

Intrinsic transverse parton momenta in deeply inelastic reactions

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Intrinsic transverse parton momenta p_T play an important role in the understanding of azimuthal/spin asymmetries in semi-inclusive deep-inelastic scattering (SIDIS) and the Drell-Yan process (DY). We review and update what is presently known about p_T from these processes. In particular, we address the question to which extent data support the popular Gauss model for the p_T -distributions. We find that the Gauss model works very well, and observe that the intrinsic transverse momenta in SIDIS and DY are compatible, which is a support for the factorization approach. As a byproduct we recover a simple but practical way of taking into account the energy dependence of p_T -distributions.

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I. INTRODUCTION

Intrinsic transverse parton momenta in hadrons can be probed in deeply inelastic reactions, such as SIDIS and DY, when adequate transverse momenta in the final state are measured. Here “transverse” means with respect to the hard momentum flow in the process, e.g. in SIDIS transverse momenta of produced hadrons with respect to the virtual photon. Transverse parton momenta are described in terms of transverse momentum dependent parton distribution functions (TMDs) or analogously generalized fragmentation functions [1–7].

Although the concept of transverse parton momenta dates back to early days of QCD [1, 2, 8–12] the field has received continuous interest from theory [13–22] and important steps in the understanding of TMDs within QCD were taken only recently [23–27]. In particular, factorization theorems have been formulated, which extend previous work [2] and ensure that the productions of hadrons in SIDIS or dileptons in DY with small transverse momenta compared to the hard scale factorize in hard scattering parts and universal non-perturbative objects: TMDs, fragmentation functions, and soft factors [26, 27].

The appealing perspective of the TMD approach is that it may explain single spin or azimuthal asymmetries in various reactions [28–30], especially in SIDIS [31–49], DY [50–57], or hadron production in e^*e^- -annihilations [58–60]. In phenomenological studies of such data [61–84] one often works in lowest order QCD (“tree level”) and neglects higher order QCD corrections, which includes soft factors as well as higher order corrections in the hard scattering. In some sense, the information on soft factors can be thought of as “reshuffled” into an effective description of involved TMDs or fragmentation functions.

As in such studies typically many novel TMDs are involved [85–88], it is popular to assume the so-called Gauss model. For example, in the case of the unpolarized TMD or fragmentation function one assumes

$$f_1^a(x, p_T) = f_1^a(x) \frac{\exp(-p_T^2/\langle p_T^2 \rangle)}{\pi \langle p_T^2 \rangle}, \quad (1)$$

$$D_1^a(z, K_T) = D_1^a(z) \frac{\exp(-K_T^2/\langle K_T^2 \rangle)}{\pi \langle K_T^2 \rangle}, \quad (2)$$

where $p_T = |\vec{p}_T|$, and the normalization is $\int d^2p_T f_1^a(x, p_T) = f_1^a(x)$ and similarly for D_1^a . Strictly speaking the Gauss widths $\langle p_T^2 \rangle$, $\langle K_T^2 \rangle$ could be flavor dependent, and x - or z -dependent, i.e., the Ansätze (1, 2) in general do not imply a factorized x - or z - and transverse momentum dependence.

Although in this way the unknown p_T -dependence is reduced to “one number,” a Gauss width which “characterizes the p_T -dependence” of a TMD, and although the Ansatz has not the correct large- p_T asymptotics [91], it nevertheless proves to be useful in many processes sensitive to intrinsic transverse momenta [61–63]. In fact, it was shown in [61] that the Gauss model provides a useful approximation in many processes sensitive to intrinsic transverse momenta.

The information on the Gauss model parameters in (1, 2) used in many recent works was due to two independent studies of SIDIS data [62, 63]. In [62] the EMC data [32] on the azimuthal asymmetry $A_{UU}^{\cos\phi}$ in unpolarized SIDIS were used to fix the parameters in (1, 2). In [63] the widths were determined using (uncorrected for acceptance effects) mean values from HERMES on average transverse momenta $\langle P_{h\perp} \rangle$ of produced hadrons. The results were found

$$\langle p_T^2 \rangle = \begin{cases} 0.25 \text{ GeV}^2 & \text{in [62],} \\ 0.33 \text{ GeV}^2 & \text{in [63],} \end{cases} \quad \langle K_T^2 \rangle = \begin{cases} 0.20 \text{ GeV}^2 & \text{in [62],} \\ 0.16 \text{ GeV}^2 & \text{in [63],} \end{cases} \quad (3)$$

which are consistent within the unestimated uncertainties involved in the analyses. Both studies were probably the best one could do at that time, but are not free of criticism. In [62] it was *assumed* the observable $A_{UU}^{\cos\phi}$ is due to the Cahn effect only, omitting effects of other TMDs. In [63] it was *assumed* the acceptance corrections were not large. In both works the very applicability of the Ansatz (1, 2) was *presumed*.

Meanwhile new data emerged, which improve the situation and allow to test the Gauss Ansatz in SIDIS more thoroughly than it was possible in [61–63]. One of the aims of this study is to provide such tests. In Sect. II after convincing ourselves that the Gauss model passes the tests imposed by the new data, we will be in the position to update the information (3) on the Gauss model parameters. We shall also discuss the EMC data on azimuthal asymmetries in the light of the updated information, and comment on the Cahn- and Boer-Mulders effect.

Another purpose of this work is to test the Gauss Ansatz in the Drell-Yan process, and to determine the Gauss width of f_1^a . Also here we shall discuss azimuthal asymmetries in unpolarized DY, and comment on the Cahn- and Boer-Mulders effect, see Sec. III.

Finally, having established the applicability of the Gauss model in SIDIS and DY, we will address the question whether the descriptions of p_T -dependences in the two processes are compatible. This is what one expects on the basis of the factorization approach and universality arguments, but a meaningful comparison of the effects requires to carefully take into consideration the different energies typically probed in SIDIS and DY, see Sec. IV. Our conclusions are contained in Sec. V.

II. INTRINSIC TRANSVERSE MOMENTA IN SIDIS

In this Section we study the distributions of transverse hadron momenta $P_{h\perp}$ in SIDIS. Two effects play a role, namely intrinsic transverse parton momenta from the target which we model by (1), and transverse momenta the hadrons acquire in the fragmentation process which we model by (2). After a brief introduction to SIDIS, we shall study in detail data from CLAS and HERMES, and comment then on the Cahn effect at EMC and forthcoming experiments.

Let P , l (l') and P_h denote (respectively) the momenta of the proton, incoming (outgoing) lepton and produced hadron. The relevant kinematic variables are $q = l - l'$ with $Q^2 = -q^2$, $W^2 = (P + q)^2$, $x = Q^2/(2P \cdot q)$, $y = (P \cdot q)/(P \cdot l)$, and $z = (P \cdot P_h)/(P \cdot q)$.

In the following we are interested in the transverse momentum $P_{h\perp}$ of the produced hadron, which is defined with respect to the momentum of the virtual photon, see Fig. 1. Assuming the Gauss model (1, 2) the cross section differential in x , y , z , $P_{h\perp}^2$ (but averaged over the azimuthal angle ϕ of the produced hadron) reads

$$\frac{d^4\sigma_{UU}(x, y, z, P_{h\perp})}{dx dy dz dP_{h\perp}^2} = \frac{4\pi^2\alpha^2 s}{Q^4} \left(1 - y + \frac{1}{2}y^2\right) \sum_a e_a^2 x f_1^a(x) D_1^a(z) \mathcal{G}(P_{h\perp}) \quad (4)$$

where the subscript “ UU ” indicates that both leptons and nucleons are unpolarized. The function $\mathcal{G}(P_{h\perp})$ is given by

$$\mathcal{G}(P_{h\perp}) = \frac{1}{\pi\kappa_T^2(z)} \exp\left(-\frac{P_{h\perp}^2}{\kappa_T^2(z)}\right), \quad \kappa_T^2(z) = z^2\langle p_T^2 \rangle + \langle K_T^2 \rangle, \quad (5)$$

with the normalization $\int d^2P_{h\perp} \mathcal{G}(P_{h\perp}) = 1$, while the cross section (4) is normalized such that

$$\frac{d^3\sigma_{UU}(x, y, z)}{dx dy dz} = \frac{4\pi\alpha^2 s}{Q^4} \left(1 - y + \frac{1}{2}y^2\right) \sum_a e_a^2 x f_1^a(x) D_1^a(z) \quad (6)$$

In the cross sections (4) and (6) we neglect power suppressed terms of the order $\mathcal{O}(M^2/Q^2)$ where M is the nucleon mass. The neglected terms include purely kinematic factors, as well as a structure function ($F_{UU,L}$ in the notation of [88]) which has no partonic description. Depending on the kinematics these contributions of $\mathcal{O}(M^2/Q^2)$ need not to be small. We will recall this when necessary, but we will not need their explicit forms here.

For later convenience we introduce also the notion of average transverse hadron momenta and their squares which are defined, and in the Gauss model given, as

$$\langle P_{h\perp}(z) \rangle = \left\langle \frac{P_{h\perp} d\sigma_{UU}(z, P_{h\perp})/dz dP_{h\perp}^2}{d\sigma_{UU}(z, P_{h\perp})/dz dP_{h\perp}^2} \right\rangle \stackrel{\text{Gauss}}{=} \frac{\sqrt{\pi}}{2} \sqrt{\kappa_T^2(z)}, \quad (7)$$

$$\langle P_{h\perp}^2(z) \rangle = \left\langle \frac{P_{h\perp}^2 d\sigma_{UU}(z, P_{h\perp})/dz dP_{h\perp}^2}{d\sigma_{UU}(z, P_{h\perp})/dz dP_{h\perp}^2} \right\rangle \stackrel{\text{Gauss}}{=} \kappa_T^2(z), \quad (8)$$

where $\langle \dots \rangle$ denotes average over $P_{h\perp}$. Analogously one could define mean transverse momenta as functions of other kinematic variables, for example $\langle P_{h\perp}(x) \rangle$. Strictly speaking, in obtaining the Gauss model results in (7, 8) we assume the Gauss widths to be flavor- and x -independent. As we shall see below, these are reasonable approximations.

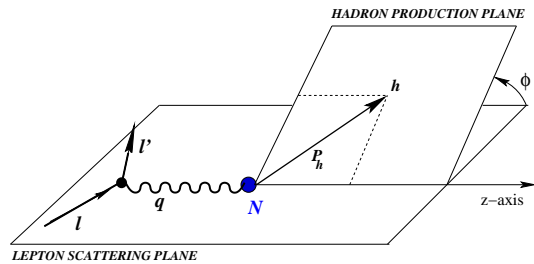


FIG. 1: Kinematics of the SIDIS process $lp \rightarrow l'hX$.

A. Lessons from CLAS and Hall-C

In the CLAS experiment [33] the semi-inclusive π^+ electro-production off a proton target was studied with a 5.75 GeV beam. Among others the following quantity was measured:

$$R(P_{h\perp}) \equiv \frac{d^4\sigma_{UU}(x, y, z, P_{h\perp})/dx dy dz dP_{h\perp}^2}{d^4\sigma_{UU}(x, y, z, 0)/dx dy dz dP_{h\perp}^2} = \exp\left(-\frac{P_{h\perp}^2}{\kappa_T^2(z)}\right), \quad (9)$$

at $x = 0.24$ and $z = 0.30$ for three different values of Q^2 . In the last step of (9) we assumed flavor-independent Gauss widths. For all three values of Q^2 the data are remarkably well described by the Gauss model [33]. In Fig. 2a we show the data on the ratio (9) for the highest $Q^2 = 2.37 \text{ GeV}^2$, which is very well described with the parameter

$$\kappa_T^2(z) \Big|_{z=0.30} = 0.17 \text{ GeV}^2. \quad (10)$$

The agreement of the data with the Gauss Ansatz is astonishing. At lower $Q^2 = 1.74 \text{ GeV}^2$ and 2 GeV^2 the situation is equally impressive, with somewhat lower values for $\kappa_T^2(z)$ [33].

However, several reservations need to be made. First, in the CLAS kinematics [33] the contributions of $\mathcal{O}(M^2/Q^2)$ we mentioned in the context of Eqs. (4) and (6) are not negligible, i.e., the description of CLAS data in Eqs. (9, 10) effectively parametrizes also these contributions. Second, at $z = 0.30$ the measured hadrons are not only due to the fragmentation of the struck quark (“current fragmentation”) but can also originate from the hadronization of the target remnant (“target fragmentation”). The latter is described in terms of so-called fracture functions, which — while being a fascinating topic by themselves — are an undesired contamination from the point of view of DIS.

Nevertheless, although one has to keep in mind these reservations, presently the 5.75 GeV beam CLAS data [33] provide the best support for the applicability of the Gauss model in SIDIS. It would be desirable to solidify this observation with data taken at higher energies at CLAS12, HERMES and COMPASS.

Another conclusion one can draw from the CLAS data [33], modulo the above-mentioned reservations, is that the Gauss width $\langle p_T^2 \rangle$ of $f_1^a(x, p_T)$ is only moderately x -dependent, see Fig. 2b which shows the average transverse momentum square $\langle P_{h\perp}^2 \rangle$ of π^+ produced at $z = 0.34$ and $Q^2 = 2.37 \text{ GeV}^2$ at CLAS [33] as function of x . In fact, for $0.2 < x < 0.5$ we find $\langle P_{h\perp}^2 \rangle = 0.15 \text{ GeV}^2$ within 20%, which is demonstrated by the shaded region in Fig. 2b.

In the Gauss model, the z -dependence of the average hadron transverse momentum square $\langle P_{h\perp}^2(z) \rangle$ in Eq. (8) allows in principle to fix the Gauss widths $\langle K_T^2 \rangle$ and $\langle p_T^2 \rangle$ of D_1^a and f_1^a . Such data were presented in [33], but here we will not use them for a quantitative determination of the Gauss model parameters. In fact, at the moderate beam energies for $z \lesssim 0.4$ [41] the produced hadrons receive also contributions from target fragmentation, see above, and at still lower z “threshold effects” play a role [33]. Therefore we refrain here from using these data quantitatively, and will come back to them later for a qualitative comparison in Sec. II C.

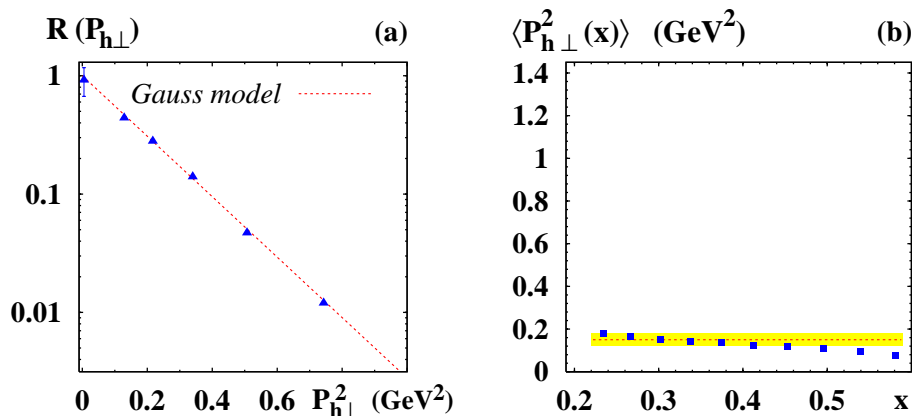


FIG. 2: (a) The ratio $R(P_{h\perp})$ as defined in Eq. (9) as function of the hadron transverse momentum square $P_{h\perp}^2$. The data are for π^+ from CLAS [33]. The dotted line is an effective description in the Gauss model, see text. (b) The average transverse momentum square $\langle P_{h\perp}^2 \rangle$ of π^+ produced at $z = 0.34$ and $Q^2 = 2.37 \text{ GeV}^2$ in SIDIS at CLAS [33] as function of x . The dotted line is an effective description in the Gauss model assuming the Gauss width of $f_1^a(x, p_T)$ to be x -independent. This describes data within 20% in the region $0.2 < x < 0.5$ as the shaded region shows.

Next we turn our attention to the Jefferson Lab data from the Hall-C collaboration [34] where 5.5 GeV electrons were scattered off proton and deuterium targets in the kinematics $0.2 < x < 0.5$, $2 \text{ GeV}^2 < Q^2 < 4 \text{ GeV}^2$, $0.3 < z < 1$, and π^\pm with transverse momenta up to $P_{h\perp}^2 < 0.2 \text{ GeV}^2$ were measured. In spite of the narrow $P_{h\perp}^2$ -range covered, the results on the differential cross-section (Ω and E denote solid angle and energy of produced the hadron h with $h = \pi^\pm$)

$$\frac{d^5\sigma(P_{h\perp}^2)_{et\rightarrow e'hX}}{d\Omega dE dz dP_{h\perp}^2} \stackrel{\text{Gauss}}{=} \frac{d^5\sigma(0)_{et\rightarrow e'hX}}{d\Omega dE dz dP_{h\perp}^2} \exp\left(-\frac{P_{h\perp}^2}{\kappa_T^2(z)}\right) \quad (11)$$

allow a valuable cross-check at $\langle x \rangle = 0.32$ and $\langle z \rangle = 0.55$ ($t = p, d$ denotes the proton, deuteron target).

As the kinematics is similar to CLAS, we expect the Hall-C data, which refer to $\langle z \rangle = 0.55$, to be described in the Gauss model by the parameter $\kappa_T^2(z) = 0.24 \text{ GeV}^2$. This value is taken from CLAS data on $\langle P_{h\perp}^2(z) \rangle$ [33] (see below, Fig. 5b). In this way we obtain a very good description of the $P_{h\perp}^2$ -dependence of the Hall-C data, see Fig. 3.

At this point it is interesting to stress that we assumed $\kappa_T^2(z) = 0.24 \text{ GeV}^2$ to be the same for any hadron h from any target t . This means, we assumed flavor-independent Gauss widths $\langle p_T^2 \rangle$ and $\langle K_T^2 \rangle$. Thus, although a dedicated analysis in [34] indicated a preference for slightly flavor-dependent Gauss widths, we conclude from Fig. 3 that the assumption of flavor-independent Gauss widths $\langle p_T^2 \rangle$ and $\langle K_T^2 \rangle$ is reasonable — in the valence- x region for $\langle z \rangle = 0.55$.

To summarize, the data from Jefferson Lab [33, 34] are very well described assuming a Gauss distribution of intrinsic transverse parton momenta in the unpolarized distribution and fragmentation functions, and suggest a weak flavor- and x -dependence of the Gauss width $\langle p_T^2 \rangle$ of $f_1^a(x, p_T)$, and a weak flavor-dependence of the Gauss widths $\langle K_T^2 \rangle$ of $D_1^a(z, K_T)$. However, one has to bear in mind that these data contain contributions from fracture functions and/or terms of $\mathcal{O}(M^2/Q^2)$, such that we shall limit ourselves here to these qualitative conclusions, and use for a quantitative analysis data from HERMES [35], see Secs. II B and II C.

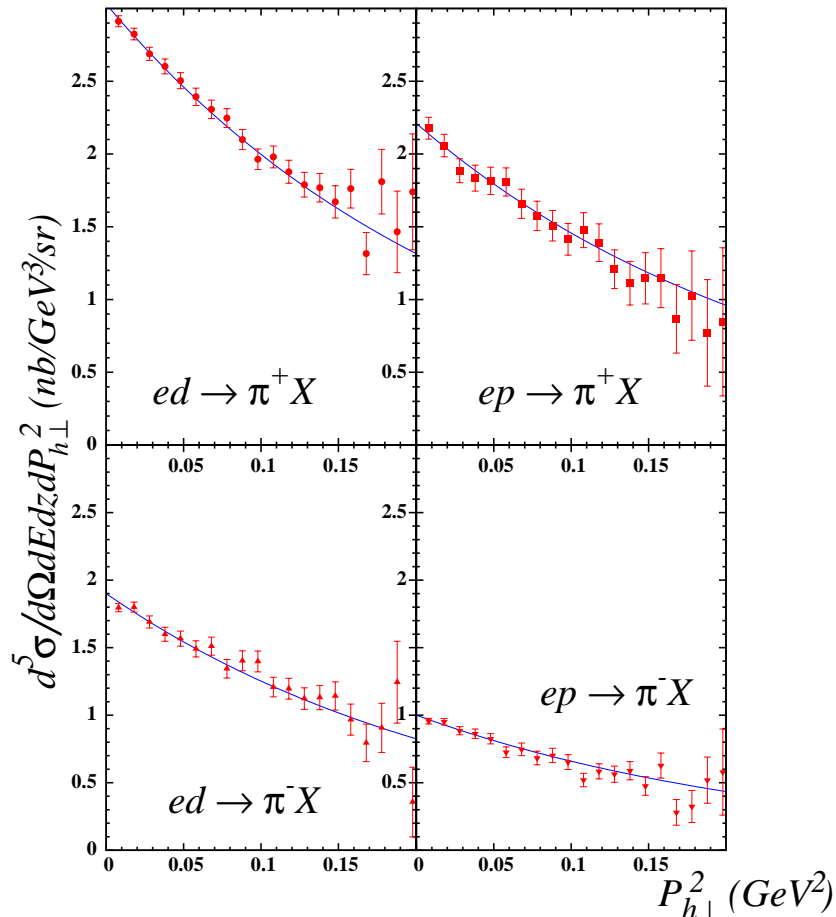


FIG. 3: The differential cross-section $d^5\sigma/d\Omega dE dz dP_{h\perp}^2$ for pion production off proton and deuterium targets at $\langle x \rangle = 0.32$ and $\langle z \rangle = 0.55$ as function of $P_{h\perp}^2$ from Hall-C [34]. The theoretical curves are from the Gauss model with the Gauss width fixed from CLAS [33]. The overall normalization of the cross sections is fixed by hand.

B. Testing the Gauss model at HERMES

Assuming the Gauss model (1, 2) and flavor- and x -independent Gauss widths, we obtained for the mean transverse hadron momenta and their squares the results quoted in (7, 8). In particular, these two quantities are related in the Gauss model according to

$$\langle P_{h\perp}(z) \rangle^2 = \frac{\pi}{4} \langle P_{h\perp}^2(z) \rangle. \quad (12)$$

Meanwhile data from HERMES on SIDIS of electrons or positrons with $E_{\text{beam}} = 27.6 \text{ GeV}$ off a deuteron target allow to test the Gauss model prediction (12) in the kinematics $Q^2 > 1 \text{ GeV}^2$, $W^2 > 10 \text{ GeV}^2$, $y < 0.85$, $z > 0.2$ and $0.023 < x < 0.4$ with $\langle Q^2 \rangle = 2.4 \text{ GeV}^2$, $\langle x \rangle = 0.09$ [35, 39].

In Fig. 4 we show $\langle P_{h\perp}(z) \rangle$ as function of z from [39] (triangles). These mean values refer to pions and kaons and were not corrected for acceptance effects. We compare them with $\frac{1}{2}\sqrt{\pi}\langle P_{h\perp}^2(z) \rangle^{1/2}$ using the data from [35] on $\langle P_{h\perp}^2(z) \rangle$ for positive pions. At HERMES the SIDIS events are subject to the cuts $0.2 < z < 0.7$, as indicated in Fig. 4, and in this range π^\pm and K^+ have very similar $\langle P_{h\perp}(z) \rangle$ [35]. We shall come back to this observation in the next section. Let us add that the actual aim of [35] was to study nuclear $P_{h\perp}$ -broadening effects, the so-called Cronin-effect, and various nuclear targets were used besides deuteron, namely He, Ne, Kr, Xe.

We conclude from the exercise in Fig. 4 that the SIDIS data from HERMES support very well the Gauss model relation (12), even though one should keep in mind possible effects due to acceptance corrections in the case of the $\langle P_{h\perp}(z) \rangle$ results from [39]. Of course, data on the cross section differential in $P_{h\perp}$ from HERMES, of the type as presented by CLAS [33], are required to provide fully conclusive support for the Gauss model. However, we still may view the picture in Fig. 4 as an encouraging indication in favor of the applicability of the Gauss model in SIDIS at HERMES.

C. Determining parameters from HERMES, and cross check with CLAS

The HERMES data [35] allow two further insights. The first is that the $\langle P_{h\perp}^2(z) \rangle$ for π^+ , π^- , K^+ are very similar in the SIDIS region, i.e., below the cut $z = 0.7$ which excludes exclusive effects at HERMES, see Fig. 5a. Thus, there is no evidence of a strong flavor-dependence of the Gauss widths $\langle p_T^2 \rangle$ and $\langle K_T^2 \rangle$ at HERMES. The second insight is that the data in Fig. 5a allow to fix the Gauss widths $\langle p_T^2 \rangle$ and $\langle K_T^2 \rangle$. A fit in the region $0.2 < z < 0.7$ yields

$$\begin{aligned} \langle p_T^2 \rangle &= (0.38 \pm 0.06) \text{ GeV}^2, \\ \langle K_T^2 \rangle &= (0.16 \pm 0.01) \text{ GeV}^2, \end{aligned} \quad (13)$$

with a χ^2 per degree of freedom of 0.44. The best fit and its $1-\sigma$ region are shown in Fig. 5a as (respectively) dotted line and shaded region.

The values of the new fit in Eq. (13) are in good agreement with the results from [62, 63] quoted in Eq. (3), especially if one recalls that those numbers have unestimated systematic uncertainties. This is the main point, in which our new result (13) constitutes an improvement over the old numbers (3): it has a well-estimated uncertainty.

In any case, the good news is that the numbers for the Gauss widths from the works [62, 63] are confirmed numerically. This means that, in this respect, phenomenological results for fixed target experiments obtained on the basis of those numbers remain valid.

Finally, let us turn back to the CLAS data [33] on the z -dependence of $\langle P_{h\perp}^2(z) \rangle$ which refer to $\langle Q^2 \rangle = 2.37 \text{ GeV}^2$ which is similar to HERMES, but $\langle x \rangle = 0.27$ which is substantially higher than HERMES. In Sec. II A we refrained from using these data to determine quantitatively Gauss model parameters for reasons discussed there in detail. At this point it is instructive to compare the fit result obtained from HERMES to the CLAS data, see Fig. 5b. Clearly, in the region above $z > 0.4$ where the produced pions are predominantly from current fragmentation, we observe a good agreement. This indicates that it is the same non-perturbative mechanism which generates intrinsic transverse momenta in the two experiments.

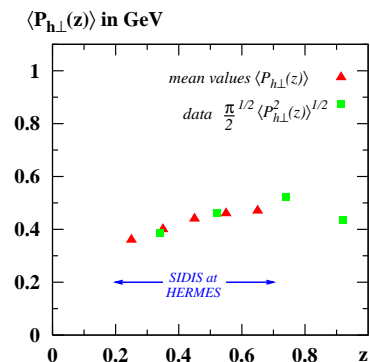


FIG. 4: Transverse momenta of hadrons in SIDIS off deuterium at HERMES vs. z . We compare $\langle P_{h\perp}(z) \rangle$ (triangles) from [39], with $\frac{1}{2}\sqrt{\pi}\langle P_{h\perp}^2(z) \rangle^{1/2}$ (squares) from [35]. In the indicated SIDIS range of HERMES these quantities are predicted to coincide in the Gauss model, see Eq. (12).

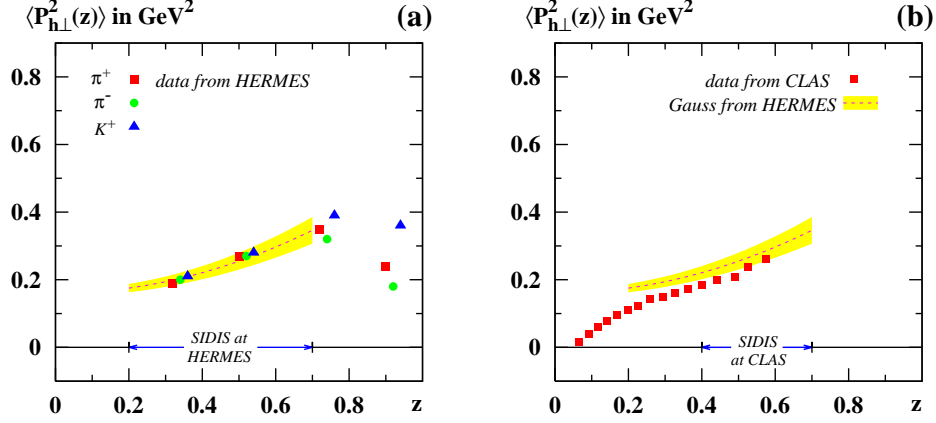


FIG. 5: (a) $\langle P_{h\perp}^2(z) \rangle$ of hadrons in SIDIS off deuterium at HERMES vs. z from [35]. The dotted line and the shaded region are the best fit and its $1-\sigma$ region from Eq. (13), see text. (b) $\langle P_{h\perp}^2(z) \rangle$ of π^+ in SIDIS off proton at CLAS vs. z [33]. Dotted line (shaded region) are the best fit (its $1-\sigma$ region) to HERMES data from Fig. 5a, see text.

D. Cahn effect at EMC

In SIDIS of unpolarized leptons off unpolarized nucleons the cross section differential in the azimuthal angle of the produced hadrons, see Fig. 1, is — for purely electromagnetic interactions — given by [89]

$$\frac{d^5\sigma(x, y, z, P_{h\perp}, \phi)}{dx dy dz dP_{h\perp}^2 d\phi} = \frac{d^4\sigma_{UU}(x, y, z, P_{h\perp})}{(2\pi) dx dy dz dP_{h\perp}^2 d\phi} \left[1 + \frac{(2-y)\sqrt{1-y}}{1+(1-y)^2} \cos(\phi) A_{UU}^{\cos\phi} + \frac{1-y}{1+(1-y)^2} \cos(2\phi) A_{UU}^{\cos 2\phi} \right], \quad (14)$$

where $d^4\sigma_{UU}(x, y, z, P_{h\perp})$ denotes the differential cross section $d^5\sigma_{UU}(x, y, z, P_{h\perp}, \phi)$ after averaging over ϕ , as defined in Eq. (4). $A_{UU}^{\cos\phi}$ and $A_{UU}^{\cos 2\phi}$ in (14) depend on $x, z, P_{h\perp}$ and are ratios of adequately defined structure functions. The meaning of the subscript “UU” is as in (4), the superscript recalls the type of ϕ -modulation.

At low $P_{h\perp}$, the observable $A_{UU}^{\cos\phi}$ is suppressed by $1/Q$, and in this sense a “twist-3” effect. At present it is not clear whether there is factorization in SIDIS at subleading twist [90, 91]. This $\cos\phi$ -modulation is sometimes referred to as the Cahn effect. In Ref. [9] it was shown that the existence of intrinsic transverse parton momenta in the unpolarized distribution and fragmentation functions can generate such a modulation. Later it became clear that, if one assumes factorization and restricts oneself to a lowest order QCD (“tree-level”) description, there are four different contributions to this asymmetry from several TMDs and K_T -dependent fragmentation functions [17–19, 88]. More precisely, in the notation of [88] the asymmetry is given by

$$A_{UU}^{\cos\phi} = \frac{F_{UU}^{\cos\phi}}{F_{UU,T}}, \quad F_{UU,T} = \mathcal{C} \left[f_1 D_1 \right],$$

$$F_{UU}^{\cos\phi} = \frac{2M}{Q} \mathcal{C} \left[\frac{\vec{K}_T \cdot \vec{P}_{h\perp}}{m_h P_{h\perp}} \left(x h H_1^\perp + \frac{m_h}{M} f_1 \frac{\tilde{D}^\perp}{z} \right) - \frac{\vec{p}_T \cdot \vec{P}_{h\perp}}{M P_{h\perp}} \left(x f^\perp D_1 + \frac{m_h}{M} h_1^\perp \frac{\tilde{H}}{z} \right) \right], \quad (15)$$

where m_h denotes the hadron mass, and the meaning of the convolution symbol is, for generic functions f and D ,

$$\mathcal{C} \left[\dots f D \right] = \int d^2 p_T \int d^2 K_T \delta^{(2)}(z\vec{p}_T + \vec{K}_T - \vec{P}_{h\perp}) \dots \sum_a e_a^2 x f^a(x, p_T) D^a(z, K_T). \quad (16)$$

The original “Cahn-effect-only” description of $A_{UU}^{\cos\phi}$ [9] can be “rederived” from the TMD formalism [18, 19, 88] under two conditions. The first is that quark-gluon-quark correlators, which can be separated off by exploring QCD equations of motion, can be neglected with respect to quark-quark correlators. The quality of such approximations was discussed in [92–95]. One of these approximations is $x f^{\perp q} \approx f_1^q$, and is supported by results from the bag model [96]. The second condition is that in the experiment the type of hadron h is not detected, i.e., the observed hadron with momentum specified by $z, \phi, P_{h\perp}$ is a sum over pions, kaons, etc., and in such sums the Collins effect tends to cancel. Strictly speaking, an exact cancellation occurs only if one weighs the Collins function H_1^\perp adequately, integrates over $0 \leq z \leq 1$, and sums over *all* hadrons [97]. However, string fragmentation models [98] and phenomenology [99–102] indicate that cancellations occur in practice already if one sums over charged hadrons only, which are mostly π^\pm in experiments such as EMC, where the asymmetry $A_{UU}^{\cos\phi}$ in charged hadron production was measured [31, 32].

In fact, the EMC data on $A_{UU}^{\cos\phi}$ [31, 32] were used in [62] to determine in the “Cahn-effect-only” approximation the Gauss model parameters quoted in Eq. (3). It is therefore instructive to check, whether the revised numbers from (13) still give a good description of these data.

In the “Cahn-effect-only” approximation, i.e., neglecting in (15) the “pure twist-3” tilde-functions and the Collins effect, we obtain for the asymmetry with the Gauss model (1, 2) the result

$$A_{UU,\text{Cahn}}^{\cos\phi}(z) = -\frac{z\sqrt{\pi}\langle p_T^2 \rangle}{\langle Q \rangle \sqrt{z^2 \langle p_T^2 \rangle + \langle K_T^2 \rangle}}, \quad (17)$$

where we assumed flavor-independent Gauss widths. In the EMC [31] experiment 280 GeV muons were scattered off protons and the covered kinematics was: $y < 0.8$, $z > 0.15$, $Q^2 > 10 \text{ GeV}^2$ with about $\langle Q \rangle = 4.8 \text{ GeV}$, and $160 < W^2/\text{GeV}^2 < 360$. Also the cut $P_{h\perp} > 200 \text{ MeV}$ was imposed which is ignored in (17) for simplicity. Numerically it has a negligible effect. In this kinematics we obtain with the Gauss model parameters inferred from HERMES, Eq. (13), the result shown in Fig. 6. We observe a very good agreement.

At this point, this not only demonstrates the compatibility of the EMC and HERMES data. Since we have fixed the details of the Gauss model in an independent experiment, the excellent agreement we observe in Fig. 6 can also be read in opposite direction.

The approximations we used in order to relate $A_{UU}^{\cos\phi}$ to the Cahn effect, namely the cancellation of the Collins effect in charged hadron production and the neglect of tilde-terms, are justified — within the experimental error bars and the theoretical uncertainty of our study. In this sense, the EMC data on $A_{UU}^{\cos\phi}$ support the “Wandzura-Wilczek-type” approximations discussed critically in [92–95]. (However, we shall come back to this point at the end of Sec. IV.)

To draw an intermediate summary, in the Sections II A–II D we have seen that data from EMC [31, 32], Jefferson Lab [33, 34] and HERMES [35] support the Gauss model with flavor- and x - or z -independent Gauss widths. In the future one may need to refine the description by allowing for flavor- and x - or z -dependent Gauss widths, when more precise data will make it necessary to introduce and possible to constrain further parameters.

The Gauss model has an important principle limitation though. It may work only if the transverse momenta of the produced hadrons are of the order of magnitude of the hadronic scale, i.e., much smaller than the hard scale in the process. This condition is fulfilled in the case of EMC, Jefferson Lab, and HERMES data [31–35] where typically $\langle P_{h\perp} \rangle \simeq 0.4 \text{ GeV} \ll \langle Q \rangle = 2\text{--}5 \text{ GeV}$ in these experiments. When transverse hadron momenta become substantially larger than the hadronic scale or even become so large that they set the hard scale in the process, one does not deal with non-perturbative intrinsic p_T anymore, but can apply perturbative QCD [8]. SIDIS data from high energy experiments, for example E665 at Fermilab [36] or ZEUS at DESY [37, 38], are sensitive to such perturbative p_T -effects. In practice, in order to describe these data it is necessary to include both, perturbative and non-perturbative effects. We refer to the pioneering study [10], see also the recent work [103].

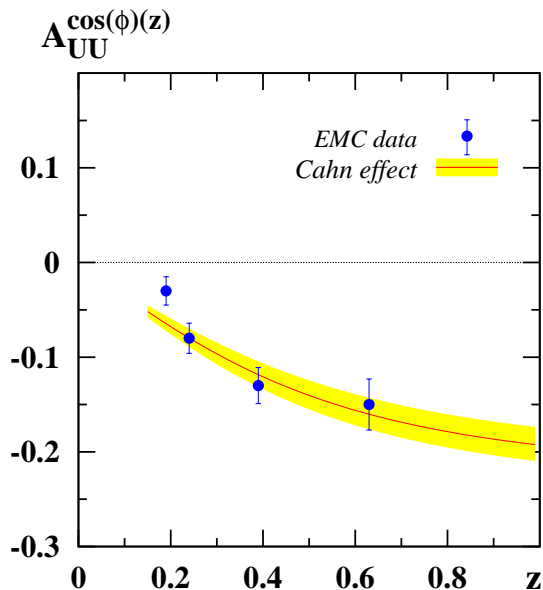


FIG. 6: Azimuthal asymmetry $A_{UU}^{\cos\phi}$ in charged hadron production vs. z . The data are from the EMC experiment [31]. The theoretical curve is the “Cahn-effect-only” approximation for this observable, which is justified under certain assumptions (see text), using the Gauss model with parameters fixed from HERMES, Eq. (13).

E. Cahn and Boer-Mulders effect, and new data

It would be interesting to repeat the analysis presented in the previous section with recent results on $A_{UU}^{\cos\phi}$ from Jefferson Lab [33, 34] (final), HERMES and COMPASS [48, 49] (preliminary data). Since $\langle Q \rangle$ in these experiments is smaller compared to EMC, one would expect this subleading-twist observable to be there larger than at EMC.

A careful comparison of the Cahn effect prediction for $A_{UU}^{\cos\phi}$ to these new data will be instructive for two reasons. First, at Jefferson Lab, COMPASS, HERMES the hadrons are identified, i.e., the Collins effect in (15) does not cancel out — in contrast to EMC, where we could argue it may (at least approximately) cancel out. Second, the new or forthcoming data are far more precise compared to EMC, such that deviations of data from the “Cahn-effect-only” approximation will have a chance to pop up more easily, and provide insights on quark-gluon correlations. In view of the preliminary status of the HERMES and COMPASS data [48, 49], however, we shall not pursue this study here.

Let us also comment on the Boer-Mulders effect responsible for the $\cos(2\phi)$ -modulation in (14) and given by

$$A_{UU}^{\cos 2\phi} = \frac{F_{UU}^{\cos 2\phi}}{F_{UU,T}} + \mathcal{O}\left(\frac{M^2}{Q^2}\right), \quad F_{UU}^{\cos 2\phi} = \mathcal{C} \left[\frac{2(\vec{K}_T \cdot \vec{P}_{h\perp})(\vec{p}_T \cdot \vec{P}_{h\perp}) - (\vec{K}_T \cdot \vec{p}_T) P_{h\perp}^2}{M m_h P_{h\perp}^2} h_1^\perp H_1^\perp \right], \quad (18)$$

with $F_{UU,T}$ and the convolution integral as defined in (15, 16). The Boer-Mulders function h_1^\perp [19] is one of the so-called “naively time-reversal odd” (T-odd) TMDs [23–25]. The observable was discussed in [64–71].

In Eq. (18) we have indicated the power corrections modulo which the factorization theorem is formulated, e.g. [2, 11, 12, 26]. Only if such corrections are sufficiently small the factorization approach is justified, and can develop its predictive power with universal non-perturbative objects [27], though the concept of universality had to be extended in order to accommodate T-odd TMDs [23–25], which we will discuss in more detail below in Sec. III C.

In general such non-factorizable corrections are theoretically not under control and must be excluded experimentally by studying the Q -behavior of the cross section or asymmetry. In the case of $A_{UU}^{\cos 2\phi}$, however, the parton model provides a way to estimate one of the possible power corrections in (18). Namely, the Cahn effect [9] yields

$$A_{UU,\text{Cahn}}^{\cos 2\phi}(z) = - \frac{\langle p_T^2 \rangle}{\langle Q^2 \rangle} \frac{z^2 \langle p_T^2 \rangle}{z^2 \langle p_T^2 \rangle + \langle K_T^2 \rangle}, \quad (19)$$

if we assume the Gauss model with flavor-independent Gauss widths. We stress that this is only one among many *non-factorizable* power-corrections to the $\cos(2\phi)$ -modulation. Ideally, one should choose the cuts in Q in the experiment such that one does not feel the Cahn- (19) or any other $1/Q^2$ -power correction in (18).

In practice, this is difficult. At HERMES and COMPASS the DIS cut $Q^2 > 1 \text{ GeV}^2$ is imposed which leads to $\langle Q^2 \rangle \simeq 2.5 \text{ GeV}^2$. At Jefferson Lab similar $\langle Q^2 \rangle$ (but lower W) is achieved, in spite of lower beam energies, because there one can afford to choose the cut $Q^2 > 2 \text{ GeV}^2$ [34] thanks to the high luminosity. In such kinematics, at let us say $z = 0.5$, the Cahn-power correction (19) is, with Gauss widths from (13), about

$$A_{UU,\text{Cahn}}^{\cos 2\phi} \simeq -\mathcal{O}(5\%). \quad (20)$$

Let us compare this number to what one may expect from the leading twist Boer-Mulders contribution in (18). If we generously allow the Boer-Mulders function to saturate its positivity bound $\frac{p_T}{M} |h_1^{\perp a}(x, p_T)| \leq f_1^a(x, p_T)$ [21], then

$$|F_{UU}^{\cos 2\phi}(x, z)| \leq \mathcal{C} \left[\omega_{\text{BM}} f_1 \frac{K_T}{m_h} |H_1^\perp| \right] = \alpha_G \sum_a e_a^2 x f_1^a(x) |H_1^{\perp(1/2)^a}(z)|, \quad (21)$$

where $\omega_{\text{BM}} = 2(\hat{K}_T \cdot \hat{P}_{h\perp})(\hat{p}_T \cdot \hat{P}_{h\perp}) - (\hat{K}_T \cdot \hat{p}_T)$, with the “hat” denoting unit vectors and the transverse (1/2)-moment defined as $H_1^{\perp(1/2)^a}(z) = \int d^2 K_T \frac{K_T}{m_h} H_1^{\perp a}(z, K_T)$. The numerical factor $\alpha_G = \mathcal{O}(1)$ contains the dependence on the model for the transverse parton momenta. Phenomenological studies indicate that $|H_1^{\perp(1/2)^a}/D_1^a| = \mathcal{O}(10\%)$ around $z = 0.5$. Thus, the leading-twist Boer-Mulders contribution in (18) can be expected to be at most

$$|A_{UU,\text{BM}}^{\cos 2\phi}| \lesssim \mathcal{O}(10\%). \quad (22)$$

Hence, the power correction estimated on the basis of the Cahn effect in (20) is substantial in this kinematics. Studies of the Boer-Mulders effect in SIDIS [64–70] and first analyses of preliminary data [71] yield similar results. We conclude therefore that, unless it will be possible to suppress such power corrections by applying sufficiently large cuts in Q , or separate experimentally the Boer-Mulders part and power corrections by their different Q -behavior, the task of gaining insights on the Boer-Mulders function from present SIDIS data will be highly demanding — and feasible only on a basis of a thorough quantitative understanding of intrinsic transverse momentum effects in unpolarized distribution and fragmentation functions, which is what this work aims at. Interestingly, as we shall see in Section III, the situation is less demanding for the available DY data.

III. INTRINSIC TRANSVERSE MOMENTA IN DRELL-YAN

In this Section we discuss the DY process [104], i.e., the inclusive production of large-invariant-mass $\mu^+\mu^-$ pairs in hadron-hadron collisions $h_1 h_2 \rightarrow \mu^+ \mu^- X$, first observed in 1970 at the Brookhaven AGS [105], for reviews see [106–110]. Let $p_{1,2}$ and $k_{1,2}$ denote (respectively) the momenta of the incoming hadrons $h_{1,2}$ and the outgoing lepton pair. The kinematics of the process is described by the center of mass energy square s , invariant mass of the lepton pair Q , rapidity y or the Feynman variable x_F , and the variable τ which are defined as

$$s = (p_1 + p_2)^2, \quad Q^2 = (k_1 + k_2)^2, \quad y = \frac{1}{2} \ln \frac{p_2 \cdot (k_1 + k_2)}{p_1 \cdot (k_1 + k_2)} \equiv \frac{1}{2} \ln \frac{x_1}{x_2},$$

$$x_F \equiv x_1 - x_2, \quad \tau \equiv \frac{Q^2}{s} = x_1 x_2. \quad (23)$$

The three-momentum of the virtual photon $\vec{q} = \vec{k}_1 + \vec{k}_2$ can be decomposed in the center of mass frame of the incoming hadrons into a longitudinal and transverse component with respect to the collision axis as $\vec{q} = (q_L, \vec{q}_T)$. In this frame $y = \frac{1}{2} \ln(Q + p_L)/(Q - p_L)$ and $x_F = 2q_L/\sqrt{s}$.

In the parton model x_i denotes the fraction of the hadron momentum p_i carried by (respectively) the annihilating parton or anti-parton, and the cross section differential in x_1 , x_2 and $q_T = |\vec{q}_T|$ is given by (we write $f_1^a(x_i) \equiv f_1^{a/h_i}(x_i)$ for brevity, i.e., the parton distributions in the possibly distinct hadrons h_i are unambiguously labelled by the x_i)

$$\frac{d^3 \sigma_{UU}}{dx_1 dx_2 q_T dq_T} = \frac{4\pi\alpha^2}{9Q^2} \sum_{a=u,d,\bar{u},\dots} e_a^2 \int_0^{2\pi} d\phi \int d^2 p_{1T} \int d^2 p_{2T} \delta^{(2)}(\vec{p}_{1T} + \vec{p}_{2T} - \vec{q}_T) f_1^a(x_1, p_{1T}) f_1^{\bar{a}}(x_2, p_{2T}). \quad (24)$$

For an accurate, quantitative description of the q_T -dependence of the DY cross section it is necessary to go beyond the parton model expression (24) and use the Collins-Soper-Sterman formalism [12].

However, it turns out that a very useful effective description can be obtained by assuming the Gauss model (1). Inserting (1) into (24), and assuming that the Gauss widths ($\langle p_{1T}^2 \rangle$ in hadron 1, and $\langle p_{2T}^2 \rangle$ in hadron 2) are flavor-independent, yields

$$\frac{d^3 \sigma_{UU}}{dx_1 dx_2 dq_T} = \frac{4\pi\alpha^2}{9Q^2} \sum_{a=u,d,\bar{u},\dots} e_a^2 f_1^a(x_1) f_1^{\bar{a}}(x_2) 2q_T \frac{\exp(-q_T^2/\kappa_{DY}^2)}{\kappa_{DY}^2}, \quad \kappa_{DY}^2 \equiv \langle p_{1T}^2 \rangle + \langle p_{2T}^2 \rangle. \quad (25)$$

We define the mean transverse momentum $\langle q_T \rangle$ of the muon pair, and the mean transverse momentum square $\langle q_T^2 \rangle$ as

$$\langle q_T^2 \rangle = \left\langle \frac{q_T^2 d\sigma_{UU}/dq_T}{d\sigma_{UU}/dq_T} \right\rangle \stackrel{\text{Gauss}}{=} \kappa_{DY}^2,$$

$$\langle q_T \rangle = \left\langle \frac{q_T d\sigma_{UU}/dq_T}{d\sigma_{UU}/dq_T} \right\rangle \stackrel{\text{Gauss}}{=} \frac{\sqrt{\pi}}{2} \sqrt{\kappa_{DY}^2}. \quad (26)$$

When integrating (25) over q_T the transverse momentum model dependence drops out and one obtains

$$\frac{d^2 \sigma_{UU}}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9Q^2} \sum_{a=u,d,\bar{u},\dots} e_a^2 f_1^a(x_1) f_1^{\bar{a}}(x_2). \quad (27)$$

Using for $f_1^a(x)$ leading order parametrizations from DIS, the leading order QCD formula (27) underestimates data by a (“K”-factor $\sim (1.1-1.7)$ depending on Q^2 (and moderately also on $x_{1,2}$) [106].

One gets the overall normalization of the DY cross section correct only by including higher order QCD corrections. Throughout we shall work here in leading order. It is a remarkable observation that, whenever corrections to leading order DY cross sections were considered, absolute cross sections were found to be considerably altered (typically enhanced), but ratios of observables in DY were found to be only moderately altered. This is in particular the case for collinear double spin asymmetries, for example the double transverse spin asymmetry $A_{TT} \propto \sum_a e_a^2 h_1^a(x_1) h_1^{\bar{a}}(x_2) / \sum_a e_a^2 f_1^a(x_1) f_1^{\bar{a}}(x_2)$ [111–116].

A. Testing the Gauss model in DY

The Gauss model unambiguously connects the average lepton pair transverse momenta and their squares. In fact, from (26) we obtain

$$\langle q_T^2 \rangle \stackrel{\text{Gauss}}{=} \frac{4}{\pi} \langle q_T \rangle^2, \quad (28)$$

which is a parameter-free prediction of the Gauss model, c.f. Eq. (12) in SIDIS. Data allowing to check (28) were reported in [52], and are shown in Fig. 7. Several interesting conclusions can be drawn from Fig. 7. First, it demonstrates that in the range $0.2 < x_F < 1$ covered in the experiment [52] the relation (28) is satisfied within the accuracy of the data. Second, it demonstrates that for $0.2 < x_F < 0.9$ both $\langle q_T^2 \rangle$ and $\langle q_T \rangle$ are x_F -independent within error bars. Therefore, we conclude that for $x_F < 0.9$ the Gauss model with x -independent Gauss widths provides an excellent description for the q_T -dependence of the DY process, which is useful at least for the description of data with an accuracy comparable to that of [52]. More precise data may reveal deviations from the Gauss Ansatz.

It should be noticed that the parton model is not adequate at large x_F , where higher twist effects are expected to be relevant [117]. Interestingly, the Gauss model relation (28) holds even in the region $x_F > 0.9$ within error bars, see Fig. 7.

In Ref. [118] $\langle q_T^2 \rangle$ was measured for DY lepton pairs induced by various hadrons on a platinum target. Within experimental accuracy it was found that $\langle q_T^2 \rangle_{\pi^- Pt} \approx \langle q_T^2 \rangle_{\pi^+ Pt}$, etc. Since these reactions are dominated by differently flavored valence quarks, one may conclude that it is indeed approximately justified to use flavor-independent Gauss widths. It is also important to notice, that the dependence on the nuclear target [119] is moderate for transverse dilepton momenta $q_T \lesssim 3$ GeV, i.e., nuclear effects are not relevant.

The comparison in Fig. 7 is very instructive, and allowed us to draw many interesting conclusions. Yet, it is not a proof that the Gauss model really works in DY. For that it is necessary to consider data on cross sections, which we will do in the next section.

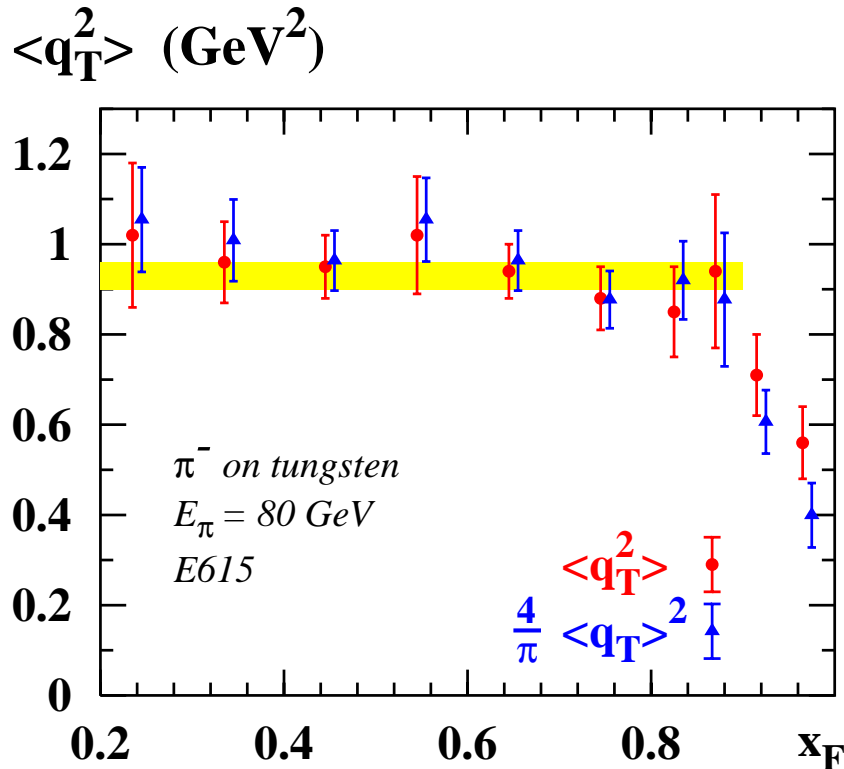


FIG. 7: The mean dimuon transverse momentum square $\langle q_T^2 \rangle$ vs. x_F as measured in the Fermilab E615 experiment [52]. The data points for $\langle q_T^2 \rangle$ are marked by circles, the data points for $\frac{4}{\pi} \langle q_T \rangle^2$ are marked by triangles. Both quantities are predicted to be equal in the Gauss model, see Eq. (28), which is the case within the statistical accuracy of the data.

B. q_T -dependence of the cross section

The q_T -dependence of the cross sections allows to fix the parameters of the Gauss model. It is convenient to rewrite the cross section (25) as (here σ_{UU} denotes the total q_T -integrated unpolarized DY cross section)

$$\frac{d\sigma_{UU}(q_T)}{dq_T} = \sigma_{UU} \frac{2 q_T \exp(-q_T^2/\kappa_{DY}^2)}{\kappa_{DY}^2}, \quad \text{or} \quad Q \frac{d\sigma_{UU}(q_T)}{d^3q} = \frac{2}{\sqrt{s}} \frac{d\sigma_{UU}}{dx_F} \frac{\exp(-q_T^2/\kappa_{DY}^2)}{\kappa_{DY}^2}. \quad (29)$$

Fig. 8 (left) shows FNAL-288 data on the q_T -dependence of the invariant differential cross section $Q \frac{d^3\sigma}{d^3q}$ for $\mu^+\mu^-$ production from scattering a 300 GeV proton beam off platinum at $\langle y \rangle = 0.21$ for two different bins in Q [50]. Clearly, the Gauss model provides a good description of the data for

$$\kappa_{DY}^2 \stackrel{\text{here}}{=} 2 \langle p_{NT}^2 \rangle = 1.4 \text{ GeV}^2, \quad (30)$$

where $\langle p_{NT}^2 \rangle$ denotes the mean parton momentum square in the nucleon.

The limitations of the Gauss Ansatz become more evident by using a logarithmic scale for the cross section. We do this in Fig. 8b that shows FNAL-E615 data [55] for the differential cross section $\frac{d\sigma}{dq_T}$ for $\mu^+\mu^-$ production from scattering a 256 GeV π^- beam on a tungsten target for $0 < x_F < 1$ and $4.05 \text{ GeV} < Q < 8.55 \text{ GeV}$. The Gauss model is applicable for $q_T \lesssim 3 \text{ GeV}$ with the parameter

$$\kappa_{DY}^2 \stackrel{\text{here}}{=} \langle p_{\pi T}^2 \rangle + \langle p_{NT}^2 \rangle = 1.7 \text{ GeV}^2, \quad (31)$$

where $\langle p_{\pi T}^2 \rangle$ denotes the mean parton momentum square in the pion.

The result (31) corresponds precisely to the experimental average $\langle q_T^2 \rangle = (1.71 \pm 0.02) \text{ GeV}^2$ [55]. This is so because the differential cross section in the region $0 \leq q_T \lesssim 3 \text{ GeV}$ (where the Gauss Ansatz is applicable) is about two orders of magnitude larger than in the region $q_T \gtrsim 3 \text{ GeV}$ where it is not. Therefore the mean transverse momentum square is dominated by that region of q_T where the Gauss model works. This means that in practice, if the Ansatz works, the respective Gauss widths can be read off directly from the experimental results for $\langle q_T^2 \rangle$.

Since the kinematics is comparable in the FNAL-288 and FNAL-E615 experiments, we can use (30, 31) to deduce

$$\langle p_{NT}^2 \rangle = 0.7 \text{ GeV}^2, \quad \langle p_{\pi T}^2 \rangle = 1.0 \text{ GeV}^2 \quad \text{at} \quad \sqrt{s} \sim 23 \text{ GeV}. \quad (32)$$

Thus, transverse parton momenta appear to be larger in the pion than in the nucleon, as already noticed in [53]. The nucleon width in (32) is in good agreement with the value $\langle p_{NT}^2 \rangle \equiv 1/\beta_0^2 = 0.64 \text{ GeV}^2$ obtained at $\sqrt{s} \simeq 20 \text{ GeV}$ in [61].

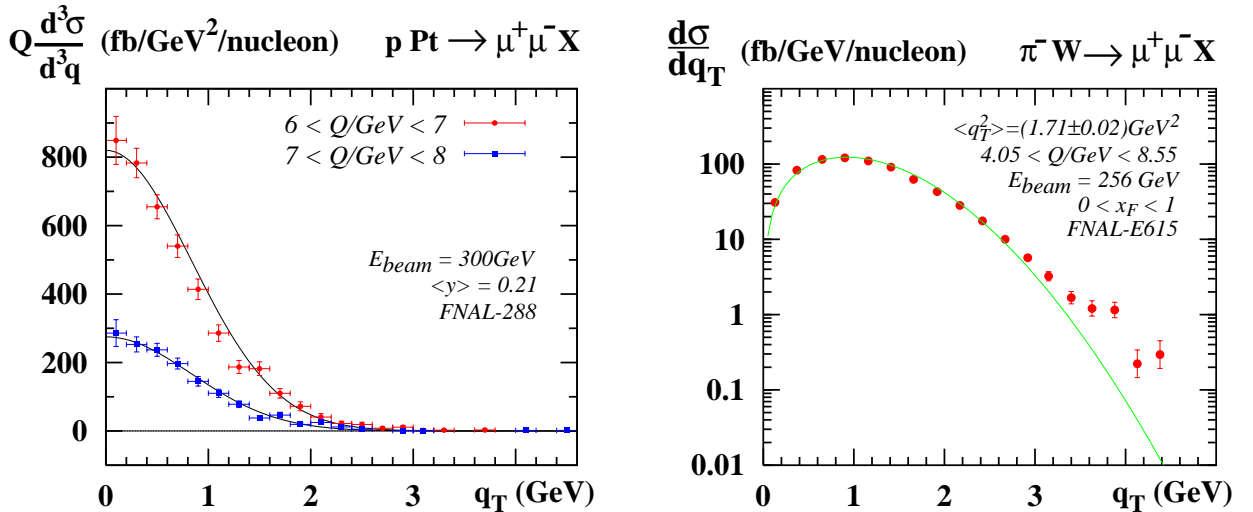


FIG. 8: Left: The invariant differential cross section $Q \frac{d^3\sigma}{d^3q}$ for $p Pt \rightarrow \mu^+\mu^- X$ at $\langle y \rangle = 0.21$ for two different Q -bins from FNAL-288 [50]. The Gauss model, Eq. (29), provides a good description of the data for $\langle q_T^2 \rangle = 1.4 \text{ GeV}^2$.

Right: The differential cross section $\frac{d\sigma}{dq_T}$ for $\pi^- W \rightarrow \mu^+\mu^- X$ from FNAL-E615 [55]. Here the Gauss model, Eq. (29), provides a good description of the data up to $q_T \lesssim 3 \text{ GeV}$ with $\langle q_T^2 \rangle = 1.7 \text{ GeV}^2$.

C. Cahn and Boer-Mulders effect in DY

Next we turn our attention to the azimuthal dependence of the DY cross section in unpolarized hadron collisions, which was measured in various experiments [51–57]. The general expression for the angular differential cross section in the Collins-Soper frame [120] is traditionally written as

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left(1 + \lambda \cos^2 \theta + \mu \sin(2\theta) \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos(2\phi) \right). \quad (33)$$

In collinear QCD factorization to $\mathcal{O}(\alpha_s)$ the coefficients λ, μ, ν are expected to obey the Lam-Tung relation [121, 122]

$$2\nu + \lambda = 1, \quad (34)$$

which is preserved as a good approximation at $\mathcal{O}(\alpha_s^2)$ [123]. But in experiments with pion beams on nuclear targets the relation (34) was found to be strongly violated: while $\lambda = \mathcal{O}(1)$ was observed, large values $\nu \neq 0$ were found [51–55]. (However, in proton-proton and deuteron-proton collisions [56, 57] the relation (34) was found satisfied.)

An attractive explanation for the violation of the Lam-Tung relation (34) is provided in the framework of TMDs [20] in terms of the Boer-Mulders function h_1^\perp [19]. In this approach ν is related to $\sum_a e_a^2 h_1^{\perp q}(x_1) h_1^{\perp \bar{q}}(x_2)$ [20].

It is of interest to learn about the Boer-Mulders function from DY and SIDIS because this T-odd TMD was predicted to have unusual universality properties. On the basis of time-reversal arguments it was predicted [24] that h_1^\perp in semi-inclusive deeply inelastic scattering (SIDIS) and in the Drell-Yan process (DY) have opposite signs,

$$h_1^\perp(x, \mathbf{p}_T^2)_{DIS} = -h_1^\perp(x, \mathbf{p}_T^2)_{DY}. \quad (35)$$

Analogous relations are expected to hold also for the Sivers function and other T-odd TMDs [24, 25]. The experimental check of such universality relations for T-odd TMDs would provide a thorough test of our understanding of the factorization approach to transverse momentum dependent processes in terms of p_T -dependent correlators [24–27]. Perspectives to test the universality relation for the Sivers function were discussed in [72–80], and for the Boer-Mulders function in [77–83].

As we have seen in Sec. II E the extraction of the Boer-Mulders function from SIDIS is hampered by substantial power-corrections due to the Cahn-effect. It is interesting to ask whether the same difficulties occur also in DY. In order to address this question a quantitative understanding of intrinsic p_T -effects in DY is necessary, and on the basis of the results from Section III B we are prepared to have a look at that.

As in SIDIS, in the LO QCD (“tree level”) approach at low q_T [110] the coefficient of the $\cos \phi$ modulation in (33) is suppressed by one power of the large scale Q , but ν which is related to the Boer-Mulders effect is leading twist. And, as in SIDIS, the Cahn effect generates a $1/Q$ -contribution to μ and a $1/Q^2$ -power-correction to ν . These contributions are given by

$$\mu_{\text{Cahn}} = A \frac{q_T}{Q} \frac{\langle p_{1T}^2 \rangle - \langle p_{2T}^2 \rangle}{\langle p_{1T}^2 \rangle + \langle p_{2T}^2 \rangle}, \quad (36)$$

$$\nu_{\text{Cahn}} = A \frac{q_T^2}{Q^2} \left(\frac{\langle p_{1T}^2 \rangle - \langle p_{2T}^2 \rangle}{\langle p_{1T}^2 \rangle + \langle p_{2T}^2 \rangle} \right)^2, \quad (37)$$

where

$$A = \left[1 + \frac{1}{2Q^2} \left(\frac{4\langle p_{1T}^2 \rangle \langle p_{2T}^2 \rangle}{\langle p_{1T}^2 \rangle + \langle p_{2T}^2 \rangle} + \left(\frac{\langle p_{1T}^2 \rangle - \langle p_{2T}^2 \rangle}{\langle p_{1T}^2 \rangle + \langle p_{2T}^2 \rangle} \right)^2 q_T^2 \right) \right]^{-1}. \quad (38)$$

Notice that in both cases the effect vanishes, if the Gauss widths are equal. Which does not mean that it vanishes, if the colliding hadrons are the same. For example, in proton-proton collisions the Gauss widths of quarks and antiquarks enter, which could be different. It is true that in Sec. II we saw no evidence for a flavor dependence of the Gauss widths, but we should keep in mind that in SIDIS at Jefferson Lab and HERMES sea quarks do not play a dominant role.

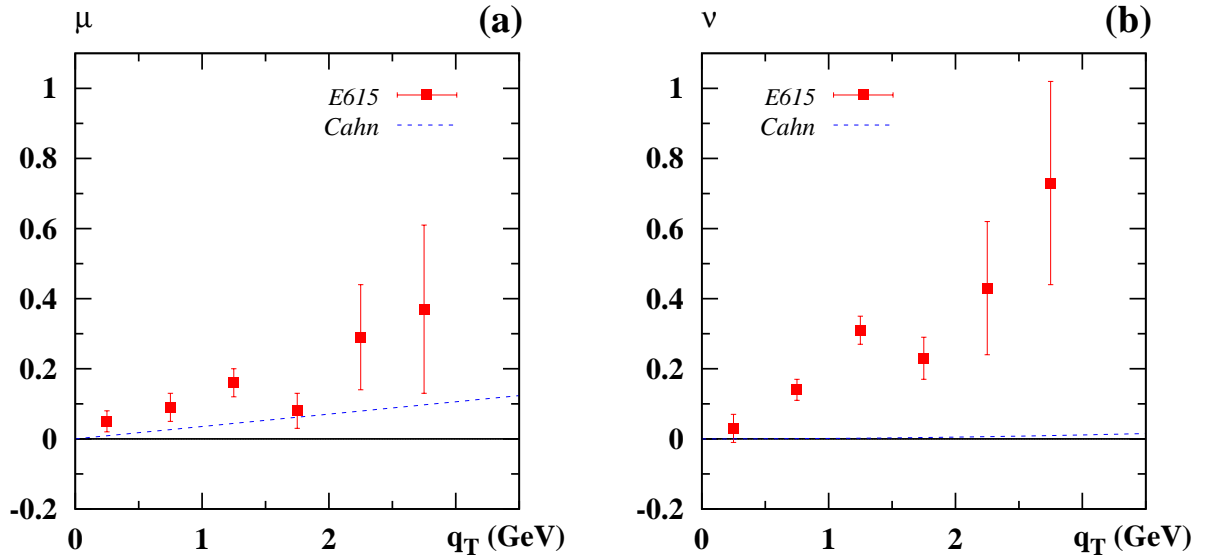


FIG. 9: The coefficient μ and ν in the azimuthal distribution (33) of the dileptons in DY as function of the dilepton transverse momentum q_T in the Collins-Soper frame. Data are from the Fermilab E615 experiment [55]. The curves represent an estimate of the Cahn effect contribution to these coefficients, see text.

With the results on the Gauss widths for pion and nucleon inferred in Eq. (32) we obtain the estimates for the contributions of the Cahn effect to the coefficients μ , ν shown in Fig. 9 in comparison to Fermilab E615 data taken from π^- -nucleus collisions [55]. According to the convention what is hadron 1 and hadron 2 in [55] we have to identify $\langle p_{1T}^2 \rangle = \langle p_{\pi T}^2 \rangle$ and $\langle p_{2T}^2 \rangle = \langle p_{NT}^2 \rangle$ in Eq. (32).

We see in Fig. 9a that the Cahn effect is able to account partly for the power-suppressed coefficient μ of the $\cos \phi$ -modulation in (33). At this point we should not be worried too much about the fact, that in SIDIS at EMC the Cahn effect was able to account for the entire $\cos \phi$ -modulation. Here we deal with a different kinematics and TMDs replacing the role of fragmentation functions. (For the latter reason one also cannot conclude anything on the WW-type-approximations, see Sec. IID.) Also we have to keep in mind that the result for μ is strongly sensitive to the difference of the Gauss widths for pion and nucleon, which we estimated crudely in Eq. (32). In fact, if one was interested in that, one could describe the μ -coefficient entirely in terms of the Cahn effect with the choice $\langle p_{\pi T}^2 \rangle = 1.3 \text{ GeV}^2$ and $\langle p_{NT}^2 \rangle = 0.3 \text{ GeV}^2$, but we shall refrain from doing this here.

What is important at this point is the observation, that the Cahn-effect-power-correction to the ν -coefficient is negligible. We conclude from this exercise that the present DY data provide a “safer” way of accessing information on the Boer-Mulders effect, in the sense that they are far less sensitive to power-corrections as compared to available SIDIS data. In practice, of course, data from both processes need to be explored in order to test the universality prediction (35). Studies of the Boer-Mulders effect were presented in SIDIS [64–71] and DY [77–84], see also [124].

IV. ENERGY DEPENDENCE OF INTRINSIC TRANSVERSE MOMENTA IN DY AND SIDIS

In Eqs. (30, 31) we were able to combine information on the q_T -dependence of DY cross sections from different experiments to arrive at (32), because of comparable kinematics. In general experiments performed at different energies have to be compared with care. In DY the mean lepton momentum square $\langle q_T^2 \rangle$ is energy (s -)dependent. This dependence is different for $i = \pi N$ or pN induced Drell-Yan. For $50 \text{ GeV}^2 < s < 600 \text{ GeV}^2$ it can roughly be described as [125]

$$\begin{aligned} \langle q_T^2(s) \rangle_i &= A_i + B_i s, & A_{\pi N} &= (0.59 \pm 0.05) \text{ GeV}^2, & B_{\pi N} &= (2.8 \pm 0.2) \cdot 10^{-3}, \\ & & A_{pN} &= (0.52 \pm 0.11) \text{ GeV}^2, & B_{pN} &= (1.4 \pm 0.2) \cdot 10^{-3}. \end{aligned} \quad (39)$$

This is just one way of parametrizing the s -dependence. The data are compatible also with a linear in \sqrt{s} increase of $\langle q_T^2 \rangle$ [125], while in QCD one would rather expect a logarithmic increase. But in a limited s -range an effective parametrization of the type (39) works reasonably well. In any case, $\langle q_T^2 \rangle$ increases with energy, which reflects the transverse momentum broadening due to gluon radiation [12].

Fig. 10 shows $\langle q_T^2 \rangle$ in πN induced reactions and the original fit from [125]. The values of $\langle q_T^2 \rangle_{\pi N}$ deduced from Figs. 7 and 8 are shown for comparison but not included in the fits. Assuming the Gauss model (26) we obtain from (39) the following effective s -dependence of the intrinsic transverse parton momenta in the hadron h :

$$\langle p_T^2(s) \rangle_h \approx \langle p_T^2(0) \rangle + C_h s \quad (40)$$

$$\langle p_T^2(0) \rangle = 0.3 \text{ GeV}^2 \quad (41)$$

$$C_h = 10^{-3} \times \begin{cases} 2.1 & \text{for } h = \pi, \\ 0.7 & \text{for } h = p. \end{cases} \quad (42)$$

At this point two questions arise. First, are transverse parton momenta in DY and SIDIS compatible? Second, if so, is there any indication of transverse momentum broadening in SIDIS too?

Concerning the first question, it is encouraging to observe that (40) “predicts” for HERMES ($s = 52 \text{ GeV}^2$) the result $\langle p_T^2(s) \rangle|_{\text{HERMES}} = 0.34 \text{ GeV}^2$ which is within the error bars of the Gauss width in (13). Probably it would be more consistent to use in SIDIS the photon-hadron center of mass energy square W^2 instead of s . However, in view of the uncertainties in (13) and (40–42) this is numerically of little relevance.

Concerning the second question, let us compare the mean square transverse momenta $\langle P_{h\perp}^2(z) \rangle$ in SIDIS from various experiments at a common value of z , let us say $0.5 < z < 0.6$ (as different ranges are covered we cannot compare averages over z). For Jefferson Lab we use $\langle P_{h\perp}^2(z) \rangle = 0.24 \text{ GeV}^2$ at $z = 0.55$ from CLAS [33], which describes well the Hall C data [34], see Fig. 3. For HERMES we take $\langle P_{h\perp}^2(z) \rangle = 0.27 \text{ GeV}^2$ at $z = 0.52$ [35]. For COMPASS we use $\langle P_{h\perp}^2(z) \rangle = 0.55 \text{ GeV}^2$ at $z = 0.566$ from [40] which we convert by means of (12).

Fig. 11 must be interpreted with care. The shown $\langle P_{h\perp}^2(z) \rangle$ were obtained in different ways, span a small s -range, and have systematic uncertainties except for the HERMES value. For a conclusive comparison acceptance corrected data are needed from all experiments. Nevertheless, we see a tendency for an increase in s with a slope which is 20% lower than (42), see Fig. 11. We made no effort to estimate the uncertainty of (42) but it is presumably not smaller than 20%. So the s -slopes in SIDIS and DY are compatible.

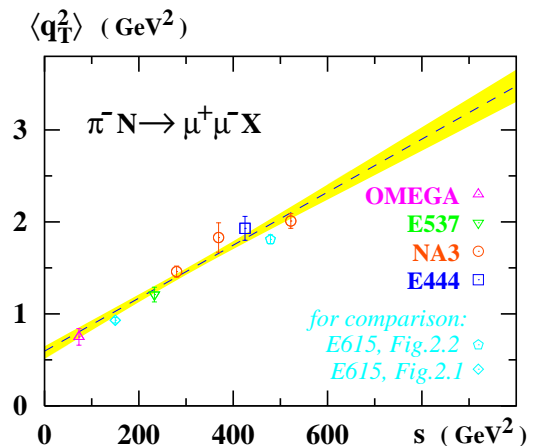


FIG. 10: Mean dimuon transverse momentum square $\langle q_T^2 \rangle$ as function of the center of mass energy square, s , in $\pi^- N$ induced Drell-Yan. Following [125].

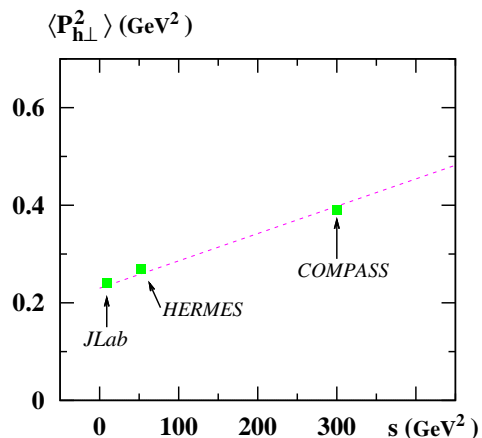


FIG. 11: Mean square transverse momenta $\langle P_{h\perp}^2(z) \rangle$ in SIDIS around $z \sim 0.5$ as function of s from Jefferson Lab [33, 34], HERMES [35], COMPASS [40].

Notice that in SIDIS also the “broadening” of the width of the transverse momenta in the fragmentation function contributes, and we above tacitly assumed that both $\langle p_T^2 \rangle$ in $f_1^a(x, p_T)$ and $\langle K_T^2 \rangle$ in $D_1^a(z, K_T)$ increase with s at a similar rate. In fact, would we extrapolate from HERMES at $s = 52 \text{ GeV}^2$, Eq. (13), assuming that $\langle p_T^2 \rangle$ increases with s at the rate (42) but keeping $\langle K_T^2 \rangle$ at its initial value in (13), we would “predict” $\langle P_{h\perp}^2(z) \rangle \sim (0.30 \pm 0.03) \text{ GeV}^2$ at $z = 0.566$ for COMPASS which strongly underestimates the measured value $\langle P_{h\perp}^2(z) \rangle \simeq 0.39 \text{ GeV}^2$ [40]. Thus, the result in Fig. 11 also indicates $\langle K_T^2 \rangle$ -broadening in the fragmentation function.

Finally, let us comment on EMC data on $A_{UU}^{\cos\phi}$. In Sec. IID we have seen that the Cahn effect can explain these data with the Gauss widths from HERMES. At first glance, this seems to imply that the Gauss widths and EMC (where $s = 525 \text{ GeV}^2$) are as large as at HERMES, while Fig. 11 would suggest that they should be substantially larger. Indeed, if we extrapolate from Fig. 11 to EMC energies then we obtain for EMC about 2 times larger widths resulting in an about (30–40)% larger Cahn contribution to the $\cos\phi$ -modulation. At this point, however, we have to recall that the Cahn effect description of this observable requires the neglect of quark-gluon-correlations in (15). Such quark-gluon-correlations could be as large as (30–40)% compared to the contributions from quark correlators, as was found in [95] in the context of a different observable. Thus, the intrinsic transverse momenta at EMC could be well compatible with the picture in Fig. 11. Future data from HERMES, COMPASS, and Jefferson Lab on $A_{UU}^{\cos\phi}$ will clarify the situation.

To summarize: we conclude that the non-perturbative mechanisms responsible for intrinsic transverse parton momenta in DY and SIDIS are apparently compatible. On the basis of TMD-factorization and universality this is expected, but in view of the typically different energy scales probed in these processes it is not straight-forward to see.

V. CONCLUSIONS

In this work we discussed and reviewed the present understanding of intrinsic transverse parton momenta in SIDIS and DY. An important aim was to demonstrate that the popular Gauss model works very well in these processes. A similar in spirit study was presented in [61]. But meanwhile many more data especially from SIDIS emerged, which allow more conclusive tests of the Gauss model, and make an update of previous results [61–63] possible.

More precisely, in the case of DY we have seen that the Gauss model is applicable if the transverse (dilepton) momenta much smaller than the hard scale. In SIDIS at energies probed at Jefferson Lab or HERMES we have seen that the Gauss model well describes all available data on cross sections or transverse hadron momenta. It is interesting to remark that the Gauss model has received certain support from models [96, 129] and lattice QCD [130].

We discussed the known fact that the Gauss widths increase with energy in DY. By comparing transverse hadron momenta at Jefferson Lab, HERMES and COMPASS we found indications for the energy dependence of the Gauss widths in SIDIS too. More precise data from SIDIS are needed, but on the basis of what is available now, we conclude that the intrinsic transverse parton momenta in SIDIS and DY are compatible. This demonstrates the universality of the p_T -dependence of $f_1^a(x, p_T)$ in SIDIS and DY and supports the factorization approach in terms of unintegrated correlators, although the support has presently a qualitative character.

Our results are of importance for many practical applications. First, the Gauss model can now be used in SIDIS as an effective tool for the description of transverse parton momenta with more confidence. On the basis of presently available data we have seen no evidence for a worthwhile mentioning flavor- or x - or z -dependence of the Gauss widths. Second, we learned that the Gauss widths are energy dependent also in SIDIS, which is of importance when quantitatively comparing results from Jefferson Lab, HERMES and COMPASS. The energy dependence of the Gauss widths in DY was well known before [125]. Third, a solid understanding of p_T -effects and their energy dependence is indispensable in order to study the azimuthal and spin asymmetries in DY and SIDIS. Our results will in particular be helpful to make estimates for the planned or proposed DY physics programs at COMPASS, GSI, J-PARC, U-70 where the process will be probed at different energies.

Especially in the context of the azimuthal asymmetries in unpolarized SIDIS a good quantitative understanding of intrinsic transverse momenta is of importance in the context of the Boer-Mulders effect. We have estimated that the associated $\cos(2\phi)$ -modulation of the SIDIS cross section receives at Jefferson Lab, HERMES, and COMPASS energies sizeable $1/Q^2$ power corrections from the Cahn effect — which is just one of the possible non-factorizing power corrections — in agreement with results from other studies [70, 71]. Due to the larger Q in the available DY data the Cahn effect does not hamper the extraction of the Boer-Mulders function.

As a byproduct, we confirmed the observation [62] that the Cahn effect can account for the EMC data [31, 32] on the “twist-3” $\cos\phi$ -modulation in the unpolarized SIDIS cross section, using the Gauss model parameters from HERMES. However, we were lead to the suspicion that this good description is likely to be due to a cancellation of effects due to the Gauss width broadening at the larger EMC energies and terms neglected in the so-called Wandzura-Wilczek-type approximation needed to justify here the “Cahn-effect-only” approximation.

To conclude, the Gauss model with carefully taken into account energy dependence of the Gauss widths is not only a convenient but also within a good accuracy well justified tool to describe intrinsic transverse parton momenta. This approach, especially now at the early state of art of studies of azimuthal and spin asymmetries in SIDIS and DY, represents a sufficient approximation for many practical purposes. The Gauss Ansatz remains to be tested, in particular, when polarization phenomena are included, and future data may demand to refine it, which will improve our understanding of intrinsic transverse parton momenta.

Future steps, necessary when one will be interested in high precision and/or in going to high energies, for example DY at RHIC with $\sqrt{s} = 200$ GeV, will include the treatment within the Collins-Soper-Sterman formalism [12], as implemented for instance in [126] and consideration of scale dependence [127, 128].

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