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Lorentz invariance relations among parton distributions revisited

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Abstract

We revisit the derivation of the so-called Lorentz invariance relations between parton distributions. In the most important cases these relations involve twist-3 and transverse momentum dependent parton distributions. It is shown that these relations are violated if the path-ordered exponential is taken into account in the quark correlator.

1. Parton distributions which are of higher twist and (or) dependent on transverse parton momenta ($k_\perp$-dependent) contain important information on the structure of the nucleon which is complementary to that encoded in the usual twist-2 distributions. Certain spin asymmetries in inclusive and semi-inclusive deep inelastic scattering (DIS) as well as in the Drell–Yan process are governed by twist-3 distributions [1–3]. The $k_\perp$-dependent correlation functions typically give rise to azimuthal asymmetries. Very recently significant efforts have been devoted to measure such asymmetries in semi-inclusive DIS [4,5].

In Refs. [6–9] several relations between twist-3 and (moments of) $k_\perp$-dependent parton distributions have been proposed. The derivation of these relations (called LI-relations in the following) is based upon the general, Lorentz invariant decomposition of the correlator of two quark fields, where the fields are located at arbitrary space–time positions. The LI-relations impose important constraints on the distribution functions, which allow one to eliminate unknown structure functions in favor of the known ones, whenever applicable. Two specific LI-relations have been doubted in Ref. [10] by an explicit calculation of the involved parton distributions in light front Hamiltonian QCD using a dressed quark target. Although the arguments given in Ref. [10] are not complete, that work motivated us to revisit the derivation of the LI-relations. (Compare also the discussion in Ref. [11].)

It is the purpose of the present Letter to study the validity of the LI-relations in a model-independent way. We find that they are violated if the proper path-ordered exponential is taken into account in the quark correlation function. The reason for this result lies in the fact that the gauge link requires a decomposition of the correlator which contains more terms than the ones given in Refs. [6,7]. Our result provides an explanation of the outcome of the model-calculation presented in Ref. [10].

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2. To begin with, we specify the correlation function through which the $k_\perp$-dependent parton distributions are defined,

$$\Phi_{ij}(x,\vec{k}_\perp,S)=\int\frac{d\xi^- d^2\vec{\xi}_\perp}{(2\pi)^3}e^{ik^+\xi^- - i\vec{k}_\perp\cdot\vec{\xi}_\perp}\langle P,S|\bar{\psi}_j(0)\mathcal{W}_1(0,\xi)\psi_i(\xi),|P,S\rangle|_{\xi^+=0}. \tag{1}$$

The target state is characterized by its four-momentum $P=P^+p+(M^2/2P^+)n$ and the covariant spin vector S ($P^2=M^2$, $S^2=-1$, $P\cdot S=0$), where the two light-like vectors p and n satisfying $p^2=n^2=0$ and $p\cdot n=1$ have been used. The variable x defines the plus-momentum of the quark via $k^+=xP^+$. A contour leading to a proper definition of the $k_\perp$-dependent parton distributions was given in Refs. [12–15]:[1]

$$\mathcal{W}_1(0,\xi)=\mathcal{W}(0,\xi|n)|_{\xi^+=0}, \tag{2}$$

with

$$\mathcal{W}(0,\xi|n)=[0,0,\vec{0};0,\infty,\vec{0}]\times[0,\infty,\vec{0};\xi^+,\infty,\vec{\xi}_\perp]\times[\xi^+,\infty,\vec{\xi}_\perp;\xi^+,\xi^-,\vec{\xi}_\perp]. \tag{3}$$

In this equation, $[a^+,a^-,\vec{a}_\perp;b^+,b^-,\vec{b}_\perp]$ denotes the Wilson line connecting the points $a^\mu=(a^+,a^-,\vec{a}_\perp)$ and $b^\mu=(b^+,b^-,\vec{b}_\perp)$ along a straight line. It is important to note that the Wilson contour in Eq. (3) not only depends on the coordinates of the initial and the final points but also on the light-cone direction n.[2]

The $k_\perp$-dependent parton distributions are defined by the correlator in Eq. (1) using suitable projections. For instance, the unpolarized quark distribution is given by $f_1(x,\vec{k}_\perp^2)=\mathrm{Tr}(\Phi\gamma^+)/2$.

Before dealing with the derivation of the LI-relations, we list the most important examples [8], which will be shown to be not correct:

$$g_T(x)=g_1(x)+\frac{d}{dx}g_{1T}^{(1)}(x), \tag{4}$$

$$h_L(x)=h_1(x)-\frac{d}{dx}h_{1L}^{\perp(1)}(x), \tag{5}$$

$$f_T(x)=-\frac{d}{dx}f_{1T}^{\perp(1)}(x), \tag{6}$$

$$h(x)=-\frac{d}{dx}h_1^{\perp(1)}(x), \tag{7}$$

with

$$g_{1T}^{(1)}(x)=\int d^2\vec{k}_\perp\frac{\vec{k}_\perp^2}{2M^2}g_{1T}\big(x,\vec{k}_\perp^2\big),\quad \text{etc.}, \tag{8}$$

specifying the $k_\perp$-moments [7]. All distributions on the l.h.s. of Eqs. (4)–(7) are of twist-3, while the functions on the r.h.s. appear unsuppressed in the observables. For instance, g_T is the well known structure function measurable via inclusive DIS on a transversely polarized target. The functions g_1 and h_1, respectively, represent the quark helicity and transversity distribution. The distributions in Eqs. (4), (5) are time-reversal even (T-even), while the ones in (6), (7) are T-odd. Only recently has it been explicitly shown that the $k_\perp$-dependent T-odd distributions are non-vanishing in general [13,16].

[1] We note that the choice of the contour depends on the process considered. Here we restrict ourselves to the case of semi-inclusive DIS, although all our arguments are valid for other processes as well.

[2] In fact Wilson lines that are near the light-cone rather than those exactly light-like are more appropriate in connection with $k_\perp$-dependent parton distributions [12,13]. However, our general reasoning remains valid if we use a near light-cone direction instead of n.

3. The discussion of the LI-relations starts with the most general correlator which, upon integration over k^-, reduces to the correlator in Eq. (1):

$$\Phi_{ij}(P,k,S|n) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi} \langle P,S|\bar{\psi}_j(0)\mathcal{W}(0,\xi|n)\psi_i(\xi)|P,S\rangle. \tag{9}$$

We emphasize that the correlator (9) (like the one in (1)) not only depends on the four-vectors P, k and S but also, through the gauge link, on the light cone direction n, which we have indicated now explicitly. As we shall show in the following, it is precisely the presence of this additional vector that spoils the LI-relations.

To write down the most general expression of the correlator in (9), we impose the following constraints due to hermiticity and parity,[3]

$$\Phi^\dagger(P,k,S|n) = \gamma_0\Phi(P,k,S|n)\gamma_0, \tag{10}$$

$$\Phi(P,k,S|n) = \gamma_0\Phi(\bar{P},-\bar{S},\bar{k}|\bar{n})\gamma_0, \tag{11}$$

where $\bar{P}^\mu = (P^0, -\vec{P})$, etc. With these constraints the most general form of the correlator reads

$$\begin{aligned}\Phi(P,k,S|n) = {}& MA_1 + \not{P}A_2 + \not{k}A_3 + \frac{i}{2M}[\not{P},\not{k}]A_4 + \cdots \\ & + \frac{M^2}{P\cdot n}\not{n}B_1 + \frac{iM}{2P\cdot n}[\not{P},\not{n}]B_2 + \frac{iM}{2P\cdot n}[\not{k},\not{n}]B_3 + \cdots,\end{aligned} \tag{12}$$

where we have not listed those terms which only appear in the case of target-polarization. The structures in the second line in Eq. (12) containing the vector n are absent in the decomposition given in Refs. [6,7]. Note that, in order to specify the Wilson line in (9), a rescaled vector λn with some parameter λ could be used instead of n. By construction, the terms in (12) are not affected by such a rescaling.

Next one makes use of the fact that integrating the correlator (9) upon k^- necessarily leads to the correlator given in (1), i.e.,

$$\Phi(x,\vec{k}_\perp,S) = \int dk^-\,\Phi(P,k,S|n). \tag{13}$$

This identity has been used to derive the LI-relations. As an explicit example, we consider the relation (7) which does not require target-polarization. In this case, Eq. (13) allows one to express the involved distributions according to

$$h_1^\perp(x,\vec{k}_\perp^2) = 2P^+\int dk^-(-A_4), \tag{14}$$

$$h(x,\vec{k}_\perp^2) = 2P^+\int dk^-\left(\frac{k\cdot P - xM^2}{M^2}A_4 + (B_2 + xB_3)\right). \tag{15}$$

If the structures in the second line in Eq. (12) and, hence, the amplitudes B_i were absent then both $h_1^\perp$ and h would be given as an integral over the same amplitude A_4, which is the origin of the LI-relation (see also in particular Eq. (2.30) in Ref. [6] and Ref. [9]). However, as we have discussed, the amplitudes B_2 and B_3 need to be taken into account as a direct consequence of gauge invariance.[4] Accordingly, the relation (7) is violated. One can easily extend our analysis to show that the relations (4)–(6) involving target-polarization are violated also.

[3] This implies a proper choice of the operator ordering in the correlator (9). The specific choice of this ordering is inessential for our discussion.

[4] Note that the amplitudes B_i do not show up if one connects the quark fields in the correlators in Eqs. (1), (9) by a single straight Wilson line. However, one cannot define $k_\perp$-dependent parton distributions through such correlators.

In summary we have shown that the so-called Lorentz invariance relations between parton distributions are violated due to the path-ordered exponential in the quark correlator. We note that this result applies to the corresponding relations among fragmentation functions as well.

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