following:

3. Does professional development support reform oriented teaching practices?
4. Do teachers' beliefs vary with respect to the grade level they teach?
5. Do teachers' beliefs vary with respect to their levels of education?

The subjects were mathematics teachers from three grade levels, elementary, middle and high school selected from three school districts in northeastern United States. One hundred seventy -four mathematics teachers participated in the main study. Ten of the teachers who completed the Questionnaire voluntarily participated in in-class observations and post-observation interviews. The Reformed Teaching Observation Protocol (RTOP) was used for the observation. All 10 teachers were interviewed individually immediately either after the in-class observation took place or a day later.

The most salient finding of the study was that while teachers express a strong belief in the major tenets of reform oriented mathematics teaching, their actual demonstration of this type of teaching is far less evident. Pearson correlation analysis demonstrated only marginal relationships between teachers' demographic characteristics and their beliefs. A multiple regression analysis found that only 6% of the variance in beliefs is accounted for by the demographic variables. One of the major conclusions of the research is that teachers feel compelled to teach in ways that are discrepant from their beliefs in order to prepare their students for the standardized tests, which are now a critical component of educational accountability. Educational implications, limitations of the study and suggestions for future research are discussed.

ACKNOWLEDGEMENTS

I came alone but I could not have done it alone. First, I thank God for you Dr. Zebulon Kendrick. You were there just at the time when I needed you. You epitomizes what a Dean of Student should be. Of course Dr. DuCette, you were always there and it was so good to hear you say several times, “we are making progress”, and progress we have made. You were also very instrumental in helping me conceptualize my study topic. Thank you so much for the guidance you gave me in the many aspects of my dissertation.

Thanks to the superintendents, principals, mathematics supervisors, coordinators and teachers in the State for their willingness to support me in this endeavor. Thank you Ms. Skrabala for granting permission for me to use questions from the Rand instrument to construct the Questionnaire I used for this study. I also thank the authors of the Reform Teaching Observation Protocol (RTOP) for making the instrument publicly available to researchers like myself. The use of the RTOP was helpful for the in class observations I did.

I am very grateful to Dr. Snelbecker, Dr. Stahler, Dr. Fullard and Dr. Connell who serve as members of my dissertation committee. Dr. DuCette, you in particular, have been my advisor, my editor, my Chair, my supervisor and my mentor. I am very grateful to you for helping me overcame my weakness, for believing in me and for giving me the opportunity to develop my own style of writing. I am grateful for the challenges you gave to me and I have learned from each one of them. I sincerely believe that my education and training at Temple University would not be as complete had I not been given those challenges, and for that I say, thank you.

I especially like to thank Dr. Helmut Bartel, my initial advisor for his early direction, encouragement, and continued support. Thanks to Dr. Camille Bell-Hutchinson, Dr. Coleen Clay, Dr. Jerome Epstein, Dr. Rossi Hassad, Dr. Edison O. Jackson, Dr. Mrytle Jonas, Dr. Bart Pontecorvo, my aunt and confidante Dr. Ena Richards, Dr. Kaowle Soyibo, and my supervisor Dr. Ellen Sperber, for their moral support at different stages of my dissertation.

To the various members of the Temple University community, Linda Pryor and Marge Pippet in particular, and others whose names are too many to mention, I want to say a special thanks to you. To the librarians at Princeton University's Firestone Library thank you for allowing me access to invaluable research resources.

To the members of my church family you all have been a constant source of strength on this remarkable journey.

Finally and by no means least, I want to say thanks to my husband Robert Baird Paterson for his diligence and an eye for the details. Thank you for your love and support.

DEDICATION

I dedicate this dissertation in loving memory of my late father, Mr. George Alexander Barrett who insisted that his children should grow up and have a better life than he had, and my dearly beloved step-mother, friend and confidant, Mrs. Ada Barrett, whom I lovingly called Meada. The sacrifices you made for me in the early years of my development gave me the foundation on which to build. Because of that foundation I am able to earn a doctorate today. Your spirits have guided me through some of the most difficult times in this process. Meada and Dada, I know you would be very proud if you were able to witness this event-Vinie receiving her doctorate.

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CHAPTER 1

INTRODUCTION

Statement of the Problem

Despite a growing consensus among professionals and professional organizations about how to teach mathematics (Askey, 2001; Battista, 1994; Boaler, 1997, 2002a; Ellis & Berry III, 2005; Klein, 2003; Mason, 2000, 2002; Schoenfeld, 1994; Skemp, 1976; Watson, 2002a), several studies have shown that mathematics instruction in most American public schools is too teacher centered, with too much emphasis placed on lecturing and the use of textbooks. Those reports have also shown that teachers do not place enough emphasis on helping students think critically about mathematics or use their mathematical knowledge in real life situations (Cobb, Wood, Yackel & McNeal, 1992; Cohen, Mclaughlin & Talbert, 1993). In view of these problems, The National Council of Teachers of Mathematics (NCTM) has addressed the teaching of mathematics as a critical issue and has proposed a series of reforms to improve the way mathematics is taught at the elementary, middle and high school levels. For example, in 1989, 1991 and 1995, the NCTM published three standards documents, in addition to the more recently published Principles and Standards for School Mathematics*,* which suggested a set of guidelines for improving the teaching of mathematics (Romberg, 2001). Their efforts have helped to not only make the emphasis on traditional teacher-centered approach to teaching mathematics somewhat less favored, but have also made the teaching of mathematics into a national issue. In that regard, many researchers (e.g., NCTM 1989, 1991, 1995) including the Rand Corporation (Le et al., 2004) have developed a theory about how to appropriately teach mathematics. This theory is commonly referred to as the reform oriented instruction method.

According to Cruickshank (1990) and Reynolds (1992), in classrooms where mathematics instruction is reform oriented, teachers "promote active teaching" (p.21) and guard against students' disengagement in mathematics (Yair, 1999). In these classrooms, teachers demonstrate good knowledge of the subject matter, provide clear explanations of concepts and assignments, and interact with their students rather than leaving them to work in isolation. Brophy and Good (1986) said that in reform oriented mathematics classrooms teachers use short lectures and demonstrations to present ideas and information. They then do follow-up with students by giving them feedback based on their responses to discussion questions. They prepare their students to take part in follow-up seatwork activities. They provide students with clear instructions, review practice examples, and monitor students' progress on assignments after freeing them to work independently. Finally, these researchers said that teachers' who actively use reform oriented teaching follow-up with the appropriate feedback and reteach when necessary to make sure students understand (Brophy & Good, 1986). In that same vein, Slavin (1995a) said cooperative learning is encouraged, as students work together in small groups face-to-face.

Despite all the things that the NCTM, the Rand Corporation and advocates of the reform movement say teachers should be doing in terms of reform oriented mathematics teaching, there is no available database to indicate whether or not the teachers' practices are aligned with these guidelines. We also do not know whether or not the teachers who are teaching mathematics are (a) aware of the NCTM core principles, (b) if they agree with them and (c) if they actually practice them.

Hence the core question of this study is: to what extent do teachers agree with, and actually practice, reform oriented teaching methods? This study examined the practices and beliefs of elementary, middle and high school mathematics teachers from different public schools in the northeastern region of the United States. These mathematics teachers were asked to complete a survey, which assessed the extent to which they believed that the tenets of reform oriented mathematics teaching are appropriate. A subset of these teachers were then observed, to see how much of reform oriented practices they actually exhibit.

Purpose of the Study

The purpose of the study was to see whether mathematics teachers support reform oriented teaching practices and whether there is correspondence between what classroom mathematics teachers say they should do when they teach mathematics and what they really do in the classroom. To carry out this investigation, elementary, middle and high school mathematics teachers responded to survey questions about their beliefs and practices and were observed. There are two major research questions that underlie this research and several secondary questions. The primary questions are:

1. Do in-service mathematics teachers support the major principles of reform oriented mathematics instruction?
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Theoretical Framework

This study began by acknowledging the different historical, philosophical and pedagogical perspectives that led to the movement toward reform oriented mathematics instruction. It is guided by the beliefs and pedagogical approaches advanced by the educational theory of constructivism (Ernest, 2001 Cobb, Yackel, & Wood, 1992; Lerman, 1996), and complimented with views of mathematics that are fallibilistic (Ernest, 2001). It is also complimented with hermeneutic principles (Brown, 1991; 1994; 1996) with respect to the essential requirements and practices of mathematics teaching (Watson & Mason, 1998). Such terms of reference are acknowledged because much of the current reform efforts in mathematics education have grown out of constructivist approaches to teaching and learning. Constructivist epistemology focuses on “knowledge and learning” (Jaworski, 1994, p.70). More specifically, for mathematics learning, constructivism asserts that students learn best when provided with opportunities to engage in problem solving activities and to discuss, explain, and resolve conflicting interpretations (Ernest, 1994; Yackel, Cobb & Wood, 1991).

The fallibilistic view of mathematics is an important theoretical corollary to the constructivist approach and a more consistent substitute for the procedural formalist or traditional approach to mathematics teaching. The fallibilist accepts the ever expanding, variable, and non-deterministic view of mathematics as well as the historically, socially and culturally changing way in which the discipline and the science of mathematics are viewed. Hermeneutics explains the social, cultural and interpretive dimensions of mathematics. Both the fallibilistic view of mathematics and the principles of hermeneutics when applied to mathematics provide additional lenses through which teachers can view and expand notions of how to teach mathematics and a mechanism for interpreting mathematical word problems.

The main emphasis is that regardless of grade level, teachers' beliefs ([1]information correct or incorrect that a person has . . .) about mathematics influence their teaching of the subject. This assertion is made based on my agreement with researchers who posit that teachers' beliefs and practices have profound implications for the reform oriented mathematics teaching in the classroom (Battista, 1994; Manswell Butty, 2001; Timmerman, 2003). Thus, it becomes plausible that in an effort to understand what teachers believe and how well their beliefs are combined with what they say they do, I have joined other researchers who are beginning to look for other ways that the constructivist foundation of mathematics teaching can be complemented (Whiteaker, 2003). In that regard, I have advanced such understanding by including fallibilism and hermeneutic principles as essential parts of my study's basic framework.

Definition of Terms

For the purpose of this study the following terms are defined:

Hermeneutics. Hermeneutics refers to a view of mathematics that emphasizes the use of verbal, non-verbal and other social processes in the understanding of mathematical languages and symbols (Brown, 1991, 1994a, 1994b; Ihde, 2000).

Mathematical Activity. Mathematical activity describes "cycles of activities" (Whiteaker, 1995, p.24) or "set of [mathematics] activities" (Whiteaker, p, 24) derived from classroom observations and teacher interviews that are centered around a particular theme or concept.

Mathematical Literacy. To be mathematically literate is to know the language of the symbols and signs used in mathematics as well as their meanings. It is having the desire to explore mathematical ideas and concepts, to know what mathematical words mean and to use them in the right context to solve problems and to make decisions. It is being able to think, write, speak, read, and communicate about mathematics using words, grammar and syntax that are unique to the subject as a whole (Gee, 1996; Romberg, 2001; Organization for Economic Corporation & Development (OECD), 2003).

Mathematical Thinking. Mathematical thinking refers to the use of critical thinking skills within the domain of mathematics. This entails the use of mathematical processes such as generalizing, conjecturing, specializing, justifying, verifying, organizing, remembering, analyzing, testing, and inquiry. It refers to the use of mathematical language in a higher order of mental processes that involve many non-mathematical but related mental activities as well as the use of uniquely creative and inventive mathematical thought processes (Fisher, 1990; Krulik & Rudnick 1999; Mason, Burton & Stacey, 1985; Orton & Frobisher, 1996; Watson & Mason, 1998; Watson, DeGeest & Prestage, 2007).

Mathematical Understanding. Mathematical understanding is the person's ability to use mathematical knowledge to think and to reason about mathematics, to connect mathematical ideas, to know mathematical rules and having the ability to know when and how to apply these rules (Pirie and Schwarzenberger, 1988; Skemp, 1978; Marton & Saljo, 1997; Watson, 2002a; 2002b).

Reform Oriented Instruction. Reform oriented instruction describes a collection of instructional practices that are designed to engage students as active participants in their own learning and to enhance the development of complex cognitive skills and processes ((Le, Stecher, Lockwood, Hamilton, Robyn, Williams, Ryan, Kerr, Martinez, Klein, 2006). It is a type of teaching approach to mathematics that allows for teaching methods that make possible a focus on helping students develop mathematical thinking, reasoning and literacy

(Draper, 2002 Stein, Grover, Hennington, 1996; Tanner & Jones, 1999). This approach to mathematics teaching practices include strategies such as:

- Using open-ended questions
- Encourage discussion or debate
- Using manipulatives to solve problems
- Explaining mathematical thinking clearly and coherently
- Using correct mathematical language to communicate mathematical ideas
- Utilizing small group instruction, and oral presentation
- Responding to students' questions during seatwork
- Managing the classroom (Le, et al, 2006).

All of the terms described above are further elaborated on in Chapter 2.

Significance of the Study

The study is significant in that there is not a good understanding of what in-service mathematics teachers actually do, when viewed from the lens of how closely they are approaching what is called reform oriented mathematics teaching. Some teachers have "beliefs about mathematics that are incompatible" (Battista, 1994, p. 1) with those underlying the National Council of Teachers of Mathematics (NCTM) reform efforts. Indeed, according to Battista (1994), because teachers' beliefs play an important role in what they teach as well as in how they teach, such inconsistencies prevent reform and allow for the continued use of curricula and teaching [practices] that are detrimental "to the mathematical health of . . . children" (p.1). This then speaks to teachers' professional development, which also has implications for the way mathematics teachers are trained.

In Chapter Two, I examine research that addresses issues of a well-structured pedagogy which, taken together, would be best described as a reform oriented approach to the teaching of mathematics. These form the major theoretical framework that surrounds the nucleus of the review of literature and this study as a whole.

CHAPTER 2

REVIEW OF LITERATURE

This review of the literature is organized around three main themes that address various theoretical, philosophical and pedagogical issues associated with the teaching of mathematics. These include: beliefs about the nature of mathematics teaching; constructivism; and reform oriented instruction. These three themes encompass research that spans more than half a century of investigation. Most, if not all, of the information discussed in the research, when taken together, can be best described as a reform oriented teaching approach to classroom mathematics.

Context

While this study is not specifically about educational reforms, it would be remiss not to provide some historical information concerning pedagogical and curriculum advances in the reform of K-12 mathematics teaching and learning in America, during the last 30 years. This information is relevant because in terms of the literature review for this study it is important to know what some of the concerns regarding mathematics teaching and learning were like in the past compared to what they are now. The most important of these discussions centered on mathematics content and pedagogy from the early 1990's to the present. This section will therefore provide some account of the advances that took place in K-12 mathematics education before and during that period in the United States from an historical perspective.

Mathematics education in American schools has been the focus of many reform movements since the 1920s. Throughout and after World War II, the early Civil Rights era, and throughout the ideological and intellectual competition of the "Cold War" toward the end of the 80's, reform in mathematics education remained constant if only partially practiced in the classrooms. However, this movement became much more widespread and widely practiced in the 1990's. Around this time it also triggered what many in the educational arena pejoratively described as a continuation of the "math wars" (Klein, 2003, p.1). Klein (2003) said the math wars were the culmination of a widespread dissatisfaction by some policy makers, educators and parents over what should constitute the established standard for the teaching and learning of mathematics. As such, he indicated that the 1990's were perhaps the most "tumultuous period" (Klein, 2003, p.2) in the history of mathematics education in America.

Much of the tumult he said stemmed from a carryover of the disagreement over the introduction of new mathematics textbooks that contained significantly diminished content and a dearth of basic skills. Mathematics pedagogy was also greatly scrutinized. Klein said the public's scrutiny was due in large part to what Kilpatrick (1992) and later Usiskin (1997) had presented as the growing concerns of some public school administrators, and instructors' reaction to the weakened support for the behaviorists' (direct) approach to the teaching of mathematics. The behaviorists' approach viewed mathematics as a set of discrete knowledge and skills, which could be transmitted to the students in a seemingly top down manner. Similarly, Klein (2003) said there was beginning to be greater regard for the pedagogical principles of the progressive reform movement that were applied to the teaching and learning of mathematics. Progressive reformers, he said, advocated an approach that emphasized discovery learning, was student-centered with less focus on operational and computational content, and placed greater emphasis on developing mathematics as an important everyday resource that has practical utility. Furthermore, he said it was the shift in focus that led to the use of the adage, “we teach children not subject matter” (Klein, p.5).

Along these lines, Klein (2003) said that those charged with the responsibility for developing an effective mathematics curriculum (which was considered by most to be the first step toward a change in emphasis) began to change the way they taught and viewed mathematics. Such changes, he said, were necessary because there were still many at that time who didn’t accept the general need to broaden mathematical thinking by making it a part of all subject areas. Some even argued against the need to establish curriculum and teaching standards for pre-school and early childhood education that emphasized proficiency in reading and comprehension as important learning skills to have in place in order to do mathematics. At that time, neither the teaching nor the learning of mathematics was undertaken as an “activity” (Brown, 1994a, p. 148). Brown (1994b; 1996) would later explain the concept of mathematics as an activity (1994b; 1996) in his discussion on hermeneutics and the *doing* of mathematics. (Further discussion of Brown’s ideas on the issue of mathematics as an activity is presented later in this chapter and in the discussion in Appendix A).

Notwithstanding the critics’ lack of acceptance, Klein (2003) posited that this change in curriculum was what reformers hoped to achieve. In addition, he said that the early institutional advocates of this progressive reform movement-The National Commission on Excellence in Education: A Nation at Risk (NCEE, 1983) and the National Council of Teachers’ of Mathematics (NCTM, 1989)- in their standards and criteria for teaching mathematics, began to stress the need for many more mathematics teachers who were highly capable facilitators, who possessed the necessary mathematical knowledge and skills, and who could effectively use them to present the subject matter. These institutional advocates of reform in the teaching of mathematics also believed there was a need for basic pedagogical reforms that would make the development of mathematical concepts the most important academic and critical thinking activity, and thus make way for the fulfillment of their recommendation for greater student involvement and less “teacher directed instruction” (Klein, 2003, Pp.8-11).

Despite the public’s support for the new progressive teaching reforms, Klein said there were still many who considered the behaviorist or “teacher directed instruction” (p.21) (that promoted limited and even restricted approach to the teaching and learning of mathematics) to be the appropriate and justifiable method of instruction. Despite their objections, he said, the new approach offered by the progressive movement became the dominant approach at the time because it was seen as having a broader and more inclusive utility that could go beyond simply furthering the mathematical needs of a few, toward transforming mathematics instruction as a way of thinking. Most importantly, Klein (2003) said that mathematics teaching and learning would now be undertaken not only for the purpose of developing basic math and computational skills, but also as a way to broaden opportunities for students to take more rigorous and advanced mathematics (Klein, p.11). In fact, it is the kind of mathematics that many researchers (e.g., Abedi & Lord 2001; Chubb & Loveless, 2002; Kennedy, 2005; Powell, 1990; Ryan & Ryan, 2005) and some policymakers now see as the gateway to better educational opportunities.

Still another important advancement that drew support from the progressive movement was a new approach to teaching and learning mathematics that led to widespread modification and expansion of K-12 mathematics curricular content and pedagogy. Klein

(2003) even posited that this move followed Askey's (2001) Great Curriculum Debate on how to teach reading and math. Years later, Ellis and Berry (2005) would describe such debate as a major shift in the paradigm that would determine what should constitute mathematics teaching and learning. Indeed, the authors said that such a shift in paradigm created what many had hoped for: a mathematics curriculum that reflected not only the needs of students but also the larger desire for societal change that would improve opportunities for advancement. Ellis et al. (2005) said what was most needed was a greater focus toward pedagogy. Consequently, they said a variety of strategies and interventions must be explored toward achieving the objectives of the "New Math" (p.8). The National Science Foundation (NSF) assumed the principal role in facilitating the change in curricular reform and pedagogy.

The National Science Foundation (NSF) for a long time had been funding projects aimed at developing new textbooks, and creating experimental pedagogy. All of these efforts however, did not bring about what the reformers had hoped for. Thus, the NSF became more actively engaged in supporting the development of new materials, centered on promoting this new approach that resulted in the widespread reform of curricula in the teaching and learning of mathematics.

Ellis et al (2005) said that central to the National Science Foundation projects were activities that facilitated the development of a discovery-based learning approach to the teaching and learning of mathematics. One such project was the project developed by Max Beberman and his colleagues at the University of Illinois. Teachers who participated in this project received several weeks of intensive and costly training before and after the project was implemented. Indeed, according to the authors, the philosophy of the training workshops in which the teachers participated was guided by discovery learning principles. One of the basic tenets of discovery learning is the notion that if mathematics is to be successfully taught and learned there should be a strong connection to students' real world experiences and interests. Klein reported that the discovery learning approach has been adapted and promoted by the National Council of Teachers of Mathematics under the name "constructivism" (Klein, 2003, p.13).

Hirsch Jr. (1996) reported several years ago that constructivist or discovery learning approaches were first used by cognitive psychologists but later became popular in the educational arena as a new approach to instructional pedagogy. He said that constructivism was also greatly criticized but mostly by supporters of the old behaviorist paradigm. Some writers later construed the old paradigm as the "Traditionalist Formalist Paradigm" (Ellis et al., p. 11). Despite this benign sounding name, the criticisms made about constructivism were more ferocious, but they did not dampen the resolve for reform held by some notable mathematicians and cognitive scientists, including John Anderson, Lynn Reder and Herb Simon of Carnegie Mellon. These mathematicians who had been early supporters of the adoption of constructivism as the new pedagogy now began to strengthen their resolve by drawing on the principles of constructivism found in the developmental research of Jean Piaget and Lev Semenovich Vygotsky. It was Klein (2003) who earlier said that both sets of ideas (Piaget's developmental stages of learning and Vygotsky's Zone of Proximal Development) were, at that time, consistent with the new instructional orientation of most colleges of education. By this time, these institutions, he said, had long embraced constructivist learning approaches and, for the most part, practiced teaching pedagogies that stressed the need for teachers to develop the capability of individual students within a broad learning environment.

In the remaining paragraphs, I will attend briefly to what I perceive as the motives for these reform initiatives in mathematics education, which took place during the period beginning with the last decade of the 20th century and have continued into the first decade of the current century.

At the end of the twentieth century leaders from business and industry, politicians (particularly those responsible for education policy), concerned citizens, scientists and mathematicians as well as a few notable citizens (some involved in advancing the effectiveness of mathematics and science research, some experienced in the reform of curriculum instruction and science and some with specific expertise in computer science and technology) all continued to express disappointment over the general academic and mathematical under-preparedness and under-performance of America's students in mathematics. Ball, Hill and Bass (2005) suggest that their prominent expressions of disappointment were centered mainly around reports provided by the TIMSS (Third International Mathematics and Science Study) and PISA (Program of International Students Assessment) studies of specific scores of American students on tests of performance in mathematics. These reports compared American students' relatively poor performance to the level of performance and rate of improvement of their international peers from countries such as China, Singapore, Belgium, India and Japan. In a later study, Wang and Lin (2006) suggest that the better performance of students from these other countries could be attributed to their countries' investment in developing the mathematical and scientific capability of their teachers. By using a better aligned and more stratified curriculum, teachers were not only able to spend more time explaining mathematical concepts to their students, but were also able to encourage more teacher- to- teacher collaboration. This entire movement, therefore, has been focused on teacher preparation and training as the critical element to make the reform movement successful. As the President of America's Education Trust, Kati Haylock, said:

> 'We're headed in the right direction, but not quickly enough.' Results simply signal what we already know: We need to focus far more energy on getting strong teachers to the children who need them and on providing those teachers with quality curriculum and support, because accountability is not enough' (p.1).

Beliefs about the Nature of Mathematics Teaching

Several researchers (Bell-Hutchinson, 2005; Clark & Peterson, 1986; Ernest, 1996; Ollerton & Watson, 2001; Prawat, Remillard, Putnam & Heaton, 1992) have emphasized the critical relationship between what a teacher believes is correct in terms of pedagogy, and what the teacher actually does in the classroom. Bell-Hutchinson (2005) takes this general observation and focuses it on mathematics by pointing out that teachers' beliefs, concerning the essentials of mathematics are likely to determine how they present the subject to students.

What, then, are these beliefs and what impact do these beliefs have on the way mathematics is taught? As pointed out in Chapter 1, the traditional (Procedural Formalist and the Social Efficiency, or behaviorist) approaches that emphasized rote teaching and memorization, were the dominant pedagogical approaches used in teaching mathematics in America's classrooms for many years. As also pointed out in Chapter 1, the last decade of the 20th century saw a turning away from the traditional belief toward a more constructivist approach. This reform movement, however, saw a major hindrance to its advance with the enactment of the reform movement under President George W. Bush. As this dissertation will show, the current focus on achieving annual yearly progress as determined by the No Child Left Behind Act (NCLB) has forced many teachers to revert to a traditional approach to teaching mathematics through necessity. In the current educational climate, where attaining yearly progress goals is determined by students' performance on standardized tests, Ernest (2001) and Andrew (2006) point to the obvious fact that the main goal of teaching mathematics is to teach the students to obtain the "right" answers. These right answers, they posited, are typically attained through the manipulation of abstract symbols following established and sometimes well-rehearsed algorithms. Borasi (1992) in expanding on those views captured the essence of what happened in those mathematics classrooms years ago and what is increasingly happening today, when he wrote the following:

> *Arithmetic computation* is entrenched as the basis of the mathematics curriculum, with the four rules gradually being developed to handle more and more complicated numbers-natural, integer, fractions, decimal, and later, matrices and vectors. *Algebraic work* develops the skills of solving more and more complicated equations and of rearranging complicated expressions so that they can be solved. *Geometry,* if taken seriously at all, is developed as an area to which one can apply arithmetical and algebraic techniques, be it thereby trigonometry or coordinate geometry. And for those who have succeeded at, or survived, that diet, the gateway to further delight is *the calculus*, with its myriad of integrals and differential equations waiting to be recognized, classified, and of course, solved (Borasi, p.1)

The issue facing the teaching of mathematics today, then, is to somehow balance the idea that the nature of mathematics represents much more than simply a set of established facts to be learned and a group of problems to be solved. The entire issue of the inherent nature of mathematics is fundamental to a deeper understanding of several critical issues to the mathematical teaching reform movement. Since this discussion is somewhat peripheral to the main topic of this dissertation, a presentation of how mathematics is viewed from a fallibilistic and hermeneutic perspective is presented also in Appendix A.

Why Teachers Teach Mathematics

Is mathematics important? Why do some teachers hold "students' feet to the fire" (Bullock, 1994, p.735) when they fail to perform? What is it that drives some teachers to want to teach mathematics while others approach the subject with some degree of trepidation? For the purpose of the review of literature for this study, I will limit my focus on one aspect of the question. Put simply, why do teachers teach mathematics?

Most people, even those who don't do well in it, would agree that mathematics is one of the few highly respected courses in the curricula of many American schools. In fact, in the United States I believe there is no other subject, with the exception of reading, that attracts the attention and the emotional concern of as many people as mathematics does. At every level of education, as well as in the business world, mathematics holds a special place above the other disciplines and no one bothers to question the academic pedestal upon which it is put. Our society seems to have such high regard for the subject that it is included in "high stakes assessment" (National Research Council, 2003) and looked upon as the "gatekeeper" (Ryan & Ryan, 2005, p.53), the mastery of which will open up many educational and economic opportunities (Viadero, 2005). Ball, Hill and Bass (2005) best describe the importance people in our society place on this subject very well by saying that with the release of every new international mathematics assessment, concern over U.S. students' mathematics achievement has grown even greater. Is the need for such concern justified?

Policymakers, educators, parents and students seem to agree that mathematics deserves the heightened level of concern as well as the emphasis on reform that it has received. Many of them had based their conclusions on what they perceived as the practical usefulness and the importance of mathematics as a subject in the curriculum. The National

Curriculum of England and Wales (Department of Education and Employment, (DfEE), 1999), the Principles and Standards for School Mathematics (2000) published by the National Council of Teachers of Mathematics in the USA, and the Revised National Curriculum Statement published by the Department of Education in South Africa (Department of Education, 2002) are examples of curricula which, in different ways, allude to the importance given to mathematics.

It is clear from these examples, however, that people around the world and in America perceive usefulness in different ways. Despite the differences, the research (PISA, 2003) seems to establish that mathematics is important for all aspects of life including the workplace (Ben-Yehuda, Lavy, Linchevski & Sfard (2005). Paradoxically though, there are many who are still being told that much of the school mathematics they learn will be of no value to them in later life. It was Devlin (2000), for example who said few citizens in modern society ever need to make real use of any appreciable knowledge of, or any particular skill in mathematics. Devlin (2000) claimed that much of the mathematics, which most people really need and use has already been acquired by the time they are twelve years old. Devlin's (2000) statement gives rise to several important questions. If, indeed, the average citizen does not use most of school mathematics, then would it not be prudent and responsible for school administrators and teachers to reduce the content, the rigor, and the amount of the school mathematics? Also, if competitive attainment of a higher quantitative competence and improvement in international assessment are the only reason for teaching mathematics, should we really care how American students perform on these assessments?

There are several reasons for posing these questions. The first question is to bring to the fore the ever-present tension of trying to reconcile the needs of students who will go on to study mathematics at a higher level, with those whose ambitions do not necessitate more than the basic skills in mathematics. Furthermore, a common sense approach would be to provide to all students as broad a base in mathematics as is feasible, in order to meet the needs of all, even those persons who may not choose to make use of it. That being said, I am convinced that there must be a place for the nurturing of those who see and teach mathematics as a lifetime skill (PISA, 2003). On the other hand, what type of mathematics is considered appropriate and how much of it should be offered to the average person will always be difficult and controversial questions to resolve and the correct answer may not always find consensus (Orton, 1994a). Orton (1994a) said Smith (1928) several years ago succinctly addressed this dilemma when he wrote:

> A subject even so essential as [mathematics] in our world economy today need not be mastered by every citizen . . . [but] every educated man or woman should know what mathematics means, what its greatest uses are, and something of its soul, and should thus be able to decide whether or not he or she cares to pursue its study beyond the point of acquiring this elementary knowledge . . . everyone should know . . . what mathematics means, at least for the reason that the world uses it so extensively (Smith 1928, cited in Orton, 1994a).

I believe this tension is one that is strongly felt in the mathematics community today, especially in poor urban school districts where there is a high concentration of uncertified mathematics teachers (U.S. Department of Education, 2003). Many of these teachers do not have the academic background or the professional development training to teach basic mathematics (U.S. Department of Education, 2003) much less at an advanced level. Earlier in this chapter, I presented research that suggests that the basis for wanting to advance mathematics teaching and learning is to make “rigorous and advanced mathematics” (American Institute for Research, 2005, p.11) available to all students. But based on Orton’s assessment it is possible to argue that mathematics is not necessary for some students to learn, especially if there seems to be little reason to assume that they make use of it in their careers.

Goldenburg (1998) believes the tension created over who should and should not learn mathematics [and in what quantity] can be overcome. In that regard, he proposed that the mathematics curriculum should be organized around 'habits of mind' (p.8). Goldenburg, said:

> 'habits of mind' are mathematical ways of thinking which increases the coherence students see in mathematics. . .connect mathematics to the rest of students' experiences; and bring a culture of mathematical exploration into the classroom. Consequently, it is not the **content** of mathematics that should be the focus of attention but rather the "habits of mind" (Goldenburg, 1998, p. 8).

Thus, the author said, in so doing, the question shifts from what content is at the core of mathematics teaching, to what habits of mind are at its core. When viewed in this manner, the two questions posed earlier are easily answered. Further, Goldenberg (1998) said:

> . . . we must look for mathematical ways of thinking that support almost any vocation or avocation . . . These ways of thinking – despite the fact that they serve people outside of mathematics – deserve to be called mathematical ways of thinking because they are absolutely central to mathematics, particularly apparent and well refined within mathematics, and readily learned in mathematical study (Goldenberg, p.7).

Goldenberg's (1998) central claim is that by considering the concept of "habits of mind" (p.8) as the organizing principle for the mathematics classroom, the needs of both those who will pursue rigorous and advanced mathematics and those who will not are adequately served. That is because it is possible to choose those "habits of mind" (p.8) that best serve the future needs of most students while also meeting a school's immediate general education objectives. Below is a list of ten mathematical ways of thinking to which Goldenberg (1998) refers. I have included them here because they add an additional dimension to this review and for this study. They include:

- The inclination to visualize;
- The inclination to interpret diagrams;
- The inclination to describe formally and informally
- The inclination to translate between visually and verbally presented information;
- The inclination to tinker (to tinker is to explore different possibilities when working with mathematical entities. For example, a problem posed in two dimensions may be re-examined in one or three . . . .)
- The inclination to mix experiment with deduction;
- The inclination to build systematic explanations and proof for observed invariants;
- The inclination to construct and reason about algorithm;
- The inclination to reason by continuity (one of many connections with analysis) (Goldenberg, p.8).

This view resonates well with the discussion on reform oriented instruction because the "habits of mind" approach as Goldenberg (1998, p.8) enunciates it . . . with the emphasis placed on mathematical ways of thinking, in my view, bears a relationship to the reform oriented method of teaching mathematics. This is because these approaches have at their core the development of both the learner's thinking ability and problem solving skills (Lockwood, Le, Stecher & Hamilton, 2005).

Problem Solving

Another reason for the central place mathematics holds in the curriculum over and above other academic subjects is its unique potential to enable the development of students' word problem-solving skills. Solving problems has been part of the school mathematics curriculum for ages although word problem-solving or application problems are more recent (Stanic & Kilpatrick (1989). In fact, the change in emphasis on problem solving in mathematics curricula began in the 1980's and was the result of lobbying interest groups such as "Agenda for Action" which at that time was associated with the National Council of Teachers' of Mathematics (NCTM). This group recommended that word problem solving or solving application problems presented in word forms to be the focus of school mathematics in the 1980s and beyond. Hence it is believed that the growth of students' word problem-solving skills is an important outcome of the mathematics instruction to which the students are exposed (American Institute for Research, 2003).

That being said, I will now discuss the second question posed earlier. That question can be summed up in this way: Is gaining competence in mathematical content the only reason for teaching mathematics? As Boaler (2002a; 2002b) indicates gaining competency in mathematical content is certainly not the only reason for teaching mathematics. For me, Boaler's conclusion is plausible. Likewise, Bell-Hutchinson (2005) said even without holding such a perspective there are a number of other important reasons for teaching mathematics aside from acquiring quantitative competence. Some of the well-acknowledged ones the author describes are:

- mathematics facilitates the development of mathematical thinking (including spatial thinking), of reasoning skills, and of concepts such as ideas of proof;
- mathematics can be used to predict, explain and describe phenomena;
- mathematics is the foundation for science and technology;
- mathematics provides opportunities to promote cultural development
- mathematics provides a powerful means of communication (Bell-Hutchinson, 2005, p.63).

I believe Friel's (1998) earlier report reflects well on Bell-Hutchinson's (2005) statement above when he said that the importance of students having strong conceptual understanding of mathematical ideas couldn't be overstressed. In what seems like an attempt to recognize this view, Bell-Hutchinson (2005) said many curricula documents now refer to 'data-handling' (p.63) rather than 'statistics' (p.63), thus shifting the emphasis from the mere computation of measures to conceptual understanding of these measures and the collecting, organizing, analyzing and interpretation of different types of data (Bell-Hutchinson, 2005). In the same way, if mathematics learning is to be truly empowering in the ways Friel (1998) described earlier, then teaching approaches, which facilitate these activities, must be employed. I believe that reform oriented instruction can be seen as one such approach. As such, it may allow teachers to see greater results of their teaching and enable learners (students) to begin to realize the many benefits of their mathematics education.

The Principles and Standards for School Mathematics (NCTM, 2000), puts forward yet another important benefit to be derived from mathematics learning. It posits that all students need an education in mathematics in order to be prepared for a future where the underpinnings of everyday life are increasingly mathematical and technological. *The Standards* indicate that a society in which only a few have the mathematical knowledge needed to fill the varying critical roles is neither consistent with the values of a just democratic system nor its economic needs. The Standards state:

> In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps these doors closed (p.5).

Though not directly stated, implicit in this view are the notions of equity and access, which bring to the fore the socio-cultural issues centered around mathematics teaching and learning.

Constructivism

The philosophical, theoretical and pedagogical underpinning that underlies the reform movement in mathematics education is called constructivism (von Glasersfeld, 1989). Constructivism is concerned with "how people come to know. It is about knowledge and learning" (Jaworski, 2002, p.70). There are many different views of constructivism

(Carpenter, 2003; Cobb, Yackel & Wood, 1992; Ernest, 1994). In fact, Carpenter's (2003) view about constructivism is that "it is a group of theories about learning that can be used to guide teaching" (Carpenter, 2003 p. 29). In terms of its application to mathematics teaching, Reys, Suydam, Linquist, and Smith (1998) said that teachers who have adopted constructivist theories believe that children construct their own mathematical knowledge, rather than receive it in finished form from the teacher or textbook. So, rather than simply accepting new information, the authors said students interpret what they see, hear or do in relation to what they already know.

Jaworski (1994) and Simon (1995) described constructivism as a philosophical perspective on knowledge and learning which has had a significant impact on mathematics pedagogy. They argued that a constructivist view of mathematics is derived from a philosophical position that as human beings we have no access to an objective reality, that is, a reality independent of our way of knowing it. Putting it more simply, Confrey (1990) posits that constructivism can be described as:

> . . . . a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our cognitive acts. We can have no direct or unmediated knowledge of any external or objective reality. We construct our understanding through our experiences, and the character of our experience is influenced . . . by our cognitive lenses (Confrey, 1990, p. 110).

From time to time, many people have supported these philosophies over the years though they may differ in perspectives. Consequently, some have adopted different names for what they believe in as a way of distinguishing their philosophical positions. Such names include, 'radical constructivism' (more often just described as constructivism) inspired by Piaget's epistemological ideas, and 'social constructivism', which Lerman (1996) said is based on Vygotsky's psychological theories. The remaining discussion focuses on radical constructivism philosophy.

Radical Constructivism

In building on Piaget's work, von Glasersfeld (1995) claims that radical constructivism starts from the assumption that knowledge, no matter how it be defined, is in the heads of persons, and that the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience. Along these lines, von Glasersfeld (1995) said two basic tenets of constructivism are:

- Knowledge is not passively received but built up by the cognizing subject;
- The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality (von Glasersfeld, 1995, p.1).

Orton (1994b) places these principles within the context of the classroom and suggests the following hypotheses:

- Knowledge is actively constructed by the learner, not passively received from the environment;
- Coming to know is an adaptive process that organizes one's experiential world; it does not discover an independent, pre-existing world outside the mind of the knower (p. 38).

The author said a major implication of the first assertion for teaching is that one cannot assume that learners will benefit from being told anything. In other words, the traditional transmission mode of teaching has no place from a constructivist perspective and one must reject the assumption that one can simply pass on information to a set of learners and expect that understanding will result (Orton, 1994b). He argues that the second hypothesis is a much more radical one since it forces one to think of reality. When taken at face value the proposition suggests there is no reality. In that regard, Jaworski (1994) said that it is not that constructivists believe that no reality exists; rather, they only claim that one cannot directly know this reality. In that same vein, von Glasersfeld (1995) argued that in keeping with the biological metaphor, "fit or viability" (von Glasersfeld, p. 1) is all that we can aim for in coming to know any construct. Taken to the level of the classroom, Begg (1995) refers to this need for fit as an adaptive process, which occurs when students intentionally take on new ideas and attempt to fit them in by linking them with their prior experiences and understandings. These prior ideas, according to Begg (1995), progressively influence learning by continuously adjusting the lens through which new ideas are viewed.

In fact, with regards to the teaching and learning of mathematics, Towers (1998) said radical constructivists hold that there can be no way of knowing that a problem or mathematical concept has the same structure for different individuals, not because it might be found that each person constructs his or her own knowledge differently, but rather because radical constructivist epistemology does not ever permit us to conclude that two individuals have the same knowledge (Towers, 1998).

What does this way of thinking about the development of knowledge and understanding hold for the mathematics classroom? The implications are profound and very significant for how instruction is pursued. How should mathematical understanding be interpreted within such a perspective? How should learning be conceived? As mentioned previously, in the current educational environment it is critical that students arrive at the correct answer to questions asked on the high-stakes standardized tests that underlie NCLB. In other words, the teacher must have some way by which it is possible to accurately interpret when the student's "fit" is in full alignment with the "fit" established by the objectives set by the teacher in the classroom, objectives which are assumedly in alignment with state standards.

There is no easy way to do this within radical constructivist philosophy. The outcome of this is a mismatch between the philosophical principles that teachers have been taught and the reality they face in their classrooms. Faced with a demand to make sure that their students pass tests, teachers find it increasingly difficult to teach in a way they believe is best philosophically, but which possibly places their students, as well as themselves, at risk for failing to meet annual yearly progress goals.

Social Constructivism and Socio-cultural Perspective

Neyland (1995) posits that one of the basic tenets of social constructivism is the belief that all mathematical knowledge is socially constructed, and classroom pedagogy should reflect this way of thinking. Another important tenet of social constructivism is Ernest's (1994) notion that both social processes and the way the individual constructs meaning are central, if not essential, to the learning of mathematics. Along these lines, Nickson (2000) said:

> . . . because mathematical meaning is inherently dependent on the construction of consensual domains, the activities of teaching and learning must necessarily be guided by obligations that are created and regenerated through social interaction (p. 230).

This review recognizes that social constructivism has been given a number of different interpretations over the years (Lerman, 1994). Nevertheless, the most popular view is that the social domain does indeed have an impact on the individual. It may do so in several different ways. Its importance depends on the meaning that the individual constructs. This meaning originates from experiences gained in a social context. Lerman (1994) said that by embracing this view it is always possible to recognize and appreciate the meaning making of the individual. In assuming there is a middle ground between individualistic and collectivist perspectives, Bauresfeld (1988, cited by Lerman) posits that it is plausible to assume that a teacher and his/her students communicating cohesively and interactively together constitute the culture of the classroom. Indeed, according to Tanner and Jones (1999) culture of the classroom is characterized by the subjective reconstruction of knowledge through negotiation of meaning making in social interaction.

The view taken by Tanner and Jones (1999) to some extent does speak to earlier views of ‘fit’ and ‘viability’ (Goldenberg, 1998, p.8) discussed in this review. Cobb, Yackel and Wood (1992) also support this view. For example, these authors said it is through the process of negotiation of meaning making that ‘fit’ is achieved through taken-as-shared meanings. In terms of application to classroom mathematics teaching and learning they argue that;

> . . . it is potentially more fruitful for our purposes as mathematics educators to view students as actively constructing mathematical ways of knowing that make it possible for them to participate increasingly in taken-as-shared mathematical practices. From this perspective, mathematical truth is accounted for in terms of the taken-as-shared mathematical interpretations, meanings, and practices institutionalized by the wider society. The notion of mathematical truth is therefore dealt with pragmatically (Cobb, Yackel & Wood, 1992, p.16).

This kind of classroom learning environment, in my view, is typically represented by the concepts and methodologies and instructional strategies of reform oriented teaching. I also believe reform oriented teaching is one method for teaching mathematics that can show how the principles of constructivism, social constructivism and socio-cultural perspectives can be effectively expressed in the mathematics classroom. But there are other views still being applied in the mathematics classroom that the research generally discredits as the most effective ways to teach the new understanding of the nature of mathematics. The next section explores some of these other views that still persists in the mathematics classroom despite the research that shows constructivism to be the most effective way to teach such an understanding.

Another View of Constructivism

In the preceding sections, I have offered some positive views of constructivism. The purpose of this section is to present another view of constructivism by Fox (2001) that contradicts claims uphold by this theory. I will begin with what Fox (2001) described as "one of the major weaknesses" (p.23) of constructivism. He said constructivism claims, "effective learning requires meaningful, open-ended, challenging problems for the learner to solve"(p.24). Fox (2001) opposed this claim, saying:

> "One may well agree with this [claim] as a general prescription for the curriculum, though noting that some rather less challenging kinds of instruction and practice may also be helpful, [to students] but it is difficult to see why it should follow from any of the earlier claims of constructivism, any more than from any other view of learning . . . motivating learners to engage with the topic requires more than simply facing them with new learning to do" (Fox, 2001, p. 33).

The notion of . . . "some rather less challenging kinds of instruction . . . " that is being promoted here by Fox, in my view, would again lead to a segregated classroom where the teaching and learning of mathematics is pursued by both the teacher and the student on the basis of some ascribed notion of capacity to learn mathematics (e.g. Johnny can learn math but Mary will not get it so there is little need to try). In addition, I believe like any other theory of learning, constructivism may present problems to some teachers in the application of key principles of the reform oriented approach to mathematics teaching (e.g. problems in managing the classroom, or with the use of open-ended problems). This is especially true if teachers are asked to apply these principles without adequate support and resources or the kind of training they would need to effectively convey to students the new understanding of the nature of mathematics. Therefore, what Fox (2001) has indicated as a weakness for constructivism, I see as merely a problematic condition. In that regard, I believe it is essential that teachers adhere to the scientific undertakings of the mathematics community that supports constructivism as the new approach to the teaching and learning of mathematics. I also believe that at the present time, there is no reasonable alternative to the reform oriented constructivist approach to mathematics teaching and learning presented here.

That being said, there are two principal examples of such alternatives, the Singapore Math Method of teaching mathematics that is promoted in the United States in some texts as "a good alternative to reform mathematics [teaching] (http://Wiki/Singapore_Math, p. 1). It is offered as a method that could bring about better mathematics teaching outcomes. The other is the Saxon Math method (http://education-reform/saxon, p.1), which is largely promoted by "members of the home schooling community". Neither of these methods should be viewed as reasonable substitutes for the reform oriented constructivist approach, as some would dare to suggest. The Singapore Math Method, in my view, is yet another example of the traditional formalist paradigmatic or instructionist approach applied to the extreme. That approach to instruction in Singapore is, in my view, an approach to mathematics instruction that works from the top down, whereas, constructivism advocates an approach that works from the bottom up. Constructivism also emphasizes the social, cultural milieu of the classroom environment as one of the key elements in the teaching of the new understanding of the nature of mathematics. The recognition of such an emphasis could become problematic for homeschoolers who may have differing views on the importance of the social context for teaching and learning the new understanding of the nature of mathematics.

Reform Oriented Instruction

The literature on the teaching and learning of mathematics contains a variety of views on what pedagogical reforms are needed to improve performance in mathematics, what adjustments and reorientations are needed in teacher preparation and curriculum content, and what constitutes an effective approach to mathematics instruction (American Institute on Research, 2003; Draper, 2002; Le, Lockwood, Stecher, Hamilton, Williams, Robyn, Ryan & Alonzo, 2004; National Assessment on Educational Progress (NAEP), 2005, 2006, 2007). In the discussion of the reform-oriented approach that follows, I will pinpoint those views that have direct relevance to this study.

Silver and Stein (1996) said, that in contrast to a conventional or traditional approach, reform oriented instruction allows for pedagogical approaches that focus on helping students develop more meaningful mathematical understanding. They found that focusing instructional tasks on helping students gain a deeper understanding of mathematics was particularly important in assuring that students acquire a broader, more useful and interdisciplinary way of thinking through an expanded knowledge of new mathematical ideas.

This deeper understanding was typically achieved through one of the fundamental strategies that Le et al. (2004) say is aligned with reform oriented instruction in mathematics which is the full and effective embrace of the active engagement form of instructional pedagogy. Mathematics classrooms that use reform oriented instruction usually unfold within a sequentially challenging series of mathematical tasks. This approach to the teaching and learning of mathematics allows both teachers and students to place primary emphasis on developing mathematical thinking, mathematical understanding, and mathematical literacy.

This, in turn, enhances mathematical performance and mathematical achievement, "through sequential activities of mathematical inquiry" (Le et al., 2004, p. 2).

These researchers have also argued that to be effective reform oriented instruction often depends on the progressive development and use of thinking and reasoning skills essential for communicating and acquiring basic, standard and advanced mathematical ideas. The National Council of Teachers' of Mathematics (NCTM) and the American Association for the Advancement of Science (AAAS) believe that a reform oriented approach to instruction allows teachers and students to place less emphasis, initially, on the mathematical tasks typically geared toward just the acquisition of discrete numeracy and computational skills. This approach was the usual focus of instruction that sought to achieve only a general operational knowledge of mathematics (e.g. computation skills). However, in mathematics classrooms where teachers use reform oriented instruction these tasks typically follow from a basis of numeracy that opens up opportunities for a deeper understanding of mathematics that students first acquire after they gain a firm grasp of the fundamental concepts of mathematics.

It is not surprising, then, that the National Research Council (1996), the American Association for the Advancement of Science (1993) and the National Council of Teachers' of Mathematics (1989; 2000) emphasize that the major objective of reform oriented instruction is the engagement of students as active participants in their own learning, including the learning of mathematics. This is accomplished through the promotion, development and use of complex cognitive skills. Thus, these agencies emphasize that the advocates of a reform oriented instruction approach to teaching mathematics do not negate the importance of acquiring computational skills (purely factual and operational mathematical knowledge).

Rather, these organizations contend that a firm conceptual foundation of mathematical understanding is added to render these skills more meaningful and useful. In other words, these organizations (the NCTM and The AAAS, and others) argue that what they view to be problematic is what the traditional or conventional approach typically offers. Indeed, these approaches offer mathematics instruction that very often just emphasizes students' development of computational skills and factual knowledge of mathematics operations while sometimes minimizing or excluding their need to grasp more complex understanding of conceptual thinking and creative problem solving (AAAS, 1993; NCTM, 1989, 2000; Le, et al., 2004).

Such exclusions, these organizations argue, result in having students who are poorly prepared for situations and/or careers that will require critical thinking (being able to understand why the process works; being able to transfer skills from one problem to another that is unfamiliar; being able to recognize similarities and relationships) as well as higher levels of mathematical and other higher order thinking. These higher order levels go beyond the mathematical skills and knowledge typically acquired through traditional or conventional mathematics instructions.

Reports of Research

Kim, Crasco, Blank, and Smithson (2001) reported that although some teachers have begun to incorporate reform oriented instruction in their classrooms, statistical evidence supporting the use of these practices is weak. Le et al. (2004) said other studies that reported the relationship between students' achievement and the frequency with which teachers report their practice of reform oriented instruction showed only somewhat positive relationships, but actual effect sizes were quite small. Mayer (1998), in examining the relationship between instructions based on a reform-oriented strategy and students' scores on standardized multiple-choice tests, reported a small but noticeable positive relationship.

Hamilton, McCaffrey, Stecher, Klein, Robyn and Bugliari (2003) utilized data from twelve National Science Foundation Funded Systematic Reform Initiatives (FSRI) and reported both null and positive results when different assessments (multiple choice and open ended) were used. "Can reform oriented instruction alone improve scientific or mathematical communication, problem solving, or higher order thinking skills" (Le et al., 2004, p. 2). These are crucial questions and Le et al. believe reform oriented instructional strategies can be effective in improving mathematical performance. In fact, they cited Cohen and Hill's (2000) study where the authors presented findings of a relationship between scores on the California Learning Assessment Systems (CLAS) Mathematics Tests, a performance based assessment that measured students' understanding of mathematics problems, and their acquisition of mathematical procedural knowledge. Thompson and Senk's (2001) study also provided support for Cohen and Hill and reported research findings that indeed found that reform oriented instructional strategies and new classroom initiatives (such as using mathematics as an integrated whole; bringing meaning to mathematics; using prior knowledge), when they are strongly aligned with a reform-oriented curriculum, have a positive correlation with improving mathematics achievement. These findings were clearly demonstrated with respect to solving multi-step problems and those that involve application or graphical representations of more complex mathematics (Le et al., 2004).

In terms of the small effect sizes that these studies report, Le et al. (2004) posit that they may be due in part to the instruments that were used, particularly since most of these instruments have been judged to be inadequate. Many of these studies, they say, used surveys in which teachers are only asked to self report on the frequency of their engagement in expected reform oriented practices in their teaching of mathematics. Indeed, according to the authors, while the data gathered from the teachers' use of these instruments have been successfully used by several researchers in the past (e.g., Cohen & Hill, 2000; Gamoran, Poter, Smithson, & White, 1997), the authors said, Wenglinsky (2002) and to some extent Rowan, Corenti, Miller (2002)) argue that self reported data from surveys are inherently problematic because of their limited utility. They argued that aside from the limitations, the usefulness of these surveys has been further limited by their design either for use in very specific settings or for focusing on long-term patterns of behavior. Along these same lines Mullens and Gayler (1999) had previously reported that survey instruments often ignore the subtleties of specific behaviors, as well as individual variations in instructional strategies that are used by different teachers in conveying the particular content either for the same or various subjects at and within particular grade levels. They also argued that such surveys alone cannot collect the nuances of how well individual teachers understand particular mathematical terminologies or how appropriately they present and implement their proper mathematical practice.

Summary

The history and underlying philosophical basis of reform oriented mathematics teaching, has been presented in this chapter. Throughout this presentation it is clear that writers and theoreticians in the field of mathematics are in general agreement that this type of teaching should be the norm in today's mathematics classroom. What is not clear, however, is whether in-service teachers of mathematics understand and support these ideas. More critically, little evidence has been presented in the literature concerning whether such teachers actually demonstrate these reform oriented practices. As mentioned several times in the review, there is growing evidence that teachers feel more and more pressured to teach in a manner that will facilitate their students' passing the standardized tests that under gird the NCLB requirements. As also pointed out in this review teaching in a way that solely focuses on helping students to pass standardized tests is more than likely not the type of teaching recommended through the reform movement. Little documented evidence of this, however, currently exists. Thus the purpose of the present study is two-fold: to ascertain if in-service mathematics teachers support the tenets of reform oriented teaching, and whether they actually practice these types of activities.

CHAPTER 3

METHODOLOGY

Research Design

The core question of this study is: to what extent do teachers agree with, and actually practice, reform oriented teaching methods? The study employed an explanatory mixed method design (Creswell, 2002). This approach consisted of a primary quantitative component (survey) and two secondary qualitative components (in-class observations and post-observation teacher interviews).

I chose this explanatory mixed method approach for several reasons. The principal reason was that it allowed me to look carefully at a small group of teachers, rather than choosing an approach that would have had me look more broadly at mathematics teachers or teachers in general. By choosing this mixed method, I was able to focus on the correspondence between what teachers say they do and what they actually do in teaching mathematics. This choice of methodology allowed me to overcome the limitations of simply using survey data. By their nature, survey data are limited by the ability or desire of participants to report accurately what they actually do. By adopting this mixed method design approach, I was able to address this threat to the validity of my research and gathered data that more accurately depict the actual teaching practices of in-service teachers of mathematics.

The Research Setting

The intent of the study was to collect data in a variety of school districts that varied on relevant demographic characteristics. As an initial effort, letters of inquiry were sent to 14 school districts. These school districts were selected based on their geographical location, my accessibility to them, and my familiarity with the schools in the districts. The letters of requests and prior notice emails were sent to the Superintendents and Assistant Superintendents. Copies of the letters and email are shown in Appendix B and B-1. Ultimately, approval to conduct the research was granted in three school districts in New Jersey. An elementary school, a middle school and a high school from each of the districts were included in the various phases of the research. At the time of the study, the schools and school districts used in this study were located in geographical areas that would be defined as midsize cities (NCES, 2000). The 2000 decennial Census defined a midsize city as a territory inside an urbanized area and inside a principal city with a population less than 250,000 and greater than or equal to 100,000. All three of the districts included schools with grades from kindergarten to 12th grade.

A brief demographic overview of each of the school districts follows. The names of the school districts have been changed for the sake of anonymity.

The Hewing School District

The Hewing School District is located in a county in north central New Jersey. At the time of the study, there were five schools in the district: three elementary schools, one middle school and one high school. The overall student population at the time of the study was 3,949. Of this total, 136 were English Language Learners (ELL)(formerly Limited English Proficiency -LEP), and 1,319 had Individualized Education Plans (IEP's). The teacher student ratio was reported as 12.8 students for every full-time equivalent teacher employee.

The total number of full-time equivalent employees at the time the study was conducted was 483.6 persons, including 307.7 full-time teachers and 187.9 full-time support staff and administrators. Almost all of the teachers in the district are white.

The Addison School District

The Addison School District is located in approximately the same geographical area as the Hewing District. At the time of the study, there were 16 schools in the district, including 10 elementary schools, four middle schools and two high schools. The overall student population at the time the study was conducted was 13,689. Of this total, 279 were English Language Learners (ELL) and 2,639 had Individualized Education Plans (IEP's). The teacher student ratio was reported to be 12.4:1 students per full time teacher employee.

The total number of full-time employees at the time of the study was 1,505 persons. Of this number, 1,101.5 were full-time teacher employees. The remaining 400 or more, were support staff and administrators. The overall racial distribution of the teachers in the district at the time of the study appeared to be a diverse mix of Asian and Caucasian Americans (approximately 50% each).

The Triton School District

The Triton School District is located in the same county as the Hewing School District. It is classified as an Abbot school district, a designation given to school districts in New Jersey where many of the residents are from lower social-economic levels. At the time of the study, there were 22 schools in the district, including 17 elementary schools, 2 middle schools, and 2 high schools. The overall student population at the time of the study was 12,513. Of this number, 1,076 were English Language Learners (ELL), and 2,981 had Individualized Education Plans (IEP's). The teacher student ratio was reported to be 11.9:1 students per full-time teacher employee.

The total number of full-time employees at the time of the study was 1,702 including 1,049, full-time teacher employees and 653 support staff and administrators. The overall racial distribution of the teachers in the schools when the study was conducted was observed to be predominantly African American (approximately, 95 %).

Pilot Study

A pilot study was conducted to ascertain if the survey (described below) was appropriate for use in the main phase of the research. To conduct both the pilot study and the main study, an information packet was sent to the mathematics coordinators or mathematics supervisors in all of the schools in the selected districts. These coordinators arranged a series of brief meetings with small groups of mixed elementary, middle and high school mathematics teachers. The numbers that attended the meetings varied from district to district. Meetings were convened sometimes at the end of the day during the teacher's prep period and during break time. The intent was to secure the cooperation of a group of teachers for the pilot study who would be representative of mathematics teachers across the different types of schools, grade levels and districts, and who would also be representative of the main sample.

At the meetings I elicited questions from the teachers to determine their willingness to participate in the study and to get their assessment of the feasibility of what they were being asked to do. I assured the teachers that the identity of their schools as well as their participation in the study would be kept confidential. I assured them that the information gathered would be used for the purpose of the research only. After the meetings, several teachers provided me with information on their class schedules as well as cell phone and home numbers. That information was helpful for me because I was able to contact the teachers outside of the regular school hours when they could discuss their availability for participation in different phases of the study at their own leisure.

The purpose of the pilot study was also to ascertain if the instrument was clear, if the teachers understood what was being asked, if it met my expectations, if the time to complete the survey was adequate, and if there were any ambiguity in the items. Eighteen elementary, middle and high school mathematics teachers participated in the pilot. A description of these teachers is provided in Table 3.1.

Table 3.1: Demographic Description of the Pilot Sample

	Frequency *N* = 18	Percent %
Age		
20-35	16	89%
35 and above	2	11%
Total	18	100%
Gender		
Male	6	33%
Female	12	67%
Total	18	100%
Ethnicity		
African American	3	17%
American Indian/Alaskan	0	0%
Hispanic/Latino	1	5%
Oriental/Asian	2	11%
White/Not Hispanic	12	67%
Other	0	0%
Total	18	100%
School		
Elementary	1	17%
Middle	3	50%
High	2	33%
Total	6	100%
District		
Urban	1	33%
Suburban	2	67%
Total	3	100%
Grade		
Five	2	11%
Six to Eight	10	55%
9 - 16	3	17%
Other	3	17%
Total	18	100%

Degree		
B.A./B.Sc.	9	41%
M.A./M.Sc.	7	32%
Ph.D./Ed.D.	1	5%
Other	5	22%
Total	22	100%
Mathematics Major		
Yes	8	44%
No	10	56%
Total	18	100%
Mathematics Minor		
Yes	5	28%
No	13	72%
Total	18	100%
Teaching Certificate		
None	0	0%
Temporary	1	6%
Probationary	0	0%
Regular	17	94%
Total	18	100%
Specific Endorsement		
Yes	11	61%
No	7	39%
Total	18	100%
Years Full-Time		
Zero - five	7	39%
6 - 19	7	39%
20 - 30	2	11%
31 and above	2	11%
Total	18	100%
LEP/ELL Learners		
Zero - five percent	11	61%
Six - 10 percent	5	28%
11 percent and above	0	0%
Don't know/Not sure	2	11%
Total	18	100%
Confidence		
Not confident	0	0%
Somewhat confident	0	0%
Moderately confident	0	0%
Very confident	18	100%
Total	18	100%

Ability Groups		
Fairly homogeneous/ Low ability	3	17%
Fairly homogeneous/ average ability	6	33%
Fairly homogeneous/ high ability	0	0%
Heterogeneous two or more abilities	5	28%
Combination abilities	4	22%
Total	18	100%
Professional Development (1)		
None	4	22%
Less than four hours	8	44%
Four to eight hours	2	11%
9-16 hours	0	0%
More than 16 hours	1	6%
Missing	3	17%
Total	18	100%
Professional Development (2)		
None	2	11%
Less than four hours	4	22%
Four to eight hours	6	33%
9-16 hours	5	28%
More than 16 hours	1	6%
Missing	0	0%
Total	18	100%
Professional Development (3)		
None	1	6%
Less than four hours	5	28%
Four to eight hours	9	50%
9-16 hours	1	6%
More than 16 hours	2	11%
Missing	0	0%
Total	18	100%
Professional Development (4)		
None	1	6%
Less than four hours	8	44%
Four to eight hours	4	22%
9-16 hours	3	17%
More than 16 hours	1	6%
Missing	1	6%
Total	18	100%

The modified Rand Survey (described below) was administered to the 18 teachers that constituted the pilot study sample. The administration was done individually and in groups during their study period or at the end of the school day immediately following the dismissal period. After completing the modified Rand Survey, the teachers provided feedback about their understanding of the instrument, of reform-oriented teaching and on the clarity of the items. Analysis of the pilot data revealed that the survey questions were clearly presented, the teachers understood what was asked of them, and the 10-15 minutes time allotted for the completion of the survey was adequate. The pilot data revealed that the instrument was appropriate and that the teachers could complete it in an appropriate time period. As a consequence, the surveys for the main phase of the study were distributed to the participating school districts.

Main Study Phase I- Survey Administration

When it came time to administer the main survey, the mathematics supervisors, coordinators and assistant superintendents responsible for the oversight of the mathematics teachers in their respective school districts asked that the surveys be given to them for distribution to their teachers. I delivered the surveys to these administrators in 10"x13" white envelops with the name of each school type clearly marked. I included a "dear colleague" letter in the envelope that provided instructions to the administrators on how the survey should be administered and collected. That letter also thanked them for their support (see letter in Appendix C) and instructed them to exempt all participants who completed surveys for the pilot study.

The distribution of the surveys varied in the different schools and districts depending on the method that the supervisor thought to be most efficient. In some cases this involved placing the survey in the teacher's mailbox; in others the teachers were reminded of the availability of the survey by email.

I asked the administrators to remind their teachers of the confidentiality of the process and that the survey results would be used for the purpose of the research project only. Teacher Consent Forms were attached to the surveys (See Appendix D for Teacher Consent Forms). Administrators were also reminded to ask the teachers to sign the Consent Forms prior to completing the survey.

Three weeks after the surveys were distributed to all three school districts I made phone calls and visited the mathematics supervisors and the assistant superintendents at their respective schools to collect the completed surveys. The responses from the various districts varied considerably due to a number of issues that had not been foreseen. In addition, it became apparent that administering the surveys at the end of the school year was problematic in some cases. As a consequence, it was decided to ask the districts for permission to administer the surveys again in the early fall when the teachers returned from summer vacation. This permission was granted and surveys were again sent to the appropriate administrator during the first month of the school year. A description of these teachers from the original distribution is presented in Table E1 in Appendix E. Table E2 in Appendix E describes the teachers from the second distribution. As a consequence of these activities, a total of 174 questionnaires were eventually returned. A description of these teachers is contained in Table 3.2.

Table 3.2: Demographic Description of the Main Sample

	Frequency N = 174	Percent %
Age		
Less than 25	12	8%
26-30	30	17%
31-35	21	12%
36-40	16	9%
41-45	24	14%
46-50	20	11%
More than 50	48	27%
Missing	3	2%
Total	174	100%
Gender		
Male	40	23%
Female	122	70%
Unidentified	12	7%
Total	174	100%
Ethnicity		
African American	29	17%
American Indian/	2	1%
Hispanic/Latino	11	6%
Oriental/Asian	6	3%
White (Not Hispanic)	119	68%
Other	5	4%
Missing	2	1%
Total	174	100%
Type of School		
Elementary	95	55%
Middle	40	23%
High	39	22%
Total	174	100%
District		
Urban	55	31%
Suburban	118	68%
Missing	1	1%
Total	174	100%
Grade		
Five	35	20%
Six – eight	34	19%
9-12	28	16%
Other/combination	74	42%
Missing	3	3%
Total	174	100%

Degree		
BA/BSc	94	50%
MA/MSc	63	40%
Multiple MA/MSc	9	6%
Ph.D. or Ed.D.	4	2%
Other degrees	3	2%
Missing	1	0%
Total	174	100%
Mathematics major		
Yes	49	28%
No	122	70%
Missing	3	2%
Total	174	100%
Mathematics minor		
Yes	23	13%
No	142	82%
Missing	9	5%
Total	174	100%
Teaching Certificate		
None	2	1%
Temporary	11	6%
Probationary	2	1%
Regular	157	91%
Missing	2	1%
Total	174	100%
Specific endorsement		
Yes	57	33%
No	109	63%
Missing	8	4%
Total	174	100%
Years teaching full-time		
Zero - five	48	28%
Six - 19	78	45%
20 - 30	38	22%
31 and above	4	2%
Missing	6	3%
Total	174	100%
Years teaching mathematics		
Zero - five	59	34%
Six - 19	73	42%
20 - 30	38	22%
31 and above	1	0%
Missing	3	2%
Total	174	100%

LEP/ELL		
Zero – five	124	71%
Six – 10	8	6%
11 and above	23	13%
Other /don't know	5	2%
Missing	14	8%
Total	174	100%
Confidence		
None	2	1%
Somewhat	5	3%
Moderately	50	29%
Very	114	66%
Missing	3	1%
Total	174	100%
Mixed Ability Groups		
Fairly homogeneous/low ability	28	16%
Fairly homogeneous/average ability	32	19%
Fairly homogeneous/high ability	7	4%
Heterogeneous two or more Abilities	94	54%
Combination ability levels	2	1%
Missing	11	6%
Total	174	100%
Professional Development (1)		
None	83	48%
Less than four hours	36	21%
Four – eight hours	20	11%
9 – 15 hours	11	6%
More than 16 hours	12	7%
Missing	12	7%
Total	174	100%
Professional Development (2)		
None	39	23%
Less than four hours	61	35%
Four – eight hours	35	20%
9 – 15 hours	16	9%
More than 16 hours	14	8%
Missing	9	5%
Total	174	100%
Professional Development (3)		
None	31	18%
Less than four hours	54	31%
Four – eight hours	47	27%
9 – 15 hours	16	9%
More than 16 hours	17	10%
Missing	9	5%
Total	174	100%

Professional Development (4)		
None	47	27%
Less than four hours	63	36%
Four - eight hours	26	15%
9 - 15 hours	11	6%
More than 16 hours	15	9%
Missing	12	7%
Total	174	100%

Main Study Phase 2- Classroom Observation and Interviews

The last question in the survey asked the teachers if they would agree to have their teaching observed. Thirty-two (5.6%) teachers responded positively to that question. Because the group was large, I selected teachers that represented the three different types of school districts and the three different types of schools in the study. Because I also wanted to look at patterns, I made the effort to observe younger and more senior teachers within the same grade, in the same type of school and the same type of school district. For this phase of the study, I used the Reformed Teaching Observation Protocol (RTOP) and Training Manual developed by the Arizona Collaborative for Excellence in the Preparation of Teachers (ACEPT) to conduct the observations (RTOP). These instruments will be described below.

A total of 10 teachers were ultimately observed. A description of these teachers is contained in Table 3.3.

Table 3.3: Demographic Description of the Observed Sample

	Frequency N = 10	Percent %
	N	%
Age		
Less than 25	1	10%
26-30	3	30%
31-35	0	0%
36-40	2	20%
41-45	1	10%
46-50	0	0%
More than 50	3	30%
Missing	0	0%
Gender		
Male	2	20%
Female	7	70%
Missing	1	10%
Ethnicity		
African American	3	30%
American Indian/ Alaskan Native	1	10%
Hispanic/Latino	1	10%
Oriental/Asian	0	0%
White (Not of Hispanic origin)	4	40%
Other	1	10%
School		
Elementary	6	60%
Middle	2	20%
High	2	20%
District		
Urban	5	50%
Suburban	5	50%
Grade		
Five	2	20%
Six - Eight	2	20%
9 - 12	2	20%
Other	4	40%

Degree		
B.A./B.Sc.	5	50%
M.A./M.Sc.	5	50%
Ph.D/Ed.D	0	0%
Other	0	0%
Mathematics Major		
Yes	3	30%
No	7	70%
Mathematics Minor		
Yes	0	0%
No	9	90%
Missing	1	10%
Teaching Certificate		
None	0	0%
Temporary	1	10%
Probationary	0	0%
Regular	9	90%
Specific Endorsement		
Yes	2	20%
No	8	80%
Years Teaching Full-time		
Zero - five	3	30%
6 - 19	6	60%
20 - 30	1	10%
31 and above	0	0%
Years Teaching Mathematics		
Zero - five	4	40%
6 - 19	5	50%
20 - 30	1	10%
31 and above	0	0%
Confidence		
None	0	0%
Somewhat	1	10%
Moderately	3	30%
Very	6	60%

Professional Development (1)		
None	2	20%
Less than 4 hours	3	30%
Four - Eight hours	2	20%
9 - 16 hours	1	10%
More than 16 hours	2	20%
Professional Development (2)		
None	1	10%
Less than 4 hours	3	30%
Four - Eight hours	3	30%
9 - 16 hours	1	10%
More than 16 hours	2	20%
Missing	0	0%
Professional Development (3)		
None	1	10%
Less than 4 hours	2	20%
Four - Eight hours	5	50%
9 - 16 hours	1	10%
More than 16 hours	1	10%
Missing	0	0%
Professional Development (4)		
None	1	10%
Less than 4 hours	3	30%
Four - Eight hours	2	20%
9 - 16 hours	2	20%
More than 16 hours	1	10%
Missing	1	10%

Two external observers and I conducted in-class observations of the teachers. We used The Reformed Teaching Observation Protocol (RTOP) Training Guide to rate the teachers on specific criteria of reformed teaching (See Appendix F for description of the RTOP Training Guide). Prior to using the RTOP Guide, external observers received training on how to rate the items in the Protocol. This was followed up with the engagement of each observer in practical activities in which they independently coded and recorded several interactions before carrying out the actual observation.

The additional exercise was to develop skill in using the instrument and provided an opportunity for the external observers to develop familiarity with the items.

All observers were educators with experience in teaching mathematics. One of the external observers is a Harvard graduate with several years of experience in the public schools. The other has a degree in Business Management and is also an educator with several years of teaching as a substitute teacher working with a variety of student populations (See Appendix G for a brief biography of external observers).

As part of the classroom observation efforts were taken to have a good understanding of reformed constructivist mathematics practice the way the teachers conceived of such practice. As such, all observations and interview data were carefully analyzed to establish what mathematics activity was taught during each observation, how it was taught and the length of time that was devoted to each lesson.

In other words we wanted to get a good understanding of what was done by both teachers and students that could be described as reformed constructivist mathematics teaching. In that regard, the external observers were instructed to look for specific actions by the teacher. These actions included such things as lesson design and implementation, content-propositional knowledge, content-procedural knowledge, classroom culture and teacher-student relationship, that Sawada, Piburn, Turley, Falconer, Benford, Bloom and Judson (2000) said are characteristic of reformed constructivist mathematics teaching.

As soon as the external observers and I were in agreement as to what the lesson activities were for each teacher in each of the different classrooms, we made every effort to understand what each teacher considered to be reformed teaching of mathematics throughout the course of each lesson. In other words, we wanted to know what type of content knowledge characterized the lesson, what was the culture of the classroom environment and the student teacher relationships etcetera that were taking place during the lessons that could be construed as practicing reformed constructivist mathematics teaching. In analyzing observation data I adapted an approach developed by members of the Arizona Collaborative for Excellence in the Preparation of Teachers (see Sawada, Piburn, Turley, Falconer, Benford, Bloom & Judson, 2000) to identify specific and related mathematics reformed activities that teachers and students were engaged in during the different lessons observed. For this study, "lesson" is used synonymously with Whiteaker's (2003) definition, which the author described as "a set of events grouped together around a common activity, concept, or objective" (p. 25). Appendix H provides examples of several lesson activities teachers gave to their students during or after a lesson.

Samples of students' work were also collected to augment the observation data and for triangulation. These work samples are also included in Appendix H.

Instrumentation and Materials

Teacher Survey

The major instrument used in the study was a modified 16-item Teacher Survey questionnaire (See Appendix I for description of the teacher survey). Items for the Survey (Personal Communication, Skrabala January 24th, 2008) were taken from the Teacher Survey and elements of reform statements developed by the Rand Corporation (2003). The initial "survey items were piloted locally and with teachers" (Le, Stecher, Lockwood, Hamilton, Robyn, Williams, et al., 2006, p. 17). The Survey items are rated on a 6-point Likert scale (agree strongly to disagree strongly) to which participants were asked to indicate their response to questions about their beliefs and practices concerning reform oriented mathematics teaching. Participants indicated how often they engaged in specific instructional activities (e.g., requiring students to explain their reasoning for arriving at an answer). They rated what they considered to be important mathematics activities that promoted students' learning of mathematics (e.g., answering worksheet questions, working on extended investigations). These types of questions were selected because the research suggests that they have been used successfully in past mathematics research (e.g., Cohen & Hill, 2000; Hamilton et al., 2003; Wenglinsky, 2002).

The survey consists of two parts—one part measuring beliefs and one part measuring practices. Beliefs about mathematics teaching are composed of ten items measuring the extent to which teachers embrace the principles underlying reform oriented mathematics teaching as described by the Rand Corporation. Examples of beliefs about reform oriented mathematics teaching include a focus on fewer topics which are taught more deeply, relating concepts, exploring mathematical rigor, focusing on problem solving and reasoning, identifying different ways to solve problems, communicating mathematically, focusing on literacy skills, using open-ended questions, and identifying multiple strategies and tests for understanding.

Mathematics teaching practice is composed of six items that measure the frequency with which teachers believe they should implement reform-oriented practices when teaching mathematics. Examples of reform oriented teaching practice include helping students to develop and use mathematical thinking, interpret and solve mathematical problems, communicate mathematically, use appropriate mathematical language, encourage mathematical thinking, communicate mathematically, connect mathematical ideas, solve various mathematical word problems, use mathematical English words, and manage and encourage mathematical discourse.

The Teacher Survey questionnaire also assessed a variety of demographic and background factors including, age, ethnicity, years of teaching, highest degree earned (Ph.D., or Ed.D. etc.) (See also Appendix I for demographic information). The survey also assessed the extent to which teachers participated in professional development activities that included training in mathematics teaching methods, particularly those in line with the National Council of Teachers of Mathematics (NCTM) Standards.

Reformed Teaching Observation Protocol (ROTP)

The *Reformed Teaching Observation Protocol* was developed by a group of teachers (Sawada, Pibum, Falconer, Turley, Benford, & Bloom, 1999) from Arizona State University, in conjunction with the Arizona Collaborative for Excellence in the Preparation of Teachers. It was used in this study to obtain an independent measure of the extent to which participants' actual mathematics teaching was a reflection of what they believe about reform oriented instruction. The Reformed Teaching Observation Protocol (RTOP) measures the extent to which mathematics teaching is "reformed" (Sawada et al., 1999). "The instrument is reported to be criterion-referenced, and observers' judgments should not reflect a comparison with any other instructional setting than the one being evaluated" (Sawada & Piburn, 2000, p.32). The instrument consists of 25 items (See Appendix J for a description of the Reformed Teaching Observation Protocol), that are rated on a 4-point Likert scale where a score of "0" was assigned if that particular behavior was *not observed* and a score of "4" was assigned if the behavior was *very descriptive* of the individual being observed. Scores ranged from 0-100 points with higher scores reflecting more reform-oriented teaching practices.

The instrument was designed, piloted and validated by the Evaluation Facilitation Group of the Arizona Collaborative for Excellence in the Preparation of Teachers. The instrument draws on the following sources:

- National Council for the Teaching of Mathematics. Curriculum and Evaluation Standards (1989), Professional Teaching Standards (1991), and Assessment Standards (1995).
- National Academy of Science, National Research Council. National Science Education Standards (1995).
- National Association for the Advancement of Science, Project 2061. Science for All Americans (1990) Benchmarks for Scientific Literacy (1993) (Sawada & Piburn, 2000).

A specific Training Guide accompanied the instrument, and provided pertinent information on the interpretation of individual terms in the protocol. The Guide was also used as part of the formal training in which the external raters observed actual classrooms and independently scored and discussed their observations (See Appendix F for description of Training Guide). Observers wrote comments after each item that further described the practice. Teachers were observed once during a five-day observation period. Periods were forty-five (45) minutes long.

Post-observation Teacher-Interviews

Post-observation one-on-one teacher interviews were conducted with the eight teachers who were observed, in order to have a clearer grasp of the “contextual influences on the teachers’ practice” (Le Stecher et al., 2006, p.20) and their beliefs. The seven items that made up the post-observation interview questions were formulated from a variety of scales developed by the Rand Corporation (Skrabala, Personal Communication, January 24, 2008) and which had been used in previous research (Le Stecher et al., 2006) (See Appendix K for post-observation interview questions). This was done in order to establish reliability of the information the teachers provided in the survey and for data triangulation. Interviews were audio taped. A written permission to audiotape was obtained from each teacher (See Appendix L for description of permission). During the process teachers were informed of their rights and assured confidentiality of the data collected. Efforts were made to interview a senior and a younger teacher at the same grade level. For this study, various sources and types of data were used. Table 3.4, describes the overall summary of the sources and types of data collected.

Table 3.4: Sources and Types of Data Collected

DATA SOURCE	DATA TYPE
Teacher Self-Reported Data Related to Reform Oriented Teaching	Beliefs about Teaching Mathematics (10 items) 1. Explore fewer topics deeply 2. Relate mathematical concepts 3. Use multiple problems 4. Emphasize problem solving 5. Identify different ways to solve problems 6. Use mathematical language to communicate 7. Teach literacy skills 8. Use open-ended questions 9. Identify multiple problem solving strategy 10.Test for understanding Mathematics Teaching Practice (6 items) 1. Encourage critical thinking 2. Communicate clearly 3. Connect mathematical ideas 4. Solve mathematical word problems 5. Give proper meaning to English words 6. Mange and encourage discussion Demographic Information (12 items) 1. Age 2. Gender 3. Grade level 4. Ethnicity Teacher Educational Background

	5. Highest degree earned Mathematics Degree 5. Major 6. Minor Teacher Certification 7. Specific certification Classroom Experience 8. Full-time 9. Grade level Subject Matter Confidence 10.Level of confidence Class Composition 11.Percentage non speakers of English Types of Groups 12. Mixed ability Amount of Professional Development (4 items) 1.In-depth study of mathematics 2.Methods of teaching mathematics 3.Use of particular mathematics curricula or curriculum 4.Use of Mathematics standards or framework – (e.g. NCTM, state and or district
Observation Data	Overall Reform 1. Lesson design (5 items) 2. Content (5 items) 3. Level of mathematical interactions (5 items) 4. Classroom dialogue (10 items)
Post Observation Interview Data	Overall Reform 1. Response to training (1 item) 2. Method of engagement (1 item) 3. Level of confidence (1 item) 4. Mathematical thinking /Understanding (1 item) 5. Response to reformed instruction (1 item) 6. Revising mathematics instructions (1 item) 7. Attitude to reform

CHAPTER 4

RESULTS OF THE TEACHER QUESTIONNAIRE

This chapter reports analyses that were done on the data obtained from the 174 mathematics teachers' responses to the Questionnaire. These data were collected to answer the first major research question and all of the secondary research questions.

Descriptive Statistics

A total of 174 mathematics teachers participated in the study. Descriptive data on these teachers were presented in Chapter 3 (Table 3.2).

<u>Research Question # 1</u>: Do in-service mathematics teachers support the major principles of reform oriented mathematics instruction?

In order to answer this question the responses of the teachers to the items in the questionnaire were analyzed. Descriptive data on these responses are reported in Tables 4.1a and 4.1b. These tables present the distribution for all of the items of the teachers' responses to the 16 items on the questionnaire, and also present the mean for each item on the 6-point Likert scale used.

Table 4.1a: Frequency of Response for the Belief Items on the Questionnaire

Belief Items (N = 10)	1 SD	2 DM	3 DS	4 AS	5 AM	6 AS	Mean
1. Explore fewer topics in greater depth rather than covering more topics quickly or superficially	3	2	10	24	67	67	5.03
2. Select topics that help students connect Mathematics to their own experience and The larger community rather than under-Standing mathematics as isolated skills and procedures		1	5	10	65	92	5.40
3. Explore complex problems rather than only Simple problems that emphasize specific Skills.	1	1	10	34	76	50	4.94
4. Place greater emphasis on reasoning and problem solving rather than on operations and computation.	1	2	7	38	73	52	4.94
5. Focus lessons on the reasoning process Rather than only on obtaining the right Answers.	2		6	26	62	77	5.18
6. Use the language of mathematics to express mathematical ideas		2	1	20	64	86	5.34
7. Attend to the literacy needs of the students in their mathematics classroom	1	1	11	39	52	66	4.99
8. Use open-ended questions			1	12	54	105	5.53
9. Emphasize the process through which students arrive at solutions			1	12	65	92	5.46
10. Guide students to generalize from a specific instance to a larger concept or relationship				20	71	80	5.35

Note: SD 1 = strongly disagree; DM 2 = disagree moderately; DS 3 = disagree slightly; AS 4 = agree slightly; AM = agree moderately; AS = agree strongly

Table 4.1 b: Frequency of the Responses for the Practices Items on the Questionnaire

Practice Items (*N* = 6)	1 SD	2 DM	3 DS	4 AS	5 AM	6 AS	Mean
11. Help students monitor and evaluate their own problem solving and evolve more sophisticated mathematics thinking rather than leaving thinking procedures unexamined.			1	9	52	110	5.58
12. Help students communicate their mathematics thinking clearly and coherently to others				6	47	119	5.66
13. Help students see connections between mathematics and other disciplines				6	47	119	5.66
14. Help students translate mathematical word problems			2	5	47	118	5.63
15. Help students ascribe the appropriate mathematical meaning to English words			4	12	53	102	5.49
16. Manage the classroom, keeping all students engaged and on task				4	26	142	5.80

Note: SD 1 = strongly disagree; DM 2 = disagree moderately; DS 3 = disagree slightly; AS 4 = agree slightly; AM = agree moderately; AS = agree strongly

It is evident from Tables 4.1a and 4.1b, that teachers in this sample are generally supportive of all aspects of reform-oriented teaching. Specifically, a majority of the responses for all questions are in the “agree moderately” or “agree strongly” categories. Moreover, all of the means are either close to or above “5” on the 6-point Likert scale used. With respect to the major principles that teachers say they believe should underlie mathematics teaching, the teachers strongly support placing emphasis on the process through which students arrive at solutions (item 9)(92 or 53% responding with “Agree Strongly”), selecting topics that help students connect mathematics to their own experience and their community rather than understanding mathematics as isolated skills and procedures (item 2) (92 or 53%), and focus lessons on reasoning process rather than only on obtaining the right answers (item 5) (77 or 44%). Teachers also moderately support principles to explore complex problems rather than focusing only on simple problems that emphasize specific skills (76 or 44%). They support principles that place greater emphasis on reasoning and problem solving (73 or 42%), and for guiding students to generalize from a specific instance to a larger concept or relationship (conceptual understanding) (71 or 41%). Teachers strongly support the use of open-ended questions (item 8)(105 or 61%).

Secondary Research Questions

- Does professional development support reform oriented teaching practices?
- Do teachers’ beliefs vary with respect to the grade level they teach?
- Do teachers’ beliefs vary with respect to their levels of education?

In essence the three questions presented above can be summed up by asking: Are teachers’ beliefs about reform practices affected by demographic variables? To answer this question, Pearson’s correlations were computed between the teachers’ responses to the questionnaire items and those demographic variables that refer to the three questions presented above. Since grade level taught was conceptualized as ordinal, it was not included in the analysis. In addition, to broaden the analysis, all of the remaining variables that could be analyzed parametrically were included in this analysis. The results of this analysis are presented in Table 4.2a and Table 4.2b. Only correlations that are significant at the .05 levels or beyond are included in the table.

It is evident from Table 4.2a and Table 4.2b that there are only a small number of significant correlations and that almost all of the correlations that are significant are modest. In reference to the three specific research questions listed above, only professional development correlates with certain of the items on the questionnaire, and even these correlations account for less than 5% of the variance. It is perhaps more interesting that certain variables that would logically seem to be related to these beliefs are not, in fact, related. For example, the numbers of years that the teacher had been teaching and the teacher's confidence in his or her mathematical knowledge have no relationship with the responses. Contrary to what might be expected, the more professional development the teachers have the less support was given for item 1, which led them away from believing that they should explore fewer topics in greater depth, rather than covering more topics quickly or superficially.

As mentioned above, since the grade level taught could not be analyzed parametrically, this variable was analyzed separately. One-way ANOVAs were conducted on each of the 16 questions from the questionnaire. There were two significant findings: *Question number 7 and Question number 16*. For question number 7 which asked if teachers should attend to the literacy needs of their students, the teachers in the 5^{th} and 6th and 7^{th}

grades had significantly higher means than teachers in the other groups. Since literacy is more typical an issue in the lower middle grades, this result is not surprising.

Table 4.2a: Correlations Among Selected Variables for Beliefs About Mathematics Teaching Items

	Variable	BTM1	BTM2	BTM3	BTM4	BTM5	BTM6	BTM7	BTM8	BTM9	BTM10
1	Age										
2	Gender							-.247**			
3	Highest Degree						.200**				
4	Mathematics Major							.251*			
5	Special Certificate Endorsement							.207**			
6	Years of Teaching Full-time										
7	Years Teaching Math to Grade Level										
8	Confidence Teaching Mathematics										
9	Percentage English Language										
10	Variations in mathematics Ability										
11	Professional Development 1	-.168*					.215**				
12	Professional Development 2	-.189*									
13	Professional Development 3	-.244**					.153*				
14	Professional Development 4	-.278**									

** Correlation is significant at the 0.01 level (2 tailed test).
* Correlation is significant at the 0.05 level(2 tailed test).

Table 4.2b (Continued): Correlations Among Selected Variables for Mathematics Teaching Practices Items

	Variable	MTP11	MTP12	MTP13	MTP14	MTP15	MTP16
1	Age						
2	Gender						-.233**
3	Highest Degree	-.210**					
4	Mathematics Major						
5	Special Certificate Endorsement					.167*	.227*
6	Years of Teaching Full-time						
7	Years Teaching Math to Grade Level						
8	Confidence Teaching Mathematics						
9	Percentage English Language			-.165*			
10	Variations in math Ability						
11	Professional Development 1	.168*					
12	Professional Development 2						
13	Professional Development 3						.157*
14	Professional Development 4	.164*					

*. Correlation is significant at the 0.05 level (2-tailed)
**. Correlation is significant at the 0.01 level (2-tailes)

For question 16, which talks about managing the classroom so that all students are engaged, teachers in the 10th, 11th and 12th grades had a significantly lower mean. This indicates that teachers in the upper three high school years are less concerned about classroom management issues. As before, this result is probably not surprising. In both cases, however, the result while statistically significant was not large. For both analyses, the partial eta squared statistic, which is a measure of effect size, was less than 10%.

As an additional analysis, the teachers' total score from the questionnaire was computed by summing the responses to the 16 questions. This composite was then used as the criterion variable in a multiple regression with the variables listed in Table 4.2 as the predictors. The R computed for this analysis was not significant with only 6% of the variance accounted for by the predictors.

In summary, data from the questionnaire are clear in indicating that teachers in general, are in strong support of all the tenets of reform-oriented teaching of mathematics. Moreover, this support is not strongly related to any of the demographic characteristics of the teachers.

CHAPTER 5

RESULTS OF THE FIELD OBSERVATIONS

This chapter presents the data obtained from the in-class observations and the interviews of the 10 teachers selected from the pool of mathematics teachers who completed the questionnaire. The information in this chapter is intended to answer the second major research question: To what extent do in-service mathematics teachers exhibit reform-oriented teaching in their classrooms? The chapter is divided into two parts. In part one, the teaching practices of each of the 10 teachers will be described by presenting a description of the classroom and the pedagogical approaches taken by each of the teachers in presenting a mathematics lesson. Each of these will be presented as a brief case study. In part two, the data derived from the observation will be presented and compared to the data generated by the questionnaire. These data will be summarized at the end of the chapter.

In Class Observations

The classroom observations lasted a minimum of forty-five minutes and were conducted from October 2008 through the end of November 2008. The observation lengths varied from teacher to teacher based on the school district's policy, the school's daily schedules and the teachers' availability.

The in-class observations were neither audiotaped nor videotaped. This was due to strict protocols protecting the privacy rights of the teacher participants as well as the students in the class. Observers worked in triads and rated teachers based on a set reformed teaching criteria. Field notes were assembled during the observation session and post-observation interview data.

Post Observation Interviews

Teacher interviews were conducted each day throughout the observation period that lasted from October to November 2008.

Pre-observation interviews were conducted during the week prior to the observation to gather background information about the teachers and their classes. As part of the interview process an agreed upon date was established for in-class observation of the teacher and a fully executed letter of permission providing audiotape consent was subsequently obtained from each teacher (See Appendix L for description of permission to audiotape). Post observation interviews took place mainly, after each classroom observation. Seventy percent of the interviews were done face-to-face and the remaining 30% were done over the telephone. Telephone interviews were scheduled to accommodate the teacher's schedule and to avoid interruptions to the school's activities.

Interviews were formal and many of the same questions were asked each day (See Appendix K for post-observation interview questions). The reason for the post-observation interviews was to understand from the teachers' point of view how well they believe in and practice reform oriented instruction and their views of mathematics teaching and learning. Interviews were audio taped and transcribed as agreed.

Analyzing Mathematical Beliefs

I looked at interview and observation data to identify how each teacher conceived her/his role as a mathematics teacher and what she/he conceived to be the best way to teach mathematics. These analyses zeroed in on the field data source collected from specific mathematics teachers as the summary of the teachers' own words describing their actual beliefs, concerns and practices during the post-observation interviews.

Mathematical Reform Practice

As part of the observation conducted each day, efforts were taken to have a good understanding of reformed constructivist mathematics practice the way the teachers' conceived of such practice. As such, all observations and interview data were carefully analyzed to establish what mathematics activity was taught during each observation, how it was taught and the length of time that was devoted to each lesson. In other words, I wanted to get a good understanding of what was done by both teachers and students that could be described as reformed constructivist mathematics teaching. In that regard, external observers were instructed to look for specific actions by the teacher. For example, observers were asked to focus on the specific reform practices that characterized reformed constructivist mathematics teaching practices (See Appendix M for description of the Reformed Teaching Observation Protocol items). It is important for the reader to know that the reformed constructivist practice as is used in these observations does not imply that the mathematics presented to the students were changed in any way. It does, however, imply, that the teachers would use their knowledge of texts (mathematics) and of mathematics, to inform, and they were informed by what took place in their classrooms. It was also about how the activities fitted into that particular classroom environment.

For us to clearly identify the different activities that met the conditions described as *reformed* mathematics teaching, two sets of data were used. First I analyzed transcripts from classroom observations to identify common themes among the ten lessons during the 10-day observation period. Second, post -observation interviews conducted with the teachers were also analyzed which provided me with better insights into what the teachers' aims and objectives were for each of the lessons taught during the observation period. These data helped me also to better understand how the teachers intended to build on their lessons, and not necessarily how reformed mathematics teaching was eventually defined and presented by them in their daily classroom interactions.

Analyzing Individual Classroom Practice

The classroom observations focused on what type of content knowledge characterized the lesson, what was the culture of the classroom environment and the student teacher relationships that were taking place during the lessons, what was the design of each lesson and how were they implemented. In analyzing observation data I used the Reformed Teaching Observation Protocol approach to identify specific and related mathematics reformed activities that teachers and students were engaged in during the different lessons observed. Here lesson is used synonymously to Whiteaker's (2003) definition, which the author described as "a set of events grouped together around a common activity, concept, or objective" (p. 25).

This definition is in line with other views put forward by Brilliant-Mills' (1994, p.310) of lesson as "a bounded set of activities about a common theme on a given day" (p.310). For these observations lessons were identified and analyzed by carefully looking at my field notes gathered during classroom observations. Appendix H provides examples of several lesson activities teachers gave to their students during or after a lesson. A case-by-case description of the 10 classes observed in relation to each teacher's mathematical practice is described in the narratives that follow in Part One. The teachers' level of experience is presented after their names. The names of the schools and school districts as well as the names of the teachers discussed in these case studies have been changed for the sake of anonymity.

Part One: Teaching Practices of 10 Teachers

Ms. Karen (Senior Teacher)

Days Middle School is one of several middle schools in The Triton school district with a total full time equivalent teaching staff of 64 teachers and a total student population of 696 students. Grades ranged from six to eight. The school district has been identified by the State Department of Education as one of several districts entitled to additional financial support for learning, in compliance with a court ordered mandate, in order to provide students with adequate and efficient education. The student teacher ratio is 11:1 compared with the average student teacher ratio of 14:1 in similar schools in the State. The racial/ethnic make-up of the student body in this school is zero percent American Indian, one percent Asian, 57 percent Hispanic, 36 percent Black and six percent White. Approximately 59 percent of the students are eligible for free lunch. As of this interview, Ms. Karen said she had been teaching mathematics for more than 16 years and felt very confident in her ability to teach the subject.

Upon entering the room, I found Ms. Karen's classroom to be well lit and well fit. Students' and teacher's completed mathematics work and various types of math charts and mathematical stimulus posters decorated nearly every inch of available space on the classroom walls. The students' desks were fittingly arranged in the "traditional style", (as one external observer described it), with two groups of four desks at the center of the classroom, one group of three desks off to the side and another group of two desks adjacent to the center row, which was closer to the main entrance of the classroom. On top of the desks were supplemental guides, manipulatives and warm-up folders. There were computer tables, and other tables covered with math books off to one side but in easy reach of the students sitting next to them. At the back of the room was a table covered with various colored notebooks and ring binders around which were small metal chairs. Students deposited homework into a red box on a table designated for that purpose. Right next to this homework depository box was an orderly arrangement of mathematics textbooks, most of which were reading resources that appeared to be at grade level. There were three small metal chairs around this table. They provided a place for observers to sit and observe Ms. Karen's teaching.

Ms. Karen spent approximately three minutes playing "at place" money using monopoly cards, which she placed on each student's desk after each one was seated. This I later learned was to reward the students for being in their designated seats.

Those students who entered the room after us seemed to know the routine of commencing with the warm-up activities from their binders located in the rear of the classroom. After all the students were settled, Ms. Karen spent approximately another 18 minutes returning previous graded quizzes to the class, directing attention to the "warm-up" folders on the students' desks, and answering questions about the warm-up quizzes from students who worked independently.

The following excerpt is typical of Ms. Karen's action during this observation.

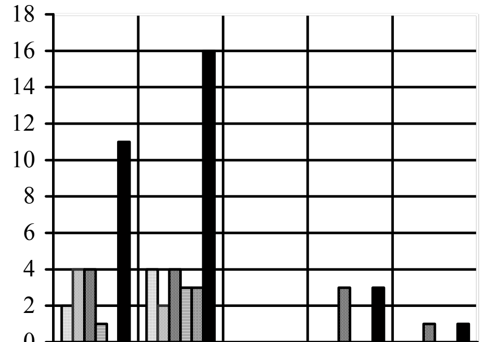
Ms. Karen: Come on guys' speed up. You're coming in too late. No talking, if you have a question ask me. Hurry up guys' you have quizzes . . . Put your quizzes away, that's how they got loss. Pull out your spiral notebooks.

Ms. Karen: (*Transition point)(Ms Karen introduces lesson*) now we are going to talk about factors. How many of you heard about GCF? (*A number of students in the front row responded in a chorus*) No! Well, she said, if you didn't, you'll know today. *(She asks class)* How many of you go to Columbus market? (Pauses, *then says*) At the flea market they have tables, and every one of them is measured (*She writes 12 sq ft" on graph chart mounted on easel in front of the class and again asked*) "What does this mean?"(She p*oints to the 12-sq-ft boxes she outlines on the graph paper mounted on an easel in front of the class. (Class is silent. Ms Karen says)* if this were a square each side would be 12 feet. (*She continues to address the class*) Open your boxes and pick out 12 of those squares . . . . When you have twelve, you make a rectangle. You must have two stipulation [s], no doughnuts in the middle.

Ms. Karen: (*Transition #2.Ms. Karen addresses the class*) now we are going to work on Area Models (*See activity HA in Appendix H*). Do your area models in pencil first. Why? You mess up, we have to give you a new paper and we start all over again . . . . . do them quickly, so we can put them up on the board. (*Approximately seven minutes into this activity, she begins to walk between rows of desks* . . . . listen guys'; you are not tracing the blocks because they are not big enough. Use the blocks on your graph papers.

The warm-up and the return of homework assignment activities continued for approximately ten minutes. As you can see from the above, Ms. Karen began the activity by establishing who would be the resource person ("If you have a question ask me"). From the beginning of this lesson, she established her authority for contributing mathematical information and for determining mathematical procedures ("You must have two stipulations, no doughnut in the middle; we are going to work on area models; do your models in pencil first; put away your quizzes we are going to talk about factors"). Ms. Karen maintained this control throughout the three different events by lecturing, giving the students problems and then checking what they reproduced and then moved on. Throughout the course of these events, the opportunities afforded to students to interact meaningfully with mathematical concepts were limited to the reproduction of procedures indicated by Ms. Karen. Figure 5.1 provides further analysis of what constitute the mathematical reform practices for Ms. Karen.

Figure 5.1: Analysis of Ms. Karen RTOP Scores



	L D I	C PP K	CPDK	C C C	S T R
Subscale Item 1	2	4	0	0	0
Subscale Item 2	4	2	0	0	0
Subscale Item 3	4	4	0	3	1
Subscale Item 4	1	3	0	0	0
Subscale Item 5	0	3	0	0	0
Total Subscale Score	11	16	0	3	1

Subscale: LDI = Lesson Design and Implementation; CPPK=Content Propositional Knowledge; CPDK=Content Procedural Knowledge; CCC=Classroom Culture Communication STR=Student Teacher Relationship (See Appendix M for description of Subscale items 1-5).

Her overall rating of 31 on all five subtests is indicative of lesser degree of reform. Her highest rating was in the area of content, prepositional knowledge.

Ms. Karen responded with "Strongly Agree" to 14 of the 16 items on the Reform Oriented Questionnaire and "Agree" to the remaining two questions. These data are reported in Table 5.11 at the end of the chapter. As described in Table 5.1, Ms. Karen asked a number of questions that required the students to recall what specific factors meant, described specific dimensions and reproduced the various ones she wrote on large graph paper mounted on easel. Students were not asked to give any mathematical explanation for the work they did.

In a post-observation interview the next day Ms. Karen commented that she did not like the way she is forced to teach mathematics to the children. She said teaching to the test and cramming for exams are not productive to students learning mathematics. She lamented that she wished she could do things differently. Allowing students more time to think and to see fun in mathematics were consistent themes for Ms. Karen during the post observation interview. It must be emphasized however, that throughout the interview process Ms. Karen expressed satisfaction with the way the students learn in her class as well as satisfaction in her ability to teach mathematics. She spoke confidently of the RAVE method, which she said she used frequently to give students the opportunity to make connections. She said, RAVE also allows students to *restate* a question to make sure they understand it, *answer* the question using as few words as possible but getting to the point, use the appropriate mathematical *vocabulary*, and show *examples* of what the vocabulary words mean.

Table 5.1, describes the procedures that unfolded from the lesson on factors. They represent Ms. Karen's understanding of what constitutes reform mathematics teaching. The minutes listed are approximations of the length of each event.

Table 5.1: Description of Ms. Karen Mathematics Activities

Time in Minutes	Students' Actions	Teachers' Actions
8	Warm- up Students worked on Quizzes	Teacher gave directions to students on quizzes
5	Engaged in homework activity on factors Worked independently	Teacher returned homework
8	Followed teachers' procedures Students listened	Transition: Introduced lesson Reviewed lesson on factors
8	Students solved problems Shared results with teacher and copied different dimensions from a chart paper	Teacher presented problems orally to students to solve Teacher quizzed students on finished work and explained the "D" word (dimension)
7	Students copied area model from chart mounted on Easel Students described area model teacher constructed	Transition: Introduced how to make area models Teacher constructed examples of area models
10	Students shared area models with the teacher Students gave completed area models to the teacher End of activities	Teacher gave directions and posed questions regarding students' area models Teacher displayed students' area models and concluded the activities

On the Reform Oriented Questionnaire Ms. Karen responded "Strongly Agree" to 14 of the 16 items and answered "Moderately Agree" twice. These data are reported in Table 5.11 at the end of the chapter.

Ms. Mellicent (Young Teacher)

Tompkins Jefinson Middle School is in the Addison school district. The total full time equivalent teaching staff is 74 teachers for a total student population of 747 students.

Grades ranged from six to eight. The teacher student ratio is 10:1 compared with the average teacher student ratio of 14:1 in similar schools in the State. The ethnic/racial composition of the student body in this school is zero percent American Indian, 30 percent Asian, 14 percent Hispanic, 13 percent Black and 43 percent White. Only 12 percent of the students are considered eligible for free lunch. In addition, six percent of the student population is considered eligible for reduced lunch. Ms. Mellicent said she decided to participate in this study because she "just feels like helping and [giving] other people the opportunity to come into her classroom to see different environments of learning". Certainly, from my observation, Ms. Mellicent's classroom setting is unlike the other sixth grade classroom I had observed a week before at another Middle School. At that school Ms. Karen had the typical classroom setting where she has taught mathematics for more than 16 years. However, at Tompkins Jefinson Middle School at the time of my observation, Ms. Mellicent had been teaching mathematics to her sixth grade class for only five years.

Ms. Mellicent began her lesson by standing at the front of her classroom while most of the students sat around clusters of four desks. One student sat by herself in one corner of the room close to the main entrance into the classroom. All the students sat facing the teacher and her Special Education Assistant. Desks and chairs were connected, and book baskets were beneath the chairs. Classroom management charts (e.g. 4 Steps to begin the day, code of conduct) were displayed on the sidewalls. A white board was mounted on the front wall of the room and was frequently used by the teacher and her Special Education Assistant to present examples to the students. Stimulus charts and other visual stimuli were minimal. A television covered with a large black plastic bag stood almost in the center of the room aligned against the back wall facing the students and the main entrance. One computer was off to the side of the room nearest to the white board and to the right of the teacher.

An overhead transparency projector was positioned next to it. Another lap top computer was on the teacher's desk positioned for her use. The desk was situated to the opposite side of the television and positioned in such a manner that the teacher could easily see anyone entering the room. The question of the day and a problem-solving chart mounted to the right of the white board near the main entrance was visible from anywhere in the room.

As soon as each individual student entered the room and was settled in his or her seat, Ms. Mellicent allowed them several minutes to work independently on 'warm up' quizzes. After she had gone around to several students (assisted by the Special Education teacher) giving each one feedback on the work they had done on the quizzes she praised them for what they had done, then she asked everyone to put their quizzes away and get the notebooks ready to take notes.

She began the class by spending several minutes reviewing note taking, and some mathematical vocabulary words such as variable, equation, expression, constants, symbols, and signs. After several lively interactions with the students on this review of words, she then transitioned into the formal mathematical lesson she had prepared for the day by announcing to the class how she and her students were going to begin to solve equations.

Ms. Mellicent: Today we are going to talk about equations
> or expressions. Equations are expressions. We want to define equations. An equation is a mathematical sentence that has variables, constants, symbols and equal signs (*Walks around the room while students copy equation definition from white board*).

In the interview that followed after the observation, Ms. Mellicent said she was positive about her students' prospect of learning mathematics but expressed concern that almost half of the class is in need of [1]special attention. She referred particularly to the four students in her class who are English language learners. She said these students are struggling with the mathematics vocabulary, but at the same time they do enjoy doing math because she encouraged them to ask for extra help. She said she worked with them to "fix their calculation problems and make sure everyone has [had] a good understanding" before she moved on. According to Ms. Mellicent, part of the reason for the students wanting to do math was because over the years she had

[1] This is an inclusion class with 12 students and an Aide

discovered easier and better ways to teach mathematics. She said:

> . . . the way I understand it in one way doesn't mean the kids are going to understand it in certain ways. So, once I started teaching I started learning shortcuts, easier ways to teach to the students.

She gave similar answers when asked about what she did on a daily basis in her class to develop students' mathematical thinking.

> In the class that you saw normally we have a new lesson almost every single day. I would say almost every single time we try to teach them some kind of, I don't want to say short cut, but some kind of method that help them to remember the skill, something fun, or just unique to help them. I would say every other lesson there is something like that to help them with their math skills and to help them like math. Cause I know a lot of kids don't like math. Math is not something they enjoy.

When asked to describe her confidence in her ability to teach mathematics, Ms. Mellicent said her confidence grew the more she developed "short cuts" which are "certain tricks" that help her to discover better ways to teach math.

> I'm very confident in teaching math. Ohm, when I first started teaching, I was not. This is 6th grade math, you know, it's not calculus! But I was still nervous . . . I like math because it is very concrete. This is what the answer is; this is how you get the answer; here are all the steps. So I like that. I also was fortunate enough to have very good teachers when I was in high school, which was really good. That is why I want to pursue being a teacher in math.

She opined, however, that because it is an inclusion class she teaches the "special needs students differently from those that are regular students".

> I repeat myself a lot when I'm in an inclusion setting a special needs setting, I make sure I repeat my directions, I make sure I repeat what I am saying, and sometimes I try to rephrase it that way in case if they didn't understand, I do go a little bit slower and I make them do more of the work that way I can asses them. I'll actually write the notes, for them it takes too much time from them to concentrate on writing and think at the same time.

In discussing her confidence as a mathematics teacher, she hinted that one of the drawbacks she experienced in teaching mathematics comes from being forced to move the students too quickly through the material before they have mastered it. As such, the students tend to easily forget what they have learned.

> I think that we are trying to cram so much into our curriculum and not just in sixth grade. I think in all grade level, I think that there is so much that is being taught and nothing is being taught for mastering. And I think that's hard for the kids. Because although we are doing it year after year they go, oh! I remember that, but they don't remember what it's for…

After this brief interview with Ms. Mellicent, I decided to look more closely at the kinds of mathematical activities that took place in the classroom during the observed lesson to better understand the mathematical context in which she and her students were doing mathematics. For this I turned to my field notes to try to understand how the lesson was organized and what she had attempted to accomplish with her students during their interaction. Table 5.2 shows how the lesson on solving equations was organized.

Table 5.2: Description of Mellicent Mathematics Activities

Time in Minutes	Students' Action	Teachers' Action
11	Warm up worked on quizzes	Reviewed and graded quizzes
13	Prepared to take notes Wrote notes on how to solve an equation and vocabulary words	Transition: Reviewed note taking on solving equations and vocabulary Randomly questioned students for comprehension
15	Copied examples and solved equation problems Used vocabulary words (signs, symbols, variables to do math problems	Transition: Presented examples and problems on white board Used vocabulary words (sign, symbol, etc. to construct math problems
12	Prepared for new activity; found answers to problems Provided explanations to teacher and to peers Thought of other ways to find answers	Transition: Introduced "solution" vocabulary and solved problems Inquired into students' problem solving technique Asked students to check each other's problem solving process

Table 5.2 (Continued)

Time in Minutes	Students' Action	Teachers' Action
10	Worked in groups independently and discussed freely	Used overhead projector to assign seatwork problems
5	Collected worksheets	Passed out worksheet homework assignment

As described in Table 5.2, the two major mathematical activities that Ms. Mellicent conducted at the fifteen (15) and twelve (12) minutes transition points during the observed lesson provided an opportunity for the students to solve problems as a class, and to interact meaningfully with the teacher and their peers during the time spent. Throughout the nearly fifty minutes of observation, Ms. Mellicent typically introduced an activity, first by reviewing previous notes, including vocabulary, on solving equations, provide examples, after which she, the students and the aide would engage in the activity as a whole class. This sequence was followed with each transition and contained three to four different activities focusing on the same lesson. The pattern Ms. Mellicent established may have been the principal routine that helped her to build a framework on which to scaffold the tasks and the concepts she presented in the lesson as she gradually gave students harder and harder problems and different types of problems to solve. Such a pattern could be more evident particularly with the special needs students in her class.

Moreover, establishing such a consistent pattern of delivery might also allow Ms. Mellicent and her students to construct an understanding of the kinds of behaviors or interactions expected during each transition point in the lesson. Which brings me to the question of how does the construction and integration of a social and academic routine influence Ms. Mellicent's teaching and learning of mathematics in this classroom?

To answer this question, I turned once again to my field notes, to provide the following brief excerpt from the first ten minutes of Ms. Mellicent's formal mathematics lesson presentation.

Ms. Melllicent: Now we have finished our quiz, its time to take notes. First thing we have on our notes is 10/16/07 [/08]. You think it's Saturday? We will solve equations.

Student: I don't like it.

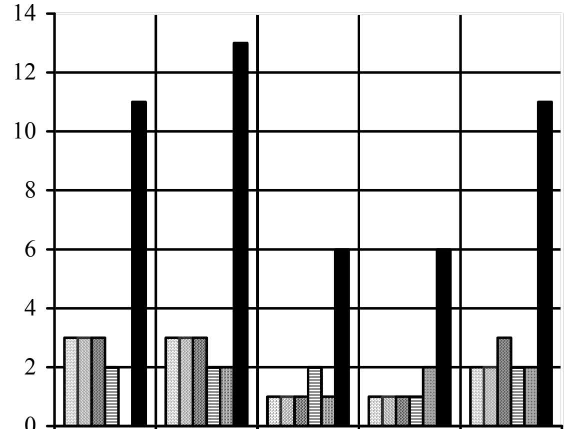
Ms. Mellicent: Why don't you like it? What did it do to you? Ok. You remember we talk about variables? Variables are mathematical symbols. Today we are going to talk about equations. (*She writes the word 'expression' beneath 'equations, then she continues*) Equations are expressions. We want to define equation; do you remember we say evaluate expression? What do you think we have to do? What do solve wants us to do?

Student: Find the answer.

Ms. Mellicent: D, you did just what we want to hear.

In this example, Ms. Mellicent discursively creates the social environment for doing mathematical activities. She does this by setting up groups, physically and cognitively (e.g., "Today, <u>we</u> are going to take notes), by complimenting behaviors (e.g., "D, you did just what we want to hear) and by establishing the seriousness of the moment (e.g., "Think today is Saturday?"). In like manner, she creates the academic environment through the framing of the activities (e.g., "We are going to solve equations; we want to define equations; what do you think we have to do?"). In addition, she constantly defines and redefines the activity (e.g., "What do solve wants us to do? "So when we talk about expression we want to solve the equation"). It is obvious from the mathematical interaction that takes place between Ms. Mellicent and the students that they were constructing an academic routine with which to teach and learn mathematics. Additional observation information derived from an analysis of Ms. Mellicent's reformed practice presented in Figure 5.2 helps me to identify the social and academic constraints that Ms. Mellicent and her students established for doing mathematics.

Figure 5.2: Analysis of Ms.Mellicent RTOP Scores



	L D I	C PP K	CPDK	C C C	S T R
Subscale Item 1	3	3	1	1	2
Subscale Item 2	3	3	1	1	2
Subscale Item 3	3	3	1	1	3
Subscale Item 4	2	2	2	1	2
Subscale Item 5	0	2	1	2	2
Total Subscale Score	11	13	6	6	11

Subscale: LDI = Lesson Design and Implementation; CPPK=Content Propositional Knowledge; CPDK=Content Procedural Knowledge; CCC=Classroom Culture Communication STR=Student Teacher Relationship (See Appendix M for description of Subscale items 1-5).

Ms. Mellicent's composite score of 41 as indicated by observers' ratings on the five subtests described in Figure 5.2, is reflective of a lower degree of reform. As described in Table 5.2 Ms. Mellicent presented her students with fundamental concepts of the topic and demonstrated good grasp of the content inherent in the lesson on solving equations. Such performance may speak to her lack of knowledge about reform oriented mathematics teaching as she herself admitted during the post-observation interview. On the Reform Oriented Questionnaire, Ms. Mellicent responded "Strongly Agree" only once, answered with "Agree Moderately" four times, and responded with "Agree" for all of the remaining questions. Her overall score on the questionnaire was the lowest of the 10 teachers observed. A report of these data is presented in Table 5.11.

Ms. Ballas (Young Teacher)

Francois Gore Elementary is one of three elementary schools in the Hewing Township school district. The total full-time equivalent teaching staff is 44 teachers and a total student body of 609 students (male 48 percent, female, 52 percent). The teacher student ratio corresponds to the states average of 14:1. The racial ethnic composition of the school is 5 percent Asian [2]*(7 %), seven percent Hispanic *(19 %), 38 percent Black *(18 %) and *51 percent White (56 %). Twelve (12%) of the students are eligible for free lunch and 8% meets the eligibility requirement for reduced lunch. The medium household income of the families of children who attend Francois Gore School is estimated at above $66,000.

At the time of the investigation Ms. Ballas had been teaching mathematics for nine years. Four of those years were spent teaching mathematics to third grade. When asked about her confidence in teaching mathematics, she said she is fairly confident because teaching mathematics is something she has been doing for five years.

In Ms. Ballas's classroom the students' desks were neatly organized in seven groups of threesomes with one group of four desks in one of the rows. There were appropriate classroom posters throughout the room. Two computers were to the right of the main entrance to the room. There were two white boards, which were used frequently with one accessible easel located in the rear of the class. This is an inclusion class with pullouts. At the time of the study, the class had 17 students. Soon after we entered the room five students left with the inclusion teacher, one for gifted and talented mathematics and four for additional support in mathematics at their levels. The remaining students corrected homework after they were guided and instructed by the teacher.

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[2]Asterisks indicates state average

Ms. Ballas began the class with routine directions for the ‘Problem of the Day’ warm up while the students work independently. Her calm soft-spoken voice complemented the silence of the remaining 12 students sitting quietly and attentively awaiting the teacher’s questions. The fact that she did not have to raise her voice might have helped her maintain the organized, structured social and academic setting Ms. Ballas creates during the warm up session.

After 15 minutes on the problem of the day activity, Ms. Ballas invited the students to come to the carpet area at the front of the room near the white board for a lesson on reading and decimals. The students sat on the floor in front of her while she stood between her table and the white board, which she used frequently to write on during the lesson. The students focused attentively on the teacher as she wrote. She began the lesson:

> Today we are going to practice write the decimal. (*She draws 0/T/H on the white board, instructs her students’ to look at number two in their workbooks, and then she says*) Number two has seven skinny guys, how do we say that?

Student: zero and seven tenths. (Ms. Ballas writes. 0.7 =. 7)

She spent the next several minutes developing the concept, introducing activities, and working with small groups at times having students use several manipulatives (e.g., small rods and flat squares) as they learned how to read and write decimals. In a post observation interview immediately following the observed lesson, Ms. Ballas stated that it is important for teachers to support students’ efforts in learning and make mathematics meaningful to them.

> I think it is important for teachers to engage students and I think that children learn by doing, so I think that for some students, where it did appear, help them a little bit because it gives them some sort of meaning and it engage them in math in such a way, where they never quite get [it] before and in seeing this box here that, you now, not, [is] three dimensional may not have been helpful, but hopefully that’s something that engage them in learning. So I think that for many students that’s what they need. Some of them pick it up and get it right away. Obviously, I wasn’t that child so I know that there are kids who need that extra support. So I think it is important as teachers to think of ways to make math meaningful, in order to help learning.

When asked about her strength as a mathematics teacher she said it is her ability to gear mathematics teaching toward her school’s goals, which are character building, responsive classroom and guided reading. She said she makes guided reading an important part of her mathematics instruction.

> We are very much into character education here, and responsive classroom and we do guided reading, I kind of gear my math toward guided reading in that, you teach to the whole group, then you pull reading groups, and then you return again. So I kind of do that with math now. I guess that's kind of a mix of what you said.

In terms of her strength, she added that there are other reading and writing activities in which she engages her students in order to develop their mathematical thinking.

> Something else that we do is create a "Word Wall" and when reading time during centers they have to put those words in alphabetical order, but that's during reading, but they are still doing some kind of math related activity so I am trying to get and we do also some kind of writing during that time as well getting them to do some explaining how we get our answers and we also get them to do some math related stories. So I try to incorporate math across curriculum.

Ms. Ballas also described her weakness in teaching mathematics.

> . . . something I am working on this year is open-ended
> questions and thinking open-ended answers, so ohm, something I tried to do is to incorporate those types of things into problem-of-the day" to keep them thinking. I'm always open to suggestions in terms of things they can do.. as far as getting them to think more as math learners.

With this general understanding of Ms. Ballas mathematics teaching background and beliefs about mathematics teaching, I decided to understand the context in which her mathematics teaching takes place. That is how the different activities were constructed moment by moment during the observed lesson and what the she and her students were actually engaged in as doing mathematics during the observed lesson. Table 5.3 provides a description of the activities that count as doing mathematics in this class.

Table 5.3: Description of Ms. Ballas Mathematics Activities

Time in Minutes	Students Action	Teacher's Action
7	Problem of the Day "warm up"	Wrote problem of the day on white board
8	Responded to teacher's questions Students used DMSB acronym to solve problems with teacher on white board as a class	Used math acronym DMSB to question understanding of the problem
10	Students followed set procedures Students read and wrote problems with decimals	Transition: Teacher initiated Teacher reviewed reading decimals and writing decimals
10	Groups used manipulatives to read and write decimals	Teacher worked with individual student at Math Center while groups worked alone
	Individual student used teacher's manipulatives	Teacher introduced manipulatives
12	Students followed Procedures. Students Practiced NJ ASK "Fast Fact" skills. Students completed "Fast Facts" activities then glued them to Ring Binders	Teacher initiated transition to NJ ASK "Fast Fact" skills. Teacher introduced "Fast Fact" activities

At both transition points in the lesson students were directed to a number in their textbooks and asked to identify the place value. In the post observation interview that followed Ms. Ballas admitted that she uses this procedure on a daily basis. The process was highly repetitive. During the interview she admitted that the routine is important because it helps to allay the students' fear of the subject, and makes learning mathematics easier for the students.

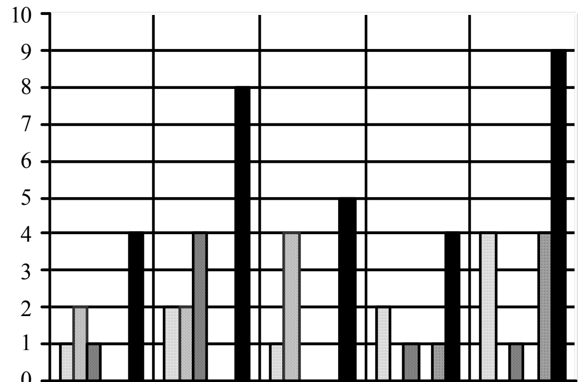
> I think it is important to have a routine so that the kids know what to expect each day . . . so that there's a lot of [Many] kids have anxiety with math. So I hope the routine sort of, eliminate [s] some of that anxiety.

Of course the context in which Ms. Ballas framed the teaching of place value with decimals was informed by her own beliefs and teaching practice concerning how teachers teach and how students learn place value.

> And I think that teachers are very guilty of just trying to plug along 'cause you have so many things to do. It's easier to say do this and do that check it off and move on, whereas, you know its ok if I'm a little bit behind, where I used to be if my students are thinking a little bit more and applying a little more effort, and that's my belief about math.

Figure 5.3, presents a further analysis of Ms. Ballas's mathematical practice. The different subtest scores gave some indication of what characterized her reform practice. As shown in Figure 5.3, Ms. Ballas overall score of 30 on the 25 reformed items on all five subtests is indicative of a lower degree of reform. During the observation there was much focus on developing the fundamental concepts of the lesson.

Figure 5.3: Analysis of Ms.Ballas RTOP Scores



	LDI	CPPK	CPDK	CCC	STR
Subscale Item 1	1	2	1	2	4
Subscale Item 2	2	2	4	0	0
Subscale Item 3	1	4	0	1	1
Subscale Item 4	0	0	0	0	0
Subscale Item 5	0	0	0	1	4
Total Subscale Score	4	8	5	4	9

Subscale: LDI = Lesson Design and Implementation; CPPK=Content Propositional Knowledge; CPDK=Content Procedural Knowledge; CCC=Classroom Culture Communication STR=Student Teacher Relationship (See Appendix M for description of Subscale items 1-5).

On the Reform Oriented Questionnaire Ms. Ballas responded with "Strongly Agree" on all 16 items. These data are presented in Table 5.11 at the end of this chapter.

Ms. Hardiman (Senior Teacher)

Ms. Hardiman teaches at a school where the teacher student ratio is 14:1, which is same as the State's average.

When Ms. Hardiman volunteered to participate in this research she had been teaching mathematics for five years. She said her third grade class is one of several other grades in her school district taking part in a mathematics pilot program called Investigation [Structure of

Investigations in Number, Data and Space] that her school district has adopted. The program is designed for K-12 students and this was her second year participating in it. When asked about her confidence level for teaching mathematics, she added that her "comfort ability level" to teach mathematics "has improved" because of the training she receives from being part of the pilot program.

> It's only my second year using the investigations program, which we are implementing so I'm becoming more comfortable with it, so the comfort ability level is increasing as I use it more.

According to Ms. Hardiman, she chose to participate in the research because it ties into her coursework for a Masters Degree in mathematics which she is currently pursuing online through Walden University.

Her classroom setting was atypical. The desks and chairs were arranged in an uneven squared shape looking horseshoe with three additional desks and chairs positioned along the longest sides of the uneven square shape two on one side and one on the other with a single desk and chair in the center of the uneven square facing the green chalkboard. These additional desks were oriented in a manner that the students who occupied them had their backs turned toward other students but facing the teacher and the chalkboard in front of the class. Behind the students' desks and chairs and close to the rear back wall were several bookshelves and piles of blankets. A globe, a cupboard, a file cabinet, an easel and piles of book bins with students' names lined the sidewalls on opposite sides of the room but close to the main entrance. There were two computers and an overhead projector positioned directly across from the main entrance to the room. Students' work was present throughout the room. Many were pasted at the top of one of the sidewalls very close to the ceiling.

As the students entered the room and were settled Ms. Hardiman exited for a brief three minutes as the Aide conducted the warm up mathematical activity on building equations to 50.

Soon after she re-entered the room, she gathered the students at the back of the room in front of an easel on which a chart hung displaying a drawing of a thermometer. The Aide followed suit. Students sat on a rug in a circle, some facing the easel and the teacher, others facing the Aide and their peers. Ms. Hardiman stood close to the easel and announced to the class that it was time to find the temperature. She spoke to the students in a voice that was soft but clear.

(*She asked*). Who knows what the temperature has to be outside for it to snow?
(*Class responding altogether as if in chorus*) Thirty-two degrees

During this observed lesson, Ms. Hardiman's third grade class participated in mathematics as a form of discovery. They shared, questioned and presented information individually and as a group. They learned how to use mathematical measurement tools to find answers to mathematical problems. The teacher guided the students to make predictions: students checked and made corrections about predictions they made; they recorded data; reflected on their experience; and were able to move from abstract to concrete presentation with help from the teacher. Table 5.4 is an attempt to outline the kind of mathematical discovery practices that Ms. Hardiman and students engaged in during the observed lesson. The minutes are approximations.

Table 5.4: Description of Ms. Hardiman Mathematics Activities

Time in Minutes	Students' Action	Teacher's Action
10	Warm-up quiz Students wrote math Equations to equal 50	Aide gave directions Teacher facilitated
15	Students lead discussion on temperature graphs designs, compared predictions and checked accuracy of predictions Students explained "wacky weather" conditions	Teacher guided and directed students' weather predictions Teacher used students knowledge of weather conditions to make predictions
10	Students followed set procedures Students used measuring tools and recorded data	Teacher initiated transition Teacher reviewed linear measurement tools
15	Students worked in groups to solve problems using specific measuring tools Students made distinctions between measurement tools Students recorded information	Teacher presented information on specific measurement tools and their uses Teacher facilitated students Discussion
5	Shared results with the teacher and with peers Students followed teacher's direction. Students wrote own in workbooks Activities ended	Teacher initiated transition Teacher directed students to measure their feet and to write their own measurements Lesson ended

Table 5.4 shows the activities from the lessons observed on data analysis and measurement of length and distance. During the lesson Ms. Hardiman and her students participated in several activities that led to basic scientific explanation for the changes in the weather conditions, identified different linear measurement tools and how they are used and investigated the use of various self made charts and rulers to present information on linear measurement. Students used specific questions and gathered data on the length of time students attend Anthill school. The teacher guided the students to use simple bar graphs to present statistical information of their data, thus moving from the abstract to the concrete in their formal

presentation to their classmates as a group. For the various activities, students used mathematics to solve the problems.

The mathematical practices that are highlighted in Table 5.4 reflect some of the similarities in which Ms. Hardiman defined and talked about mathematics in her classroom. These similarities can be seen from the way in which she talked about her mathematics teaching to the different ways she structured her lesson to how students become involved in the process.

> My instruction is student oriented . . . . . . I introduced the vocabulary words and outlier, range, mode and median to them and then we brainstorm other words, which they can use to describe the data. We have the chart up there that we created together, kind of sentence starters of how they can describe data, other than those four vocabulary words which I gave them they generated everything else. And then they were able to, anything they noticed, is how they can describe, I didn't tell them specifically what they needed to look for, they just needed to look at their data and describe something they noticed.

It's obvious from the above description that Ms. Hardiman seems to be making some changes in the ways she views and teach mathematics. For example, during the post observation interview, she boasted about her students' ability to think and do mathematics.

> I've seen a lot of their strengths; I've seen a lot of strengths in every student in every unit we have worked on so far. . . .

On the other hand, when asked about what she could do to improve mathematics instruction in her school, she indicates that more needed to be done in developing students' mathematical thinking.

> Getting, using activities [students to engage in activities] that encourage students [their mathematical] thinking rather than, and again encourage students to be mathematical thinkers and seeing math as a process not just an answer that they have to get. And math is not always about getting the right answer is about the steps that you need to use to get there.

She added that her view of constructivism helps in her efforts to develop students' mathematical understanding.

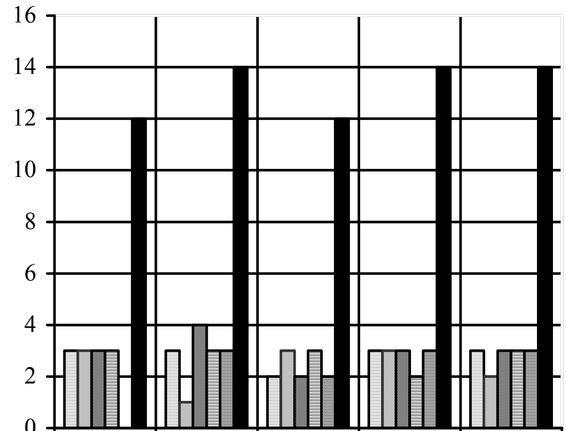
> . . . . my views are I can see the benefits of the constructivist theory in math every day with my students. Getting them to think, getting them engaged in what they are doing through inquiry and through discussion. It just help them grow as learners and rather than making them just able to do computations, it makes them able to think mathematically.

> It's not, you know they're not just civil processors now where they can just do something route (*pauses*); now there is more understanding.

The manner, in which Ms. Hardiman talked about her teaching, and the activities she used to engage her students in, portray a mathematics classroom that is unstructured. She allowed students the opportunity to interact freely and gave them the freedom to use different designs to create bar graphs to present their data. Other differences can be found in Ms. Hardiman's reform mathematical practice described in Figure 5.4.

As shown in Figure 5.4, her composite score of 66 on the five subtests of the Reform Teaching Observation Protocol described in Chapter 3, is indicative of a relative degree of reform practice. Such relatively high degree of reform may be influenced by Ms. Hardiman's exposure to the new curriculum she is piloting in her school, which, she said, allows her to use the constructivist teaching method to present mathematical information.

Figure 5.4: Analysis of Ms.Hardiman RTOP Scores



	L D I	C PP K	CPDK	C C C	S T R
Subscale Item 1	3	3	2	3	3
Subscale Item 2	3	1	3	3	2
Subscale Item 3	3	4	2	3	3
Subscale Item 4	3	3	3	2	3
Subscale Item 5	0	3	2	3	3
Total Subscale Score	12	14	12	14	14

Subscale: LDI = Lesson Design and Implementation; CPPK=Content Propositional Knowledge; CPDK=Content Procedural Knowledge; CCC=Classroom Culture Communication STR=Student Teacher Relationship (See Appendix M for description of Subscale items 1-5).

On the Reform Oriented Questionnaire Ms. Hardiman responded "Strongly Agree" to 14 of the 16 items on the questionnaire and answered "Moderately Agree" twice. Table 5.11 reports the data in more detail.

Ms. Betheus (Young Teacher)

Ms. Betheus said she has been teaching mathematics at Stokke Elementary School for six years. Stokke Elementary School, located in the Triton School District, is one of 36 public schools in the district. The school serves 340 students (54 % female; 46 % male), ranging from grades kindergarten to grade six (k-6). There are twelve (12) students for every full-time equivalent teacher. The student population is predominantly black (95 %). Other ethnic groups include Hispanics (4%), White (1%), and American Indian/Alaskan Native (< 1%). In addition, 72 % (244) of the students are eligible for discounted/free lunch.

Ms. Betheus' classroom was atypical, with teacher–made reading posters and other commercial ones all around the room. Some were mounted on easels and chalkboards, others on cupboard doors and walls. Mathematics charts were minimal. In the post observation that follows Ms. Betheus admitted that the glut of reading charts in the classroom reveals the emphasis her school places on reading. She said:

> My particular school is a reading first school so we get a lot of, when I say reading first, the concentration is on reading getting the kids to achieve in the subject of reading . . .

Ms. Betheus stood in front of the chalkboard, which she used frequently to present information to the class. Her students sat in groups of six, with the exception of one group of three, all oriented in such a manner that some students had their sides toward the teacher, but facing each other, while others faced the teacher directly with their backs facing the rear windows in the room. She began with the warm-up of the day activity, which was approximately eighteen minutes of "math minute", which students spent identifying parts of whole and writing number sentences to 10. After reviewing homework assignments, she transitioned into the formal math lesson on subtraction with regrouping using triple digit numbers minus a double-digit number.

Throughout the observed lesson, she constantly reviewed the material and then provided a model after which she and her students engaged in the activity as a class. The following is an excerpt from the first five minutes of Ms. Betheus' mathematics lesson taken from my field notes.

> Get ready for a new lesson teams. Open up to a clean sheet of paper to take notes. (*She writes, "Subtracting Triple Digit Number, then says*). Now watch Ms. Betheus as I model to [for] you. So you can do it independently.

> (*Teacher writes 147-64, she said*). I want to take 4 from 7. (*She asks*). A, can I do it? (After hearing student's correct response, t*eacher writes and verbalizes next question*). I have 4 tens and I want to take away 6 tens, A, can I do it? (*A, said*) No. (*Teacher asks A*) Why? (*A responds*) We only have 4. (*Teacher draws ten blocks on the chalkboard, and then says*). We learn earlier this semester that these are 10's. We know that base 10 blocks are 10's. If I have 14 in the 10's place it's the same as having 140. So I must ask the 100's for help. So I borrow the 100 and put it with the 40 and get 140 (*She draws 14 blocks and again reminds class that each one represents 10*)

Ms. Betheus continued with this sequence, reviewing, modeling, and solving problems as a class using six to eight different examples. She demonstrated her control of the discourse by saying:

> Look at how Ms. Betheus lines her place value up. I know that some of you will need help. I know that you want to spend time with Ms. Betheus. But you must first write the problem down.
>
> Remember Ms. Betheus is not expecting you to get it the first time. We have to practice. I will do this many more times.

She further regulated participation by calling on specific students to share information and to work on specific examples at the chalkboard.

> Teacher: A, I want to take 4 from 7 can I do it? M, do the first one, S, do the second, V, do the third, N, do the fourth.

She also established control by the kinds of questions she asked that provided known-answer types of information.

> Teacher: Five in the tens place means what class?

This control was also evident in the social practice that unfolded in the classroom.

> Teacher: Those of you who are finished create your own problem. A and R, you'll do your homework during free time . . .. The fact that you do not bring your homework doesn't give you free time.

As Ms. Betheus and her students involved themselves in the task of learning place value, they constructed a routine that defines how they do mathematics in that particular setting. Table 5.5 described the social and academic routine established by Ms. Betheus during the observed lesson. The minutes are approximations.

Table 5.5: Description of Ms. Betheus Mathematics Activities

Time in Minutes	Students' Action	Teacher's Action
18	Warm-up Math Minute	Teacher gave directions to begin warm-up activity
	Students wrote math Sentences to 10 and described parts of whole	Wrote sample math sentences for students to build on
20	Students listened	Teacher reviewed subtraction with double and single digits.
	Students followed set procedures, answered teachers questions and subtracted double digits from triple digits	Teacher initiated transition, modeled subtraction with double and triple digits
15	Students followed set	Transition: Teacher invited specific teams to the chalkboard to solve triple and single digits problems
	Students worked in teams and solved problems with double and triple digits at chalkboard	
15	Students worked independently with special teams	Teacher constructed subtraction problems with double and triple digits for students to solve independently.
	Students received individual help from teacher Lesson ended	Teacher directed small group to her table Small groups worked alone on homework assignments

When asked about her confidence in teaching mathematics, Ms. Betheus said she is "pretty confident". Part of the reason for her confidence, she believes, is because her students' have done well mathematically. According to Ms. Betheus, they all pass the State's test, and that is an indication to her that the students learn what she teaches them.

> Pretty confident. Yes I feel comfortable teaching math. The students usually do well. Ohm, when I'm teaching math, they were successful in the New Jersey ASK, so, I just assume that ohm, they pick up well what I am teaching.

A similar response was offered when Ms. Betheus was asked to describe specific things she does to help students develop mathematical thinking.

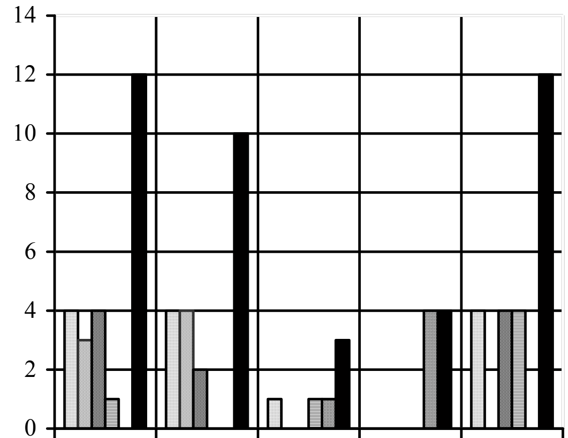
> Ohm, I have them using higher order word problems, ohm, which we do three days a week. Ohm, the Math Minutes like I share with you and then I try to even have myself doing think aloud, so they can see the process that you take when you are thinking in a higher order level. I try to situate my questions ohm, the same way.

When discussing her strength, she said it is based on how well her students do in mathematics and their enthusiasm to want to learn the subject. She said if given additional training other teachers including her would teach math more effectively. She acknowledged that reading is her strongest area.

> A lot of people are not that confident teaching math . . .. I know for me I feel reading is my stronger subject. Ohm, that's what I went to school for, but . . . when I teach math the kids seem to react well to how I teach the math, even when I feel that my stronger subject is reading.

Earlier in the post observation interview, when asked if she knew about or practiced reform oriented mathematics teaching, Ms. Betheus responded negatively ("No, I have not"). Figure 5.5, presents a description of Ms. Betheus' reform practice.

Figure 5.5: Analysis of Ms.Betheus RTOP Scores



	L D I	C PP K	CPDK	C C C	S T R
Subscale Item 1	4	4	1	0	4
Subscale Item 2	3	4	0	0	0
Subscale Item 3	4	2	0	0	4
Subscale Item 4	1	0	1	0	4
Subscale Item 5	0	0	1	4	0
Total Subscale Score	12	10	3	4	12

Subscale: LDI = Lesson Design and Implementation; CPPK=Content Propositional Knowledge; CPDK=Content Procedural Knowledge; CCC=Classroom Culture Communication STR=Student Teacher Relationship (See Appendix M for description of Subscale items 1-5).

As shown in Figure 5.5, Ms. Betheus' composite score on the twenty-five items in the subtests that comprise reform practice is rated at 41. This score is reflective of a lesser degree of reform. It is also reflective of Ms. Betheus' own report during the post-observation interview when she acknowledged that she did not know about reform teaching. On the Reform Oriented Questionnaire, Ms. Betheus responded with "Strongly Agree" on all 16 items. A description of this data is presented in Table 5.11.

Ms. Parker (Senior Teacher)

Ms. Parker has been teaching at this school for ten years. When asked in a post-observation interview why she decided to take part in the study, she said it seems like a good thing to do because she is always interested in research and wants to help with any one [research] that tries to discover how math is working. Ms. Parker was concerned about the new math program at her school in which she is participating as a pilot teacher, and so when she was asked on the survey about her views on reform oriented constructivist approach to mathematics teaching, she commented:

> As I switch programs from traditional to constructivist, there are parts I love and think encourage the students math understanding better in our new program, but there needs to be repletion [repetition] of wrote [rote] skills such as math facts for the students that need it. I find a balanced approach is always best in teaching.

Ms. Parker teaches in a school where less than 5% of the students are eligible for reduced lunch. The full-time equivalent teacher student ratio is 12:2. In Ms. Parker's classroom there were multiple commercial and teacher made math posters and other stimuli materials displayed on the walls, cupboards, chalkboards and window panes. The students' desks were arranged in three groups of six desks and one group of four desks. In a far corner of the room there were several empty plastic containers suspended from non-standard balanced scales with captions, indicating weights of objects. Close to the teacher's desk and not in close proximity to where the students sat was a computer. A mathematics number line was pasted on top of the chalkboard but was not part of the observed lesson. There were several reading centers in different areas of the room and in clear view from the teacher' tables situated in the back of the room but directly behind where the students sat.

Ms. Parker began the lesson by asking the students to sit quietly, put their hands in their laps and pay attention. For the next 35 minutes, she stood in front of the class reviewing the concept followed by individual and whole class practice, ending with some students working independently on worksheet activities removed from the textbooks, or developing their own math riddles on the lesson reviewed. During the independent activities students communicated freely with the teacher and with each other. Throughout their interaction, Ms. Parker constantly reminded the students of the need to help each other and to not rush to finish. During their interaction, one student appeared frustrated and began to cry. Ms. Parker told the student that there was “no need to cry”. “You are all in the same grade”, she said and “working on the same thing”. Table 5.6 reflects the manner in which mathematical activities were practiced in this class.

Table 5.6: Description of Ms. Parker Mathematics Activities

Time in Minutes	Students’ Actions	Teacher’s Actions
8	Students sat quietly waiting for the teacher Responded to teacher’s questions by providing answers	Teacher introduced lesson on rectangles, reviewed what students needed to know to make rectangle
20	Students listened Students built rectangles of multiple sizes guided by the teacher	Teacher introduced lesson on building Teacher used students’ ideas to build rectangles and to describe them
15	Students followed procedures and built rectangles using manipulatives given by the teacher	Teacher initiated transition. Teacher introduced manipulative (e.g. square tiles)
15	Students worked dependently in groups constructing rectangle riddles	Teacher introduced rectangle riddle
	Students used examples from textbook and manipulatives to create rectangle riddles	Teacher directed students to specific textbook page for examples Teacher used manipulatives to create rectangle riddles
	Math lesson ended Students read silently	Teacher told students to read silently Lesson ended

The above pattern of activity may be a way for Ms. Parker to reinforce the concepts and eventually allow the students to undertake more responsibility for solving the problems as they move through the activities. During the post-observation interview the same day, when she was asked about her confidence in her ability to teach mathematics, Ms. Parker described it as medium to high. In addition, she described what could be construed as her weakness in teaching mathematics.

> Sometimes the vocabulary is difficult for me, but I feel comfortable with teaching it, and I love the program that we have attend, the backup professional development so that I can look up the vocabulary.

Although Ms. Parker admitted to having these difficulties she feels the professional development program has provided the means for her to overcome these barriers. Further interaction with Ms. Parker also reveals her beliefs about mathematics and the ability of her students to learn mathematics. These beliefs may also affect the choices she makes in her lesson presentation on “Building Rectangles”. As such, when she was asked about her views on reform oriented constructivist mathematics teaching she responded:

> Reformed oriented teaching? Ah! Well I like the idea. And when you say reform oriented teaching, I think of the constructivist theory and that is new to me, but I like the way the kids are developing their own understandings and I think it will last longer with them though, instead of just showing them the rote, they’ll have a better understanding of what math is and how to use it and apply it to other problems.

During the interview, she said she often engaged her students regularly in activities that promote their mathematical thinking. She went on to describe how it was done.

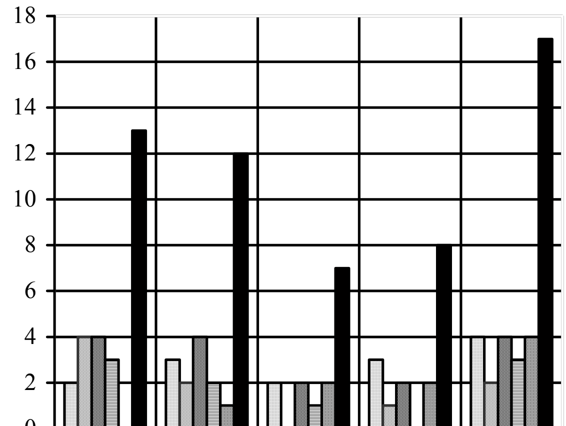
> Give them concrete examples, and then ask them to share their ideas. Give them time to explore and then listen to each other’s ideas. Very often they each come up with ideas that the other wasn’t thinking.

When asked about the frequency with which she engaged students in activities that were related to their experiences, she had this to say:

> Oh, everyday. Ohm, even when we’re doing, there’s math lesson every day and even when we do our lunch count we ask the kids to think about, if we have everybody that orders lunch and how many people didn’t order lunch. So they are always thinking how it relates to the rest of the world.

The next step in this observation was to analyze Ms. Parker's mathematical practice and what transpires between her and her students to understand how the activities were framed within the context of reform mathematical practice. The RTOP scores in Figure 5.6 provide further description of Ms. Parker's practice.

Figure 5.6: Analysis of Ms.Parker RTOP Scores



	L D I	C PP K	CPDK	C C C	S T R
Subscale Item 1	2	3	2	3	4
Subscale Item 2	4	2	0	1	2
Subscale Item 3	4	4	2	2	4
Subscale Item 4	3	2	1	0	3
Subscale Item 5	0	1	2	2	4
Total Subscale Score	13	12	7	8	17

Subscale: LDI = Lesson Design and Implementation; CPPK=Content Propositional Knowledge; CPDK=Content Procedural Knowledge; CCC=Classroom Culture Communication STR=Student Teacher Relationship (See Appendix M for description of Subscale items 1-5).

A high score reflects a greater degree of reform. As shown in Figure 6, Ms. Parker's composite score on the 25 items that comprised the five reform oriented practice subtests is 57, which is reflective of some degree of reform. This score might also be reflective of the ongoing training in implementing a new mathematics curriculum that exposes her to the constructivist approach to teaching mathematics. Further discussion of the issues that might surround Ms. Parker's degree of reform practice in relation to observers' ratings is presented at the end of this Chapter. On the Reform Oriented Questionnaire, Ms. Parker responded "Agree Moderately" five times and responded with "Strongly Agree" to all of the remaining items.

Ms. Robins (Young Teacher)

At the time of Ms. Robins' participation in the study, she had been teaching for six years. Four of those years were spent at another school. This year was her first time in six years teaching mathematics to second grade. When asked during a post-observation interview about her confidence in her ability to teach math, Ms. Robins said she is fairly confident but finds teaching mathematics to second grade students difficult. The difficulty, she said, stemmed from the many times she had to repeat the information.

> I'm fairly confident. It's a little bit difficult having second grade, because they are so ohm… I can't find the word, it's like you teach them and the next day they forget everything. So everything has to be repetitive, repetitive, and repetitive.

During this brief conversation she appeared positive about her ability and acknowledged that although she is not perfect, she is getting better at teaching math to the second grade students.

> I'm getting better at it; I don't think I'm perfect I'm getting better.

Ms. Robins teaches in a school that is comprised mostly of Hispanic students who accounted for 51 percent of the student population. Thirty seven percent of the student body is considered Limited English Proficient (LEP) students. In addition, 100 percent of the parents of the Hispanic students live in the community and own homes. The second highest ethnic group in the school is Black (48 percent). Whites and Asians comprised one percent. Seventy nine percent of the students in this school are eligible for free or reduced lunch program.

The school is one of three schools in the Triton School District that met the states annual yearly progress (AYP) the previous year. It is classified as a bilingual school. Every class has a bilingual teacher. There are 11 students to every full-time equivalent teacher compared with the State's average of 12 students per full-time equivalent teacher. According to Ms. Robins, she participated in the study because she thinks it is important that other teachers see how other individuals teach different subjects. She said, seeing how other teachers teach may be beneficial, [in that], "you always learn from other people".

Ms. Robins' classroom was atypical. The educational posters and decorations were neatly organized. There was a technology center with three computers and an overhead projector situated close to a water sink with faucet. The math center had a human skeleton with the bones numbered, a math wagon, a chart with ordinal numbers to 101, a spider web with math sentences made up of solutions to various addition and subtraction problems and a mathematics word wall. In a post-observation interview later in the day, Ms. Robins said whenever she teaches mathematics she tries to relate it to other subject areas so students can see the connections in their real lives. On the chalkboard (that was frequently used to provide information to the students) was a chart with cooperative group rules. The students' desks were arranged in groups of five clusters, yet oriented toward the chalkboard, so everyone had a clear view of it and the teacher.

After gaining the students' attention, Ms. Robins stood at the front of the class and told the students what the math objectives were for the day, and the reason for the objective.

> Our objective for the day is to review sums (answers) when given the dividends (numbers). Reason for the review, students will have test tomorrow.
>
> She then began the formal review lesson with the following:
>
> Teacher: Give me an addition sentence when two of them are added together equals 7? (*She verbalizes the statement but writes =7*)

Student: 1 + 3 = 7;

4 + 3 = 7

Teacher: What kind of problem is this?

Student: Addition.

Teacher: Now I want some subtraction problems where two numbers subtracted gives [give] 7? (*She verbalizes the statement but writes 7)*

Student: 10 – 3 = 7

4 + 3 = 7

Teacher: What kind of problem is this W?

"W": Subtraction

This pattern of requests, question, and then moving into whole class and individual practice with teacher and students solving problems together at the chalkboard and at their seats continued during the observed lesson period. The repetition that Ms. Robins described earlier was apparently her concern about her students knowing addition and subtraction well enough to pass the test. She repeatedly used similar activities giving her students' repeated practice with subtraction and addition problems.

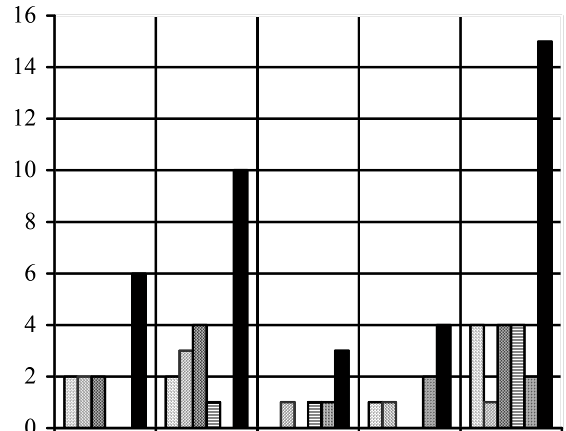
Rather than continue to view this lesson in its broad context, I decided to focus specifically on the participants' activities during the observed lesson to understand how Ms. Robins discursively established a framework for mathematics activities in her class on a daily basis. Table 5.7, provides an example of the pattern of activities that existed. The times are approximations.

Table 5.7: Description of Ms. Robins Mathematics Activities

Time in Minutes	Students' Action	Teacher's Action
15	Students listened to math objectives	Teacher told students the objectives for the math lesson
	Students constructed Addition problems	Teacher requested addition problems with double digits from students
	Students practiced to solve addition problems	Teacher wrote addition problems dictated by students
	Students worked independently and constructed more addition problems	
25	Students followed	Transition: Teacher requested subtraction problems with double digits from students
	Students practiced to solve subtraction problems independently and in groups	Teacher wrote subtraction problems students dictated
10	Students wrote addition and subtraction problems	Teacher gave students sheets of paper to write addition and subtraction problems to specific sums
	Students followed teacher's Instruction. Students wrote Problems without the use of Textbooks	Teacher asked students not to use textbooks
	Activities ended	Lesson ended

As described in Table 5.7 there seem to be some conscious efforts on the part of Ms. Robins to use her students' knowledge of mathematics concepts to construct addition and subtraction problems. During the process the students interacted freely with the teacher and responded to her questions. However, the interaction among the students was minimal. To better understand how the mathematics activities that took place in Ms. Robin's class were framed within the context of reform practice additional analysis of her practice was obtained using the 25 items on Reform Teaching Observation Protocol. That information is presented in Figure 5.7.

Figure 5.7 Analysis of Ms.Robins RTOP Scores



	L D I	C PP K	CPDK	C C C	S T R
Subscale Item 1	2	2	0	1	4
Subscale Item 2	2	3	1	1	1
Subscale Item 3	2	4	0	0	4
Subscale Item 4	0	1	1	0	4
Subscale Item 5	0	0	1	2	2
Total Subscale Score	6	10	3	4	15

Subscale: LDI = Lesson Design and Implementation; CPPK=Content Propositional Knowledge;
CPDK=Content Procedural Knowledge; CCC=Classroom Culture Communication
STR=Student Teacher Relationship (See Appendix M for description of Subscale items 1-5).

As described in Figure 5.7, Ms. Robin's score of 38 as indicated by her reform practice on the five subtests that comprised the 25 reform items, is reflective of a lower degree of reform. As shown in Table 5.7 workbooks and ditto sheets were not part of her formal lesson presentation. Students' knew what the teacher expected of them and what their role was in meeting those expectations. The students were well presented with the fundamental concepts of the lesson and there was adequate student/teacher interaction. On the Reform Oriented Questionnaire, Ms. Robins answered with "Strongly Agree" six times and responded with

"Agree Moderately" for all of the remaining questions. Her overall score on the questionnaire was above average.

Ms. Skirrotta (Senior Teacher)

Ms. Skirrotta has been teaching mathematics for 13 years. The school, in which she teaches, is predominantly white (46 percent). The second largest group is Black (38 percent) followed by Hispanics (nine percent), with the remainder a mixture of Asians (six percent), Native Americans (one percent), Hawaiian Native (one percent), and people of two or more races (one percent). Seventeen percent of the student population has Individualized Education Plans (IEP's) compared with the State's average of 12 percent. During a post-observation interview Ms. Skirrotta said she participated in the study because she wanted to give me useful information. [As such], she thinks it might be helpful if I come in and see the children.

In Ms. Skirrotta's classroom the lights in one corner of the room were completely dim, the walls were well decorated with teacher made and commercial posters, and students' work. Ms. Skirrotta stood sideways beside a chart mounted on an easel. The students sat in front of her on a carpeted floor. Groups of five desks were arranged in such a manner that all the students could see the teacher and the easel from where they were sitting. Ms. Skirrotta began by asking student J to teach the class to count from 1 to 10 in sign language. She then transitioned into the formal lesson by stating that they (she and the students) will be learning [geometry] shapes.

> Teacher: Let's start with an easy one. What is this shape? *She draws a square. Hearing no response, she asks another question while pointing to a squared shape*). How many sides does a square has [have]?
>
> Student: Four
>
> Teacher: How many sides does a triangle has [have]?
>
> Student: Three

Throughout the post-observation interview that followed, Ms. Skirrotta appeared self-assured in the way she said she teaches mathematics to her students.

In fact, she said she felt very confident in her ability to teach math because math is a subject she always liked. When asked about what she does to develop students' mathematical thinking and the frequency with which she engaged in activities that developed students thinking, she responded:

> Oh, I think by not talking as much but giving them the opportunity to investigate, I think that develops mathematical thinking.

She further pointed to specific things she did in the observed lesson that was her deliberate effort to develop students mathematical thinking. She said:

> It was done by them taking the shapes and having time to explore with them and try to see how the shapes worked in the pattern. So I think actually using the tools.

Ms. Skirrotta gave a similar response when she was asked to explain the reason behind her frequent use of the phrase to a few of the children, "oh you make connections".

> Well, you hope that when you are teaching them something, that it's not just they are learning this is a triangle, but they are learning that maybe they'll use it later on in life. So you want them to make connections to other things. And we try to do that across all subject areas. We do that in reading, we do that in writing, we want them to make connection to their world with what they are learning.

When discussing how reading was used to make connections to students' real world experience, Ms. Skirrotta said she used story problems several times each week. Students read story problems about dogs then answer questions relating to the number of white or black dogs in the story.

In light of this observation, I decided to see if I could get a picture of the kinds of activities that take place within each lesson each day to get an understanding of the context in which Ms. Skirrotta and her students did mathematics. For this I turned to my field notes and looked at how the different activities came together during the fifteen, thirteen and twelve -

minute transition points in the lesson. Table 5.8 describes the activities. The minutes are approximations.

Table 5.8: Description of Ms. Skirrotta Mathematics Activities

Time in Minutes	Students' Action	Teacher's Action
10	Students imitated peer instructor	Teacher facilitated as student led class in counting from 1-10 in Sign Language
15	Students followed teacher's directions Students told teacher correct number of sides and corners in a square and in a rectangle Students used different shaped blocks to see how many will cover the surface of a rhombus in their text-books	Transition: Teacher questioned students' knowledge of corners and sides of triangles and squares Teacher introduced unfamiliar geometric shapes (parallelogram/ rhombus and octagon)
13	Students followed directions Colored shapes on pages teacher assigned Students responded to teacher's questions by giving only numbers (e.g. 11, or 10).	Transition: Teacher asked students to color shapes they built on assigned textbook pages Teacher asked students to tell the number of blocks they used to cover a given shape in their textbooks
5	Students responded to teachers questions that they tried different ones	Transition: Teacher asked students to tell how they built their shapes
5	Students described both sad and happy feelings (e.g., it's too hard, I have trouble drawing the blocks, one student sobbed because she was not able to finish filling in all the shapes the teacher assigned) Activities ended	Transition: Teacher asked students what they liked about the activity Lesson ended

The first five minutes focused on students interacting as a class engaging meaningfully with peers. The teacher controlled the main activities at the three transition points, which focused on naming, building and tracing geometric shapes. In this kind of activity students would count the number of blocks they used to fill in a given shape. They would also try to use as many blocks as they could to cover a given shape in their textbooks. It was also important that students used the appropriate blocks to fill in shapes and know the names of the blocks they used for this activity. The number of pages students were able to fill in during the time allotted was also important. This activity continued until the teacher ended it. No effort was made to connect the activities by showing how each shape relates to the other or by showing why a particular shape block could not fill the space on a particular geometric shape other than the obvious fact that "white spaces would be left out". In terms of the actual doing of mathematics, Table 5.8 shows how the lesson was organized.

As shown in Table 5.8, at each transition point, Ms. Skirrotta asked questions of the whole class, assigned an activity, then engaged the students in the activity as a class, questioned them again and the sequence would be repeated for each new activity. This sequence may be helpful for some students but not others because as it appeared some students might be ready for the different types of problems while others may not. On the other hand by having such consistency in routine it allowed Ms. Skirrotta and her students to construct a shared understanding of the academic and social behaviors expected during each segment of the lesson. These routine behaviors are the focus of the discussion that follows and, the reformed teaching analysis presented in Figure 5.8. Presented first is an excerpt taken from the last ten minutes of the mathematics lesson. In this excerpt Ms. Skirrotta and her students engaged in a question and answer session to determine the number of blocks that is needed or were used to fill spaces of a given shape. Analysis of reformed data in Figure 5.8 provides additional insights into what Ms. Skirrotta conceived as her mathematics practice.

Teacher: Everybody takes out page 18 please. (*After she assigns the activity, minutes later she began her questioning*) Tell me how many blocks you use K? I am going to write down the number of blocks. (*She repeats several students verbatim*) K says she use 11 blocks. E uses 10. (*She again questioned the class*) Does anyone use any other number?

Students: (*Speaking one after the other*) Yes, 11, 10, 9.

Teacher: How many people did 8? (*No show of hands, she continues*). I have a tricky number for you. If you have the same number, did you have to use the same number of blocks?

Student: No.

Teacher: Are you telling me that's [that it's] more than one way to build the same block?

Student: Yes.

Teacher: Did anybody have less than five (*No response*) less than four? (*No response*)

Teacher: Did anybody did [do] this with more than eleven blocks?

Students: Yes, (*and another*) No.

Teacher: I don't see how you could do that with 15 blocks. Did anybody do so with three blocks?

Student: Yes.

Teacher: (Questioning again) Is that possible J?

Student: No

Teacher: Why?

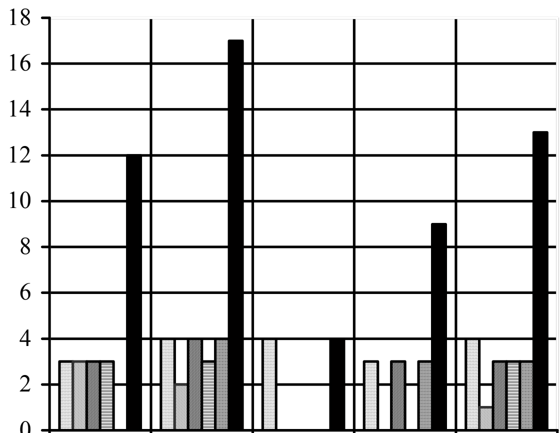
J : White spaces would be left out.

Teacher: How many students did three pages?

In this excerpt Ms. Skirrotta created the academic practice by the way she framed the activity (e.g. I have a tricky number for you) and engaged the students in question and answer (e.g. Did anybody did this with more than three blocks? Is that possible?). My next step in this observation was to analyze these academic and social practices that Ms. Skirrotta and her students developed from the standpoint of what she conceived as her mathematics reform.

Figure 5.8, describes her reform practice based on the Reform Teaching Observation Protocol subtest items.

Figure 5.8: Analysis of Ms.Skirrotta RTOP Scores



	L D I	C PP K	CPDK	C C C	S T R
Subscale Item 1	3	4	4	3	4
Subscale Item 2	3	2	0	0	1
Subscale Item 3	3	4	0	3	3
Subscale Item 4	3	3	0	0	3
Subscale Item 5	0	4	0	3	3
Total Subscale Score	12	17	4	9	13

Subscale: LDI = Lesson Design and Implementation; CPPK=Content Propositional Knowledge; CPDK=Content Procedural Knowledge; CCC=Classroom Culture Communication STR=Student Teacher Relationship (See Appendix M for description of Subscale items 1-5).

As shown in Figure 5.8, Ms. Skirrotta's composite score on the 25 reform items that comprised the five subtests is 55 and is reflective of a relatively high degree of reform. Figure 5.8 also shows that Ms. Skirrotta places much emphasis on specific items that were directly related to content-propositional knowledge.

On the Reform Oriented Questionnaire, Ms. Skirrotta responded "Agree Moderately" ten times, answered with "Agree Slightly" five times and responded once with "Disagree Moderately". Her overall score on the questionnaire was among the highest.

Mr. Tompkin (Senior Teacher)

Mr. Tompkin has been teaching mathematics for 14 years. Seventy percent of the students at the school were African Americans, 27 % Hispanic and 52 % were participating in free or reduced price lunch program. In a post-observation interview that followed the same day of the in-class observation, Mr. Tompkin said he agreed to participate in the study because he thought that my research topic was interesting. He hoped that by participating he would know how reform oriented teaching works, if only for his class. He said he believed that it is good to engage students but not all students can be engaged. However, he said if it is possible to engage all students he wants to know "how to do it".

In Mr. Tompkin's classroom the seating was arranged in the traditional manner. The teacher was seated in front of the class facing the students who were seated in orderly rows in front of the teacher and in clear view of the chalkboard perched on a wall directly behind where the teacher sat. To present mathematical information to the class, both he and his special education assistant used the chalkboard. A Math Word Bank was displayed above the chalkboard. Essential tools from The New Jersey Core Curriculum Standards and the mathematics objective of the day were also displayed on the chalkboard. The students who entered the room seemed to know the routine of signing the attendance sheet and opening their textbooks to begin solving the problem of the day. While the students worked at their desks, Mr.

Tompkin circled the room returning homework papers, which the students filed in their notebooks. Mr. Tompkin spoke in a clear, strong, commanding but friendly tone of voice when he addressed the class. Despite the occasional din and laughter coming from the students, his voice carried clearly over the chatter of his 12 students. He spent the next 10 minutes introducing the new textbook. Students were quizzed on the purpose of the table of contents, the glossary, the index and the reflection. During this time students also responded to Mr. Tompkin's inquiry about the difference between the designs of their mathematics textbooks versus the design of their English textbooks. Simultaneously, his assistant teacher coached one student silently off to the side. Throughout the discussion, Mr. Tompkin continuously called on the students to "listen up".

He began the formal lesson by sitting in his chair at the front of the classroom. He directed instructions regarding definition sheets (See Appendix H for the "Definition Sheet") that students completed for homework. The definition sheets focused on translating and writing algebraic expressions. Mr. Tompkin, (sometimes assisted by his special education assistant) spent the 20-minute period in dialogue about mathematics vocabulary related to algebraic expression. The students questioned his use of specific mathematics vocabulary and what they meant. He answered student's questions using mathematical terminologies (e.g. exponent, factor, base, variable, nth power). This pattern of question and answer and providing explanation using mathematical vocabulary words continued throughout the observed lesson.

During the post-observation interview, Mr. Tompkin said he placed much importance on word problems because this is an area where students encounter difficulty. He pointed out that as students read, understand, and translate the verbal expression into algebraic expression it makes them think mathematically.

> I also like to give word problem, causes [because] a lot of students seems [seem] to have problems understanding the word and trying to translate it to think mathematically. Now by doing the word problem, example, like the verbal expression, let them think about the

> word, try to understand it, and then actually come up with an answer with what they were asked to do. So by doing word problem, it makes them read the problem, it also encourage them to talk with their neighbor, I intend [to give them] them problem like that, not necessarily doing the problem themselves, but give them a chance to talk with their neighbor about what they get out of it.

Having some knowledge of Mr. Tompkin's belief about mathematics teaching and his views of how students learn mathematics, I decided to take a closer look at the context of the mathematics activity, how he and his students organized the translation of algebraic expression during the 50-minute time period. In the process of having such understanding, I analyzed the activities (what was done) and the interactions (teacher's actions and student's actions) of he and his students to pinpoint how the activities were conceptually arranged during this single observation.

The three main activities were extended across the first 30 minutes of class time. Mr. Tompkin introduced the concept by asking students leading questions to stimulate their interests in understanding the vocabulary for doing algebraic expression as well as understanding the main parts of the new textbook. In the first 10 minutes students were directed to read and clarify definitions. In the second ten minutes students engaged in similar activity but this time there was more dialogue with the teacher. The third ten minutes focused on the problem of the day. Students and teacher used different operations to solve word problems, which consisted of verbal and algebraic expressions. The final 10 minutes of class time included several combinations of the first three minutes of activity consisting of examples written on the chalkboard by teachers, students asking questions about the teachers' examples, students copying notes from the chalkboard and concluding with the teacher simultaneously explaining to students the meaning of mathematical vocabulary words. Table 5.9 describes the organization of the activities.

Table 5.9: Description of Mr. Tompkin Mathematics Activites

Time in Minutes	Students' Action	Teacher's Action
10	Students responded to review questions	Teacher reviewed how to use math textbook and questioned students knowledge of the material
10	Students responded to specific teacher-questions	Teacher asked leading questions leading questions on using assigned textbook and the "Problem of the Day"
	Students worked on examples based on the "Problem of the Day". Students filed away Corrected homework assign-ments.	Transition: Teacher wrote algebraic expressions on chalkboard for students to solve. Teacher returned Homework assignments.
10	Students questioned teacher's use of vocabulary words and solved verbal and algebraic word problems	Teacher wrote and solved mathematical word problems with students involving verbal and algebraic word problems
	Students worked minimally with peers but interacted well with the teacher	Teacher encouraged participation from students when solving verbal and algebraic word problems
10	Students copied meaning of mathematics vocabulary words from chalkboard and from textbook	Transition: Teacher explained to students what mathematics vocab-ulary words meant and wrote additional meanings on chalkboard
10	Students listened	Teacher discussed with students how to respond to the State's Tests
	Activities ended	Lesson ended

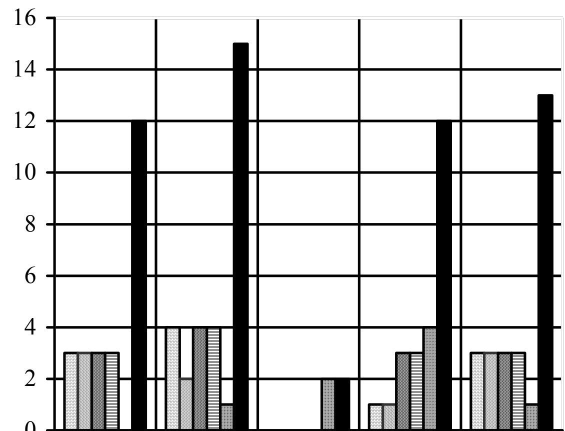
Throughout the different segments of the lesson, students were presented with a problem, either from their textbooks orally from the teacher, or both. More detailed analysis of Mr. Tompkin's interview transcripts indicated that his intent throughout the 50 minutes of instruction was to build students' mathematical vocabulary, mathematical thinking and mathematical understanding. As such, he was bent on providing practice in translating algebraic expressions. This is despite some repetition in the third and fourth segments of the lesson, which Mr. Tompkin described during the post-observation interview as a deliberate attempt to get the students to understand the "words" in word problems. Hence, he repeatedly used similar activities, giving his students more and more practice with solving problems involving algebraic expressions.

The next thing was to see how Mr. Tompkin and his students co-construct the social and academic practices from one minute to the next. I believe that the pattern described in Table 5.9 was informed by some social and academic practices with which to teach mathematics. I then tried to understand how Mr. Tompkin and his students negotiated these social and academic practices. Because Mr. Tompkins's situation was similar in many ways to the other classes I had observed before, I decided to look first to my field notes to find answers. The following passage describes what took place between Mr. Tompkin's and his students as they negotiated the social and academic practices in preparation for the activities described in Table 5.9.

> All right, I want you to listen up for a moment. Hopefully you'll read a little bit. I gave you an activity. What I want you to do now is put your pencil [s] down. Listen for a few moments. Yesterday we were trying to find some things in the book. I want you to go to the student handbook where the work begins. (*He asked the question*) Who remember? (*Pauses, hearing no response, he said*) If you don't remember where would you look? (*R-responded, "table of content". Mr. Tompkin reiterates student, by saying)* Yes, table of contents. (*He asked again*) What page does student handbook begin on? (*Hearing no audible response he repeated the question)* What page does student handbook begin on? *What were we asked to do? (R-write the algebraic expression*). (*Mr. Tompkin responded*) I want you to write them both, verbal and algebraic expression.

The above ten-minute monologue demonstrates Mr. Tompkin's effort to define and redefine his expectations for the mathematical activity described in Table 5.9. He did this by the manner in which he elicited the student's attention (I want you to listen up), by directing (*I want you to put your pencils down*). Simultaneously, he established his expectations for the academic practice by identifying how they were going to proceed (*I want you to do them both, verbal and algebraic expression*). I next looked for further evidence of how the social and academic practices were developed during the observed lesson. By performing a careful analysis of observers' ratings of Mr. Tompkin's lesson presentation, the content, the level of discourse and interaction that took place between and amongst students and teacher and the nature of the student/teacher relationship, I was able to conclude that Mr. Tompkin established himself as having the sole authority on the contribution and assessment of mathematical knowledge. Figure 5.9 describes Mr. Tompkins's mathematics practice based on each of the 25 items in the Reform Teaching Observation Protocol (See Appendix M for Description of items).

Figure 5.9: Analysis of Ms.Tompkins RTOP Scores



	L D I	C PP K	CPDK	C C C	S T R
Subscale Item 1	3	4	0	1	3
Subscale Item 2	3	2	0	1	3
Subscale Item 3	3	4	0	3	3
Subscale Item 4	3	4	0	3	3
Subscale Item 5	0	1	2	4	1
Total Subscale Score	12	15	2	12	13

Subscale: LDI = Lesson Design and Implementation; CPPK=Content Propositional Knowledge; CPDK=Content Procedural Knowledge; CCC=Classroom Culture Communication STR=Student Teacher Relationship (See Appendix M for description of Subscale items 1-5).

As shown in Figure 5.9, Mr. Tompkin's composite score of 54 reflects a relatively high degree of reform. More reformed practice appears to be in the area of content-propositional knowledge (CPP). From the descriptions presented in Figure 5.9, I began to get an idea of the manner in which Mr. Tompkin framed the academic practice through his interaction with students when viewed from a reform oriented constructivist point of view. He established the academic expectations by stating that he assigned word problems because he wanted the students to know [mathematical] words, which helped them understand word problems in order to solve them and also to develop their mathematical thinking. Then he articulated specific procedures for solving word problems. The questions he asked at the beginning were the exact replica of what he asked at the end. In fact, throughout the 50 minutes of observation the interaction was controlled. There was a high degree of questioning and manipulatives were not used during the instructional period. On the Reform Oriented Questionnaire, Mr. Tompkin responded “Strongly Agree” to nine of the 16 items, answered with “Agree Moderately” four times and with “Agree Slightly” once. These data are reported in Table 5.11 at the end of the chapter.

Mr. Zaro (Young Teacher)

At the school where Mr. Zaro is employed the majority of the students are black (70%) and more than 52% participated in free or reduced lunch program. At the time of his participation in the study, Mr. Zaro had been teaching mathematics on a full-time basis for thirteen years. During the post-observation interview he lamented that he is forced to use a different curriculum with the present cohort of freshmen students taking algebra 1, because in the previous year many of the students had failed their algebra classes before the end of the first semester. He said the reason for the failure was that the students did not have a strong enough mathematics background to do the course work in algebra 1. Regrettably, he said by the end of the second semester many of the students had lost as many as 10 credits. This was because they performed so poorly that half way through the school year it became almost impossible for them to continue in the course. In light of such poor performance, he said his school district adopted a new curriculum that allowed the current freshmen cohort who opted to take algebra 1 to instead take a pre algebra lab course during the first semester before they are allowed into algebra 1 during the second semester.

Although Mr. Zaro expressed concern regarding his students' performance during the interview, he said that, he did not mind the change because it gave the students the chance to pass at least one of the two courses and to earn as many as five credits rather than losing all 10 credits at the end of the school year if they did not do well. Mr. Zaro said he agreed to participate in the study because he liked to receive feedback from professionals, and get other people's ideas. Therefore, he said whenever he gets an opportunity for someone to come into his class to observe he always uses that opportunity.

Mr. Zaro's classroom was an atypical one. The room was decorated with lush green plants and flowers in vases well arranged on a wooden stand off to the opposite side of the room facing the main entrance to the classroom. The shutters were opened allowing additional light into an already well-lit and well-fit classroom environment. A variety of students' work was pasted on a chalkboard in the far end of the room, and on the walls. There was one poster on which was written a "quote of the day" that could be seen from every corner of the room. Mr. Zaro stood off to the side with his back facing a flower stand and a lap top computer opened on a specialized stand. An overhead projector screen was pulled half way down the chalkboard, which he used frequently to present activities to the students. Students' desks and chairs were arranged mostly in seven groups of four desks with one larger group situated near the classroom entrance. All desks and chairs were oriented in such a manner that the students had their backs toward the main entrance but facing Mr. Zaro.

Mr. Zaro spoke in a loud clear voice over the chatter of the students.

Mr. Zaro: Come, let's check your homework real quick. (*He asked one student*) Why do you have the biggest number in the exponent box? (*Hearing* no *response from students he writes* 24^6 *and* 42^6) Which is bigger?

Students: 42^6 power.

Mr. Zaro: What does 42^6 power mean?

Students: Multiply 42, six times.

An analysis of the reported demographic information revealed that Mr. Zaro happens to be the only teacher among the ten others observed with a major and a minor in mathematics. When asked about his confidence in teaching mathematics, Mr. Zaro said he was very confident. He attributed his confidence to the length of time he has been teaching mathematics and not due to his mathematics background.

Interviewer: How confident are you in your ability to teach mathematics?

Interviewee: Oh, very confident. I have been doing this
for a while, I'm pretty confident

Mr. Zaro began the formal lesson by asking students to choose their activity from among several options. The following options are excerpted from my field notes.

Option #1: 1 penny doubled every day for 25 days

Option #2: 1 penny tripled everyday for 16 days

Option #3: 1 penny quadrupled every day for 13 days

Option #4: 1 penny multiplied by 5 every day for 11 days

To understand the different ways Mr. Zaro and his students interacted and learned mathematics I tried to get a good grasp of the context in which the above activities took place. To this end, I proceeded to focus on the activities that took place during the observed lesson to see how they were connected. Table 5.10 describes the content of the lesson during the observation, related to scientific notation. The minutes are approximations.

Table 5.10: Description of Mr. Zaro Mathematics Activities

Time in Minutes	Students' Action	Teacher Action
5	Returned homework assignments Responded to teacher's closed questions	Checked and reviewed students homework Used closed questions to verify students' understanding of completed assignment
10	Copied objectives in in journals Placed digits as teacher Instructed	Transition: Introduced objectives Reviewed scientific notation to represent large and small numbers Invited students to place digits (2, 4, 6, etc) to get the largest value
20	Chose options and worked in groups to Mimicked teacher's Example, built tables and wrote appropriate algebraic expressions Followed step by step procedures to solve	Transition: Introduced new problems and options. Demonstrated how to solve problems by constructing tables and wrote algebraic expressions Verbalized step by step procedures for finding answers
15	Listened, reviewed previous homework assignment, built tables and wrote appropriate algebraic expressions with exponents in groups Activities ended	Explained how to multiply exponents to get total amount received; wrote appropriate algebraic expressions with exponents; encouraged group participation Lesson ended

By examining the array of activities described in Table 5.10 I was able to see how they were connected to the particular objective at the time of the observation. The post-observation telephone interview with Mr. Zaro the same day also provided additional insights about the lesson and what counted as doing mathematics each day.

> I sort of wanted them to organize, you know, the work into tables that they can look back on and then compare their tables with the other students in their group to find out which one is the best option. They also use the table to [see], find a pattern in the problem, and hopefully came up with an algebraic expression for the problem. . . . I have them do it their way first, then kind of gave them the idea with the table, then afterward, say ok what could be an easier way. . . . how could you write an expression to represent that, using the exponent, you know.

Further analysis of interview transcripts in addition to my field notes showed that at the two transition points in the lesson, the activities involved constructing and solving problems as a class. Mr. Zaro did not use textbooks or worksheets. Throughout the almost fifty minutes of lesson, he referred back only once to the homework assignment but only as a reminder of how the activities were conceptually connected. With this in mind I tried to ascertain from Mr. Zaro's own words how he made the connection. He had this to say.

> . . . after we did that they pretty much understand the concept of exponents you know, but I was trying to (see, you know), show them again where that was relevant, where they could use exponent, not just teach them how to evaluate an exponent. . . I gave them a number which was very large, like a trillion, and what came up on the calculator, and [asked them] what does that mean, and then showing [showed] them that was a number written in scientific notation and what that means and so on . . . . . and then I gave them a problem with a number that came up to be very small and [I did] the same thing, (you know), I gave them a problem where they, which turn out to be big, is something like, if you receive a penny every minute on the first day and it (like) tripled every day for (I don't know) 70 days, ohm, how much money would you have on the 70^{th} day.
>
> . . . then instead of making a table and listing them all out we use the expression, a penny x through to the 70^{th} power (X^{70})actually it was 69^{th} power (X^{69}), they type it on the calculator, and then they say, who! It's a lot easier, if we had an expression, then we could just plug in the numbers. . . it has the Σ and the numbers going forward, then they said, what kind of a number is that? . . . then we talk about scientific notation, and a couple of them remember scientific notation, from before . . . then we wrote out the numbers and then said, ok, what is that number then? Then they said, ok, you move the decimal places, and so on and so forth, and that's kind of what we got to when we came up with some big numbers.

On the basis of this information, I assumed that the lesson I observed was part of a larger plan of activities related to scientific notation that Mr. Zaro presented using problems he constructed from one day to the next, and from one lesson to the next. The specificity of his objective and the explicit connections with the lesson revealed more than just an understanding of what Mr. Zaro and his students considered as doing mathematics. One, his non-reliance on worksheets and textbook may be the outcome of his confidence in teaching mathematics. Two, the connections that were made across activities made it easy for the students to understand how the concepts of algebraic expression and scientific notation with very large and small numbers were linked. This might have improved how the students learn scientific notation.

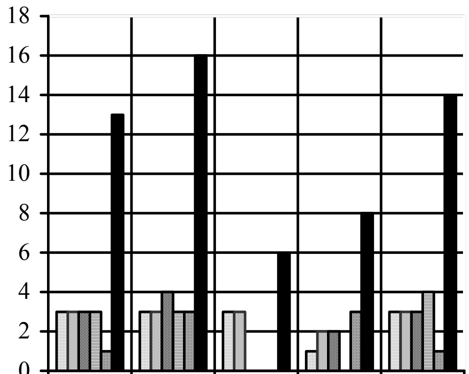
What is interesting about this class is the absence of the warm up activity and the modeling that took place in several of the other classrooms I observed. Even the introduction was part of the activity; it was not separated. It would seem as if Mr. Zaro and his students worked progressively toward defining the nature of the specific activity.

> You should come to what we are studying now. What are we studying now? Who picks what in the group? You decide whether you want to do option one, two, three or four.

Based on Mr. Zaro's classroom activities in Table 5.10, I decided to look closely at his reformed teaching practice. This information is described in Figure 5.10.

High scores of 100 or more indicate greater degree of reform. Mr. Zaro's reform teaching practice score of 57 on the 25 items on the five Subtests described in Figure 5.10 is reflective of a relatively high degree of reform.

Figure 5.10: Analysis of Ms.Zaro RTOP Scores



	L D I	C PP K	CPDK	C C C	S T R
Subscale Item 1	3	3	3	1	3
Subscale Item 2	3	3	3	2	3
Subscale Item 3	3	4	0	2	3
Subscale Item 4	3	3	0	0	4
Subscale Item 5	1	3	0	3	1
Total Subscale Score	13	16	6	8	14

Subscale: LDI = Lesson Design and Implementation; CPPK=Content Propositional Knowledge; CPDK=Content Procedural Knowledge; CCC=Classroom Culture Communication STR=Student Teacher Relationship (See Appendix M for description of Subscale items 1-5).

On the Reform Oriented Questionnaire Mr. Zaro responded “Strongly Agree” to 12 of the items, and answered with “Moderately Agree” to the remaining four. These data are reported in Table 5.11 at the end of the chapter.

Part Two

One of the purposes of the dissertation was to compare teachers' beliefs about reform-oriented mathematics teaching to their actual teaching behavior. This section of the chapter will attempt to provide some data to meet this goal. Specifically, the teachers' responses on the Reform Oriented Teaching Questionnaire will be compared to their behaviors as coded by the Reform Teaching Observation Protocol (RTOP). It should be mentioned that this analysis could be only tentative since the two sources of data are not directly comparable. Nonetheless, I believe that some insight into this research question can be gained from the analysis presented here.

One of the difficulties in using the two sources of data is that they use different scales. Specifically, the 16 items on the Questionnaire are rated on a 1-6 scale (where 1= Strongly Disagree and 6= Strongly Agree) while the coding form uses a 5-point scale (where 0 = no indication of the behavior and 4 = a clear indication). To provide a metric that can be compared, the data for the 10 teachers were converted into percentages. For example, the highest possible score on the Questionnaire is 96 (16 items times 6 points). The highest score on each section of the coding form is 20 (each section has five options with 4 as the highest possible score).

To make these two data sources comparable, each teacher's actual score was divided by the highest possible score. For example, a teacher who rated 13 items on the Questionnaire as "6" and the remaining 3 as "5" (for a total of 93 points) would receive a percentage of 96.9 (93 divided by 96). A teacher who scored 3's on all sections of the coding form would receive a percentage of 75.0 (15 divided by 20). A summary of the data for the 10 teachers is presented in Table 5.11.

Table 5.11: Summary of Questionnaire Responses and Observed Behavior

Teacher	Sum of Question-naire Items	LDI	CPPK	CPDK	CCC	STR	TOTAL OF RTOP
Karen	97.9%	55.0%	80.0%	0.0%	15.0%	5.0%	31.0%
Mellicent	78.1%	55.0%	65.0%	30.0%	30.0%	55.0%	47.0%
Ballas	95.8%	20%	25.0%	20.0%	45.0%	45.0%	31.0%
Hardiman	97.9%	60.0%	70.0%	60.0%	70.0%	70.0%	66.0%
Betheus	95.8%	60.0%	50.0%	15.0%	20.0%	60.0%	41.0%
Parker	94.8%	65.0%	60.0%	35.0%	40.0%	85.0%	57.0%
Robins	90.6%	30.0%	50.0%	15.0%	20.0%	75.0%	38.0%
Skirrotta	98.9%	60.0%	85.0%	15.0%	45.0%	65.0%	54.0%
Tompkin	89.6%	60.0%	75.0%	10.0%	60.0%	65.0%	54.0%
Zaro	96.8%	65.0%	80.0%	30.0%	40.05	70.0%	57%
TOTAL	93.6%	53.0%	64.0%	23.0%	38.5%	59.5%	

There are a number of interesting findings presented in Table 5.11. First as mentioned previously, all of the teachers' expressed strong beliefs in the tenets of reform oriented mathematics teaching. In fact the 10 teachers chosen for the observation are in even stronger agreement than the sample of teachers whose data are reported in Chapter 4. With the exception of Ms. Mellicent, Mr. Tompkin and Ms. Skirrotta, all seven teachers chose either the "5" or "6" option for the 16 questions on the questionnaire. On the other hand, the actual behavior of the teachers is considerably lower. In order, the observed behavior of the teachers was as follows:

Content-Propositional Knowledge 64%

Student/Teacher Relationships	59.5%
Lesson Design and Implementation	53.0%
Classroom Culture Communication	38.5%
Content Procedural Knowledge	23.0%

It is also of some interest that there is no relationship between the teachers' questionnaire data and their behavior as coded by the Reform Teaching Observation Protocol (RTOP). Specifically, Spearman correlations computed between the total of the questionnaire items and all aspects of the RTOP were all non-significant. (The correlation between the total of the questionnaire items and the total of the RTOP was .018)

Summary

By using a multi-step analysis of the classroom observations of 10 teachers and their classrooms, I was able to report some information on the teachers' classroom practices and their beliefs about mathematics teaching and learning. Some of these teachers were involved in a pilot as they implemented new curriculum materials and methods for teaching mathematics. Those involved considered the experience rewarding in that they were able to develop their students' interest in mathematics in a positive way. Others are still uncomfortable with the way they had been teaching mathematics over the years and are still forced to continue along those same paths.

As discussed in the case studies, each teacher constructed his or her own definition of what it means to do mathematics based on his or her contextual situation. Because of these contextual differences, making comparisons between teachers and across different classrooms is difficult. Although there were much variations among the teachers based on their practices in terms of lesson design and implementation, content-propositional knowledge, content-procedural knowledge, classroom culture and communicative interaction and student teacher relationship, the minimal presence of these practices was salient because they speak to the teachers' ability to present mathematical knowledge to students in ways that are aligned with reform oriented mathematics teaching.

Despite these similarities there were also important individual differences with respect to issues of professional development, curriculum and the way the teachers presented mathematics to their students. Three teachers (Ms. Parker, Mr. Zaro and Mr. Tompkin) participated in the study because they are looking for better ways to teach mathematics. Along this same line, one teacher (Mr. Zaro) felt that the new school curriculum had brought about changes in the way he teaches mathematics. This is because he is forced to teach mathematics without the use of textbooks. Two other teachers, Ms. Hardiman and Ms. Parker felt that the new pilot program their school district implemented is helping to make them better at teaching mathematics to their students. One teacher (Mr. Tompkin) said he is hoping that someone will come to his class and model for him what reform oriented mathematics teaching is, if it is only for his class.

Finally, three teachers - Ms. Ballas, Ms. Mellicent and Ms. Karen - complained of not having enough time to teach mathematics in ways that will maximize how they want students to learn the subject. These contextual (social, academic) variations across the different classrooms and teachers speak to the many issues teachers encounter as they grapple with the task of improving mathematics teaching and learning in their schools.

Finally, the data reported in the last section of the chapter indicate that there is a stronger belief about reform-oriented teaching of mathematics than the teachers are willing or able to demonstrate. The implications of these findings are presented in Chapter 6.

CHAPTER 6

DISCUSSION AND RECOMMENDATIONS

Beliefs and Practices in Context

The two major goals of the research were to ascertain if teachers support the major tenets of reform oriented teaching and then to observe whether their actual behavior was consistent with their beliefs. Another goal of this study was to determine if demographic factors influence teachers' beliefs and practices. Out of these two major goals came the ability to determine the extent to which in-service teachers' classroom practices support the major principles underlying reform oriented mathematics teaching. The intention was not only to record the specifics of reform oriented practices observed in each teacher's planned lesson activity but also to note how well the teacher in his or her particular situation conceived of and how they applied the principles of reform oriented practice.

As reported in Chapter 4, the sample of mathematics teachers used in this study expressed strong and consistent beliefs in the major principles of reform oriented mathematics teaching. However, as illustrated in Figures 5.1 through 5.10 in Chapter 5, the actual performance of reformed mathematics teaching and learning practices was unique to each teacher and the specific situations he or she encountered. The only consistent element of reform practice recorded among the teachers that received unanimously high ratings by all three observers was the teachers' strong emphasis on content-propositional knowledge. This emphasis was seen as one that tapped into the mathematics teachers' attempt to focus his or her lessons involving teaching of "fundamental concepts"; enhancing "conceptual understanding"; and making "connections with other contexts". This was also seen as a conscious effort on the part of the mathematics teachers to get students to share the results of their seatwork, explain their mathematical thinking about the activity and to check students' mathematical understanding at that time.

This study also recorded the efforts by teachers to place emphasis on specific reform changes in mathematics and to focus on the kind of reform practice that might allow them to try out new and better ways of teaching mathematics. As described in Table D-2 (see Appendix D), attempts at engaging students as members of their learning communities, valuing varieties of solutions to problems that often come from ideas generated by students, and finding ways to help students understand the process of inquiry, were all folded into the teacher working at developing the fundamental aspects of the subject, improving classroom climate, making connections in context and, when necessary, resorting to symbolic representation of content.

Despite the teachers' limited demonstration of reform practices, particularly in the higher grades, the 10 teachers who were observed and interviewed consistently expressed an interest in improving their mathematics teaching. This is a reflection of the fact that these teachers seemed sincere in their desire to improve their teaching and to increase their students' engagement with and performance in mathematics.

The results of this study both support and contradict previous studies (Battista, 1994) that investigated the link between teachers' knowledge of mathematics reform practices and their efforts at implementing those practices. The data from this study indicate that teachers want to practice those reform principles that they find valuable, but that context and circumstances too often stand in the way. It is evident from the interviews that teachers feel intense pressure to meet the increasing demands of accountability built into such reform measures as No Child Left Behind. When faced with demands to increase their students' scores on standardized tests, the teachers seem too often have to sacrifice principles for expediency. This finding is not new.

What this study has shown is that these pressures are felt in all three types of schools used in this study-suburban and urban and elementary as well as in both middle schools and high schools.

One of the most salient findings that came out of this study is that teachers place a great deal of emphasis on literacy in the teaching and learning of mathematics in the lower grades, a finding that was consistent across schools and school districts. Consistent with Ernest's (1991) fallibilistic view of mathematics, these teachers' mathematics concerns were aligned with those shared by mathematicians who have considered similar teaching and learning practices. These concerns are reflective of the warm, human, personal, intuitive, active, collaborative, creative, investigational, cultural and historically elegant beauty of mathematics that these teachers experienced.

There were a number of issues that arose in completing this study that deserve mention. The first issue involved data collection. During the data collection phase of the study, it became apparent that by using two different instruments that would measure teachers' behaviors (practices) in the classroom it would be possible to present them in such a manner that would make them more compatible for data collection. An additional adjustment may be to expand the instrument items in order to identify those behaviors (practice) that do not conform to specific reform measures described in the instruments. Thus rendering it possible to collect data on these items to find out how teachers used these other behaviors (that go beyond those items that were originally measured in the instruments) to their specific advantage and when teaching mathematics to their students. The second issue that pertains to data collection involves the direct observation of teachers' practices that could be related to reform in classroom mathematics activities. This study found that teachers do rely on some reform practices to achieve certain reform oriented classroom mathematics goals, such as, getting students to connect with mathematical ideas and concepts, and be able to see that these reform practices can be applied in other contexts. A disconnect, unfortunately, was observed during the process of establishing these reform practices as a standard for improving performance in classroom mathematics. Some teachers found it impossible to avoid switching to traditional methods of instruction in order to comply with the State's requirements that are mostly oriented toward a test-based curriculum. In this case, there seemed to be little support formulated in reform practices for teachers with students who must learn the fundamental concepts (algorithms and basic skills) before they can move ahead. On the other hand, there was another kind of disconnect in the teachers' reform practices that occurs between a teacher's beliefs and his or her practices. For example, in the case of Ms. Mellicent in Chapter 5, the disconnect presents itself as an inconsistency with her beliefs about mathematics teaching that she shared during the interview and how she actually teaches mathematics to her students. Previous studies (Raymond, 1997) have also reported inconsistency among elementary mathematics teachers particularly among those teachers who are beginners. It has also been reported that such inconsistency (as in the case of Ms. Mellicent) might be influenced by how the teacher learned mathematics as a student (Raymond, 1997).

As reported in Chapter 5, Ms. Mellicent had fewer years of teaching mathematics at the elementary level than many of the other teachers described. Her response to the Reform Oriented Questionnaire items in Table 5.11 shows inconsistency with how she actually teaches the subject. In light of previous findings, Ms. Mellicent's inconsistency might be a reflection of how she learned mathematics as a student. In fact, in her own words: "I always loved mathematics. I am glad I had good teachers in school. Mathematics is concrete. Here are the steps, here is the answer". Yet her actual classroom teaching does not reflect the absolute certainty that she said she believed about mathematics. This back and forth that came out in her interview may also be more fitting for the context of her classroom environment where teaching students of different ability levels requires her to switch from one approach to the next. At one time she may find she has to be more traditional in providing correct answers as she appropriately said, she “writes notes and answers for them to save time” and at other times, depending on the student she is working with, she said she can be more facilitative especially where she can engage her students in more reformed-type approaches to mathematics teaching and learning.

Nonetheless, in situations where a teacher may ask students to solve problems, the teacher might forgo just giving students the answer as feedback. Instead, he/she could then use that feedback portion of the classroom activity that addresses a key element of reform oriented mathematics teaching, which is; communicative interaction and engagement that connects with other contexts. This strategy has been shown in the research (Yackel, Cobb & Wood, 1991) to be a better way to involve the students in engagement activities. By so doing, students can have a chance to use the kinds of critical mathematical thinking (discussed in chapter 2) to assess each other’s classroom contribution to learning mathematics and to do so in a way that is meaningful. Brown (1994; 1996), Cobb, Yackel, and Wood (1992) and Lerman (1996), have addressed the social communicative aspects of mathematics teaching and learning and suggest that this kind of meaningful collaboration (which Lerman more succinctly described as intersubjectivity) is necessary for meaning making in the mathematics classroom. Other proponents (Draper, 2002; NCTM, 1989) of reform in mathematics teaching are not against teachers teaching the fundamentals, neither are they against placing the focus on the individual like radical constructivists epistemologists (e.g. Piaget) often do. What these proponents have proposed is that such thinking and interaction should be meaningful and aligned with “real life problems” (Davison & Mitchell, 2008).

Third, there are issues involving the general educational practices of teachers in the classroom as well as the period of time set for each observation. A more extended period of observation of the various possible reform practices could be used in an effort to better understand how the different classroom norms and practices described in Tables 5-10 could be more fully portrayed and how they might continue to take shape as teachers become more comfortable with reform practices.

In reference to this study's actual findings, teachers were observed only once during a 40-50 minute time period of in-class instruction. As described in Chapter 5, there was much reliance on the Reform Teaching Observation Protocol (RTOP) instrument when describing reform practice as well as the teachers' own words when describing such practices in their post-observation response to questions. Because items from both the interview and the Reformed Teaching Observation Protocol measured the same categories of reform practice, it was appropriate to combine these two data sets in order to analyze fully, the teachers' reform practices. In this regard, it was also important to consider both beliefs and practice items together because of the degree of similarities among the items. In any case, the homogeneity that was found among the teachers' practice regardless of the corresponding similarities in their beliefs, may have been due to teachers having focused most of their efforts on complying with the demands of their schools' curricula. This narrow focus by the teachers was done for the purpose of accountability in meeting annual yearly progress standards required by the guidelines for federal funding under No Child Left Behind. However, as several teachers indicated in the case studies discussed in Chapter 5, they were seeking better ways to teach mathematics other than those prescribed by their State and school districts. On the other hand, they indicated that they would actually like to find better ways to teach mathematics that are also meaningful to their students and themselves. More specifically, rather than (according to reports from several teachers) “having students cram for the test after which all is forgotten”.

Literacy in Context

This study reported that teachers in the lower middle grades support the principle of making literacy part of their mathematics instruction. Such close attention to literacy by teachers in the lower middle grades, allowed for a deeper understanding of those mathematical practices that really matter to some teachers. It is not surprising that teachers in these lower middle grades would want to incorporate literacy instructions in their mathematics lessons because such inclusion will improve students’ “ability to learn and understand mathematics” (Draper, p.1). Also, the data from this study are consistent with previous research that found a lack of support for literacy instruction in the mathematics curriculum by teachers in the upper grades. Obrien, Stewart and Moje (1995) said secondary mathematics teachers have not shown much willingness to infuse mathematics content with literacy instruction because they believe such inclusion would be incongruent with the content prescribed by mathematics educators and colleagues in their field.

Nevertheless, as mentioned in Chapter One, because of the push by policy makers, educators, and parents to reform mathematics teaching and learning, teachers (including those at the secondary level) have been challenged to change the way they teach mathematics. Indeed, according to Draper (2002), the reform movement may just be the avenue for literacy educators to help mathematics teachers at every level infuse literacy instruction into their regular classroom teaching without compromising the objectives established by “school mathematics reformers” (Draper, p.1.) Thus, it becomes important to not only focus on what the teachers say but also to critically look at their mathematical teaching practices in light of how any of the reforms in mathematics instruction is generally viewed within the context of each teacher's particular classroom environment.

Although the complexity of the classroom environment would render such analysis difficult, and, though it may seem impossible, it became a necessary challenge for the researcher to separate what teachers said about teaching mathematics from the context in which it happened. It was also a challenge to investigate how teachers engage their students during particular lesson activities such as those described in Chapter 5. This kind of analysis along with a variety of other contextual factors can help us understand why teachers do what they do. It is important for the reader to also understand that these analyses are only first attempts to delve into this expanded research endeavor. It would be helpful in the future if researchers would take a more extended and more extensive look at both what teachers of reform oriented mathematics teaching practice say they do and how they do what they say they do. Data from such inquiry could be analyzed in relation to the way students and teachers understand those reform oriented teaching practices. It would also be helpful to look at how teachers of reform oriented mathematics teaching in the lower middle grades use the major principles of reform that they support to determine the specific cognitive influences (if any) on students' mathematical thinking and mathematical understanding within the contexts of their classroom environments

Contextual Factors

My view of contextual factors involves reference to the social norms of the environment including but not limited to administration, the demographics of the school (teachers and students), the curriculum, limits or requirements for professional development and such other constraints that may fall within the areas of concern in which the teachers function. These will be discussed in light of the overall findings of this study.

The absence of a correlation between various aspects of teachers' beliefs and their practices could be explained to some extent by a number of potential intervening and mediating contextual factors such as those described above, as well as such other factors as self-efficacy, perceived behavioral control and other personal teacher characteristics that were not explored. Had these variables been explored in a multivariate model, it is likely that a higher level of variance in practice could be explained. The lack of correlation between teachers' beliefs and practice could also be explained by planned behavior. Major models that explored people's behavior have shown that although people may have a positive belief about their practice (Ajen & Fishbein, 1975), they are afraid to practice good teaching because of a lack of confidence in the efficacy of what they teach. That is, having the assurance of the usefulness and positive outcome of their reform oriented mathematics teaching. The fear may be also be driven by the social norms in operation within each teachers' classroom environment. Teachers may feel that if they do things differently they may be seen as outcasts or misfits. Hence the lack of a correlation may also be a situational adjustment. It is hopeful, however, that all 10 of the teachers interviewed like the remaining teachers who completed the questionnaire, believe in reform-oriented teaching. While beliefs do not necessarily lead to behavior (as this study has shown), they are at least a start.

To turn once again to Ms. Mellicent, there are elements of bias in the questionnaire that caused her to respond the way she did. Or, the belief items were not clear enough and so she was forced to respond in a certain way. The inconsistency that was observed with her practice and what she said she believed about the nature of mathematics, could also be influenced by the way she was taught mathematics as a student a point I made earlier in my discussion and which I now reinforce for the purpose of emphasis. A previous study by Raymond (1997) has reported similar inconsistencies among beginning elementary school teachers. The influence of a halo effect should also not be overlooked. It is possible that because she knew that she was being observed she might also have felt compelled to demonstrate what she thought was the desirable way of teaching.

This type of response has been best described as "perceived control of behavior" (PCB) and has been looked at for many years by psychologists and sociologists (Ajen & Fishbein, 1975) alike. An example in this case would be the teacher who says, "I know that reform mathematics teaching works, but our school district does not provide the materials we need to implement such practices effectively". Or, in the case of Ms. Ballas, for example, she said during the observation "I could do so much more . . . if I didn't have to wait for the teacher next door to finish with the manipulatives that I need to teach my lesson". Or, "I have not been to one professional development workshop for a whole year. We would like to interact with other teachers to see what they do. We can learn from each other". In other words, there are teachers like Ms. Ballas who feel, they do not get enough administrative support from their school district so that they can get the professional development training they need to put reform oriented mathematics teaching into practice. When they do get such training, it often does not meet their needs. Others like Mr. Zaro cannot afford to go to professional development workshops because they would be costly and the school district will not provide reimbursement, especially if it were pursued for an individual's own professional development. Also, in Mr. Zaro's particular case, he said, "if I did get the chance and went to a workshop, say at UMDNJ or so, I would have to make up the time and I don't think it's right for me to do that". Therefore it is very important to note that beliefs alone do not mediate practice. There are other variables that do mediate practice and those have yet to be considered.

Limitations of the Research

My own evaluation of this research endeavor has helped me to recognize certain limitations, of which I believe the reader should be aware.

First, this study took place in three school districts, with only a relatively small group of mathematics teachers, drawn from three different types of public schools all within the northeastern United States. As such, the findings may only be representative of those particular mathematics teachers. Since the model that forms the theoretical basis for this study is mainly supported by research that has been done with other populations the inclusion of a different population should be seen as strength for this study. Nevertheless, the study is clearly affected by an issue with external validity.

Second, the kinds of beliefs the teachers had were not fully determined. In that, the instruments did not allow for the identification of the teachers' specific beliefs about the nature of mathematics. I believe if it were possible to monitor their classroom practices for a longer period of time, and examine more extensively their beliefs about the nature of mathematics, I believe it would be possible to gain a much deeper insight into their specific beliefs about the nature of their mathematics. Therefore, the approach to measure reform practice that is heavily depended on an external observer's perception after a relatively short period of observation (one lesson) is of concern.

Third, in order to obtain subjects for this study, I relied solely on volunteers. As Borg (1981) pointed out volunteers, tend to be different from nonvolunteers since they tend to be better educated, of higher social class, and more intelligent. This issue is relevant to all aspects of the research, but it is empirically important in relationship to the 10 teachers who on their own allowed themselves to be observed. It is possible that these 10 teachers are not representative of even the teachers in the three school districts used in the research.

Implications for Future Research

Perhaps the strongest finding of this research is that there is a marked gap between what teachers say they believe and what they do. It is encouraging that the data indicate that teachers have a clear grasp of reform-oriented mathematics teaching at least in terms of their beliefs and attitudes. Perhaps this is an area of professional development that colleges of education need to address in conjunction with local school districts and States' Departments of Education. Future research might focus on how best to provide such professional development as a first step, and then to evaluate whether this training is effective as a follow up step. Another area for future research might be to develop and administer a different questionnaire to measure attitudes and beliefs. The current study used a scale developed elsewhere. While the questionnaire seemed adequate, it perhaps was stated in such a way that the preferred answer to the items was too clearly obvious. A questionnaire where the items were more nuanced might produce a somewhat different set of results.

Summary

Beliefs and practices are contextual and situational. Findings from this study indicate that these contextual variables can influence teachers' beliefs and practices negatively or positively. Efforts to improve mathematics teaching and learning in elementary, middle, and high schools must consider the contextual influences on teachers' beliefs and practices. The data from this study indicate that teachers want to improve the way they teach mathematics, and want to improve the way they apply reform based teaching practices in their classrooms. The study has also shown, however, that the constraints within which the teachers must teach strongly influence what they can do. It is unrealistic to assume that the demands for accountability will go away or that the use of standardized tests to measure achievement will decrease. What is necessary is to find a middle way where the teachers can teach in the way they believe they should, and to have this reflected in the assessments that their students must take. It is hoped that the results from this study might prove useful in this moment.

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APPENDIX A

A VIEW OF MATHEMATICS FROM A FALLIBILISTIC AND HERMENEUTIC PERSPECTIVE

It seems plausible to conclude that Borasi's (1992) view of the mathematics classroom relegated it to 'absolutist' conception, a term he credited to Ernest (1991). In describing the 'absolutist' concept Borasi (1992) said Ernest (1991) wrote:

> The absolutist perspective describes mathematical knowledge as [an] objective, and absolute, certain and incorrigible body of knowledge. As such, mathematical knowledge is seen as both value and culture-free, as 'superhuman'. . isolated [and] as having value only because of its universal validity (p. 278-279).

The fallibilist view of mathematical knowledge, in contrast, accepts that mathematics is a historically developing area of theoretical and conceptual thinking in which ideas and truths and proofs, as well as rules and results are "modified, changed and redefined over time" (Ernest, 1991, p.279). In essence, fallibilism puts forward the belief that mathematics (a natural science) and (like everything else in this rational and irrational universe) is an outcome of social processes. Further, Ernest (1991) also said:

> Fallibilism embraces as legitimate philosophical concern, the practices of mathematics, its history and applications, the place of mathematics in human culture, including issues of values and education. In short, it fully admits the human face and [human] basis of mathematics (Ernest, 1991, p.280).

In reviewing the literature on this topic, several researchers (Brown, 1996; Draper, 2002; Yehuda, Lavy, Linchevski & Sfard, 2005) including Ernest (2001) have helped me clarify and solidify my view of mathematics as they have seen it. These individuals, from my perspective, see mathematics as an activity that has social, cultural and historical components. For them, in learning mathematics meaning (making) can be derived through each of these components, especially through (social) interaction and engagement with others. This view of the importance of interaction and engagement is part of the fundamental beliefs that underpin constructivist's approach to teaching mathematics. These beliefs are ‘echoed’ throughout this review as a whole. Cobb, Yackel and Wood (1992) share these beliefs. They argued long ago that knowing is a socially and culturally situated constructive process and that while mathematics requires individual construction, its’ meaning (making) is still derived from collective human activity.

The significance of these perspectives lies in the possibility for the teacher to locate mathematics within historical, social and cultural contexts. Such possibility might be further enhanced by the teacher gaining greater insight into the meaning and interpretation of the mathematical ideas, which he/she might regularly encounter in the classroom. Barton (1995) in an earlier study also supported this view when he argued that the teaching and learning of mathematics has never been, and can never be, removed from the socio-cultural context in which it takes place. An important notion that emanates from his view is one on which I have already placed emphasis, which is, that mathematics must be viewed as [a human] activity. This is because such a perspective coincides with other constructivists’ views of classroom engagement. Such views will be more useful to this study on reform oriented teaching practice because much of the beliefs that support these views emphasize the need to focus on understanding and interpreting classroom interactions and instructions. In fact, some researchers (e.g. Brown, 1996; Cowie, 1995) have broadened their perspectives on what some teachers might believe about mathematics by describing mathematics as an interpretive activity; a thinking activity. I also would like to place particular emphasis on the belief about mathematics as a thinking activity and couple it with my emphasis on mathematics as a human activity. This is because such views not only resonate well with other views I believe to be relevant (e.g., hermeneutics principles), but they also seem to be particularly useful in a study on reform oriented mathematics teaching, where much emphasis is placed on specific classroom practices (e.g. discourse, engagement, interaction) that the research indicates is useful for promoting mathematical thinking.

Mathematics as Thinking Activity

In discussing mathematics as a thinking activity, Brown (1996) captures this idea well by saying we “do” (p.54) mathematics. In fact, Mason (2002) said our very action of doing mathematics is characterized by the norms and nuances of mathematics. For example, he said we add, we subtract, [we multiply] we divide, we solve. These ‘doings’ seem to best distinguish this subject from English (Mason, 2002) or even from the discipline of biology (Mason, 2002). Therefore, I believe that if we were to view mathematics not only as a human activity that requires thinking, but also as one that requires doing, we might understand some of the reasons for the subject of mathematics “being rendered so elusive to so many people” (Mason, p.57). This is a very strong assertion to make, however. As such, it would be beneficial here I believe, to provide the reader with an explanation that may offer a means of gaining a better appreciation of what this assertion means. The next scenario attempts to do just that.

Let me begin this explanation with the scenario used by Mason (2002) in his discussion on the discipline of ‘noticing’ on this issue.

Mason engaged the reader in the following series of tasks and role playing exercises in an attempt to have the reader experience the various facets of noticing. This is an attempt to make the reader more sensitive to his/her own gestures, postures, and argumentative positions. In one such exercise Mason asks the reader to write a short statement that is believed in passionately; and, then to adopt a number of different stances asserting the statement out loud, first in an assertive stance, then in a defensive or apologetic one. He then invites the reader to compare these feelings either when the verbal and the gestural expressions are matching, or when they are contrasting.

In using this strategy, Mason (2002) puts the reader in an enhanced position to interpret what it means to notice ones' self in particular ways because he or she has actively participated in the exercise. It is with the same desire that the reader is asked to participate in these exercises in order to gain an understanding of why a better grasp of mathematics remains elusive to so many people. In light of this, the reader is asked to do the following:

> Consider that you are asked to participate in a game whose rules are fairly familiar to you. You know a few of them but you have not played the game sufficiently to consider yourself a proficient player. In other words, you are unsure of the *nuances* of the game. Yet, you are placed in a position where you must engage in the sport. Think for a moment how this makes you feel. What are the questions, which surface in your own mind?
> The rules are quickly given to you again and the game begins.

You begin to feel extremely uncomfortable, even though you thought you knew the rules of the game. Those around you who have been playing the sport for a long time seem comfortable with the interpretation of the rules and understand how these are applied. They even understand the spoken gestures. You try desperately to interpret the actions you see and try to mimic those actions. Sometimes it seems easy enough, while at other times something seems dreadfully wrong. You are not sure if some of your interpretations are correct. In fact, from time to time something happens which makes you feel sure that at least some of your interpretations are incorrect. Yet, you play on, in spite of the waning confidence and battered self-esteem. As you make one blunder after the other you wish you had not decided to join this game. You decide silently that this game is not for you and that you will avoid participating in this sport again (Mason, 2002, p.8-9).

In the exercise, Mason (2002) then invites the reader to draw a direct analogy with the role playing scenario and what takes place daily in many mathematics classrooms. Students are expected to participate in mathematics activities before they have become sufficiently experienced with the subject. What results, then, is a discord. Citing Schoenfeld (1994) Bell-Hutchinson (2005) wrote:

> . . . . the discord surfaces because more often than not, the subject is not viewed as an activity but as a fait accompli. Rules are provided and a few exercises worked with the assumption that these, in themselves, are sufficient for proficiency to be achieved. Vital experiences are omitted therefore and the 'players' are hardly ever able to be involved in the experiences, which would enable meaning making so necessary for an understanding of the nuances of this activity. These nuances include symbolization, abstraction, symbolic manipulation, and the particular language which the activity requires- [these are] the 'tools of the trade' (Schoenfeld, 1994, cited by Bell-Hutchinson, 2005).

Can we reasonably expect someone to gain proficiency in a mathematics activity without having a basic understanding of the language required and how it is used?

Brown (1991, 1994, 1996) also supports this view of mathematics as an activity and asserts further that mathematics is an essentially interpretive activity "comprising a system of symbols that is only activated within individual human acts" (1994, p. 148).

Brown (1994) finds it reasonable to call upon ideas entrenched within the study of hermeneutics to provide a philosophical base for his assertion that mathematics is an essentially interpretive activity. Here he argues that notions of hermeneutic understanding as applied to mathematics "require a shift in emphasis, which moves from the learner focusing on mathematics as an externally created body of knowledge to be learned, to one where the learner is engaging in the mathematical activity taking place over a period of time" (p.147). This kind of engagement, I believe, is made possible through the integration of several of the important ideas put forth by Bell-Hutchinson (2005) in her observation of Mason's (2002) role playing scenario on 'noticing'.

The complexities surrounding mathematics failure and success as implicitly expressed in this exercise resonates well with the views of mathematics expressed earlier. If we accept the view that mathematics is a human thinking activity, then it is prudent at this point to examine how the integration of hermeneutical understanding can provide to teachers an additional point of view for effective mathematics teaching.

Hermeneutics and Mathematics

Hermeneutics is generally described as a theory concerned with developing a method and means by which individuals might better comprehend and understand the purpose, intent, and meaning of a particular text (Ihde, 2000). Originally, it was the principal analytical process used by biblical scholars concerned with identifying a more consistent and reliable understanding of the interpretation of obscure biblical texts. It was later applied to mathematics by nineteenth and twentieth century philosophers (Brown, 1996). In fact, Brown (1991) credited Dilthey (1900) for the extension of the earlier ideas of hermeneutics to cover the whole of human existence. I believe such a claim might be considered in concert with Mason's (2002) observation that now the use of the term hermeneutics refers to the theory of the interpretation of all manner of texts in several languages and is also used to shed light on the process through which human beings develop an understanding of the world.

There are four basic assumptions of hermeneutics mentioned in the literature that I have chosen to include here. They are:

1. Cultural products are texts (understood in a broad sense and must be interpreted as such;

2. The primary function of a text is to communicate meaning from an author to a "reader";

3. The primary aim of textual analysis is understanding, not explanation;

4. Language, understood in its broadest sense, is the primary medium of communication of meanings.
Retrieved 8/16/03 from http:/homepage.newschool.edu/

In developing a connection between these assumptions and mathematics, Brown (1994, 1996) offers four main claims. They are:

1. mathematics is essentially an interpretive activity

2. the system of symbols which characterizes mathematics is activated only within human acts;

3. mathematical expressions are necessarily contained in practice and carry meaning which transcend mathematical symbolism; and

4. as a consequence of the above, mathematical activity is a subset of social activity.

With regards to the above assumptions, Mason (2002) said that essentially the issue at hand is where meaning resides, does meaning reside within texts (could we then replace texts with mathematics)? Or, is it a social phenomenon mediated through and residing in language, independent of the individual?

These are fundamental questions. They raise serious issues that must be considered and answered when the teaching of mathematics applies hermeneutic understandings of the language of mathematics, to better facilitate the appropriate interpretation of the mathematical language (i.e., symbols, numbers) used in the classroom. Brown (1994) states that:

> . . . mathematical phenomena do not have a tangible existence outside of symbolization. The symbols -how and when they are used, and the context from which they [derive], define the nuances of the activity. They play a significant role in the creation of meaning. In fact, it is this very symbolization that essentially gives the activity meaning and provides the language through which understanding is mediated ((p.142).

Along the same vein, Brown (1994) said:

> . . . without language, the activity has no being. Mathematics only comes into being in its classification in language. If we accept this view, then in attempting to bring meaning to mathematics we must consider the implications for its classification in language since the uncovering of meaning is tied to the linguistic qualities of mathematics.
>
> . . meaning is considered to be derived from a dialogue in a continuous process of introducing linguistic and symbolic form into the socially active space. This stress on language is paramount to understanding mathematics as hermeneutic activity because one

> must employ an interpretive dimension when applying linguistic and symbolic forms to understanding ((p.144).

This brings us to where we must address some key issues in the language of mathematics. It is important to know that hermeneutical principles can also bring meaning to mathematical thinking through the competent use and the appropriate interpretation and translation of the language of mathematical symbols.

The Language of Mathematics

What role does the language of instruction and the language of mathematics play in mathematical thinking? David and Lopes (2002) said that if we accept the notion that the construction of knowledge is essentially a social process, this then entails a further recognition that language acts as a necessary mediator and a fundamental element of the socialization of mathematical thinking in the classroom. Additionally they argued that the development of mathematical thinking has to be preceded by the concurrent development of the specialized language of mathematics. Hence, the authors conclude that learning to speak and write mathematics is part and parcel of learning mathematics.

Along these lines Pimm (1987) argued years ago that mathematics has its own particular linguistic 'register' (Pimm, 1987 p.78). He used the view of mathematics as a language to clarify and illuminate activities in the mathematics classroom. In so doing, Pimm (1987) posits that the range of difficulties relating to the language of instruction used in the learning process which occurs in the mathematics classroom are usually related to mathematical meaning-making, use of symbols, the things symbolized, and syntax of the mathematical operation. With respect to meaning Pimm (1987) claims that;

> .. . the difficulty arises because ordinary English words are used in mathematics with completely different meanings. Students are therefore required to cope with the ambiguities and misunderstandings that arise from this fact. Some of the more popular

words, which carry potentially ambiguous meanings, include *product, difference, similar, face, right*, and *degree*.

Issues of interpretation are therefore critical and the teacher has the task of trying to enable students' understanding so that the appropriate mathematical meanings are ascribed to the words. In addition, the meanings ascribed to certain words such as *some*, *all* or *any* can be problematic (Pimm, 1987, p. 56).

In expanding on this issue, Pimm (1987) further reported that:

> . . . while investigating first-year mathematics undergraduates' interpretations of the words *some* and *all* discovered that for many students, the terms *some* and *all* are contrastive rather than inclusive, i.e. *some* entails *not all*. So the statement "some rational numbers are real numbers" (p. 57) was judged to be false since *all* rational numbers are real numbers (Pimm, 1987, p. 57).
>
> Jaworski, 1994) supported Pimm's (1987) position and suggested that;
>
>> . . . in order to help pupils make sense of mathematics... there must be communication between teacher and pupils ... linguistic communication becomes supremely important ... teachers encouraging children to talk and listening to them" (Jaworski, p. 183).

Indeed, because of the complexity of mathematical meaning making, teachers and students must have the appropriate mathematical tools required for mathematical thinking in order to communicate their beliefs about mathematics and to teach mathematics successfully. In fact Yehuda, Lavy, Linchevski and Sfard (2005) describe mathematical thinking as an "activity of communication and learning mathematics as an initiation to a certain type of mathematical discourse" (p.176). This, they say, is the type of discourse in which "any student can become a skilful participant, provided that a discursive mode is found that makes the best use of this person's strengths" (p.176).

There are other issues with respect to language that Pimm (1987) says have to do with whole expressions where meanings are not readily understood by knowing what individual words mean. At issue is where "the expressions function as semantic units on their own" (p.86).

Examples, he said, are in expressions such as, simultaneous equations, square root, and absolute value.

The Language of Mathematical Symbols.

In my previous discussions I have made note of several authors' (Brown, 1991, 1994, 1996; Peat, 1990; Pimm, 1987; Yehuda, Lavy, Linchevski & Sfard 2005) assertions as to what they believe to be the various connections between the language of instruction and the language of mathematics. Brown (1991, 1994, & 1996) for example claims that mathematics comes into being in its classification as language. This language of mathematics is used in all mathematical activities. It uses both the language of instruction and the language of symbols (Peat, 1990; Pimm, 1987). As such, Pimm (1987) argues strongly that the symbolic aspect of mathematics language contributes to its distinctiveness. He argues that much of the difficulty with mathematics arises from the way the language of symbols is used and interpreted. He said too much emphasis is placed on the symbols themselves instead of what the symbols really mean. An interesting example Pimm (1987) cites was where the following letter was used in the classroom as a mathematical symbol. He writes:

Teacher: Let *n* be a number

Pupil: But *n* is a letter (p.18).

Pimm (1987) said such misinterpretation arises from how the language of symbols is used in mathematics teaching in the classroom. He said, that these symbols are used as letters rather than "conventional symbols which we learn to form and can discriminate one from another" (p.18-19). In that regard, he said students see them as such, and relate to them in like manner (1987).

On the other hand, much has been discovered in this area since Pimm (1987) presented this idea. For example, Lerman (1994) said new research in the area of semiotics and socio-cultural perspective on learning has provided additional insight on the difficulty students experience with the teachers' use of the mathematical symbols in the classroom. In that same vein, the author also said a fundamental rule of semiotics is that a mark on paper becomes a symbol only when it is deliberately associated with a conceptual meaning, and, that mathematical symbols remain meaningless until that association is made. He said that meaning is carried in social practices (e.g., associating mathematics with living in the real world).

Indeed, Lerman's idea again emphasizes what was said earlier, that meaning making does not reside simply in the symbolic and linguistic expressions of mathematics. Rather, meaning (making) he said, also mediates through external sources (e.g., use of manipulatives and cultural artifacts to illustrate mathematical meaning, Pimm, 1987).

It is my belief based on the research that the syntactic errors in mathematical operations Pimm (1987) describes, occur when an analysis has to be made between the language of instruction and the language of mathematics. Indeed, according to Pimm (1987), such errors can be derived from a problem of meaning making or problems of connections between meanings, such as when a faulty association is made between meanings. Therefore the teacher should find effective ways of presenting concepts to avoid misconceptions, misinterpretation and lapses in effective communication.

Since much of what we now know about language functions and mathematics are associated with hermeneutics principles, it is important to take a look at how such lack of conceptual understanding is viewed within the hermeneutic paradigm.

Hermeneutical Understanding and

Mathematical Understanding

Brown (1994) said that the fundamental premise for understanding mathematics from a hermeneutical perspective is that such understanding is not derived from concepts with fixed meanings. Rather, Brown considers mathematical understanding to be more like an ever changing and flexible natural process rather than being a fixed state and for him this process is usually built over time. He argues that this perspective then "'softens' the notion of a human subject confronting an independent object and enables a hermeneutic process of coming to know through juxtaposing varying perspectives"(p.148). He writes:

> The intention to learn is always associated with some presupposition about that to be learned and learning is in a sense [a] revisiting [of] that already presupposed. This continual projecting forwards and backwards affirms an essential time dimension to mathematical understanding that can never be brought to a close by an arrival at a concept since the very framing of that concept modifies the space being described (Brown, p.148).

Mason (2002) also uses hermeneutical thinking to make this argument clearer. He said:

> . . .in order to make sense of a whole text, it is necessary to make sense of the components, but sense made of components is based on one's sense of [the]whole, so the two develop and change together (Mason, 2002).

For him, understanding can be temporary, a just for-the-moment, experience. With the appropriate guidance, however, Mason (2002) holds that such understanding can be built and rebuilt until that repertoire of experience (whether it is for the student or for the teacher) is fully developed. With that being said, I believe, that Watson (2002b) has a more direct approach to this notion of understanding. She asserts that understanding is dynamic, contingent and local. In that regard she reminds us that in the process of examining understanding, we must also examine how that understanding evolves, develops and emanates from the teacher to the students and vice versa as the students make sense of what they learn.

Elsewhere in this chapter, I discussed research that said a primary function of hermeneutics is to facilitate an accurate translation of mathematical meaning and to communicate that meaning from the teacher to the learner via the text. In that discussion, Brown (1996) asserts that by using hermeneutics as a mechanism, it enables the teacher to provide an appropriate interpretation and translation between the mathematical language and the language of instruction. From a hermeneutical perspective, then, the central theme here pertains to how the presuppositions and motive of the reader (learner) affect the meaning derived in text, which the author (teacher) has provided (Brown, 1994, 1996). When applied to the classroom it is the student's (reader's) point of view and presuppositions that will influence if not determine the meaning derived from what is being communicated regardless of the intent of the teacher (author). Although this idea resonates with Pimm's (1987) experience, it is more attuned to specific problems of communication that teachers face in the classroom. In that regard, Brown (1994, 1996) did not hesitate to point out that intention and meaning are not necessarily coterminous. In fact, they may be contradictory. This is less likely where the level of mathematics proficiency (the ability to exhibit competence) that is achieved as well as the demonstrated competence in the use of mathematical language by both the teacher and the student coincide.

On the basis of Brown's perspective, it would be fair to assume that an important job of the mathematics teacher is to encourage the construction of meaning by students as a way to guide their mathematical understanding. This kind of thinking, of course, would mean that teachers would need to allow for the fact that a student's personal meaning cannot be directly taught (Steffe & Tzur, 1994) but rather it is achieved through a combination of factors, including the student's overall learning experiences. Regardless of the teacher's intent, Brown (1994) said the “emphasis is not on students recreating the teacher’s intention but on the student’s production of meaning in respect to the given task” (p.54).

I have looked at mathematics teaching as an activity requiring the use of hermeneutics in a process of hermeneutical analysis of the language of mathematics and in the exercise of mathematical meaning making. The intent is that it will provide for a richer analysis of what constitutes mathematical teaching. The use of hermeneutics allows the social, cultural and interpretive dimensions of mathematics to be fully elucidated. It also gives the teacher another lens through which he/she can view the full range of factors that influence the teaching of mathematics (Brown, 1994, 1996). In that same vein, Whiteaker, (2003) said “researchers attempting to understand the social aspects of the classroom have begun to look for other theories to complement their constructivist foundation” (p.69). Because my views as well as my thinking about what is required to effectively teach mathematics is in line with what Whiteaker (2003) said about researchers, the hermeneutical perspective used here in this review has been adopted in order to complement the connections between the different views and approaches (e.g. constructivism, fallibilism) that can be taken in the teaching of mathematics. It is also used here because research (e.g. Askey, 2001; Battista, 1994; Klein, 2003) reveals that different teachers have different views about mathematics, how the subject should be taught as well as who should be taught mathematics.

APPENDIX B

LETTERS OF REQUEST AND PRIOR NOTICE EMAIL
(IRB Approved)

January 25, 2008

Mr. ____________.
Superintendent of Schools
_________________________.

Dear Mr.________________.

I am a graduate student at Temple University. I would like your permission to use your school as a site to conduct research to fulfill the necessary requirement for my Ph.D. degree in Educational Psychology.

The purpose of this qualitative investigation is to determine the relationship between teachers' beliefs and perceptions in relationship to reform oriented constructivist teaching of mathematics, and the extent to which those beliefs and perceptions correlate with their classroom practices.

The data collection instruments that I plan to use will consist of teacher surveys, teacher logs and classroom observations. By classroom observation I mean the actual observation of the teachers' teaching of the subject matter. The target population will include six mathematics teachers selected from across three grade levels, elementary, middle and high school. Within each grade level I would like to select two grade five classes, two grade eight classes and two grade nine classes. I plan to make at least six visits to your school district.

This permission is needed in order for the Internal Review Board at Temple University to allow the data collection phase to begin and for me to complete the writing of my dissertation proposal. Please note also that all information used in the research proposal and the dissertation will be anonymous and will be in compliance with FERPA protected privacy. In furtherance of this research project, each of the participants will be given an honorarium of one hundred dollars ($100) for her/his involvement in the project.

Please indicate your approval of this permission by signing the letter where indicated below and returning it to me as soon as possible. By signing this letter, you are confirming that you have given permission to use your schools for the purpose of conducting this research project and that you also will have limited right to the proposal when it is completed.

In anticipation of working with you, I wish to thank you very much for your willingness to assist with this important study. If you need to verify the above request at this time, please feel free to contact my Advisor and Dissertation Chair, Dr. Joseph DuCette, at 215-204-4998.

Sincerely,

Violet Barrett Paterson
609-771-947 or barrettuline@verizon.net

____
PERMISSION GRANTED FOR THE USE REQUESTED ABOVE

____
Signature

____
Date:

Prior Notice E-mail (IRB Approved)

March 26, 2008

Mr. ___________
Public School of ______
Office of the Superintendent
Address 1
Address 2

Dear ___________

As part two of this project we would like to distribute a questionnaire to all the mathematics teachers in your school district to get an idea of the best way to teach mathematics. If you have any questions please contact Dr. Joseph DuCette, Temple University, 1301 W. Cecil B. Moore Avenue, Philadelphia, P.A. 19122; 215-204-4998

Sincerely yours

Violet Barrett Paterson

Violet Barrett Paterson,
Student Investigator
Temple University, Philadelphia, P.A. 19122

APPENDIX C

DEAR COLLEAGUE LETTER
(IRB Approved)

May 28, 2008

Dear Colleague:

Thank you for your participation in phase II of our research project.

As we have discussed, please ask the elementary (grades 5 and up), middle and high school mathematics teachers in your school district to complete the attached survey and return it in the self-addressed envelop provided. The survey should take no more than 10 minutes to complete.

Please note that a small number of six (6) teachers have already participated in the piloting of the survey and should therefore not be asked to complete the instrument a second time.

I hope that the information we collect can be beneficial to schools and districts, as well as institutions that prepare mathematics teachers, in providing a knowledge base about the best way to teach mathematics at these levels.

If you have any questions about the survey, please feel free to call Violet Barrett Paterson at 609-771-9473, or myself at 215-204-4998.

Thank you for your cooperation

Sincerely yours

Violet Barrett Paterson, Student Investigator

Cc: Dr. Joseph DuCette, Principal Investigator
Department of Psychological Studies in Education

APPENDIX D

TEACHER CONSENT FORMS
(IRB Approved)

Principal Investigator: Dr. Joseph DuCette

Student Investigator: Violet Barrett Paterson

Department: College of Education, Psychological Studies in Education (PSE)
Temple University, Philadelphia, PA

Phone Number: 215-204-4998

Project Title: Teachers' Beliefs and Practice in Relation to Reform Oriented Mathematics Teaching

Dear Teachers:

We are currently engaged in a study to see if there is a correspondence between teachers' beliefs about reform oriented mathematics teaching and their practice. Your Superintendent, Mr. _, has granted permission to conduct this study.

To help us gain further insight into this area we will ask you to complete a short survey. For those interested, we have provided the opportunity to participate in a classroom observation, and a post-observation interview. All collected information will be held in the strictest confidence and will be coded by the researcher to protect the identity of participants. All data will be recorded anonymously and, if you would like to participate through an interview, anything you say during the session will be held in the strictest confidence.

We welcome questions about the project at any time. Your participation in this study is on a voluntary basis and you may refuse to participate at any time without consequence or prejudice.

If you have any questions about your right as a research participant, please direct them to: Mr. Richard Throm, Office of the Vice President for Research, Institutional Review Board, Temple University, 3400 N. Broad Street, Philadelphia, PA, 19140, (215), 707-8757.

Signing your name below indicates that you have read and understand the contents of this Consent Form and that you agree to take part in this study.

______________________ ____________
Participant Signature Date

______________________ ____________
Violet Barrett Paterson, Student Date

Principal Investigator: Dr. Joseph DuCette

Student Investigator: Violet Barrett Paterson

Department: College of Education, Psychological Studies in Education (PSE) Temple University, Philadelphia, PA

Phone Number: 215-204-4998

Project Title: Teachers' Beliefs and Practice in Relation to Reform Oriented Mathematics Teaching

Dear Teachers:

We are currently engaged in a study to see if there is a correspondence between teachers' beliefs about reform oriented mathematics teaching and their practice. Your Superintendent, Mr. ___has granted permission to conduct this study.

To help us gain further insight into this area we will ask you to complete a short survey. For those interested, we have provided the opportunity to participate in a classroom observation, and a post-observation interview. All collected information will be held in the strictest confidence and will be coded by the researcher to protect the identity of participants. All data will be recorded anonymously and, if you would like to participate through an interview, anything you say during the session will be held in the strictest confidence.

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Signing your name below indicates that you have read and understand the contents of this Consent Form and that you agree to take part in this study.

_____________________ ____________

Participant Signature Date

_____________________ ____________

Violet Barrett Paterson, Student Date

Principal Investigator: Dr. Joseph DuCette

Student Investigator: Violet Barrett Paterson

Department: College of Education, Psychological Studies in Education (PSE)
Temple University, Philadelphia, PA

Phone Number: 215-204-4998

Project Title: Teachers' Beliefs and Practice in Relation to Reform Oriented Mathematics Teaching

Dear Teachers:

We are currently engaged in a study to see if there is a correspondence between teachers' beliefs about reform oriented mathematics teaching and their practice. Your Assistant Superintendent, Dr. _, has been granted permission by to conduct this study.

To help us gain further insight into this area we will ask you to complete a short survey. For those interested, we have provided the opportunity to participate in a classroom observation, and a post-observation interview. All collected information will be held in the strictest confidence and will be coded by the researcher to protect the identity of participants. All data will be recorded anonymously and, if you would like to participate through an interview, anything you say during the session will be held in the strictest confidence.

We welcome questions about the project at any time. Your participation in this study is on a voluntary basis and you may refuse to participate at any time without consequence or prejudice.

If you have any questions about your right as a research participant, please direct them to: Mr. Richard Throm, Office of the Vice President for Research, Institutional Review Board, Temple University, 3400 N. Broad Street, Philadelphia, PA, 19140, (215), 707-8757.

Signing your name below indicates that you have read and understand the contents of this Consent Form and that you agree to take part in this study.

____________________ ___________
Participant Signature Date

____________________ ___________
Violet Barrett Paterson, Student Investigator Date

APPENDIX E

ADDITIONAL MAIN SAMPLE SURVEY DATA

(Original Distribution)

	Main Sample (Original Distribution) *N* = 79	Percent %
	N	%
Age		
Less than 25	6	7%
26-30	15	19%
31-35	10	13%
36-40	7	9%
41-45	9	11%
46-50	6	7%
More than 50	25	32%
Missing	1	2%
Total	79	100%
Gender		
Male	19	24%
Female	59	75%
Unidentified	1	1%
Total	79	100%
Ethnicity		
African American	18	22%
American Indian/	0	0%
Hispanic/Latino	7	9%
Oriental/Asian	1	1%
White (Not Hispanic)	51	65%
Other	2	3%
Total	79	100%
Type of School		
Elementary	29	36%
Middle	25	32%
High	25	32%
Total	79	100%
District		
Urban	2	3%
Suburban	77	97%
Total	79	100%

Grade		
Five	6	7%
Six - eight	19	24%
9-12	14	18%
Other/combination	38	48%
Missing	2	3%
Total	79	100%
Degree		
BA/BSc	40	50%
MA/MSc	30	40%
Multiple MA/MSc	5	6%
Ph.D. or Ed.D.	2	2%
Other degrees	2	2%
Missing	0	0%
Total	79	100%
Mathematics major		
Yes	32	40%
No	45	57%
Missing	2	3%
Total	79	100%
Mathematics minor		
Yes	13	16%
No	63	80%
Missing	3	4%
Total	79	100%
Teaching Certificate		
None	2	2.5%
Temporary	2	2.5%
Probationary	0	0%
Regular	74	94%
Missing	1	1%
Total	79	100%
Specific endorsement		
Yes	37	47%
No	36	45%
Missing	6	8%
Total	79	100%
Years teaching full-time		
Zero - five	23	29%
Six - 19	32	41%
20 - 30	20	25%
31 and above	1	1%
Missing	3	4%
Total	79	100%

Years teaching mathematics		
Zero – five	25	32%
Six – 19	32	40%
20 – 30	21	26%
31 and above	0	0%
Missing	1	2%
Total	79	100%
LEP/ELL		
Zero – five	62	78%
Six – 10	4	5%
11 and above	7	9%
Other /don't know	4	5%
Missing	2	3%
Total	79	100%
Confidence		
None	1	1%
Somewhat	1	1%
Moderately	24	30%
Very	53	68%
Missing	0	0%
Total	79	100%
Mixed Ability Groups		
Fairly homogeneous/low ability	13	16%
Fairly homogeneous/average ability	16	20%
Fairly homogeneous/high ability	7	9%
Heterogeneous two or more Abilities	38	48%
Combination ability levels	2	3%
Missing	3	4%
Total	79	100%
Professional Development (1)		
None	34	43%
Less than four hours	21	27%
Four – eight hours	5	6%
9 – 15 hours	8	10%
More than 16 hours	4	5%
Missing	7	9%
Total	79	100%

Note: (1)= In-depth study of mathematics

Professional Development (2)

None	12	15%
Less than four hours	29	37%
Four - eight hours	18	23%
9 - 15 hours	10	13%
More than 16 hours	8	10%
Missing	2	2%
Total	79	100%

Note: (2)=Methods of teaching mathematics

Professional Development (3)		
None	11	14%
Less than four hours	29	37%
Four - eight hours	19	24%
9 - 15 hours	8	10%
More than 16 hours	9	11%
Missing	3	4%
Total	79	100%

Note: (3)=Use of particular mathematics curricula or curriculum materials

Professional Development (4)		
None	15	19%
Less than four hours	32	41%
Four - eight hours	10	13%
9 - 15 hours	6	7%
More than 16 hours	10	13%
Missing	6	7%
Total	79	100%

Note: (4)=Mathematics standards or framework-e.g. NCTM, State and/or District

(Second Distribution)

	Frequency (Second Distribution) N = 95	Percent %
	N	%
Age		
Less than 25	6	6%
26-30	15	16%
31-35	11	12%
36-40	9	9%
41-45	15	16%
46-50	14	15%
More than 50	23	24%
Missing	2	2%
Total	95	100%
Gender		
Male	21	22%
Female	63	66%
Unidentified	11	12%
Total	95	100%
Ethnicity		
African American	11	13%
American Indian	2	2%
Hispanic/Latino	4	4%
Oriental/Asian	5	5%
White (Not Hispanic)	68	71%
Other	3	3%
Missing	2	2%
Total	95	100%
School		
Elementary	66	69%
Middle	15	16%
High	14	15%
Total	95	100%
District		
Urban	53	56%
Suburban	41	43%
Missing	1	1%
Total	95	100%

Grade		
Five	29	30%
Six - Eight	15	16%
9 - 12	14	15%
Combination	33	35%
Other	3	3%
Missing	1	1%
Total	95	100%
Degree		
B.A./B.Sc.	54	57%
M.A./M.Sc.	33	35%
Multiple Masters	4	4%
Ph.D./Ed.D.	2	2%
Other	1	1%
Missing	1	1%
Total	95	100%
Mathematics Major		
Yes	17	18%
No	77	81%
Missing	1	1%
Total	95	100%
Mathematics Minor		
Yes	10	11%
No	79	83%
Missing	6	6%
Total	95	100%
Teaching Certificate		
None	0	0%
Temporary Provisional	9	9%
Probationary	2	2%
Regular	83	88%
Missing	1	1%
Total	95	100%
Specific Endorsement		
Yes	20	21%
No	73	77%
Missing	2	2%
Total	95	100%
Years teaching full-time		
Zero - five	25	26%
6 - 19	46	48%
20 - 30	18	20%
31 and above	3	3%
Missing	3	3%
Total	95	100%

Years teaching mathematics		
Zero - five	34	36%
6 - 19	41	43%
20 - 30	17	18%
31 and above	1	1%
Missing	2	2%
Total	95	100%
LEP/ELL classified		
Zero - five percent	62	65%
6 - 10 percent	4	4%
11 percent and above	16	17%
Don't know/not sure	1	1%
Missing	12	13%
Total	95	100%
Confidence		
None	1	1%
Somewhat	4	4%
Moderately	26	28%
Very	61	64%
Missing	3	3%
Total	95	100%
Mixed Ability Groups		
Fairly homogeneous/low ability	15	16%
Fairly homogeneous/average ability	16	17%
Fairly homogeneous/high ability	0	0%
Heterogeneous two or more Abilities	56	59%
Combination ability	0	0%
Missing	8	8%
Total	95	100%
Professional Development (1)		
None	49	52%
Less than four hours	15	16%
Four - eight hours	15	16%
9 - 15 hours	3	3%
More than 16 hours	8	8%
Missing	5	5%
Total	95	100%

Note: (1)= In-depth study of mathematics

Professional Development (2)		
None	27	28%
Less than four hours	32	35%
Four - eight hours	17	18%
9 - 15 hours	6	6%

More than 16 hours	6	6%
Missing	7	7%
Total	95	100%

Note: (2)= Methods of teaching mathematics

Professional Development (3)		
None	20	21%
Less than four hours	25	27%
Four - eight hours	28	30%
9 - 15 hours	8	8%
More than 16 hours	8	8%
Missing	6	6%
Total	95	100%

Note:(3)= Use of particular mathematics curricula or curriculum materials

Professional Development (4)		
None	32	34%
Less than four hours	31	33%
Four - eight hours	16	17%
9 - 15 hours	5	5%
More than 16 hours	5	5%
Missing	6	6%
Total	95	100%

Note:(4)= Mathematics standards or framework-e.g. NCTM, State and/or District

APPENDIX F

DESCRIPTION OF THE REFORMED TEACHING OBSERVATION (RTOP) TRAINING GUIDE

(IRB Approved)

By: Sawada, D. (External Evaluator), Piburn, M. (Internal Evaluator),
and
Turley, J., Falconer, K., Benford, R., Bloom, I., & Judson, E. (The Evaluation Facilitation Group)Arizona Collaborative for Excellence in the Preparation of Teachers
Arizona State UniversityACEPT Technical Report No. IN00-2

The Reformed Teaching Observation Protocol (RTOP) is an observational instrument that can be used to assess the degree to which mathematics or science instruction is "reformed". It embodies the recommendations and standards for the teaching of mathematics and science that have been promulgated by professional societies of mathematicians, scientists and educators.

The RTOP was designed, piloted and validated by the Evaluation Facilitation Group of the Arizona Collaborative for Excellence in the Preparation of Teachers. Those most involved in that effort were Daiyo Sawada (External Evaluator), Michael Piburn (Internal Evaluator), Bryce Bartley and Russell Benford (Biology), Apple Bloom and Matt Isom (Mathematics), Kathleen Falconer (Physics), Eugene Judson (Beginning Teacher Evaluation), and Jeff Turley (Field Experiences).

The instrument draws on the following sources:

- National Council for the Teaching of Mathematics. *Curriculum and Evaluation Standards (1989). Professional Teaching Standards (1991), and Assessment Standards (1995).*
- American Association for the Advancement of Science Project 2061. *Science for All Americans (1990) Benchmarks for Scientific Literacy (1993).*

It also reflects the ideas of all ACEPT Co-principal Investigators, but especially those of Marilyn Carison and Anton Lawson, and the principles of reform underlying the ACEPT project. Its structure reflects some elements of the *Local Systemic Change Revised Classroom Observation Protocol, by Horizon Research (1997-88).*

The RTOP is criterion-referenced, and observers' judgments should not reflect a comparison with any other instructional setting than the one being evaluated. It can be used at all levels, from primary school through university. The instrument contains twenty-five items, with each rated on a scale from 0 (not observed) to 4 (very descriptive). Possible scores range from 0-100 points, with higher scores reflecting a greater degree of reform.

The RTOP was designed to be used by trained observers. This *Training Guide* provides specific information pertinent to the interpretation of individual items in the protocol. It is intended to be used as part of a formal training program in which trainees observe actual classrooms or videotapes of classrooms, and discuss their observations with others. The *Guide,* in its present form, is also designed to solicit trainee thoughts and concerns so that they feel comfortable in using the instrument. For that reason, a space is provided after each item for trainee comments. Such input helps all those being trained to achieve a higher degree of consistency in using the instrument. Please keep this in mind in making comments.

1. BACKGROUND INFORMATION

This section contains space for standard information that should be recorded by all observers. It will serve to identify the classroom, the instructor, the lesson observed, the observer, and the duration of the observation.

Comments:

II. CONTEXTUAL BACKGROUND AND ACTIVITIES

Space is provided for a brief description of the lesson observed, the setting in which the lesson took place (space, seating arrangements, etc.), and any relevant details about the students (number, gender, ethnicity, etc.) and instructor. Try to go beyond a simple description. Capture, if you can, the defining characteristics of this situation that you believe provide the most important context for understanding what you will describe in greater detail in later sections. Use diagrams if they seem appropriate.

Comments:

The next three sections contain the items to be rated. Do not feel that you have to complete them during the actual observation period. Space is provided on the facing page of every evaluation for you to make notes while observing. Immediately *after the lesson* draw upon your notes and complete the ratings. For most items, a valid judgment can be rendered after observing the entire lesson. The whole lesson provides contextual reference for rating each item.

Each of the items is to be rated on a scale ranging from 0 to 4. Choose "0" if in your judgment, the characteristic *never occurred* in the lesson, not even once. If it did occur, even if only once, "1" or higher should be chosen. Choose "4" only if the item was very descriptive of the lesson you observed. Intermediate ratings do not reflect the number of times an item occurred, but rather the degree to which that item was *characteristic* of the lesson observed.

The remainder of this Training Guide provides a clarification of each RTOP item and the subtest (there are five) of which it is a part.

III. LESSON DESIGN AND IMPLEMENTATION

1) The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.

A cornerstone of reformed teaching is taking into consideration the prior knowledge that students bring with them. The term "respected" is pivotal in this item. It suggests an attitude of curiosity on the teacher's part, an active solicitation of student ideas, and an understanding that much of what a student brings to the mathematics or science classroom is strongly shaped and conditioned by their everyday experiences.

Comments:

2) The lesson was designed to engage students as members of learning community.

Much knowledge is socially constructed. The setting within which this occurs has been called a "learning community". The use of the term community in the phrase "the scientific community" (a "self-governing" body) is similar to the way it is intended in this item. Students participate actively, their participation is integral to the actions of the community, and knowledge is negotiated within the community. It is important to remember that a group of learners does not necessarily constitute a "learning community."
Comments:

3) In this lesson, student exploration preceded formal presentation.
Reformed teaching allows students to build complex abstract knowledge from simpler, more concrete experience. This suggests that any formal presentation of content should be preceded by student exploration. This does not imply the converse. . . that all exploration should be followed by a formal presentation.
Comments:

4) This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.
Divergent thinking is an important part of mathematical and scientific reasoning. A lesson that meets this criterion would not insist on only one method of experimentation or one approach to solving a problem. A teacher who valued alternative modes of thinking would respect and actively solicit a variety of approaches, and understand that there may be more than one answer to a question.
Comments:

5) The focus and direction of the lesson was often determined by ideas originating with students.
If students are members of a true learning community, and if divergence of thinking is valued, then the direction that a lesson takes can not always be predicted in advance. Thus, planning and executing a lesson may include contingencies for building upon the unexpected. A lesson that met this criterion might not end up where it appeared to be heading at the beginning.
Comments:

IV. CONTENT

Knowledge can be thought of as having two forms: Knowledge of what is (Propositional Knowledge), and knowledge of how to (Procedural Knowledge). Both are types of content. The RTOP was designed to evaluate mathematics or science lessons in terms of both.

Propositional Knowledge

This section focuses on the level of significance and abstraction of the content, the teacher's understanding of it, and the connections made with other disciplines and with real life.

6**) The lesson involved fundamental concepts of the subject.**

The emphasis on "fundamental" concepts indicates that there were some significant scientific or mathematical ideas at the heart of the lesson. For example, a lesson on the multiplication algorithm can be anchored in the distributive property. A lesson on energy could focus on the distinction between hear and temperature.

Comments:

7) The lesson promoted strongly coherent conceptual understanding.

The word "coherent" is used to emphasize the strong inter-relatedness of mathematical and/or scientific thinking. Concepts do not stand on their own two feet. They are increasingly more meaningful as they become integrally related to and constitutive of other concepts.

8) **The teacher had a solid grasp of the subject matter content inherent in the lesson.**

This indicates that a teacher could sense the potential significance of ideas as they occurred in the lesson, even when articulated vaguely by students. A solid grasp would be an eagerness to pursue student's thoughts even if seemingly unrelated at the moment. The grade-level at which the lesson was directed should be taken into consideration when evaluating this item.

Comments:

9) Elements of abstraction (i.e., symbolic representations, theory building) were encouraged when it was important to do so.

Conceptual understanding can be facilitated when relationships or patterns are represented in abstract or symbolic ways. Not moving toward abstraction can leave students overwhelmed with trees when a forest might help them locate themselves.

10) Connections with other content disciplines and/or real world phenomenon were explored and valued.

Connecting mathematical and scientific content across the disciplines and with real world applications tends to generalize it and make it more coherent. A physics lesson on electricity might connect with the role of electricity in biological systems, or with the wiring systems of a house. A mathematics lesson proportionality might connect with the nature of light, and refer to the relationship between the height of an object and the length of its shadow.

Comments:

Procedural Knowledge

This section focuses on the kinds of processes that students are asked to use to manipulate information, arrive at conclusions, and evaluate knowledge claims. It mostly closely resembles what is often referred to as mathematical thinking or scientific reasoning.

11) Students used a variety of means (models, drawings, graphs, symbols, concrete materials, manipulatives, etc.) to represent phenomena.
Multiple forms of representation allow students to use a variety of mental processes to articulate their ideas, analyze information and to critique their ideas. A "variety" implies that at least two different means were used. Variety also occurs within a give means. For example, several different kinds of graphs could be used, not just one kind.
Comments:

(12) **Students made predictions, estimations and/or hypotheses and devised means for testing them.**
This item does not distinguish among predictions, hypotheses and estimations. All three terms are used so that the RTOP can be descriptive of both mathematical thinking and scientific reasoning. Another word that might be used in this context is "conjectures". The idea is that students explicitly stat what they think is going to happen before collecting data.
Comments:

13) **Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.**
This item implies that students were not only actively doing things, but that they were also actively thinking about how what they were doing could clarify the next steps in their investigation.
Comments:

14) Students were reflective about their learning.
Active reflection is a meta-cognitive activity that facilitates learning. It is sometimes referred to as "thinking about thinking." Teachers can facilitate reflection by providing time and suggesting strategies for students to evaluate their thoughts throughout a lesson. A review conducted by the teacher may not be reflective if it does not induce students to *re-examine* or *re-assess* their thinking.

15**) Intellectual rigor, constructive criticism, and the challenging of ideas were valued.**
At the heart of mathematical and scientific endeavors is rigorous debate. In a lesson, allowing a variety of ideas to be presented, but insisting that challenge and negotiation also occur would achieve this. Achieving intellectual rigor by following a narrow, often prescribed path of reasoning, to the exclusion of alternatives, would result in a low score on this item. Accepting a variety of proposals without accompanying evidence and argument would also result in a low score.
Comments:

V. CLASSROOM CULTURE

This section addresses a separate aspect of a lesson, and completing these items should be done independently of any judgments on preceding sections. Specifically the design of the lesson or the quality of the content should not influence ratings in this section. Classroom culture has been conceptualized in the RTOP as consisting of: (1) Communicative Interactions, and (2) Student Teacher Relationships. These are not mutually exclusive categories because all communicative interactions presuppose some kind of relationship among communicants.

Communicative Interactions

Communicative interactions in a classroom are an important window into the culture of that classroom. Lessons where teachers characteristically speak and students listen are not reformed. It is important that students be heard, and often, and that they communicate with one another, as well as with the teacher. The nature of the communication captures the dynamics of knowledge construction in that community. Recall that communication and community have the same root.

16) Students were involved in the communication of their ideas to others using a variety of means and media.
The intent of this item is to reflect the communicative richness of a lesson that encouraged students to contribute to the discourse and to do so in more than a single mode (making presentations, brainstorming, critiquing, listening, making videos, group work, etc.). Notice the difference between this item and item 11. Item 11 refers to representations. This item refers to active communication**.**
Comments:

17**) The teacher's questions triggered divergent modes of thinking.**
This item suggests that teacher questions should help to open conceptual space rather than confining it within predetermined boundaries. In its simplest form, teacher questioning triggers divergent modes of thinking by framing problems for which there may be more than one correct answer or framing phenomena that can have more than one valid interpretation.
Comments:

18**) There was a high proportion of student talk and a significant amount of it occurred between and among students.**
A lesson where a teacher does most of the talking is not reformed. This item reflects the need to increase both the amount of student talk and of talk among students. A "high proportion" means that at any point in time it was as likely that a student would be talking as that the teacher would be. A "significant amount" suggests that critical portions of the lesson were developed through discourse among students.
Comments:

19) Student questions and comments often determined the focus and direction of classroom discourse.
This item implies not only that the flow of the lesson was often influenced or shaped by student contributions, but that once a direction was in place, students were crucial in sustaining and enhancing the momentum.
Comments:

20) There was a climate of respect for what others had to say.
Respecting what others have to say is more than listening politely. Respect also indicates that what others had to say was actually heard and carefully considered. A reformed lesson would encourage and allow every member of the community to present their ideas and express their opinions without fear of censure or ridicule.
Comments:

Student/Teacher Relationship

21) Active participation of students was encouraged and valued.
This implies more than just a classroom full of active students. It also connotes their having a voice in how that activity is to occur. Simply following directions in an active manner does not meet the intent of this item. Active participation implies agenda setting as well as "minds-on- and "hands on."
Comments:

22) Students were encouraged to generate conjectures, alternative solution strategies, and/or different ways of interpreting evidence.
Reformed teaching shifts balance of responsibility for mathematical or scientific thought from the teacher to the students. A reformed teacher actively encourages this transition. For example, in a mathematics lesson, the teacher might encourage students to find more than one way to solve a problem. This encouragement would be highly rated if the whole lesson was devoted to discussing and critiquing these alternate solution strategies.
Comments:

23) In general the teacher was patient with students.
Patience is not the same thing as tolerating unexpected or unwanted student behavior. Rather there is anticipation that, when given a chance to play itself out, unanticipated behavior can lead to rich learning opportunities. A long "wait time" is a necessary but not sufficient condition for rating highly on this item.
Comments:

24) The teacher acted as a resource person, working to support and enhance student investigations.
A reformed teacher is not there to tell students what to do and how to do it. Much of the initiative is to come from students, and because students have different ideas, the teacher's support is carefully crafted to the idiosyncrasies of student thinking. The metaphor, "guide on the side" is in accord with this item.
Comments:

(25) The metaphor "teacher as listener" was very characteristic of this classroom.
This metaphor describes a teacher who is often found helping students use what they know to construct further understanding. The teacher may indeed talk a lot, but such talk is carefully

crafted around understandings reached by actively listening to what students are saying. "Teacher as listener" would be fully in place if "student as listener" were reciprocally engendered.

Comments:

V.SUMMARY

The RTOP provides an operational definition of what is meant by "reformed teaching." The items arise from a rich research-based literature that describes inquiry-oriented standards-based teaching practices in mathematics and science.

However, this training guide does not cite research evidence. Rather it describes each item in a more metaphoric way. Our experience has been that these items have richly intuitive meaning to mathematics and science educators.

Further information about the underlying conceptual and theoretical basis of the RTOP, as well as reliability and validity data and norms by grade-level and content, can be found in the *Reformed Teaching Observation Protocol* MANUAL (Sawada & Piburn, 2000).

APPENDIX G

BRIEF BIOGRAPHY OF EXTERNAL OBSERVERS

External Observer #1

Observer #1 holds an Ed.M. in Administration, Planning and Social Policy from Harvard University, a M.Ed. in Elementary Education and a B.A. in Early Childhood Education from Trenton State College and a Certificate in Economic Education from Montclair State College. She has served on several local and national committees including, the Mathematics Curriculum Committee and the Redshaw School Academic Alliance for Elementary Science. Observer #1 teaches at the elementary grade level in one of the local school districts in the State.

External Observer #2

Observer #2 holds a B.Sc. Degree in Business Administration and an Associate Degree in Applied Business. She is a licensed Day Care Provider. Observer #2 works as a Substitute Teacher in several school districts in the State.

APPENDIX H

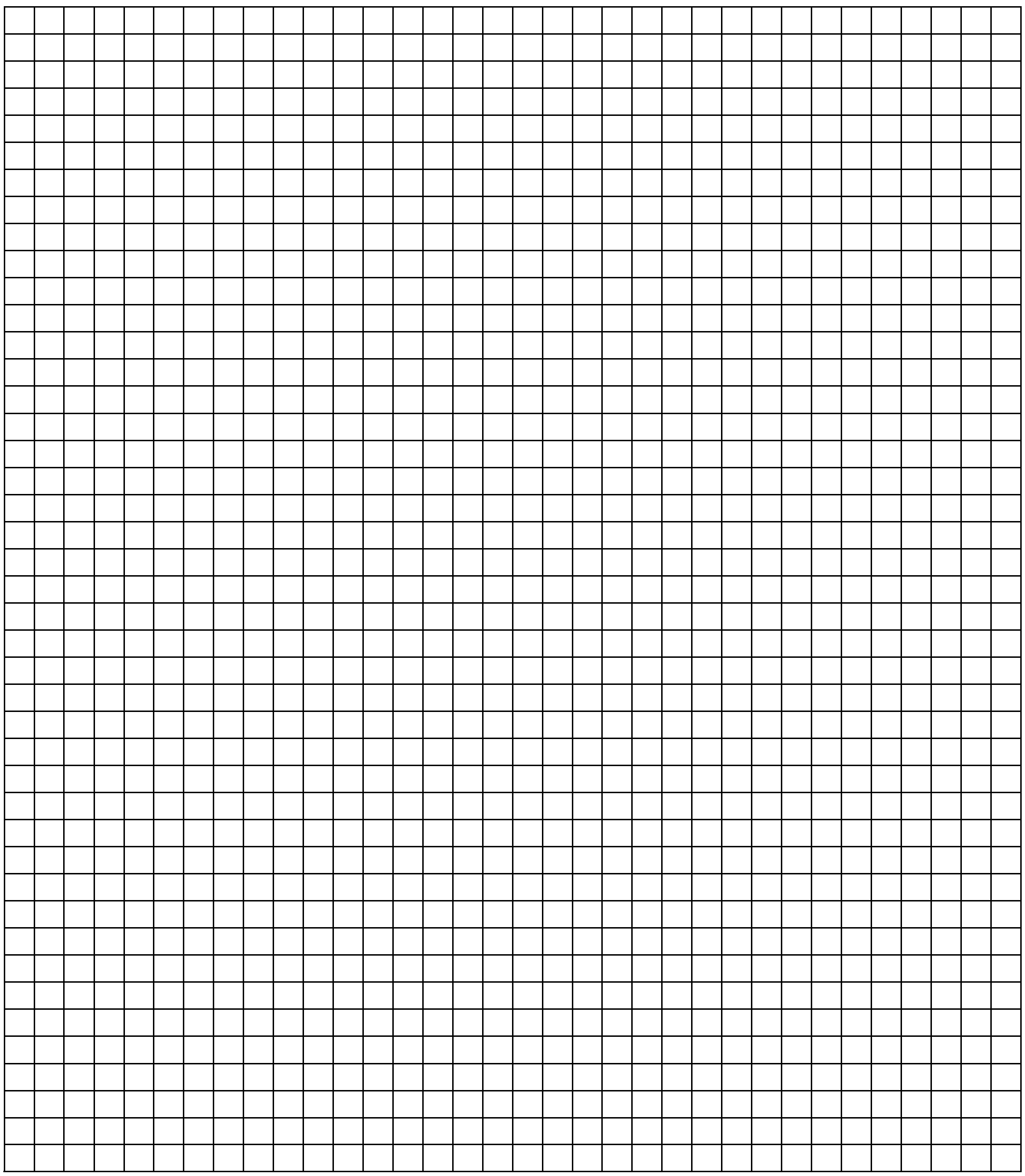
TEACHERS AND STUDENTS LESSON ACTIVITIES

Activities HA, HB and HC Given by Ms. Karen

(A) Sample Area Model
(B) Factors and Prime Factorization Activities
(C) Assessment Practice

(HA)

(HB)



Lesson 14: Factors and Prime Factorization
APPLY the

New Jersey CPIs

List the factors for each number. Find the common factors for each pair of numbers. Then find the greatest common factor.

1. Factors of 12 ____________
   Factors of 16 ____________
   Common factors ___________
   Greatest common factor __________

2. Factors of 33 _________
   Factors of 55 _________
   Common factors ________
   Greatest common factor ________

3. Factors of 36 ____________

   Factors of 16 ____________
   Common factors ___________
   Greatest common factor ___________

4. Factors of 48 _________

   Factors of 64 _________
   Common factors ________
   Greatest common factor _________

Write the prime factorization for each set of numbers. Then find the product of the common factors to find the GCF.

5. 42 = _____________

   18 = _____________
   Common prime factors: ____________
   GCF = _________________

6. 60 = _______________

   70 = _______________
   Common prime factors:
   GCF = _____________

7. 100 = _________________

   75 = _________________

   50 = _________________
   Common prime factors: ____________
   GCF = ______________

8. 24 = ____________

   40 = ______________

   Common prime factors:
   GCF = ____________

Solve each problem.

9. A florist has 21 white roses, 33 yellow roses, and 15 red roses to use in making floral arrangements. He wants to use all the flowers and place an equal number of each color rose in each arrangement.

   What is the greatest number of floral arrangements he can make?

   How many white roses will be in each arrangement?

   How many yellow roses will be in each arrangement?

   Explain how you found your answer.

   ____

____
(Source: Measuring up to the New Jersey Core Curriculum Standards)

(HC)

ASSESSMENT PRACTICE DIRECTIONS FOR QUESTIONS 1 THROUGH 6: Read each problem. Circle the letter of the answer you choose.

1. For which set of numbers below is the greatest common factor 12?

   A. 30, 48, 56
   B. 36, 48, 84
   C. 24, 60, 112
   D. 12, 102, 212

2. What is the prime factorization of 80?

   A. 2 x 2 x 2 x 2 x 5
   B. 2 x 2 x 3 x 5
   C. 2 x 3 x 5
   D. 4 x 120

3. Juan has an 80-foot wire and a 12-foot wire. For a class project, he needs to divide each wire equally to make several guy wires of the same length. What is the longest length that can be cut for each guy wire?

   A. 14ft
   B. 16ft
   C. 18ft

4. The numbers 2, 3, 5, 9, and 12 are factors of which number?
   A. 36
   B. 45
   C. 180
   D. 320

5. Bertran has 66 red marbles, 30 blue marbles, and 42 yellow marbles. What is the greatest number of bags into which Bertram can dividehis marbles so that each bag has an equal number of each color?

   A. 6
   B. 9
   C. 11
   D. 12

6. Which statement below is NOT true?
   A. All numbers that have 6 as a factor also have 2 and 3 as 12ft as factors.

   B. A pair of numbers that have 5 as a common factor also have 10 as a common factor.

Given by Ms. Ballas

(D) Decimals to Hundredths

DECIMALS

Name ____________________
Class____________________
Date ____________________

Skill 1
Sheet 1

Read each decimal numeral, then write it in words.

tens 10	units 1	tenths $\frac{1}{10}$	hun-dredths $\frac{1}{100}$	thou-sandths $\frac{1}{1000}$	ten-thou-sandths $\frac{10}{10,000}$	Answers
	4	4				1. Four and four tenths
						2.
						3.
						4.
						5.
						6.
						7.
						8.
						9.
						10.
						11.
						12.
						13.
						14.
						15.
						16.
						17.
						18.
						19.

D (1-1)
INDIVIDUALIZED COMPUTATIONAL SKILLS PROGRAM 1-60341

(HE)

Assessments

How fast can you write?

Wait for my signal to begin. You will have 1 minute to copy the numbers shown in the corner of each box. Write as quickly as you can. Ready, set, go!

3	72	8	32	9	15	1

7 boxes

94	7	80	2	28	0	63

14 boxes

4	56	6	36	5	45	8

21 boxes

27	3	81	9	55	1	64

28 boxes

2	49	6	18	4	21	7

35 boxes

24	8	48	5	75	3	35

42 boxes

Count how many boxes you completed. _____________

Given by Mr. Tompkin

(HF)

NAME ______________________________ DATE__________ PERIOD____________

Study Guide and Intervention

Order of Operations

Evaluate Rational Expressions Numerical expressions often contain more than one operation. To evaluate them, use the rules for order of operations shown below.

Order of Operations	Step 1 Evaluate expressions inside grouping symbols. Step 2 Evaluate all powers. Step 3 Do all multiplication and/or division from left to right Step 4 Do all addition and/or subtraction from left to right.

Example 1 Evaluate each expression.

a. 7 + 2 * 4 – 4

7 + 2 * 4 - 4 = 7 + 8 – 4 Multiply 2 and 4 by 3

= 15 – 4 Add 7 and 8.

= 11 Subtract 4 from 15.

b. 3(2) + 4(2 + 6)

3(2) + 4(2 + 6) = 3(2) + 4(8) Add 2 and 6.

= 6 + 32 Multiply left to right.

= 38 Add 6 and 32.

Example 2 Evaluate each expression

a. $3[2 + (12 / 3)^2]$

$3[+ (12 / 3)^2]$ Divide 12

$= 3(2 = 4^2)$

$= 3(2 + 16)$ Find 4 squared

$= 3(18)$ Add 2 and 16

$= 54$ Multiply 3 and 18

b. $\frac{3 + 2^3}{4^2 \cdot 3}$

$\frac{3 + 2^3}{4^2 \cdot 3} = \frac{3 + 8}{4^2 \cdot 3}$ Evaluate power in Numerator

$= \frac{11}{4^2 \cdot 3}$ Add 3 and 8 in the numerator

$= \frac{11}{16 \cdot 3}$ Evaluate power in the denominator

$= \frac{11}{48}$ Multiply

Exercises

1. $(8 - 4) \cdot 2$
2. $(12 + 4) \cdot 6$
3. $10 + 2 \cdot 3$
4. $10 + 8 \cdot 1$
5. $15 - 12 / 4$
6. $\frac{15 + 60}{30 - 5}$
7. $12(20 - 17) - 3 \cdot 6$
8. $24 / 3 \cdot 2 - 3^2$
9. $8^2 / (2 \cdot 8) / 2$
10. $3^2 / 3 + 2^2 \cdot 7 - 20 / 5$
11. $\frac{4 + 3^2}{12 + 1}$
12. $\frac{8(2) - 4}{8 / 4}$
13. $250 / [5(3 \cdot 7 + 4)]$
14. $\frac{2 \cdot 4^2 - 8 / 2}{(5 + 2) \cdot 2}$
15. $\frac{4 \cdot 3^2 - 3 \cdot 2}{3 \cdot 5}$

(HG)

*Alg I *Def'n* 1 - 2

Order of Operations

P

E

M

D

A

S

* Note: Special typeface is used to substitute for Mr. Tompkin's hand written work.

APPENDIX I

DESCRIPTION OF TEACHER SURVEY
(IRB Approved)

This survey contains a series of statements that ask for your beliefs about reform oriented mathematics instruction. The purpose of this survey is to gather information and find out your opinion about the best way to teach mathematics. Since any opinion is only a point of view, no opinion is right or wrong. Your responses will be held in the strictest confidence and will not affect your relationship with the school in any way.

Beliefs about Teaching Mathematics

Please respond to each statement by placing a check in the box that best expresses you opinion.

To what extent do you believe the following principles should underlie mathematics teaching?

Statement	Agree Strongly	Agree Moderately	Agree Slightly	Disagree Slightly	Disagree Moderately	Disagree Strongly
1. Explore fewer topics in greater depth rather than covering more topics quickly or superficially.						
2. Select topics that help students connect mathematics to their own experience and the larger community rather than understanding mathematics as isolated skills and procedures.						
3. Explore complex problems rather than only simple problems that emphasize specific skills.						
4. Place greater emphasis on reasoning and problem solving rather than on operations and computation.						
5. Focus lessons on the reasoning process rather than only on obtaining the right						

answers.						
6. Use the language of mathematics to express mathematical ideas.						
7. Attend to the literacy needs of the students in their mathematics classroom.						
8. Use open-ended questions.						
9. Emphasize the process through which students arrive at solutions.						
10.Guide students to generalize from a specific instance to a larger concept or relationship						

Mathematics Teaching Practice

Please respond to each statement by placing a check in the box that best expresses you opinion.

When teaching Mathematics, teachers should:

Statement	Agree Strongly	Agree Moderately	Agree Slightly	Disagree Slightly	Disagree Moderately	Disagree Strongly
11. Help students monitor and evaluate their own problem solving and evolve more sophisticated mathematical thinking rather than leaving thinking procedures unexamined.						
12. Help students communicate their mathematical thinking clearly and coherently to Others.						
13.Help students see connections between mathematics and other disciplines.						
14. Help students translate						

mathematical word problems.						
15. Help students ascribe the appropriate mathematical meaning to English words.						
16. Manage the classroom, keeping all students engaged and on task.						

Thank you for participating in the Survey

Please provide any additional comments you would like to make about reform oriented constructivist approach to mathematics teaching, or anything else that would help us understand the best way to teach mathematics.

Please indicate your willingness to have your class observed

_____ Yes, you may observe my class

Please contact me at: [ ]- [ ] – [ ] X [ ], to make
(include telephone number here)
the necessary arrangements to observe my class.

_____ No, you may not observe my class

Optional: If you would like a copy of the results, please complete the following:

Name: ________________________

Address: ________________________

____

Contact:

Dr. Joseph DuCette Phone: 215-204-4998
The College of Education
Temple University
Psychological Studies in Education
1301 W Cecil B. Moore Avenue
Philadelphia, PA 19122-6091

Demographic Information

DIRECTIONS: Your response to this survey will be held confidential. Please answer the following questions as honestly as possible by placing a check mark next to your answer.

1. Age:
   ____ a. Less than 25
   ____ b. 26 – 30
   ____ c. 31 - 35
   ____ d. 36 – 40
   ____ e. 41 – 45
   ____ f. 46 – 50
   ____ g. More than 50

2. Gender: _____ Male ____ Female

3. Please indicate the grade level at which you typically teach:

   _____ 5^{th}
   _____ 6^{th}
   _____ 7^{th}
   _____ 8^{th}
   _____ 9^{th}
   _____ 10^{th}
   _____ 11^{th}
   _____ 12^{th}
   _____ Other: Please indicate:_______________

4. Ethnicity

   ___ a. African American (Not of Hispanic Origin)
   ___ b. American Indian or Alaskan Native
   ___ c. Hispanic/Latino
   ___ d. Oriental/Asian
   ___ e. White (Not of Hispanic Origin)
   ___ f. Other (Please Describe)________________

5. What is the highest degree you hold?

   ___ a. Bachelor of Arts or Bachelor of Science
   ___ b. Master of Arts or Master of Science
   ___ c. Multiple Master's degrees
   ___ d. Ph.D. or Ed.D.
   ___ e. Other (Please Describe) __________________

6. Did you major in mathematics or mathematics intensive field (e.g. engineering, statistics, physics)?

   ___ a. Yes
   ___ b. No

7. Did you minor in mathematics or mathematics intensive field (e.g. engineering, statistics, physics)?

___ a. Yes
___ b. No

8. What type of teaching certificate do you hold?

___ a. Not certified
___ b. Temporary, provisional, or emergency certification (requires additional coursework before regular certification can be obtained)

___ c. Probationary certification (the initial certification issued after satisfying all requirements except the completion of a probationary period
___ d. Regular or standard certification

9. Do you hold a specific certificate or endorsement for teaching mathematics?

___ a. Yes
___ b. No

10. Including this year, how many years have you taught on a full-time basis?

Years ☐☐

11. Including this year, how many years have you taught mathematics?

Years ☐☐

12. With respect to the mathematics you are asked to teach how confident are you in your mathematical knowledge?

___ a. Not confident at all
___ b. Somewhat confident
___ c. Moderately confident
___ d. Very confident

13. What percentage of students in your class is classified as Limited English Proficiency (LEP) or English Language Learners (ELL)?

☐☐ percent

14. How would you describe the variation in mathematics ability of students in the class you teach?

___ a. Fairly homogeneous and low in ability
___ b. Fairly homogeneous and average in ability
___ c. Fairly homogeneous and high in ability
___ d. Heterogeneous with a mixture of two or more ability levels

Professional Development

Teachers participate in many workshops, seminars, courses, and other organized professional development activities. These programs can address many areas of mathematics, including pedagogy, content, and curriculum, but most programs have a particular focus. In the past 12 months how much time did you spend on **professional development activities** that focused on the following aspects of teaching mathematics? For activities or sessions that covered more than one topic, estimate the time for each topic covered.

Check the Box for the Answer That Best Fits Your Opinion for Each Statement

Statements	None	Less than 4 hours	4-8 hours	9-16 hours	More than 16 hours
1. In-depth study of mathematics					
2. Methods of teaching mathematics					
3. Use of particular mathematics curricula or curriculum materials					
4. Mathematics standards or framework- e.g., (NCTM), State and/or district					

Thank you for completing the demographic information. Please proceed to the next part of the questionnaire.

APPENDIX J

DESCRIPTION OF THE REFORMED TEACHING OBSERVATION PROTOCOL (RTOP)
(IRB Approved)

Principal Investigator: Dr. Joseph DuCette

Student Investigator: Violet Barrett Paterson

Department: Educational Psychology, Psychological Studies in Education
Phone Number: 215-204-4998

Project Title: Teachers' Beliefs and Practice in Relation to Reform Oriented Mathematics Teaching

________________ ____________________
External Observer (1) Participant Observer

____
External Observer (2)

1. Background Information

Name of teacher _______________________Date Agreed on To Be

Observed: ___________________________

Location of Class _____________________________________
(District, School, Room)

Years of Teaching __________ Teaching Certification ______
(K-8 or 7-12)

Subject Observed _________________ Grade Level __________

Observer _______________ Date of Observation: ____________

Start time ________________ End Time ___________________

II. Contextual Background and Activities

In the space provided below please give a brief description of the lesson observed, the classroom setting in which the lesson took place (space, seating arrangements, etc.), and any relevant details about the students (number, gender, ethnicity) and teacher that you think are important. Use diagrams if they seem appropriate.

III. Lesson Design and Implementation

Please rate the following statements concerning the teachers' lesson design and implementation, using the scale provided, where "0" represents **Never Occurred** and "4" represents **Very Descriptive.**

Statements	(0) Never Occurred	(1) Seldom Occurred	(2) Occurred Half Of the Time	(3) Usually	(4) Very Descriptive
1. The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.					
2. The lesson was designed to engage students as members of a learning community					
3. In this lesson, student exploration preceded formal presentation					
4. This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.					
5. The focus and direction of the lesson was often determined by ideas originating with students					
6. The lesson involved fundamental concepts of the subject					
7. The lesson promoted strongly coherent conceptual understanding					
8. The teacher had a solid grasp of the subject matter content inherent in the lesson					
9. Elements of abstraction (i.e., symbolic representation, theory building) were encouraged when it was important to do so					

Statements (continued)	(0) Never Occurred	(1) Seldom Occurred	(2) Occurred Half Of the Time	(3) Usually	(4) Very Descriptive
10. Connection with other content disciplines and/or real world phenomena were explored and valued					
11. Students used a variety of means (models, drawings, graphs, concrete materials, manipulative, etc.) to represent phenomena.					
12. Students made predictions, estimations and/or hypotheses and devised means for testing them					
13. Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures					
14. Students were reflective about their learning.					
15. Intellectual rigor, constructive criticism, and the challenging of ideas were valued					
16. Students were involved in the communication of their ideas to others using a variety of means and media					

Statements (Continued)	(0) Never Occurred	(1) Seldom Occurred	(2) Occurred Half Of the Time	(3) Usually	(4) Very Descriptive
17. The teacher's questions triggered divergent modes of thinking.					
18. There was a high proportion of student talk and a significant amount of it occurred between and among students					
19. Student questions and comments often determined the focus and direction of classroom discourse					
20. There was a climate of respect for what others had to say					
21. Active participation of students was encouraged and valued					
22. Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence					
23. In general the teacher was patient with students					
24. The teacher acted as a resource person, working to support and enhance student investigations					

Statements (Continued)	(0) Never Occurred	(1) Seldom Occurred	(2) Occurred Half Of the Time	(3) Usually	(4) Very Descriptive
25. The metaphor "teacher as listener" was very characteristic of this classroom					

Additional comments you may wish to make about this lesson.

Thank you for your assistance with the classroom observation

APPENDIX K

POST OBSERVATION INTERVIEW QUESTIONS
(IRB Approved)

1. How confident are you in your ability to teach mathematics?

2. What specific things can you do to help students think mathematically?

3. How frequently do you engage in activities that promote students mathematical thinking?

4. Describe your views of reform oriented teaching?

5. How do your views on reform oriented mathematics teaching influence your design of classroom instruction?

6. How do you feel about your professional development?

7. What recommendations do you have for improving school mathematics instructions?

APPENDIX L

PERMISSION TO AUDIOTAPE
(IRB Approved)

Principal Investigator: Dr. Joseph DuCette

Student Investigator: Violet Barrett Paterson

Department: Educational Psychology, Psychological Studies in Education
Phone Number: 215-204-4998

Project Title: Teachers' Beliefs and Practices in Relation to Reform Oriented Mathematics Teaching

Participant: __________________ Date: ___________

Log #:___________

I ________________________give Violet Barrett Paterson

permission to audiotape me

Signature: __________________________________

This audiotape will be used only for the following purpose (s):

_______ RESEARCH

This audiotape will be used as a part of a research project at _____________________. I have already given written consent for my participation in this research project. At no time will my name be used.

WHEN WILL I BE AUDIOTAPED?

I agree to be audiotaped during the time period: ______ to __________.

____
Signature

WHAT IF I CHANGE MY MIND?
I understand that I can withdraw my permission at any time. Upon my request, the audiotape (s) will no longer be used. This will not affect my relationship with _______________ in any way.

OTHER

I understand that I will not be paid for being audiotape or for the use of the audiotapes.

FOR FURTHER INFORMATION

If I want more information about the audiotape (s), or if I have questions or concerns at any time, I can contact:

Project Title: Teachers' Beliefs and Practices in Relation to Reform oriented Mathematics Teaching

FOR FURTHER INFORMATION

If I want more information about the audiotape (s), or if I have questions or concerns at any time, I can contact:

Principal Investigator: Dr. Joseph DuCette

Student Investigator: Violet Barrett Paterson

Department: Educational Psychology, Psychological Studies in Education

Institution: Temple University

Street Address 1301 West Cecil B. Moore Avenue

City: ________________ State: _________________

Zip Code: ___________________

Phone: Office _______________ Home: _______________

This form will be placed in my records and the person (s) named above will keep a copy. A copy will be given to me.

APPENDIX M

DESCRIPTION OF REFORMED TEACHING OBSERVATION PROTOCOL (RTOP) SUBSCALE ITEMS
(IRB Approved)

1-5: Lesson design and Implementation

- The instructional strategies and activities respected students'prior knowledge and the preconceptions inherent in them.
- The lesson was designed to engage students as members of a learning community.
- In this lesson, students' exploration preceded formal presentation.
- This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.
- The focus and direction of the lesson was often determined by ideas originating with students.

Content

1-5**:** (Propositional Knowledge)

- The lesson involved fundamental concepts of the subject.
- The lesson promoted strongly coherent conceptual understanding.
- The teacher had a solid grasp of the subject matter content inherent in the lesson.
- Elements of abstraction (i.e., symbolic representation, theory building) were encouraged when it was important to do so.
- Connection with other content disciplines and/or real world phenomena were explored and valued.

1-5 (Procedural Knowledge)

- Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives) to represent phenomena.
- Students made predictions, estimations and/or hypotheses and devised means for testing them.
- Students were actively engaged in thought provoking activity that often involved the critical assessment of procedures.
- Students were reflective about their learning.
- Intellectual rigor, constructive criticism, and the challenging of ideas were valued.

Classroom Culture

1-5**:** (Communicative Interaction)

- Students were involved in the communication of their ideas to others using a variety of means and media.
- The teachers questions triggered divergent modes of thinking.
- There was a high proportion of student talk and a significant amount of it occurred between and among students.

- Students questions and comments often determine the focus and direction of classroom discourse.
- There was a climate of respect for what others had to say.

1-5: Student/Teacher Relationships

- Active participation of students was encouraged and valued.
- Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.
- In general, the teacher was patient with the students
- The teacher acted as a resource person, working to support and enhance students investigations.
- The metaphor"teacher as listener" was very characteristic of this classroom.