DYNAMIC ENERGY MODELS AND CARBON MITIGATION POLICIES

A Dissertation
Submitted to
the Temple University Graduate Board

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in Partial Fulfillment
of the Requirements for the Degree
DOCTOR OF PHILOSOPHY

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by
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January, 2012

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ABSTRACT

In this dissertation I examine a specific class of energy models and their implications for carbon mitigation policies. The class of models includes a production function capable of reproducing the empirically observed phenomenon of short run rigidity of energy use in response to energy price changes and long run flexibility of energy use in response to energy price changes. I use a theoretical model, parameterized using empirical data, to simulate economic performance under several tax regimes where taxes are levied on capital income, investment, and energy. I also investigate transitions from one tax regime to another. I find that energy taxes intended to reduce energy use can successfully achieve those goals with minimal or even positive impacts on macroeconomic performance. But the transition paths to new steady states are lengthy, making political commitment to such policies very challenging.

ACKNOWLEDGMENTS

I must acknowledge the help of many professors, fellow students, coworkers, family, and friends in the completion of this dissertation. I thank Dr. Charles Swanson for teaching me macroeconomics, introducing me to this particular topic, and for all of the help and guidance along the way. Dr. Dimitrios Diamantaras and Dr. Mohsen Fardmanesh both offered much advice and encouragement during the process. Dr. Loretta Mester, my external reader, gave extensive outside perspective and support as well.

My fellow students were extremely helpful in the process, especially Mike Nipple, Lisa Delgado, Karen Campbell, Nurgul Ukueva, Brent Edelman, Alex Delgado, Patrick Schmid, and Natasha Muravytska. Many of my managers and coworkers at Global Insight, the U.S. Department of Housing and Urban Development, and the Federal Reserve Bank of Philadelphia helped me with content and provided inspiration, including Jim Diffley, Dr. Mohammad Iqbal, David Iaia, Phil Hopkins, Ana Orozco, Beverly Harvey, Kevin Kane, Patricia Moroz, Dr. Herb Taylor, Dr. Charles Plosser, Dr. Rick Lang, Bill Stone, Milissa Tadeo, Ardy Lynn Wurtzel, and Noelle Risser.

Lastly, my family has been a source of inspiration and support. Without them I would never have completed this dissertation. Thank you to my wife, Micki, and my children, Charlotte, Sam, and Ellie. Any many thanks go to my parents, siblings, and in-laws: Jim Tilley, Mary Lou Tilley, Katherine Sullivan, James Tilley, Matthew Tilley, Mike Carroll, Mary Carroll, Bill Carroll, and Dan Carroll.

DEDICATION

Dedicated to Micki, for making this possible,
and to Sam, Charlotte, and Ellie, for making it worthwhile.

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CHAPTER 1
INTRODUCTION AND LITERATURE REVIEW

Introduction

Recent work in macroeconomics seeks to understand the impact of energy policies designed to reduce usage of some types of energy. Specifically, policies motivated by concerns of global warming are intended to reduce consumption of fossil fuels such as coal and oil. There is a wealth of estimates in academic research as well as from professional consulting firms on the predicted macroeconomic impacts of such policies. However, nearly all existing studies use models with production functions that do not fit the empirical short- and long-run relationships between energy prices, energy use, the capital stock, and output.

In this dissertation, I examine the impact of an exogenously imposed energy price increase on investment, capital accumulation, energy use, consumption, output, and capital selection by type. Capital types differ in their energy efficiency and productivity levels. I make use of recent results in the literature to select the appropriate production function, the choice of which is central to any such analysis. The most significant challenge posed by this framework is the well-known *curse of dimensionality* arising in this case from the choice of multiple capital types. I employ an established method to deal with this issue.

I develop a model with the appropriate production function and the option for government to levy taxes on investment, capital, and energy. I conduct a steady state analysis to reveal the effects of energy taxes on the economy. I then investigate the transition dynamics of the model by imposing shocks and examining the impulse response of the economy. I also examine the impacts of revenue-neutral transitions from a capital tax to an energy tax, and from an investment tax to an energy tax.

The choice of production function is crucial to the analysis, and there are several options. Production functions differ in their flexibility of input ratios for capital and energy. At one extreme is perfect flexibility. That is, allowing firms to alter the input ratios of new and existing capital in each period in response to fluctuating relative prices. At the other extreme is a fixed input ratio for all capital. Neither extreme is very realistic. I use a production function which allows firms to choose an input ratio for new capital, but does not permit input ratio adjustments for existing capital. This choice is based on the existing literature.

The goal of this thesis is to examine both the short- and long-run economic impacts of an energy tax on an economy. It should be noted that this study is narrowly focused and, necessarily, does not include all relevant aspects of the issue. One important feature of energy is it can be derived from multiple sources including coal, petroleum, natural gas, hydropower, geothermal, and solar power. Real world economies use a combination of these sources, optimizing the mix in light of relative prices and other considerations. In this thesis, though, I model a single energy type with a single price and focus on the manner in which it combines with capital to produce output. Another aspect not considered here is the varied levels of development of nations around the world. Some countries have more mature and sophisticated energy infrastructures than others and might therefore respond differently to energy taxes. Those country-level differences are not modeled here. Lastly, labor effort would likely influence the impacts of an energy tax. In this thesis, though, labor effort is assumed to be supplied inelastically in order to isolate the impacts of taxes affecting relative prices of capital and energy.

The results of the model show significant reductions in energy use can be achieved via energy taxes with modest long-run impacts on aggregate welfare measures such as GDP and consumption. In addition, if those energy taxes are enacted along with a revenue-neutral reduction in capital or investment taxes, the energy use reductions can be achieved along with long-run net *increases* to GDP and consumption. However, the transition period from one tax regime to another is very lengthy, and requires short-run sacrifices in consumption. The short-run sacrifice combined with the lengthy transition period might make commitment to a policy difficult, especially in the presence of short political election cycles.

The remainder of the thesis is organized as follows. The next section of this chapter contains a review of the relevant literature on carbon taxation and on the modeling approach. In Chapter 2, I present a competitive equilibrium model of the economy, followed by a social planner version of the model in Chapter 3. Chapter 3 also shows the method for dealing with the curse of dimensionality.

In Chapter 4, I provide a solution to the model. I then parameterize the solution and show the steady state impacts of progressively higher energy taxes. To investigate the model's dynamics I log linearize about the steady state, and perform several simulations. I simulate the U.S. economy using the price series from my data. I conduct an impulse response to show the general behavior of the model. I then show the steady state and dynamic implications of carbon mitigation policies. Chapter 5 concludes.

Literature Review

The key challenge in macroeconomic modeling of energy use is dealing with the manner in which energy combines with capital to produce output. Specifically, one must model the relative input ratios with some care. At one extreme is the neoclassical approach, which essentially assumes that the capital-energy ratio can easily and immediately be altered in the face of changing energy prices. This is convenient for solving the model and performing analysis, but is not very accurate when one considers that existing capital has essentially a fixed energy requirement. At the other extreme is the option of fixing the capital-energy ratio for all existing and future pieces of capital at a common, permanent ratio. This is also undesirable because it is clear that changes in energy prices lead economies to adjust their capital-energy ratios.

Macroeconomic Impacts of Energy Policy

The literature on the macroeconomic impacts of carbon mitigation policies is both lengthy and inconclusive. Direct comparison is often difficult owing to the wide discretion available to each researcher for both the modeling structure as well as the scenario to be examined. The Kyoto Protocol[1] conveniently harmonized the scenario question to some degree, but ample discretion persists.

Fischer and Morgenstern (2006) identify four principal factors contributing to the wide divergence in carbon mitigation costs to the United State of adopting the protocol. These include: 1) projections of baseline emissions; 2) the policy regime implemented to achieve emissions targets, such as a carbon tax versus a cap-and-trade system; 3) structural characteristics of models; and 4) the possible macroeconomic benefits of averting climate change.[2]

The third factor is the focus of this dissertation. Surveys of macroeconomic im-

[1] The Kyoto Protocol was initially adopted in 1997 and sets targets for greenhouse gas emissions which differ by country. As of February 2010, 184 countries were participating. `http://unfccc.int/kyoto_protocol/items/2830.php` - United Nations Framework Convention on Climate Change

[2] A separate but related topic in the literature extends from (4) by attempting to establish an optimal tax on energy or emissions. For a theoretical model see Golosov et al. (2011) and for an empirical application see Parry and Small (2005).

pacts by Weyant & Hill (1999) and Lasky (2003) reveal predictions for the impact of Kyoto Protocol implementation on Gross Domestic Product (GDP) vary by a factor of more than eight. Although there is wide variation in the results, there is a crucial consistency in the models regarding the manner in which capital interacts with energy. Nearly all models allow for existing capital to be immediately malleable in its energy requirement. This allows firms and households to modify the energy requirement of their existing capital in response to energy prices.

Empirical studies of capital energy elasticities contradict this idea. The energy crisis of 1973 sparked considerable debate over whether energy and capital are substitutes or complements in production, and the magnitude of those elasticities. Berndt and Wood (1975) find very little response of energy use to energy price changes, while Griffin and Gregory (1976) and Pindyck (1979) find significant responses. The key difference is the nature of the data being used and the lengthy transition time required to respond to energy prices. Berndt and Wood use time series data which reveals small, short-run responses. Griffin & Gregory and Pindyck use international, cross section data so their results correspond to long-run equilibriums. Subsequent empirical research has also supported substitutability and confirmed the difference in short- and long-run responses. Nguyen and Streitwieser (2008) conclude substitutability in a cross-section study of more than 10,000 US manufacturing firms across industries.[3] In a meta-analysis of 34 published, empirical papers, Koetse et al (2008) derive a positive, long-run, cross price elasticity of capital and energy that is 35% higher than the short-run elasticity.[4] They find the long-run Morishima elasticity is more than twice the short-run measure.[5]

One vein of the macroeconomic literature, developed by several authors and discussed in more detail in the next section, yields a production function capable of reproducing this short-run rigidity and long-run flexibility. The solution is to introduce *capital types*. A firm may choose capital - from a wide variety of types - with the

[3]The manufacturers span 40 4-digit classification codes of the Standard Industrial Classification (SIC).

[4]The cross price elasticity they measure is demand for capital in response to energy price.

[5]The Morishima elasticity here measures the response of the energy-capital input ratio to the energy-capital price ratio.

most desirable capital-energy input ratio *ex ante*. However, once the unit of capital has been purchased, it cannot be altered *ex post*. This is an appealing solution, but also gives rise to the *curse of dimensionality*.

The curse of dimensionality is the phenomenon that economic models with increasing numbers of state variables become exceedingly difficult to solve, even at very low counts of variables. For instance, moving from a model with 3 state variables to one with 4 is a non-trivial exercise. Now consider an energy model with 25, or 50, or perhaps 100 different capital types with a different energy requirement, and one can easily see how quickly a model becomes impossible to solve directly.

Literature

Many researchers implement some kind of rigidity for input ratios like those described above. Some deal with capital-labor ratios, the issue of how many workers are required to operate a piece of equipment. Conceptually this is the same as the capital-energy ratio problem. What follows is a review of the most relevant work.

Johansen (1959) is one of the first to recognize the problems with the extreme ends of the modeling spectrum and devise a method to find a middle ground. Pointing out the unrealistic rigidity of fixed production coefficients for long term growth models and the lack of rigidity in the other setup, he models the relationship between capital and labor differently. In his model, firms invest in new capital each period, and there are many choices of production technology available. Once new capital is purchased, though, it is *ex post* rigid, in that the ratio of inputs for that capital cannot be changed. The firm can only react to a change in prices, for example, by investing in a different type of capital in subsequent periods. Capital depreciates and eventually disappears from the stock of productive resources. So an economy is able to change its production coefficients, but only at the investment margin. This setup successfully allows for evolution of input ratios but removes the option of immediate changes to existing capital. Johansen does not solve his general model, but investigates three special cases with differing depreciation assumptions.

Sheshinski (1967) establishes some equilibrium properties of Johansen's model.

Assuming that technological change is embodied in capital, savings are a fixed fraction of output, zero foresight for interest rates, and labor is paid its marginal product, he establishes the existence of a steady-state equilibrium with properties familiar to neoclassical growth models. He establishes the stationarity of the mix of capital types in the economy's capital stock on the balanced growth path. In a short section of the paper, Sheshinski is the first to examine the labor market ramifications of the model. All research to this point had assumed full employment of labor. He adjusts the model by establishing a subsistence wage rate and letting employment be determined by labor demand. He shows that unemployment is possible in equilibrium.

Struckmeyer (1986) develops a macroeconomic growth model with microeconomic foundations. Firms face an ex ante production function that is flexible in its inputs; in this case the firm chooses capital, energy, and labor, and Struckmeyer uses a CES production function. Firms form expectations about future factor prices and are therefore able to calculate the discounted expected lifetime value of marginal product and equate it with the lifetime factor payments. Put simply, the economic decision is the familiar equalization of marginal product and marginal cost, but the calculation is done for the life of the machine. Struckmeyer also endogenizes both the real wage rate and labor demand in his model. Firms encounter an exogenously determined energy price which, in turn, lets them decide the optimal labor, capital, and energy inputs for the newest vintage. Endogenous wages and labor demand ultimately affect his results significantly.

Struckmeyer examines the effects of an unexpected, permanent doubling in the price of energy. As may be expected, the most malleable variables in his model (labor, wages, and investment) react immediately to the energy shock and in the expected direction. All measurements that are inherently tied to the capital stock though (output, energy consumption, capital intensity, labor intensity, energy intensity) react slowly because the adjustment of the capital stock takes many years. Thirty years after the shock his simulation shows a massive decrease in annual energy consumption and the energy intensity of output, but only slight declines in the levels of output, labor productivity and the capital stock. Also, the steady state growth rate of output is unchanged. The message is clear: in these models, even massive changes in energy prices do not lead to catastrophic reductions in the standard of living. The result of the model is that changes in energy price lead to a change in the *nature* of the capital stock, and an economy requires a lengthy interval to make the adjustment.

Atkeson and Kehoe (1999) extend the literature with a model that allows investors to choose their capital type. Instead of assuming that technological change occurs at some arbitrary growth rate, they assume that a continuum of capital goods exist indexed along an energy efficiency spectrum.[6] That is, there is a given set of capital types whose respective energy requirements increase or decrease along the continuum. As the energy efficiency increases, though, the marginal productivity decreases. They present firms with a choice of energy efficiencies, but at a productivity cost. Atkeson and Kehoe use a social planner version of their model and solve it in a dynamic programming framework. In order to solve their model they dramatically simplify the setup by showing that i) firms invest in only one capital type per period, and ii) all capital is likely used in equilibrium.

Atkeson and Kehoe's model provides strikingly differing results compared to a neoclassical model with adjustment costs. In the latter, a doubling of energy prices leads to a long-run decrease in output of 33%, and a 78% decline in the capital stock. Their model, though, shows only a 5.5% long-run decrease in each of those measures.

In their study of business cycles, Cooley et al (1995) investigate the issue of variable capital utilization. Focusing on labor and capital (not energy), they examine whether letting a firm mothball some capital in some periods sheds any light on the properties of a business cycle over and above an RBC model with full capacity utilization. Their model uses a traditional, neoclassical production function, but they do impose some short-run restrictions by requiring firms to establish 'locations' before adding a plant. Their model does not indicate a significant difference in size or duration of an expansion or a recession. It does, though, reveal the phenomenon of payments to factors of production varying during the course of a cycle.

[6]This is analogous to one used in the international trade literature introduced by Dornbusch et al. (1977) where there is a continuum of goods with a cutoff type.

Gilchrist and Williams (2000) introduce variable capital utilization to a model with capital types. Similar to Cooley et al, they model labor and capital (not energy) as inputs. They assume that technological progress occurs at an exogenously determined rate. In each period, firms observe the current level of technology and the prices of labor and capital and decide their level of capital investment. There is also a shock that is idiosyncratic to each firm's investment in each period. So the firm can decide to invest X dollars in capital that has efficiency of Y, but sheer luck, Z, also affects how productive the capital will be. With this new investment and their existing stock, firms then decide how much of their capital to operate in each period. Theirs is a general equilibrium model so real wages and the labor supply are endogenously determined.

Simulations of this variable capital utilization yield unique results. Gilchrist and Williams present an unexpected, permanent shock to the productivity of capital (productivity increases). The near-term response of output and labor is "hump-shaped", in that growth increases slowly and steadily at first before peaking and gradually moving back to their steady state rates. They compare this to a traditional Cobb-Douglas economy where labor decreases immediately in response to the shock and continues down to its new steady state.

The authors extend their model in a later paper (Gilchrist and Williams, 2005) to consider the effects of changes to the variance of the idiosyncratic productivity shocks. They find that changes to this variance have profound effects on the time paths and equilibrium levels of labor, investment, and output. In this modified model, a permanent increase in the variance of the capital productivity shock leads to higher aggregate labor productivity and investment, but at an increased number of smaller plants. The increased uncertainty about new capital makes existing capital relatively more attractive, and its economic life is extended. New investment takes place at an increased number of plants (relative to without the shock), but individual plants are smaller because the optimal labor capital ratio is increased. There is also a '"hump-shaped" change to the path of labor productivity that peaks ten to fifteen years after the shock.

Kaboski (2005) also addresses the implications of uncertainty, but looks at stochastic variations in factor prices instead of productivity. There is a wealth of literature on investment decisions and the effects of input price uncertainty. Kaboski argues that in this new framework it is not just variations in absolute input prices that matter, but their *relative* prices as well. He shows that uncertainty of relative input prices can lead to a delay in an initial investment decision and also a delay of switching technologies for an existing producer. In this framework, firms are less likely to switch technologies, and do so less frequently than in a traditional regime.

CHAPTER 2
THE MODEL

Representative Agent

Consider an economy with a single representative agent. Let c be consumption and l be leisure, both of which enter the agent's utility function. Let $\beta \in [0, 1]$ be the time discount factor. I will set $\beta = 0.96$ for the duration of my analysis. The agent maximizes utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{2.1}$$

where E_0 is the expectations operator at time zero. The agent's utility function is assumed to be continuously differentiable, with $u(0, 0) = 0; u_c(t), u_l(t) > 0; u_{cc}(t), u_{ll}(t) < 0; \lim_{c,l \to 0} u_c(t) = \infty$; and $u_{cc}(t)u_{ll}(t) > u_{cl}(t)^2$.

The restrictions on leisure include a lower bound, l_0, which the agent must take, and an upper bound, L, which is the full time endowment. Leisure is constrained by $l_0 \leq l_t \leq L$.

The agent receives income from multiple sources, including rental income from renting his capital to firms. Let $\mathcal{V} \in \mathbb{R}_{++}$ be the set of all capital types, which are differentiated by their energy efficiency (and will be explained more fully in the subsequent section). Capital types are indexed by $v \in \mathcal{V}$. Let $k_t(v)$ be the stock of capital of type v owned by the agent at time t, with an initial endowment of $k_0(v)$ for each type. Let $r(v) \quad \forall v \in \mathcal{V}$ be rental rates earned on each capital type as he rents the capital to firms. Let $n_t = L - l_t$ be labor input, and w_t be the wage paid to the agent for that labor. Let g_t represent a government subsidy disbursement, and let τ_t^k and τ_t^n be tax rates on rental and wage income, respectively.

The agent chooses to spend his income on consumption, c_t, and on investment, $x_t(v)$, in possibly multiple capital types, with $x_t(v) \geq 0 \;\; \forall v \in \mathcal{V}$. Let τ_t^x be the tax rate on capital investment. The agent's budget constraint is

$$c_t+(1+\tau_t^x)\left[\int_v x_t(v)dv\right] \leq (1-\tau_t^k)\left[\int_v r_t(v)k_t(v)dv\right]+(1-\tau_t^n)w_t n_t+g_t, \quad t=0,1,2,...,\infty. \tag{2.2}$$

Let δ be a parameter representing depreciation of the capital stock. All capital types are assumed to depreciate at the same rate. The agent's stock of each type of capital evolves according to

$$k_{t+1}(v) = (1-\delta)k_t(v) + x_t(v) \qquad \forall v \in \mathcal{V}. \tag{2.3}$$

I set $\delta = 0.10$ which is a commonly accepted value in the literature.

Firms

Firms combine energy, capital, and labor to produce output. They rent capital from the representative agent, hire his labor services, and must import the energy from outside the local economy.

Producing output is a two-step process. First, a firm must combine energy and capital to produce *intermediate capital output* (ICO). Second, the firm combines this ICO with labor to produce final output. The production of ICO is complicated and involves the differing capital types mentioned above.

Each capital type combines with energy at a fixed ratio to produce ICO. Let e_t represent energy input. A unit of capital of type v combines with at most $1/v$ units of energy. The maximum energy input for $k(v)$ units of capital of type v is $k(v)/v$. If $e_t(v) > k(v)/v$ then the excess energy is wasted. If $e_t(v) < k(v)/v$ then the capital is underutilized. Figure 2.1 illustrates an example of the relationship between energy input and capital services for a single unit of a less energy-efficient capital type ($v = 1$) and a more efficient type ($v = 2$). The more efficient capital type is represented by the dashed line, and shows higher marginal productivity as energy is added. But at full utilization (where $e = k/v$ or, in this example $e = 1/v$) the highest output attainable from a single unit of the high capital type is less than the highest output for the less efficient capital type.

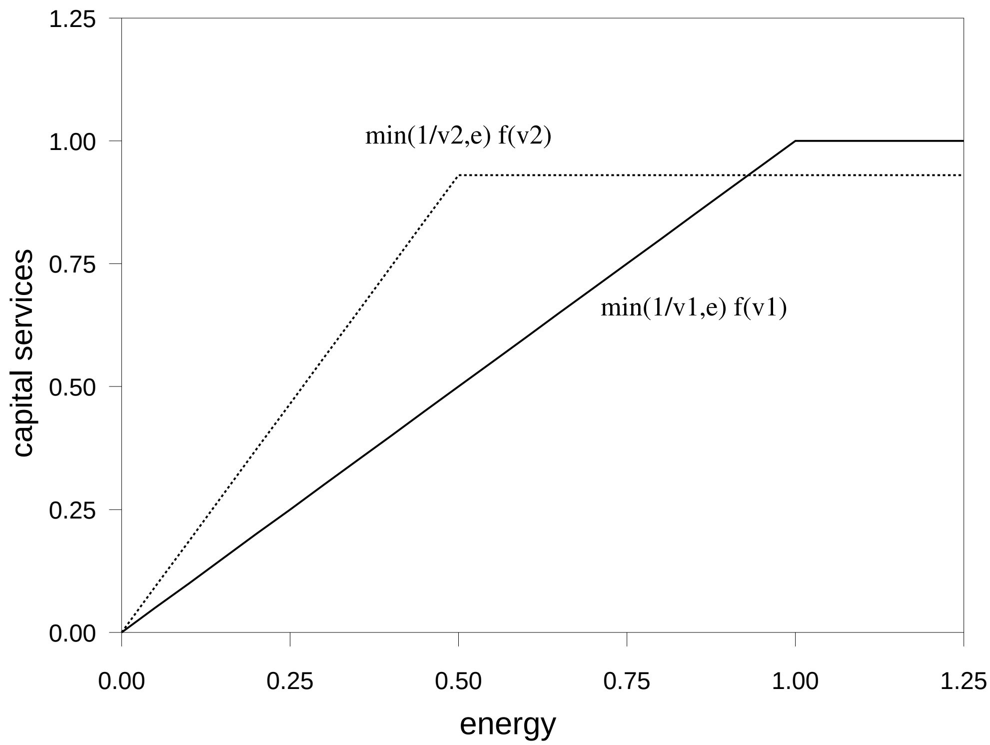

Figure 2.1: Capital Types

In this figure I have held the amount of capital constant at one unit for each type. Another way to express the relationship would be to hold output constant. To achieve some fixed level of output one must maintain a higher capital stock of the more efficient type than a less efficient type, so it is more costly. This setup is a method for making higher efficiency capital relatively more expensive without explicitly assigning prices.[1]

Expressing this unusual production function in a convenient manner requires two parts: one to capture the productive capacity of each unit of energy, and another to capture the limiting, Leontief feature. To show the productivity of each unit of energy, consider Cobb-Douglas production using capital and energy as $F(k, e)$. Rewriting in terms of *energy supplied* multiplied by *per unit of energy*, and recalling that by definition $v = k/e$, yields

$$F(k, e) = e \cdot F\left(\frac{k}{e}, \frac{e}{e}\right) = e \cdot F(v, 1) = e \cdot f(v). \tag{2.4}$$

We have $f(v)$ as the *per unit of energy* production function for k units of capital of type v, with $f(v), f'(v) \geq 0$, and $f''(v) < 0$.[2] This productive capacity of each unit of energy must be multiplied by the amount of energy supplied, which brings us to the Leontief nature of the function. The amount of energy supplied will be the *minimum* of: 1) the maximum possible capacity defined as k/v, and 2) the amount supplied. Multiplying the amount of energy supplied by its productive capacity is then expressed as

$$\min\left(\frac{k}{v}, e\right) \cdot f(v). \tag{2.5}$$

Let z_t represent total ICO from all capital types. To obtain total ICO, sum the output

[1]It is possible for strictly inferior capital types to exist. One can imagine an inefficient type that would exhibit lower marginal productivity than the two types pictured, and also have lower maximum output. This would be shown as a plot that lies completely below both pictured types with no intersection. Types such as this can be considered obsolete and would never receive positive investment and are not modeled here.

[2]In the following chapter I will assume $F(k, e) = k^\gamma e^{1-\gamma}$, so $f(v) = v^\gamma$.

from each capital type

$$z_t = \int_v \min\left(\frac{k_t(v)}{v}, e_t(v)\right) f(v)dv. \tag{2.6}$$

Let m_t be total energy supplied to all capital types at time t

$$m_t = \int_v e_t(v)dv. \tag{2.7}$$

Firms are assumed to import all energy from abroad, and the price of energy is determined exogenously in the international market. Let p_t be the exogenously determined price of energy. It is stochastic and follows an ARMA(1,1) process, following Atkeson & Kehoe (1999). The error term, ϵ_t is assumed to be `iid` with a normal distribution $\epsilon \sim \mathcal{N}(0, \sigma^2)$. The process is

$$\log(p_{t+1}) = (1-\rho)\log(\bar{p}) + \rho\log(p_t) + \eta\epsilon_{t-1} + \epsilon_t, \tag{2.8}$$

where $\bar{p}$ is the mean value of the series.

Additionally there is a tax, τ_t^m, on energy. The full cost of energy imports can be expressed as total energy use multiplied by the price and tax: $p_t m_t (1 + \tau_t^m)$.

Firms combine ICO with labor effort to generate gross final output. Let $G(z, n)$ be the production function for final output. $G(\cdot)$ is assumed to be continuously differentiable with $G(0,0) = 0; G_z(t), G_n(t) > 0; G_{zz}(t), G_{nn}(t) < 0;$ and $\lim_{z,n\to 0} G'(t) = \infty$.

The firm maximizes profit, π_t, which is gross final output minus the costs of renting capital, labor, and importing energy

$$\pi_t = G(z_t, n_t) - \left[\int_v r_t(v)k_t(v)dv\right] - w_t n_t - p_t m_t (1 + \tau_t^m). \tag{2.9}$$

Feasibility

Lastly, there is an economy-wide feasibility constraint. This simply states that consumption and investment in any period may not exceed the net amount of output, after subtracting energy expenditures.[3] Let y_t be net output, commonly referred to as Gross Domestic Product, or GDP. GDP is equal to gross output minus energy expenditures, and it is the constraint on consumption and investment.

$$y_t = G(z_t, n_t) - m_t p_t, \tag{2.10}$$

$$y_t = c_t + \int_v x_t(v) dv. \tag{2.11}$$

Comments

Comment 1. *The production function is taken from Atkeson & Kehoe (1999). The setup achieves the goal of forcing energy input requirements for capital to be ex ante flexible, but ex post rigid.*

Comment 2. *There is no restriction on the number of elements in the set* $\mathcal{V}$*. There may be many, possibly infinite, capital types available for investment. This leads to the curse of dimensionality in this model.*

Comment 3. *One main achievement of Atkeson & Kehoe (1999) is a method to deal with the curse of dimensionality. They show that a model with possibly infinite state variables can be reduced to a much simpler, solvable model.*

Comment 4. *The general setup of the model makes the leisure-labor decision endogenous. Labor is indeed an important component of the economy, but I will focus in this thesis on the choice of capital type and investment in response to energy price fluctuations and taxes. Therefore I will give the following functional form for utility and set* $b = 0$*:* $u(c, l) = \log(c) + b\frac{l^{1-\theta}}{1-\theta}$.

Comment 5. *I will solve this model by converting it to a Social Planner problem.*

Comment 6. *Values for parameters other than those given above* (β, δ) *will be assigned in Chapter 4.*

Comment 7. *There are multiple options for how to deal with government tax revenues. The government could 1) return all tax revenues to the agent as a subsidy, 2)*

[3] This includes an implicit assumption of no international borrowing or lending.

purchase a public good which would enter the agent's utility function as consumption, or 3) purchase some good which does not enter the utility function, essentially throwing it away. I am interested in the substitution effects of the tax rates and not the wealth effects, so I will use option (1), returning all tax revenues to the agent.

a. *Total tax revenues,* T_t*, are simply the sum of revenues from the individual taxes:* $\tau_t^k \left[\int_v r_t(v)k_t(v)dv\right] + \tau_t^n n_t w_t + \tau_t^x \left[\int_v x_t(v)dv\right] + \tau_t^m m_t p_t$.

b. *Total tax subsidy returned to the agent is some share,* φ*, of total revenues:* $g_t = \varphi T_t$.

c. *I choose to return all revenues to the agents so I set* $\varphi = 1$*. The agent perceives this as a lump-sum transfer.*

CHAPTER 3
SOCIAL PLANNER VERSION

The model described in the previous chapter is fairly complex. Fortunately it can be converted into a social planner framework and further simplified which enables me to apply standard dynamic programming techniques. In this chapter I show how the conversion is achieved.

The first step is to convert the problem to a social planner framework. Then I reduce the complexity of the problem by reducing the number of state variables. In the process, I make an assumption that all capital is fully used in each period. I provide support for that assumption in Chapter 4 by way of simulation.

From Competitive Equilibrium to Social Planner

To convert from the competitive equilibrium problem to that of a social planner, I first make an adjustment to the assumptions regarding capital types. In the previous chapter, types were drawn from a continuum. In this section the capital types are discrete. Let v_i for $i = 1, 2, \ldots, N$ be the available types. This is an approximation of the continuous case. The identities for ICO and total energy use, respectively, are now

$$z_t = \sum_{i=1}^{N} \left[\min\left(\frac{k_t(v_i)}{v_i}, e_t(v_i) \right) \cdot f(v_i) \right],$$

$$m_t = \sum_{i=1}^{N} e_t(v_i).$$

Representative Agent

The representative agent's problem now is to maximize expected discounted utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{3.1}$$

subject to a slightly different budget constraint and set of transition equations,

$$c_t + (1+\tau_t^x)\sum_{i=1}^{N}\Big[x_t(v_i)\Big] \leq (1-\tau_t^k)\sum_{i=1}^{N}\Big[r_t(v_i)k_t(v_i)\Big] + (1-\tau_t^n)w_t n_t + g_t, \tag{3.2}$$

$$k_{t+1}(v_i) = (1-\delta)k_t(v_i) + x_t(v_i) \qquad v_i = 1,2,\ldots,N. \tag{3.3}$$

Because of the assumptions on the utility function, no output will be wasted, so the first constraint will be binding. Define $V(k(v_1),\ldots,k(v_N))$ as the maximized value of (3.1). Removing time subscripts and letting the prime symbol represent one period forward, the agent's value function is

$$V(k(v_1),\ldots,k(v_N)) = \max_{x(v_i),k(v_i)}\Big\{u(c) + \beta E_t V'\big(k'(v_1),\ldots,k'(v_N)\big)\Big\}, \tag{3.4}$$

subject to constraints

$$\begin{gathered} c + (1+\tau^x)\sum_{i=1}^{N}\Big[x(v_i)\Big] = (1-\tau^k)\sum_{i=1}^{N}\Big[r(v_i)k(v_i)\Big] + (1-\tau^n)wn + g, \\ k'(v_i) = (1-\delta)k(v_i) + x(v_i) \;\text{ for }\; i = 1,2,\ldots,N. \end{gathered} \tag{3.5}$$

Define the Lagrangian as

$$\begin{aligned} &L(c, x(v_1),\ldots,x(v_N), k'(v_1),\ldots,k'(v_N)) \equiv u(c) + \beta E_t V'\big(k'(v_1),\ldots,k'(v_N)\big) \\ &+ \quad \varphi\Bigg((1-\tau^k)\sum_{i=1}^{N}\Big[r(v_i)k(v_i)\Big] + (1-\tau^n)wn + g - c - (1+\tau^x)\sum_{i=1}^{N}\Big[x(v_i)\Big]\Bigg) \\ &+ \quad \sum_{i=1}^{N}\Bigg[\zeta_i\Bigg((1-\delta)k(v_i) + x(v_i) - k'(v_i)\Bigg)\Bigg], \end{aligned} \tag{3.6}$$

where φ is the Lagrange multiplier for the budget constraint and ζ_i for $i = 1,2,\ldots,N$ are the multipliers for each of the capital type transition equations. The first order conditions for consumption, investment in each capital type, and the Benveniste-

Scheinkman condition for each type are

$$\frac{\partial L}{\partial c} = u_c - \varphi = 0, \tag{3.7}$$

$$\frac{\partial L}{\partial x(v_i)} = -\varphi(1+\tau^x) + \zeta_i = 0 \quad \text{for} \quad i = 1, 2, \ldots, N, \tag{3.8}$$

$$\frac{\partial L}{\partial k'(v_i)} = \beta E_t V'_{k'(v_i)} - \zeta_i = 0 \quad \text{for} \quad i = 1, 2, \ldots, N, \tag{3.9}$$

$$V_{k(v_i)} = u_c(1-\tau^k)r(v_i) + \beta E_t V'_{k(v_i)}(1-\delta) \quad \text{for} \quad i = 1, 2, \ldots, N. \tag{3.10}$$

Combining equations (3.7),(3.8),(3.9), and (3.10), and reinserting time scripts, the optimization condition for each capital type can be shown to be

$$u_c(t) = \beta E_t\left[u_c(t+1)\left[r_{t+1}(v_i)\frac{(1-\tau^k_{t+1})}{(1+\tau^x_t)} + (1-\delta)\frac{(1+\tau^x_{t+1})}{(1+\tau^x_t)}\right]\right]. \tag{3.11}$$

This argument establishes the next Proposition.

Proposition 1. *(A) Given an initial capital stock $k_0(v_i)$ for $i = 1, 2, \ldots, N$ and tax rates $\{\tau^k_t, \tau^x_t, \tau^w_t\}$ for $t = 0, 1, 2, ...\infty$, let Ω be an allocation $\Omega = \{c_t, x_t(v_1), k_{t+1}(v_i), \ldots, k_{t+1}(v_N), n_t\}$ for $t = 0, 1, 2, ...\infty$. If Ω solves the agent's problem it will satisfy Equations (3.2), (3.3), and (3.11).*

Firms

Firms maximize profit by choosing to rent capital, hire labor, and purchase energy to generate output. Their profit function is

$$\pi_t = G(z_t, n_t) - \sum_{i=1}^{N}\left[r_t(v_i)k_t(v_i)\right] - w_t n_t - p_t m_t(1+\tau^m_t). \tag{3.12}$$

Recall that m_t is the summation of energy use e_t across all capital types and that z_t is a function of the capital stock $k_t(v_i)$ and energy use $e_t(v_i)$ of all types. Leisure does not enter the utility function, so labor is supplied inelastically, and the firm chooses capital and energy for each type to maximize π_t:

$$\frac{\partial \pi}{\partial k_t(v_i)} = G_z(t) f_{k(v_i)}(t) - r_t(v_i) = 0, \tag{3.13}$$

$$\frac{\partial \pi}{\partial e_t(v_i)} = G_z(t) f_{e(v_i)}(t) - p_t(1 + \tau_t^m) = 0. \tag{3.14}$$

Equation (3.13) shows that firms are willing to rent capital types from households and pay a rental rate up to their respective marginal products. Equation (3.14) shows that firms are willing to purchase energy at an after-tax rate up to its marginal product for each respective type. These results combined with the assumption of perfectly inelastic labor prove the following Proposition.

Proposition 2. *(A) Given a tax rate* $\{\tau_t^m\}$ *and an exogenous price vector* $\{p_t\}$*, let* $\mathcal{J}$ *be an allocation* $\mathcal{J} = \{k_t(v_1), \ldots, k_t(v_N), e_t(v_i), \ldots, e_t(v_N), n_t\}$ *for* $t = 0, 1, 2, \ldots\infty$*. If* $\mathcal{J}$ *solves the firm's problem it will satisfy Equations (3.12), (3.13), and (3.14).*

Competitive Equilibrium

In the optimizations above, both the agent and firms are maximizing subject to their respective constraints. In order to describe a competitive equilibrium, two additional conditions are needed. The first is a description of the wage rate. The second is the feasibility constraint.

As described above, labor is supplied inelastically because leisure does not enter the household utility function. Firms pay for capital and imported energy. The remaining output, the surplus, is paid in the form of wages to the households for their labor. Because labor is supplied inelastically, I can normalize $n_t = 1$ for all t, and the wage is equal to that surplus

$$w_t = G(z_t, n_t) - \sum_{i=1}^{N} \Big[r_t(v_i) k_t(v_i) \Big] - p_t m_t (1 + \tau_t^m). \tag{3.15}$$

Lastly, there is a feasibility constraint. It restricts total consumption and investment in any period from being larger than output, net of imported energy and is expressed as

$$c_t + \sum_{i=1}^{N} x_t(v_i) \le G(z_t, 1) - p_t m_t. \tag{3.16}$$

Definition 1 (Competitive Equilibrium)**.** *Let* $\mathcal{A} = \{c_t, n_t, x_t(v_i), \ldots, x_t(v_N),$ $k_{t+1}(v_1), \ldots, k_{t+1}(v_N), z_t, m_t\}$ *be an allocation of choice and state variables, and let* $\mathcal{Q} = \{r_t(v_i), \ldots, r_t(v_N), w_t, \tau_t^k, \tau_t^x, \tau_t^w, \tau_t^m\}$ *be a vector of prices and tax rates for* $t = 0, 1, 2, \ldots\infty$*. Given an initial capital stock* $k_0(v_1), \ldots, k_0(v_N)$ *and an exogenous price vector* p_t*, the allocation* $\mathcal{A}$ *is a competitive equilibrium if it: satisfies Equations (3.2), (3.3), and (3.11), as stated in the agent's maximization problem; satisfies Equations (3.12), (3.13) and (3.14) as stated in the firm's profit maximization problem; and satisfies Equations (3.15) and (3.16).*

Social Planner Problem

In this section I show the social planner version of the competitive problem above. The planner is tasked with maximizing utility subject to the capital stock transition equations (3.3) and a modified version of the feasibility constraint. I insert the following terms to emulate the taxes imposed in the competitive problem. Modify the feasibility constraint, adding ϕ_t^1, ϕ_t^2, and ϕ_t^3

$$c_t + \phi_t^3 \left[\sum_{i=1}^{N} x_t(v_i) \right] \le G(z_t, n_t)\phi_t^1 - p_t m_t \phi_t^1 \phi_t^2. \tag{3.17}$$

Removing time subscripts, letting the prime symbol represent one period forward, the social planner's value function is

$$V(k(v_1), \ldots, k(v_N), p, \epsilon) = \max_{x(v_i), e(v_i), k'(v_i)} \left\{ u(c) + \beta E_t V(k'(v_1), \ldots, k'(v_N), p', \epsilon') \right\}, \tag{3.18}$$

subject to constraints

$$
\begin{aligned}
G(z,n)\phi^1 - pm\phi^1\phi^2 &= \phi^3 \sum_{i=1}^{N} x(v_i) + c, \\
k'(v_i) &= (1-\delta)k(v_i) + x(v_i) \quad \text{for} \quad i = 1, 2, \ldots, N, \\
\log(p') &= (1-\rho)\log(\bar{p}) + \rho\log(p) + \eta\epsilon^- + \epsilon.
\end{aligned} \tag{3.19}
$$

Define the Lagrangian as

$$
\begin{aligned}
& L(c, x(v_1), \ldots, x(v_N), k'(v_1), \ldots, k'(v_N), p', \epsilon) \equiv u(c) + \beta E_t V(k'(v_1), \ldots, k'(v_N), p', \epsilon) \\
+ \quad & \varphi\Big(G(z,n)\phi^1 - pm\phi^1\phi^2 - \phi^3 \sum_{i=1}^{N} x(v_i) - c\Big) \\
+ \quad & \sum_{i=1}^{N} \Big[\zeta_i\Big((1-\delta)k(v_i) + x(v_i) - k'(v_i)\Big)\Big] \\
+ \quad & \mu\Big((1-\rho)\log(\bar{p}) + \rho\log(p) + \eta\epsilon^- + \epsilon - \log(p')\Big),
\end{aligned} \tag{3.20}
$$

where φ is the Lagrange multiplier for the feasibility constraint and ζ_i for $i = 1, 2, \ldots, N$ are the multipliers for each of the capital type transition equations. Recall that energy use for each type $e(v_i)$ enters both the production function G and and total energy use m. The first-order conditions are

$$\frac{\partial L}{\partial c} = u_c - \varphi = 0, \tag{3.21}$$

$$\frac{\partial L}{\partial x(v_i)} = -\varphi\phi^3 + \zeta_i = 0 \quad \text{for} \quad i = 1, 2, \ldots, N, \tag{3.22}$$

$$\frac{\partial L}{\partial k'(v_i)} = E_t V'_{k(v_i)} - \zeta_i = 0 \quad \text{for} \quad i = 1, 2, \ldots, N, \tag{3.23}$$

$$\frac{\partial V}{\partial e(v_i)} = \varphi\big(G_z f_{e(v_i)}\phi^1 - p\phi^1\phi^2\big) = 0 \quad \text{for} \quad i = 1, 2, \ldots, N. \tag{3.24}$$

The Benveniste-Scheinkman condition is

$$V_{k(v_i)} = u_c G_z f_{k(v_i)}\phi^1 + \beta E_t V'_{k(v_i)}(1-\delta) \quad \text{for} \quad i = 1, 2, \ldots, N. \tag{3.25}$$

Combining equations (3.21), (3.22), and (3.25), and reinserting time scripts, the Euler equation can be shown to be

$$u_c(t) = \beta E_t\left[u_c(t+1)\left[G_z(t+1)f_{k(v_i)}(t+1)\frac{\phi^1_{t+1}}{\phi^3_t} + (1-\delta)\frac{\phi^3_{t+1}}{\phi^3_t}\right]\right]. \tag{3.26}$$

Comparing the Competitive Equilibrium and Social Planner

To examine the results of the proposed social planner version of the problem, combine (3.11) and (3.13) from the competitive problem

$$u_c(t) = \beta E_t\left[u_c(t+1)\left[G_z(t+1)f_{k(v_i)}(t+1)\frac{(1-\tau^k_{t+1})}{(1+\tau^x_t)} + (1-\delta)\frac{(1+\tau^x_{t+1})}{(1+\tau^x_t)}\right]\right], \tag{3.27}$$

and compare to (3.26) above. The expressions are made equivalent by setting $\phi^1 = (1-\tau^k)$ and $\phi^3 = (1+\tau^x)$. Similarly, set $\phi^2 = (1+\tau^m)$ to make (3.14) equivalent to (3.24). The remainder of the required conditions are ensured by the setup of the maximization in (3.18). This establishes the social planner problem presented above as an accurate representation of the competitive version of the model. From this point forward the social planner version is used exclusively.

Consolidation of State Variables

The next step in simplifying the model is to reduce the number of state variables. This is made possible by first assuming that all existing capital is always fully utilized in equilibrium.[1]

Assumption 1 (Full Capital Use - FCU)**.** *Assume that at any date, t, capital with a nonzero stock, $k_t(v) > 0$, is fully utilized such that $e_t(v) = k_t(v)/v \quad \forall v \in \mathcal{V}$.*

The FCU assumption is necessary to simplify the model, but at the cost of possibly losing some reality. One can imagine a situation where energy prices were high enough that it would be optimal to leave some less efficient capital types unused. However, in a later chapter, I provide support for the FCU assumption by simulating

[1] In this section, I return to showing capital types as lying along a continuum.

a parameterized version of the model. The simulation shows the probability of ever leaving capital unused is vanishingly small.

The FCU assumption above enables the important simplification of reducing the number of state variables in the model. The variables for ICO, z_t, and total energy use, m_t, can be viewed as state variables that summarize the total productive value and energy requirement, respectively, of the total capital stock. They can be assigned transition equations to replace Equations (3.3) which track the evolution of each capital type.

Initial capital stocks $k_0(v)$ are given $\forall v \in \mathcal{V}$, and let $k_0 = \int_v k_0(v)dv$, or more generally $k_t = \int_v k_t(v)dv$ for $t = 1, 2, ..., \infty$. Aggregate ICO and aggregate energy use follow directly from their definitions in the preceding chapter, and can be expressed as

$$z_t = \int_v k_t(v)\frac{f(v)}{v}dv, \tag{3.28}$$

$$m_t = \int_v k_t(v)\frac{1}{v}dv. \tag{3.29}$$

The cost and benefit of differing capital types can be seen in (3.28) and (3.29) in the expressions $\frac{f(v)}{v}$ and $\frac{1}{v}$. A higher capital type carries the benefit of reduced energy use but at the same time reduces ICO. Determining the optimal capital type requires balancing these two effects.

It is straightforward to construct how these two state variables will evolve. For any following period, existing capital $k_t(v)$ will depreciate by δ, thus reducing z_{t+1} and m_{t+1}. But investment will increase each by factors of $x_t\frac{f(v)}{v}$ and $x_t\frac{1}{v}$, respectively. This transition from one period to the next can be expressed as

$$z_{t+1} = \int_v (1-\delta)k_t(v)\frac{f(v)}{v}dv + \int_v x_t(v)\frac{f(v)}{v}dv,$$
$$m_{t+1} = \int_v (1-\delta)k_t(v)\frac{1}{v}dv + \int_v x_t(v)\frac{1}{v}dv.$$

Using the identities (3.28) and (3.29) to simplify the right hand side of each and the condition that $x_t(v) \geq 0 \;\; \forall v \in \mathcal{V}$, the new transition equations are

$$z_{t+1} = z_t(1-\delta) + \int_v x_t(v)\frac{f(v)}{v}dv, \tag{3.30}$$

$$m_{t+1} = m_t(1-\delta) + \int_v x_t(v)\frac{1}{v}dv. \tag{3.31}$$

Investment in One Capital Type

The last step in simplifying the model is to propose that in any given period, the social planner will only invest in one capital type.

Claim 1. *Given the realization of the state $\{z_t, m_t, p_t\}$, there is a single capital type v_i for which $x(v_i) > 0$.*

This proposition is used in the following Chapter to solve the model. The planner chooses a single optimal type in each period in which to invest. The implication is that the social planner will not diversify his investment in multiple capital types. If there is no uncertainty regarding future energy prices, the proposition can be proven. Atkeson and Kehoe (1999) prove this non-stochastic case although their model contains a stochastic price variable. In the presence of uncertainty, consider the following in support of Proposition (1).

Lemma 1. *Let $q_t(v_i)$ be the present value of the expected, discounted, value of investment of one unit of capital in type i at time t. In equilibrium, if there is positive investment in two types of capital $\{v_a, v_b\}$ at time t, then $q_t(v_a) = q_t(v_b)$.*

Proof. If there is positive investment in equilibrium in any type at time t, then it must be the case that the present value of that investment is at least as high as all other types, so $q_t(v_a) \geq q_t(v_i) \;\; \forall \;\; i \neq a \in \mathcal{V}$ and $q_t(v_b) \geq q_t(v_i) \;\; \forall \;\; i \neq b \in \mathcal{V}$. It follows that $q_t(v_a) \geq q_t(v_b)$ and $q_t(v_a) \leq q_t(v_b)$. Therefore $q_t(v_a) = q_t(v_b)$. □

To proceed, an expression for $q_t(v_i)$ is needed. Let E be the expectations operator, Φ_t be a discounting term, and y_t be net output as defined above. Then $q_t(v_i)$ can be expressed as

$$q_t(v_i) \equiv \frac{\partial y_{t+j}}{\partial k_{t+j}(v_i)} \frac{\partial k_{t+j}(v_i)}{\partial x_t(v_i)} E_t\Bigg[\sum_{j=1}^{T} \Phi_{t+j} y_{t+j}\Bigg]. \tag{3.32}$$

Recall $y_t = G(z_t, n_t) - m_t p_t$ and n is normalized to unity. With the FCU assumption z and m can be substituted for using expressions (3.28) and (3.29). Investment of $x_t(v_i)$ will augment the capital stock of type i in any later period j by $(1-\delta)^{j-t}$. Let s_t be the state at time t and $\Pi(s)$ be the probability of realizing each state. Then (3.32) can be rewritten as a weighted sum of all possible future states

$$q_t(v_i) = \sum_{j=1}^{T} \sum_{s_{t+j}} \left[\Phi_{t+j}(s_{t+j}) \Pi_{t+j}(s_{t+j}) (1-\delta)^{j-t} \left(G_{z_{t+1}} \frac{f(v_i)}{v_i} - \frac{1}{v_i} p_{t+j}(s_{t+j}) \right) \right]. \quad (3.33)$$

To simplify the expression define the following

$$\Omega \equiv \sum_{s_{t+j}} \Phi_{t+j}(s_{t+j}) \Pi_{t+j}(s_{t+j}) (1-\delta)^{j-1} G_{z_{t+1}}, \quad (3.34)$$

$$P \equiv \sum_{s_{t+j}} \Phi_{t+j}(s_{t+j}) \Pi_{t+j}(s_{t+j}) (1-\delta)^{j-1} p_{t+j}(s_{t+j}), \quad (3.35)$$

and rewrite (3.33) as

$$q_t(v_i) = \Omega \frac{f(v_i)}{v_i} - \frac{1}{v_i} P. \quad (3.36)$$

Let $f(v_i)$ have the functional form v^γ

$$q_t(v_i) = \Omega \frac{v_i^\gamma}{v_i} - \frac{1}{v_i} P. \quad (3.37)$$

Suppose there is positive investment in v_a and v_b. Then $q_t(v_a) = q_t(v_b)$ which can also be written as

$$\Omega \frac{v_a^\gamma}{v_a} - \frac{1}{v_a} P = \Omega \frac{v_b^\gamma}{v_b} - \frac{1}{v_b} P. \quad (3.38)$$

Let $\lambda \in (-\infty, \infty)$ and $v_0 = \lambda v_a + (1-\lambda) v_b$. If $\lambda = 1$, then $v_0 = v_a$ and if $\lambda = 0$, then $v_0 = v_b$. The value of v_0 can be expressed as

$$q_t(v_0) = q_t(v_a, v_b, \lambda) = \Omega \frac{(\lambda v_a + (1-\lambda) v_b)^\gamma}{\lambda v_a + (1-\lambda) v_b} - \frac{1}{\lambda v_a + (1-\lambda) v_b} P. \quad (3.39)$$

If there exists a value for λ which yields a $q(v_0)$ higher than $q(v_a)$ and $q(v_b)$, then it is suboptimal to invest in those two types and there could never be investment in multiple types. In the case of certainty, it is straightforward to show this because $q(v)$ can be shown to be quasiconcave with a unique global maximum on $\mathbb{R}_{++}$. But with uncertainty, Equation (3.33) is the summation of a large, finite number of quasiconcave functions, which is not necessarily quasiconcave. Additionally, if Equation (3.33) was the summation of a finite number of *concave* functions, it would be a concave function and Proposition (1) would be proven.[2] For Proposition (1) to be violated, Equation (3.33) must have multiple, equal global maxima.

Lacking a proof, I run a simulation of the parameterized version of the model in the following chapter. I calculate the $q(v)$ for a range of reasonable capital types and a range of possible values for G_z. The simulation supports the proposition of a single, global maximum for Equation (3.33).

Proposition (1) is used to further simplify the transition equations for z and m shown by equations (3.30) and (3.31).

$$z_{t+1} = z_t(1-\delta)+x_t\frac{f(v_t)}{v_t}, \tag{3.40}$$

$$m_{t+1} = m_t(1-\delta) + x_t\frac{1}{v_t}. \tag{3.41}$$

Simpler Social Planner's Problem

With the modifications carried out in this chapter, the Social Planner's Problem is now to maximize

$$E_0\sum_{t=0}^{\infty}\beta^t u(c_t), \tag{3.42}$$

subject to

$$c_t + (1+\tau_t^x)x_t = G(z_t, n_t)(1-\tau_t^k) - p_t m_t(1-\tau_t^k)(1+\tau_t^m), \tag{3.43}$$

$$z_{t+1} = z_t(1-\delta) + x_t\frac{f(v_t)}{v_t}, \tag{3.44}$$

$$m_{t+1} = m_t(1-\delta) + x_t\frac{1}{v_t}. \tag{3.45}$$

[2]Simon & Blume (1994) provide the properties of concave functions.

Proposition 3. *Given an initial capital stock* k_0*, corresponding scalars* z_0, m_0*, and a vector of tax rates* $\{\tau_t^k, \tau_t^x, \tau_t^m\}$*, let* $\{x_t, z_{t+1}, m_{t+1}, c_t\}$ *be the solution to the simpler social planner's problem. Use* $\{x_t, z_{t+1}, m_{t+1}\}$ *to derive the corresponding allocations for* $\{k_{t+1}, e_t, z_t, m_t\}$*. Gathering all variables we have a candidate solution* $\{k_{t+1}, x_t, e_t, z_t, m_t, c_t\}$ *for the original planning problem. Under the FCU assumption the candidate solution solves the original planning problem and is an equilibrium of the model.*

Proof. Under the FCU assumption, the constraints of the original planning problem and the Simpler Social Planner's Problem are identical. The utility functions are also identical. Thus, the candidate solution solves the original planning problem. □

A Cutoff Capital Type

In the following chapter I conduct a simulation to provide support for the FCU assumption. To do so, I must establish a cutoff capital type, v^*, which would be fully utilized in any period given a realized price of energy, p_t.

Proposition 4. *Given a capital stock function* k_t *and energy price* p_t*, there is a cutoff type of capital* v_t^* *such that capital of types* $v \geq v_t^*$ *is fully utilized at time, t, and capital of types* $v < v_t^*$ *is fully unutilized. The cutoff type* v_t^* *is an increasing function of* p_t*.*

PROOF. Given a positive stock of any capital type v and a realization of the energy price p, the decision of whether to utilize it any time is the problem of maximizing

$$
\begin{aligned}
G(z(v), n) - e(v)p(1+\tau^m) \;\; s.t. \\
k(v) \geq e(v) \cdot v, \\
e(v) \geq 0.
\end{aligned}
$$

The first constraint is a rearrangement of the definition of capital type, and as written states that one would never supply more energy than the capital stock's capacity. The second restricts energy input to be nonnegative. Define λ_t and ψ_t as Lagrange multipliers and form the Lagrangian

$$L \equiv G(z(v), n) - p \cdot e(v)(1 + \tau^m) + \lambda(v)[k(v) - e(v) \cdot v] + \psi(v)e(v). \tag{3.46}$$

Recall that if FCU is assumed, then $z(v) = e(v)f(v)$. Taking the derivative with respect to $e(v)$, the choice of optimal energy input yields the first-order condition

$$G_z \cdot f(v) - p(1 + \tau^m) = \lambda(v) \cdot v - \psi(v), \tag{3.47}$$

and Karush-Kuhn-Tucker (KKT) conditions

$$[k(v) - e(v)v] \geq 0, \quad \lambda(v) \geq 0, \quad [k(v) - e(v)v]\lambda(v) = 0,$$
$$\psi(v) \geq 0, \quad e(v) \geq 0, \quad \psi(v)e(v) = 0.$$

To examine the implications, first consider the case where the left hand side of the equality in (3.47) is negative. On the right hand side, all three terms, the capital type v must be positive and the Lagrange multipliers are nonnegative by assumption. For the right hand side to be negative, then, the second multiplier must be positive. From KKT, this means that $e(v) = 0$, so no energy is supplied.

Next examine the case where the left hand side is positive. For that to be the case, some level of energy is being supplied to make the left hand side positive. Additionally, we know from KKT that $\psi = 0$. For the right hand side to be positive, the term $\lambda(v)v > 0$, and that constraint must be binding, so the capital type is *fully utilized*.

It is established that there is a cutoff capital type, v^*, which solves the equation

$$G_z f(v^*) = p(1 + \tau^m). \tag{3.48}$$

In addition to establishing the cutoff type, it is established

a. $G_z f(v^*)$ is increasing in p

b. Any capital type $\hat{v} \neq v^*$ is fully utilized if $G_z f(\hat{v}) > p(1 + \tau^m)$

c. Any capital type $\hat{v} \neq v^*$ is not utilized if $G_z f(\hat{v}) < p(1 + \tau^m)$

□

CHAPTER 4
MODEL SOLUTION AND ANALYSIS

In this chapter I solve the simplified social planner's problem developed in Chapter 2. I then calibrate the model using established parameters from the literature and estimated parameters from my dataset. Following the solution and parameterization, I examine the impacts of energy taxes by considering both steady states and the dynamics of transitions between steady states. First, I find a numerical steady state for the model economy with zero taxes as well as steady states for progressively increased energy taxes on the interval from 0% to 100%. The model economy responds to higher taxes by investing in more efficient capital and decreasing its energy use. GDP declines as taxes increase, but at a decreasing rate. GDP is reduced by 3.0% in the face of a 100% tax on energy, and energy use is halved.

To examine the transition dynamics of the model I use a standard method of log linearizing around the steady state and finding linear approximations of the optimal decision functions. Using those decision functions I conduct a simulation of the U.S. economy and find the model replicates the U.S. experience from 1960 to 2008 well. Energy expenditures fluctuate greatly with fluctuations in energy prices, but energy use responds only slowly and gradually. To further investigate the model's dynamics I show the impulse response to a one-time energy price shock. The results are revealing in that they show the significant time required for the economy to fully recover from the shock.

Lastly I examine the implications of carbon mitigation policies for the model economy, using the Kyoto Protocol as a rough guide. The Protocol calls for a reduction in greenhouse gas emissions to some fraction of the levels produced in each country in the base year of 1990.[1] Countries were assigned differing targets ranging from 92.0% to 110.0%. For this analysis I use 93% of 1990 emissions, which is the target reduction assigned to the United States. Data from the Environmental Protection Agency EPA

[1] Some countries were assigned different base years. For a full presentation refer to the Kyoto Protocol Reference Manual, available online from the United Nations Framework Convention on Climate Change: `http://unfccc.int/resource/docs/publications/08_unfccc_kp_ref_manual.pdf`

(2011) show the U.S. would need to reduce total emissions by 20% from 2008 levels to reach that target.

Solving the Model

The value function is

$$V(z_t, m_t, p_t, \epsilon) = \max_{\{v,c,x\}} u(c_t) + \beta E_t V\big(z_{t+1}, m_{t+1}, p_{t+1}, \epsilon_{t+1}\big),$$

where

$$\begin{aligned} m_{t+1} &= m_t(1-\delta) + x_t \frac{1}{v_t}, \\ z_{t+1} &= z_t(1-\delta) + x_t \frac{f(v_t)}{v_t}, \\ \log(p_{t+1}) &= (1-\rho_p)\log(\overline{p}) + \rho_t \log(p_t) + \eta\epsilon_{p,t-1} + \epsilon_{p,t}. \end{aligned}$$

Define the Lagrangian

$$\begin{aligned} L(c_t, x_t, v_t) \equiv\ & u(c_t) + \beta E_t V\big(z_{t+1}, m_{t+1}, p_{t+1}, \epsilon_{t+1}\big) \\ & + \varphi\Bigg(G(z_t, n_t)(1-\tau_t^k) - p_t m_t (1-\tau_t^k)(1+\tau_t^m) - x_t(1+\tau_t^x) - c_t\Bigg). \end{aligned}$$

The first-order conditions (FOCs) for the control variables are

$$\frac{\partial L}{\partial c} = u_c(t) - \varphi = 0, \tag{4.1}$$

$$\frac{\partial L}{\partial x_t} = \varphi(-(1+\tau_t^x)) + \beta E_t[V_z(t+1)] \cdot \frac{f(v_t)}{v_t} + \beta E_t[V_m(t+1)] \cdot \frac{1}{v_t} = 0, \tag{4.2}$$

$$\begin{aligned} \frac{\partial L}{\partial v_t} =\ & \beta E_t[V_z(t+1)] x_t \left[\frac{f_v(t)v_t - f(v_t)(1)}{v_t^2}\right] \\ & + \beta E_t[V_m(t+1)] x_t \left(-\frac{1}{v_t^2}\right) = 0. \end{aligned} \tag{4.3}$$

The Benveniste-Scheinkman conditions for the state variables are

$$V_z(t) = u_c(t)G_z(t)(1-\tau_t^k) + \beta E_t[V_z(t+1)](1-\delta), \tag{4.4}$$

$$V_m(t) = u_c(t)(-p_t(1-\tau_t^k)(1+\tau^m)) + \beta E_t[V_m(t+1)](1-\delta). \tag{4.5}$$

First Euler Equation

Combine (4.1),(4.2), and rearrange to get

$$u_c(t)v_t(1+\tau_t^x) = \beta E_t[V_z(t+1)]f(v_t) + \beta E_t[V_m(t+1)]. \tag{4.6}$$

Rearrange (4.3) and combine the result with (4.6)

$$\begin{aligned} \beta E_t[V_z(t+1)]f_v(t)v_t &= \beta E_t[V_z(t+1)]f(v_t) + \beta E_t[V_m(t+1)] \Rightarrow \\ \beta E_t[V_z(t+1)] &= \frac{u_c(t)(1+\tau_t^x)}{f_v(t)}. \end{aligned} \tag{4.7}$$

Substitute (4.7) into (4.4)

$$V_z(t) = u_c(t)G_z(t)(1-\tau_t^k) + \frac{u_c(t)(1+\tau_t^x)}{f_v(t)}(1-\delta).$$

Advance one period

$$V_z(t+1) = u_c(t+1)\left[G_z(t+1)(1-\tau_{t+1}^k) + \frac{(1+\tau_{t+1}^x)}{f_v(t+1)}(1-\delta)\right].$$

Substitute the result into into the left-hand side of (4.7) to obtain the first Euler equation

$$u_c(t) = \beta E_t\left[u_c(t+1)\left[G_z(t+1)(1-\tau_{t+1}^k) + \frac{(1+\tau_{t+1}^x)}{f_v(t+1)}(1-\delta)\right]\right]\frac{f_v(t)}{(1+\tau_t^x)}. \tag{4.8}$$

Second Euler Equation

Substitute (4.7) into the right-hand side of (4.6) and rearrange

$$\beta E_t[V_m(t+1)] = u_c(t)v_t(1+\tau_t^x) - u_c(t)\frac{f(v_t)(1+\tau_t^x)}{f_v(t)} \Rightarrow$$

$$\beta E_t[V_m(t+1)] = u_c(t)(1+\tau_t^x)\left[v_t - \frac{f(v_t)}{f_v(t)}\right].$$

Substitute the result into (4.5) and advance one period

$$V_m(t) = u_c(t)\left[\left[v_t - \frac{f(v_t)}{f_v(t)}\right](1-\delta)(1+\tau_t^x) - p_t(1+\tau_t^m)(1-\tau_t^k)\right],$$

$$V_m(t+1) = u_c(t+1)\left[\left[v_{t+1} - \frac{f(v_{t+1})}{f_v(t+1)}\right](1-\delta)(1+\tau_{t+1}^x) - p_{t+1}(1+\tau_{t+1}^m)(1-\tau_{t+1}^k)\right].$$

Substitute the result and (4.7) into the right-hand side of (4.6)

$$\begin{aligned} u_c(t)v_t(1+\tau_t^x) &= u_c(t)(1+\tau_t^x)\frac{f(v_t)}{f_v(t)} \\ &+ \beta E_t\left[u_c(t+1)\left[\left[v_{t+1} - \frac{f(v_{t+1})}{f_v(t+1)}\right](1-\delta)(1+\tau_{t+1}^x) - p_{t+1}(1+\tau_{t+1}^m)(1-\tau_{t+1}^k)\right]\right]. \end{aligned}$$

Rearrange to obtain the second Euler equation

$$\begin{aligned} & u_c(t)(1+\tau_t^x)\left[v_t - \frac{f(v_t)}{f_v(t)}\right] \\ &= \beta E_t\left[u_c(t+1)\left[\left[v_{t+1} - \frac{f(v_{t+1})}{f_v(t+1)}\right](1-\delta)(1+\tau_{t+1}^x) - p_{t+1}(1+\tau_{t+1}^m)(1-\tau_{t+1}^k)\right]\right]. \end{aligned} \tag{4.9}$$

Calibration and Steady State

Calibration

To proceed I assign functional forms for the utility function and production functions. Utility is the natural log of consumption $u(c_t) = \log(c_t)$. The intermediate capital output (ICO) production function is Cobb-Douglas. Converting ICO production to terms of per-unit-of-energy, and using the definition of capital type yields:

$$F(k_t, e_t) = k_t^\gamma e_t^{1-\gamma},$$
$$f(v_t, 1) = (\frac{k_t}{e_t})^\gamma (\frac{e_t}{e_t})^{1-\gamma} = v_t^\gamma.$$

Gross output is also Cobb-Douglas, and labor is supplied inelastically so $n_t = 1$.

$$G(z_t, n_t) = z_t^\alpha n_t^{1-\alpha},$$
$$G(z_t, 1) = z_t^\alpha.$$

I follow Atkeson & Kehoe (1999) and estimate an ARMA(1,1) process for the energy price

$$\log(p_{t+1}) = (1-\rho)\log(\bar{p}) + \rho \log(p_t) + \eta\epsilon_{t-1} + \epsilon_t, \tag{4.10}$$

where $\bar{p}$ is the mean value of the series. I obtain values for ρ and η by estimating Equation (4.10) using my dataset.[2] Values for other parameters in the model (β, α, γ, and δ) take values consistent with those common in the literature:

$$\beta = 0.96, \quad \alpha = 0.43, \quad \gamma = 0.90,$$
$$\delta = 0.10, \quad \rho = 0.94, \quad \eta = 0.42.$$

Steady State with Zero Taxes

I derive a steady-state solution to the model by removing time subscripts from the Euler equations, transition equations, and identities. The first Euler equation (4.8) yields a steady state expression for z. The second Euler equation (4.9) yields an expression for v. Using bars above variables to denote steady state values, the steady state values for gross output and capital type are[3]

$$\bar{z} = \left(\frac{\alpha\gamma\bar{v}^{\gamma-1}(1-\tau^k)}{[\frac{1}{\beta} - (1-\delta)](1+\tau^x)} \right)^{\frac{1}{1-\alpha}}, \tag{4.11}$$

[2] A full description of the dataset is contained in the Appendix.

[3] It is helpful to note here the algebraic symmetry of τ^k and τ^x in each equation. For any tax rate τ^k there is a corresponding τ^x which would impact each condition identically.

$$\bar{v} = \frac{\beta\bar{p}\ (1+\tau^m)(1-\tau^k)}{[\beta(1-\delta)-1]\big(1-\frac{1}{\gamma}\big)(1+\tau^x)}. \tag{4.12}$$

The remaining steady state equations are simple algebraic rearrangements of the transition equations and constraints after removing time subscripts. In addition to equations (4.11) and (4.12) above for $\bar{z}$ and $\bar{v}$, I have steady-state expressions for $\bar{e}$, $\bar{k}$, $\bar{x}$, $\bar{m}$, $\bar{y}$, and $\bar{c}$ This gives me eight equations with eight unknowns. The steady

$$\begin{array}{lll} \bar{e} = \frac{\bar{z}}{\bar{v}^{\gamma}}, & \bar{m} = \frac{\bar{x}}{\bar{v}\delta}, & \bar{y} = \bar{z}^{\alpha} - \bar{p}\bar{m}, \\ \bar{k} = \bar{e}\cdot\bar{v}, & \bar{x} = \bar{k}\delta, & \bar{c} = \bar{y} - \bar{x}. \end{array}$$

state price of energy $\bar{p}$ is the ninth and final variable, but it is not unknown. It is determined exogenously from the data and fed into this system of equations like a parameter. In fact, Equation (4.12) needs only $\bar{p}$ and parameter values for a solution. Using the results of my dataset, I set $\bar{p} = 1.063$.[4] In this first steady state solution, I set all three tax rates $\{\tau_t^k, \tau_t^x, \tau_t^m\}$ equal to zero. Solving the system of equations yields steady values for each variable

$$\begin{array}{llll} \bar{p} = 1.063, & \bar{v} = 67.51, & \bar{c} = 1.062, & \bar{x} = 0.424, \\ \bar{z} = 2.784, & \bar{m} = 0.063, & \bar{y} = 1.486, & \bar{k} = 4.243. \end{array}$$

Log Linearization and Approximation

Decision Functions

The next step is to establish decision functions for the choice variables in the model. The model cannot be solved explicitly, so I derive a log-linear approximation around the steady state and corresponding linear decision functions for v, c, and x. I employ the method of undetermined coefficients as presented by Uhlig (1995), and the method of log-linearization developed by King, Plosser, and Rebelo (1988).

The linearization is applied to the Euler equations (4.8) and (4.9) and also to the economy-wide feasibility constraint Equation (3.16), giving me three equations.[5] The decision functions are linear functions of each of the three state variables p, z, and

[4] The standard deviation is 0.35.

[5] The linearization is shown in the Appendix.

m. They are independent of time, so time subscripts are not needed. In the log-linearized model, the relevant measurement is the log-deviation of each state variable from its steady state. Let a tilde symbol above a variable represent that variable's log-deviation from its steady state value. And let a coefficient a_{ij} represent the elasticity of choice variable i in response to state variable j. The decision functions can then be expressed as

$$\begin{aligned}
\tilde{c} &= a_{cp}\tilde{p} + a_{cz}\tilde{z} + a_{cm}\tilde{m}, \\
\tilde{x} &= a_{xp}\tilde{p} + a_{xz}\tilde{z} + a_{xm}\tilde{m}, \\
\tilde{v} &= a_{vp}\tilde{p} + a_{vz}\tilde{z} + a_{vm}\tilde{m}.
\end{aligned} \tag{4.13}$$

The decision functions contain nine coefficients. To solve for them I need nine equations. To achieve this using the method of undetermined coefficients, the key is to recognize that if each coefficient a_{ij} is true for any value of the state variables, then it must be true when, for example: $\tilde{p}$=1, $\tilde{z}$=0, and $\tilde{m}$=0. The coefficients must also be true when $\tilde{p}$=0, $\tilde{z}$=1, and $\tilde{m}$=0, and lastly, when $\tilde{p}$=0, $\tilde{z}$=0, and $\tilde{m}$=1.

Using this logic, I create three different versions of each log-linearized Euler equation, with each having a different state variable log deviation set to one and the two others set to zero. I also create three versions of the resource constraint using the same method. This generates nine equations with nine unknown a_{ij}'s. I then solve the system of equations to find the parameter values. The solution for the coefficients is presented in Table 4.1. The signs and magnitudes of the coefficients are as would

	$\tilde{p}$	$\tilde{z}$	$\tilde{m}$
$\tilde{c}$	-0.011	0.865	-0.043
$\tilde{v}$	0.757	0.363	-0.008
$\tilde{x}$	-0.130	-0.590	-0.049

Table 4.1: Decision Function Coefficients

be expected. In particular, the first column shows the response of all three choice variables to a one percent deviation in the energy price above its steady state value. The higher price impacts consumption and investment, because GDP is reduced by the more expensive, imported energy. The response of investment is significantly larger than that of consumption. The higher prices also prompt a move to a higher capital type v, as shown by the value of $a_{vp} = 0.757$.

The responses to deviations in $\bar{z}$ also fit intuition. When it is above its steady state value, all other things equal GDP will be high too, resulting in higher consumption ($a_{cz} = 0.865$) and lower investment ($a_{xz} = -0.590$) which will push the economy back toward its steady state. When energy use is above its steady state, consumption is reduced ($a_{cm} = -0.043$) as is investment ($a_{xm} = -0.049$).

Simulations

In this section I carry out simulations of the log-linearized model. I use the coefficients presented in Table (4.1) in conjunction with the postulated linear decision functions shown in Equations (4.13). In each period, these functions give the optimal values of choice variables in response to deviations of state variables from their steady state values. The state variables are then allowed to evolve according to their transition equations to generate values for the next period. In each period, the process repeats.

Verifying Full Use of Existing Capital

With the ability to run simulations, it is important to revisit the assumption of full capital use *(FCU)*, which enabled the simplification of the model. Recall from Chapter 3 that a cutoff capital type, v^*, is established in Equation (3.48), presented here again for convenience

$$G_{z_t} \cdot f(v_t^*) = p_t(1 + \tau_t^m). \tag{4.14}$$

Using the functional forms above, the steady state cutoff capital type is

$$\bar{v}^* = \left[\frac{\bar{p}(1 + \tau^m)}{\alpha \bar{z}^{\alpha - 1}}\right]^{\frac{1}{\gamma}}. \tag{4.15}$$

As the energy price p_t and z_t fluctuate, so does v^*. At a sufficiently high price, high tax, or low level of ICO (a function of z_t), the value of v^* could in principle be driven so high that some capital would be left idle. As parameterized here, the value is $\bar{v}^* = 5.22$. This is well below the steady state capital type of $\bar{v} = 67.51$. In fact, holding z constant, the price of energy would need to rise by a factor of 10 for v_t^* to be equal to $\bar{v}$. That would be a departure of 28 standard deviations from the mean energy price, the probability of which is minuscule. The reason for this result is both straightforward and intuitive. As parameterized, energy expenditures amount to just 10% of the cost of producing ICO. With such a small share of input costs, it would require a very high price for the marginal costs of energy to exceed its marginal benefits.

In a full simulation with taxes, however, both z_t and τ_t^m depart from their steady state values, so it is important to confirm that the assumption holds in a dynamic setting.

I run a simulation with 10,000 periods using the estimated parameters for the ARMA(1,1) price process presented in Equation (4.10). The results are presented graphically in Figure 4.1.[6] As is made clear in the figure, the optimal capital type, v_t, is volatile in response to energy price fluctuations. The cutoff type, v_t^*, is also volatile for the same reason, but those fluctuations are muted by the scale of this figure.

I also include the plot of the lowest value of v_t realized to date during the simulation. This keeps track of the most inefficient units of capital that could conceivably be available to the economy in any period. The cutoff type never approaches the lowest capital type in any meaningful way. In Figure 4 the minimum realized value of v_t is 31.95, approximately 4.5 times higher than the highest realized value for v^* of 6.96. Later in the simulation but not pictured, v_t does indeed reach a lower value, pushing v_{min} down to 26.30, but that is still 3.6 times higher than the highest realized value of v^* of 7.23. This simulation lends strong support to the *FCU* assumption.

[6]For readability, I present only the first 1,000 periods of the simulation. The assumption of full use is never violated in any of the 10,000 periods.

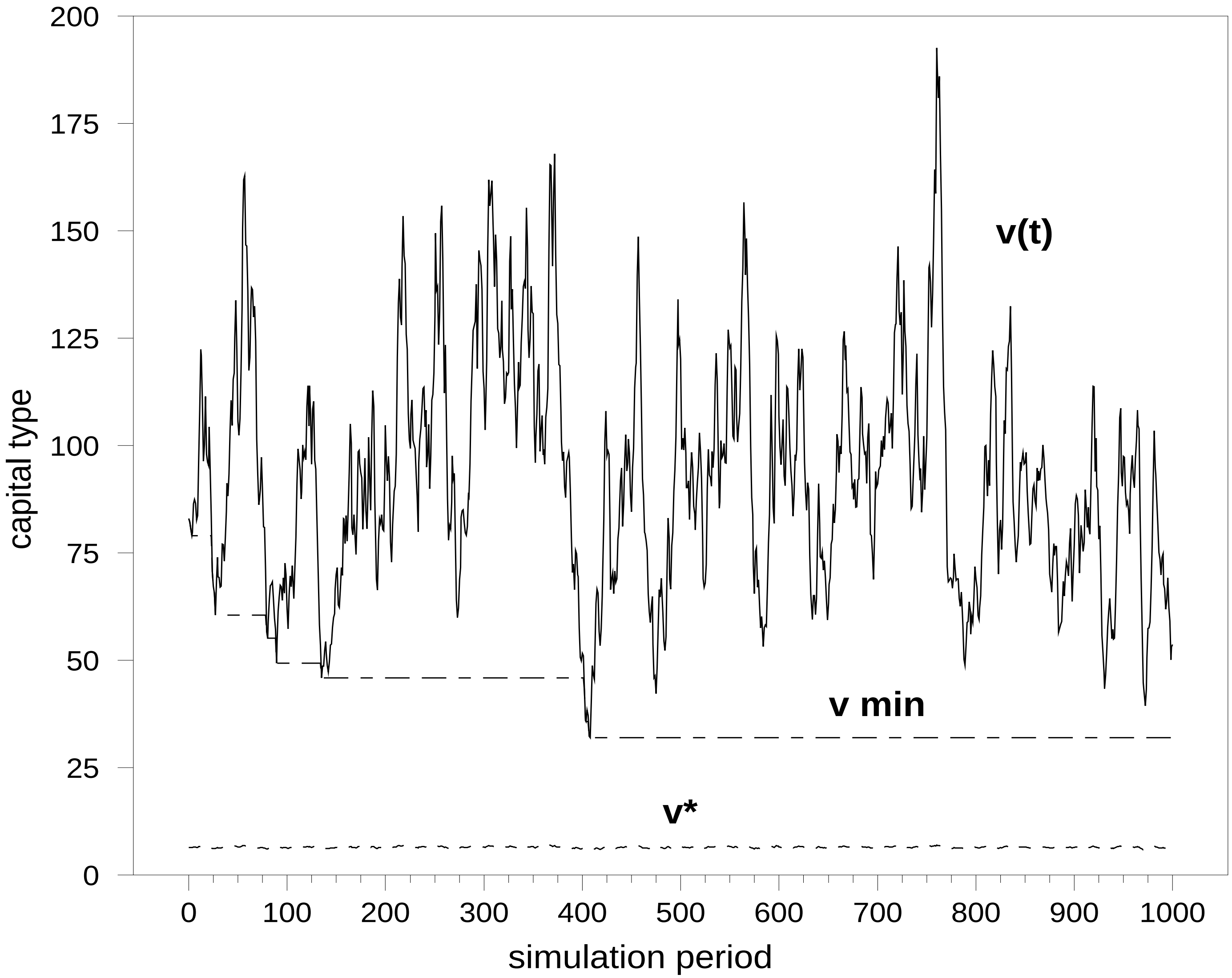

Figure 4.1: Capital Type and Cutoff Capital Type

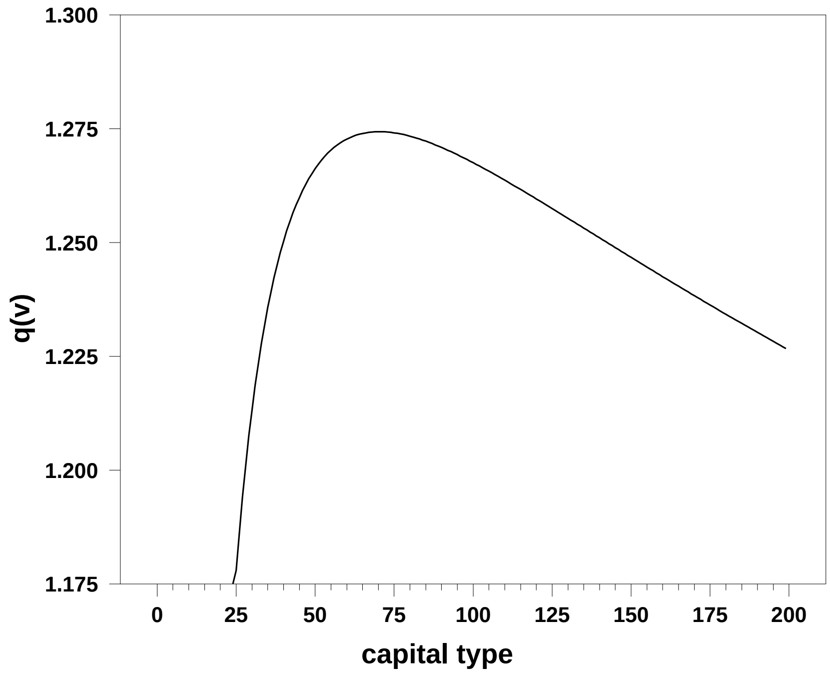

Figure 4.2: Expected Lifetime Value of Investment in Capital Types

Verifying Investment in One Capital Type

Proposition (1) in the previous chapter asserted there would be investment in just one type of capital in any given period. To support the proposition, I run a simulation of Equation (3.33) which expressed the expected lifetime value of investment in any capital type.

For the simulation, I generate 100 stochastic price series which are each 100 periods in length. Due to the depreciation parameter, the marginal change to the value of a capital past 100 periods is minute. For each stochastic price series I calculate $q(v)$ for capital types $v = [1, 200]$ and for values of $z = [2.2, 3.4]$ in increments of 0.1.

The choice of ranges for v and z come from the simulation above. From Figure 4.1 it is clear that the optimal capital type never exceeds $v = 200$. Though it is not pictured, the value of z fluctuates between 2.4 and 3.2.[7] I have extended the range on each side by 0.2 to ensure I have not restricted the simulation too much. Figure 4.2 shows the results of the simulation and the presence of a unique maximum. The

[7]Recall the steady state value is 2.784.

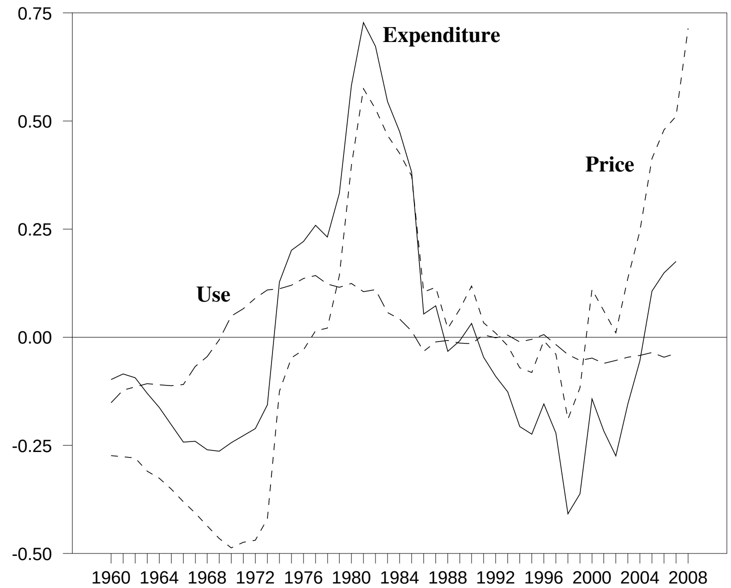

Figure 4.3: Energy Price, Use, and Expenditure 1960-2008

Notes: All series are displayed as logarithms and normalized to their mean value. The Use series is shown as a ratio to real GDP. The Expenditure series is shown as a ratio to nominal GDP. The Price series is the implicit energy price deflator in 2000 dollars divided by the GDP implicit price deflator.

maximum of 1.2744 is nearly identical to the $q(v)$ calculated for the optimal capital type with no uncertainty, which is 1.2732. Also, the associated capital type is nearly identical to the steady state optimum capital type.

Simulation of U.S. Economy

To examine the effectiveness of the model, I run a simulation using energy price data for the US, and compare the model result to US data. The data includes the well-known volatility of energy prices, shown in Figure 4.3, including the dramatic increases in 1973 and again in 1979. Those price shocks led to large increases in total expenditure. The subsequent collapse in energy prices for nearly two decades coincides with lower energy expenditures. The last portion of the figure, from 2000 to 2008, shows the impact on expenditures of the recent resurgence in energy prices. As shown in Figure 4.4, the model does a reasonably good job of modeling the real U.S. *expenditure* response to those energy price movements. As would be expected, energy expenditure is volatile, and moves very similarly to price variable. The capital stock is slow to adjust, so firms and consumers are unable to react quickly to price changes.

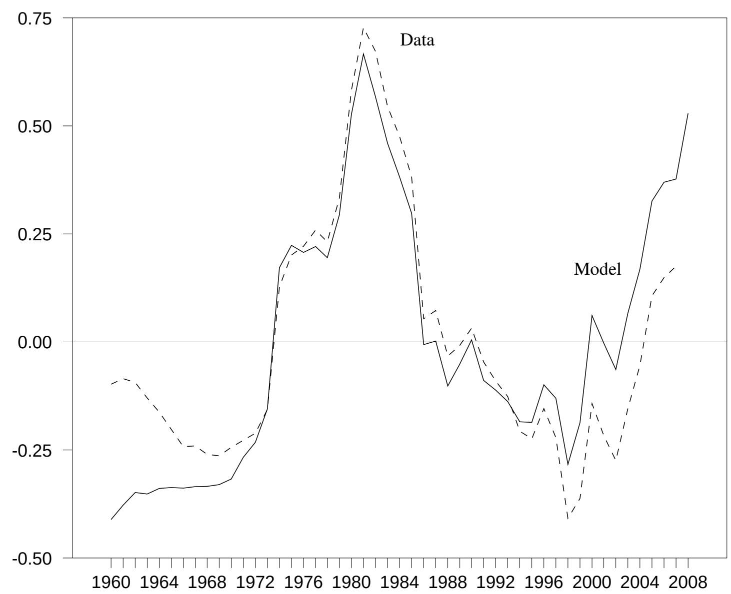

Figure 4.4: Energy Expenditure

This leads to energy expenditures moving similarly to price. Figure 4.5 compares the model response of energy *use* to U.S. data. Both the data series and the model series show energy use responding slowly to energy prices. Energy use rose as a share of GDP until the price shocks of the 1970s. Those price surges led to investment in more efficient capital. It is clear that energy use responds to sustained deviations in price from its steady state, but is not nearly as responsive to short term fluctuations. The model successfully replicates that phenomenon. Also notable is the scale of the response compared with Figure 4.4. Energy use fluctuations are smaller in magnitude than are fluctuations in energy expenditure. The model replicates that aspect as well.

Impulse Response

The next simulation is an impulse response to get a clearer picture of how each variable responds to an exogenous price change. Starting with the economy in its steady state, I implement a price shock at $t = 0$ which increases the price by one positive standard deviation, a 33% increase. The price shock is exacerbated in the second period because of the ARMA process described in Equation 4.10. The price

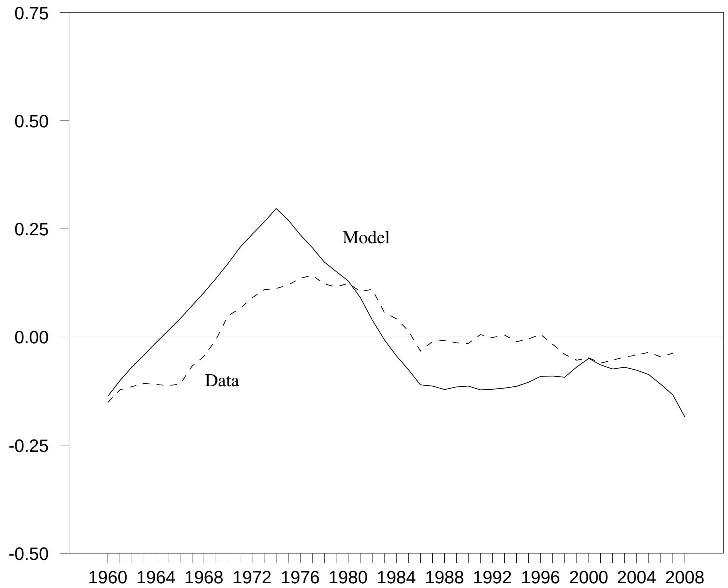

Figure 4.5: Energy Use

hits a maximum of nearly 50% above the steady state price level in that second period, and falls toward the steady state value thereafter. The responses of GDP and energy use are shown in Figures 4.6 and 4.7, and the remainder of response graphs are presented in an appendix. I use annual data so each period is one year. As would be expected, the impact on consumption is negative. Consumption reaches a trough of 2.1% below its steady state level nine periods after the shock. Capital investment reacts more sharply than consumption, as is expected due to the a_{xp} coefficient. The trough comes much earlier than consumption though, in the second period (-4.5%) at the same time as the price peak. Investment returns to its steady state level much more quickly. By the time consumption has reached its trough after nine periods, investment is just 1.2% away from its steady state, and fully recovers after fifteen periods.

The impact on GDP is both immediate and persistent. The higher energy price is a direct drain because all energy is imported in this economy. GDP drops sharply immediately in tandem with the price increase, and reaches a trough shortly thereafter at 1.9% after four periods. Energy use sees the longest impact in this simulation. The price shock induces investment in higher efficiency capital and drives down energy

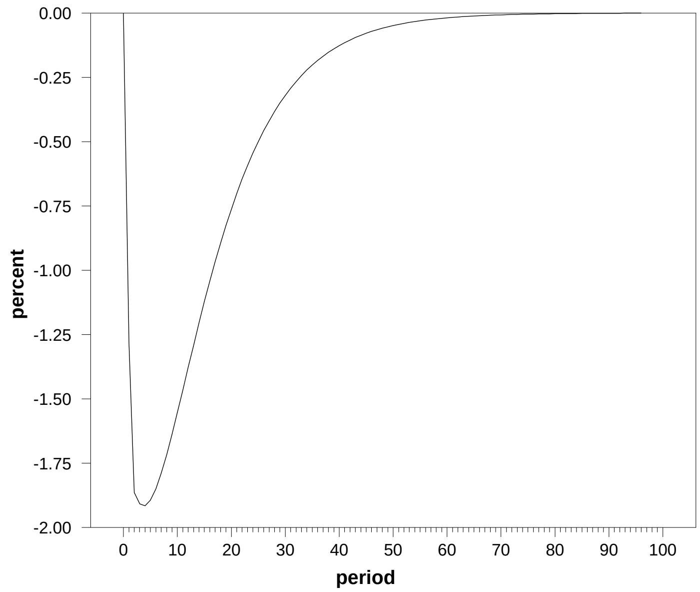

Figure 4.6: GDP Response to a 33% Price Shock

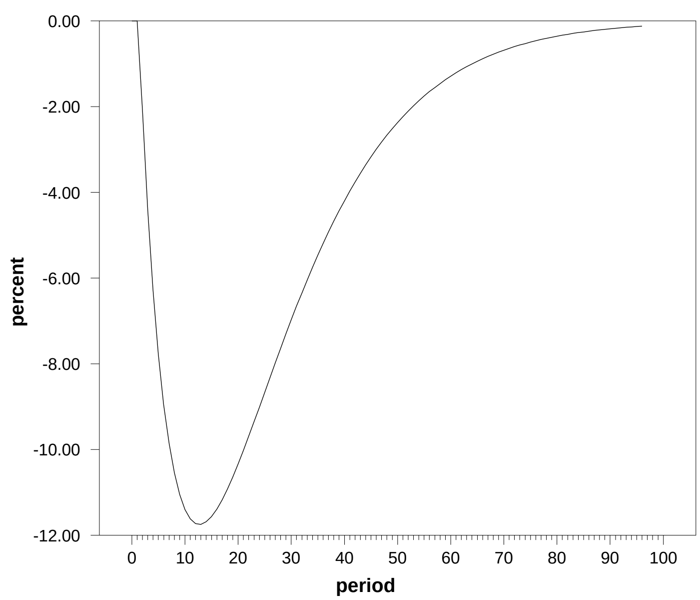

Figure 4.7: Energy Use Response to a 33% Price Shock

use. The trough comes fourteen periods after the initial shock and is 11.7% below the steady state level. After 100 periods energy use is still slightly below its steady state level. Because of the rigid nature of the capital stock in the model, energy use is slow to react to the price shock, and slow to return to its normal level.

The Impact of Energy Taxes

In this section I examine the steady state and dynamic impacts of energy tax policy. First, I show the impact of energy taxes on steady state values of GDP, energy use, capital types, and tax revenue. The model shows that energy taxes can achieve the goal of significantly reducing energy use, with much smaller impacts on GDP.

As described at the start of the chapter, I then investigate the implications of pursuing the energy reductions agreed to under the Kyoto Protocol. The provisions of that agreement required the U.S. to reduce greenhouse gas emissions to 93% of the amount generated in 1990. The most recent data are for 2008 and show this amounts to a 20% reduction. The present model uses a single energy type and does not allow for interfuel substitution (i.e. from coal to nuclear power), so a reduction in emissions is achieved only by reducing overall energy use. This can be achieved by levying taxes on energy, capital income, investment, or some combination.

Steady State Impact of an Energy Tax

To examine the impact of energy taxes on the model economy, I re-solve the steady state for τ^m in the [0,1] interval, in increments of 0.1. Figures (4.8) and (4.9) show the impact of the tax on GDP, capital type, energy use, and tax revenue.

The GDP plot shows the impact of an energy tax on GDP as percentage differences from the zero tax level. As the tax rate increases, GDP decreases. At $\tau^m = 1.0$, the net cost of energy is doubled, and steady state GDP is reduced by nearly 3.0%.[8] The effect of the tax on GDP is nonlinear, though it is not immediately obvious in the figure. As the tax is increased from zero to $\tau^m = 0.5$, GDP is reduced by 1.6%.

[8] This is less than the reduction reported by Atkeson & Kehoe (1999) of 5.5%. The reason behind the discrepancy is that I return the tax revenues to the agents in the economy.

But doubling that tax rate again to $\tau^m = 1.0$ results in a further reduction of just 1.4%. The decreasing effect is due to the substitution of capital for energy. As the tax increases the net cost of energy, a higher capital type v is optimal, and energy use is reduced. With no energy tax, steady state energy use is approximately 4.2% of GDP. As the energy tax increases, energy's share of output falls. At $\tau^m = 1.0$ when the cost is doubled, usage falls to 2.1% of GDP. Tax revenue is pictured as a share of GDP. Obviously at a zero tax there are no revenues. As the tax rate increases, higher revenues are generated. There are clearly diminishing marginal revenues, though, as the economy adjusts and uses less energy in the face of higher costs. At $\tau^m = 1.0$ when the net cost of energy is doubled, revenues are 2.2% of GDP.

Comparing Tax Rates

As noted at the start of this chapter, compliance with the Kyoto Protocol would require the U.S. to reduce greenhouse gas emissions by 20% from levels emitted in 2008. In the current model, there is a single energy type and the only method for reducing emissions is to reduce energy use. There are three tax rates in the model, and each of them could be used to achieve the reduction. As shown in Figure 4.9 the energy tax would need to be just higher than 20%. The precise figure is 22.9%, or $\tau^m = 0.229$.

To find the other tax rates, I reset $\tau^m = 0$ and solve the model two more times, the first using only τ^k. I find that a a capital income tax rate $\tau^k = .279$ reduces energy use by 20% in the steady state. In order to reduce energy use by 20% using only the investment tax, a rate of $\tau^x = .387$ is required. Table 4 shows the impact of those options as three separate steady states for comparison with the baseline, zero tax case. The first item to note is the identical impacts of using a capital income tax or an investment tax. This is not surprising given Equations (4.11) and (4.12). As noted above, for any τ^k there is a corresponding τ^x which affects those conditions identically. Here we see that, in the steady state, there is no difference between taxing capital income or investment.

The variation of economic impacts between the different taxes is dramatic. While

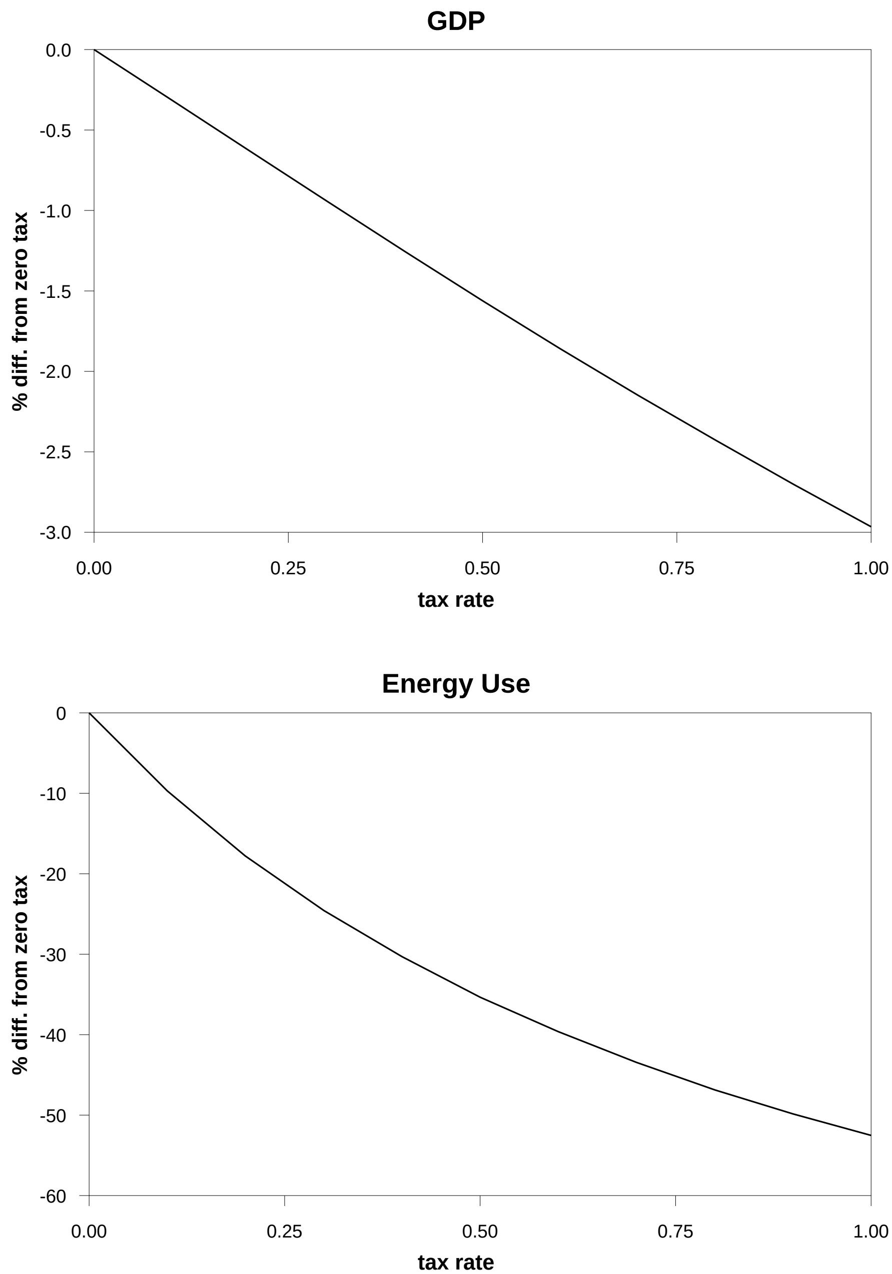

Figure 4.8: Steady State Impacts of an Energy Tax

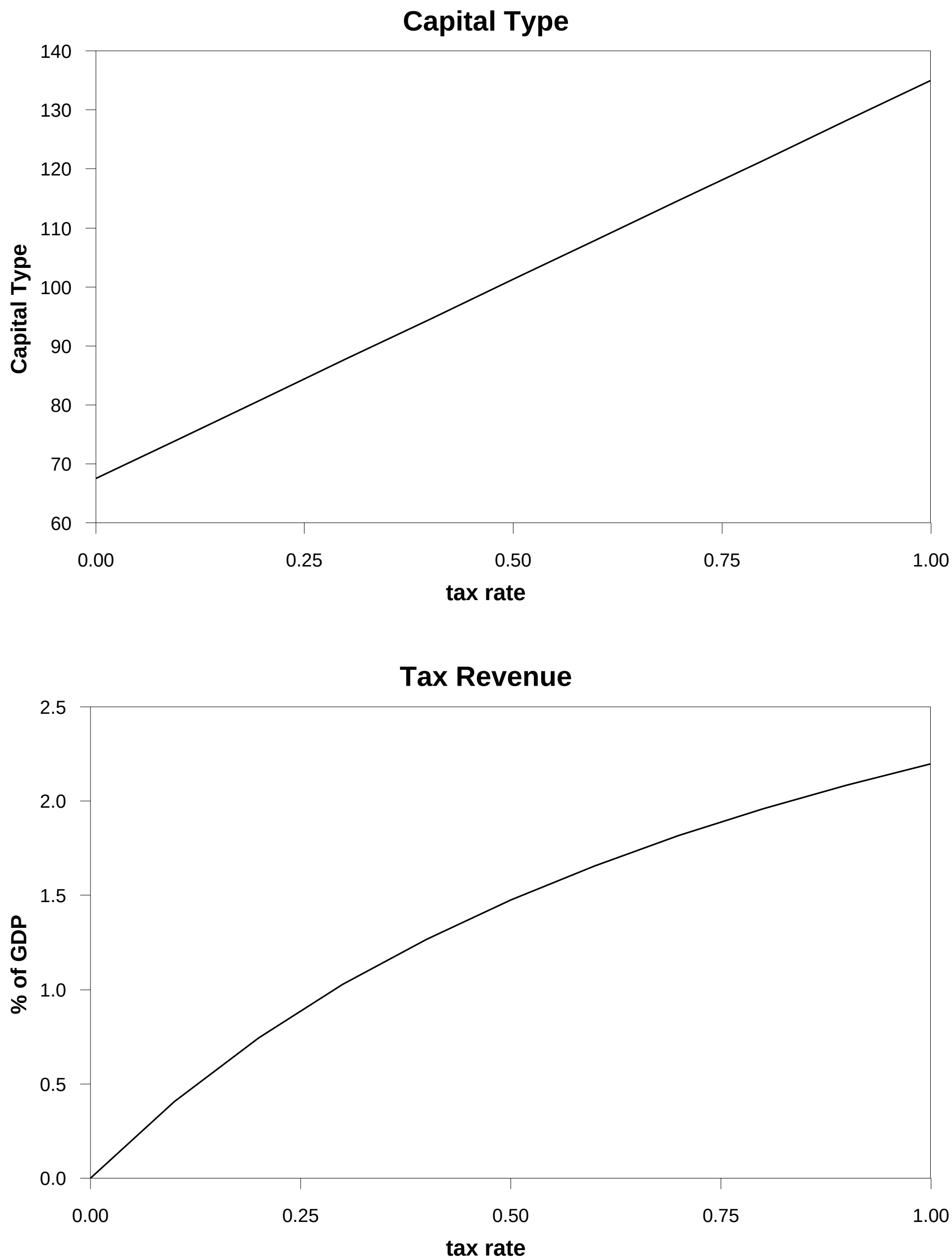

Figure 4.9: Steady State Impacts of an Energy Tax

	Baseline	$\tau^m = 0.229$	$\tau^k = 0.279$	$\tau^x = 0.387$
	level	% from base		
$\bar{p}(1+\tau^m)$	1.063	*22.9*	*0.0*	*0.0*
$\bar{v}$	67.51	*22.9*	*-27.9*	*-27.9*
$\bar{c}$	1.062	*-0.4*	*-11.0*	*-11.0*
$\bar{x}$	0.424	*-1.5*	*-42.3*	*-42.3*
$\bar{z}$	2.784	*-3.6*	*-40.3*	*-40.3*
$\bar{m}$	0.063	*-20.0*	*-20.0*	*-20.0*
$\bar{y}$	1.486	*-0.7*	*-20.0*	*-20.0*
k	4.243	*-1.5*	*-42.2*	*-42.2*

Table 4.2: Reducing Energy Use by 20% - Baseline levels and % Impacts

the tax on energy reduces GDP and consumption by 0.7% and 0.4%, respectively, those measures are reduced by 20.0% and 11.0% when capital income or investment is taxed. The impact on nearly all variables is similar. There is also a sharp difference between the optimal capital type under each tax. With $\tau^m = 0.229$, it is optimal to invest in capital with much higher energy efficiency compared to the baseline case. When taxes are applied to capital income or investment, the optimal choice is inefficient capital.

The reason for the result is straightforward. The most direct way to achieve a reduction in energy use is to make it more expensive by discouraging it through taxation, with τ^m. This prompts the use of more efficient capital but the distortion leads to some welfare loss. Discouraging energy use by making capital more expensive is indirect. A high tax is necessary to essentially drive down the use of capital and overall economic activity, including energy use.

Revenue Neutral Tax Rates

The results in Table 4 make clear that an energy tax is the least disruptive option among the three to achieve significant energy use reductions. In this section I examine the implementation of such a policy. In particular, the dynamic effects of the policy will be of interest to policymakers.

The stated goal of the policy is to reduce energy use, not to increase government revenues. It is sensible, then, to devise a revenue-neutral approach by offsetting the energy tax with a reduction in the capital income tax or a reduction in the investment tax (a subsidy).[9] To do this I derive three separate steady state solutions. In each steady state only one tax is used. Total tax revenues are equivalent across the three solutions. I use a 22.9% tax on energy to establish the baseline tax revenue level, as presented in the previous section. I then find the corresponding capital income tax ($\tau^k = 0.0268$) and investment tax ($\tau^k = 0.0303$) which provide revenue equivalent to an energy tax of $\tau^m = 0.229$. The associated steady state solutions are presented in Table 4. As can be calculated using values from the table, the total tax revenues under each rate is 0.012, or approximately 0.8% of GDP.

In the first column, the energy tax pushes the optimal capital type dramatically higher than when energy is not taxed. Welfare is best measured by GDP and consumption. Both measures are highest in the first column. When energy is taxed GDP is approximately 0.7% higher than when capital income is taxed and 1.3% higher than when investment is taxed. Consumption is 0.2% and 0.4% higher, respectively. Investment is highest when energy is taxed. The reason for this is two-fold. First, investment is discouraged when capital income or investment is taxed. Second, when energy is taxed, the optimal capital type is significantly higher. With a higher capital type, a higher level of investment is needed to achieve comparable levels of output. Reviewing the state variables, that higher level of investment when energy is taxed

	$\tau^m = 0.229$	$\tau^k = 0.02068$	$\tau^x = 0.0303$
$\bar{p}(1+\tau^m)$	1.306	1.063	1.063
$\bar{v}$	82.97	66.12	65.53
$\bar{c}$	1.058	1.056	1.053
$\bar{x}$	0.4178	0.4097	0.4036
$\bar{z}$	2.686	2.694	2.656
$\bar{m}$	0.0503	0.0620	0.0616
$\bar{y}$	1.476	1.466	1.457
$\bar{k}$	4.178	4.097	4.036

Table 4.3: Revenue Neutral Tax Rates and Steady States

understandably leads to a higher capital stock. The stock is 2.0% higher than when

[9] Note that all tax rate manipulations in the model can be viewed as marginal changes from existing taxes. That is, when $\{\tau^m, \tau^k, \tau^x\} = \{0, 0, 0\}$ in the current model, these can be interpreted as *zero deviations* from existing tax rates in the U.S. which are embodied in the data and parameters.

capital income is taxed, and 3.5% higher than when investment is taxed. The largest observable difference is energy use. When it is taxed at the high rate of 22.9%, energy use is nearly 20% lower than both of the other scenarios, achieving the reduction desired by the policy.

Transition from Capital Tax to Energy Tax

Next I investigate the transition dynamics when the economy shifts from one tax to another. To model the transition, I repeat the process of constructing log linear approximations of the decision functions for each choice variable. The assumed form of the linear decision functions is identical, but the results may differ with each tax rates because the steady state levels of each variable is different. This is to be expected. For example, imagine one economy where all taxes are levied on capital and another where all taxes are levied on energy use. Now suppose each of those two economies find themselves with a capital stock 1% below their respective steady state levels. It is natural to expect their optimal investment strategies to be slightly different in their effort to return to their steady state levels. This would manifest itself here as different a_{ij}'s for each tax regime.

Table 4 shows the decision function coefficients for an economy under the energy tax of 22.9%. These are very similar to the coefficients associate with no taxes in Table 4.1, with two exceptions: a_{cp} and a_{vm}. The first measures the reaction of consumption to deviations in price and the second shows how the optimal capital type reacts to deviations in energy use. Both show less sensitivity under an energy tax regime compared with no taxes. These are the natural results coming from an economy which, under higher energy taxes, is not as reliant on energy use and less elastic to fluctuations in its price and use. To simulate an economy's transition from

	$\tilde{p}$	$\tilde{z}$	$\tilde{m}$
$\tilde{c}$	-0.002	0.855	-0.035
$\tilde{v}$	0.759	0.362	-0.007
$\tilde{x}$	-0.121	-0.592	-0.039

Table 4.4: Decision Function Coefficients $\tau^m = 0.229$, $\tau^k = 0.0$, $\tau^x = 0.0$

a capital income tax to an energy tax, I set up the economy in such a way that the steady state will correspond to the first column of Table 4 where there is a tax on energy. I set the a_{ij}'s to those shown in Table 4. The last settings for the simulation are the initial conditions of the state variables z, m, and k. I set the initial conditions corresponding with the *second column* of Table 4, when there is a tax on capital income. Once the simulation starts, the economy "recognizes" that it is away from the optimal steady state under an energy tax. The economy then chooses v, c, and x as described by equations (4.13) and the coefficients in Table 4 until the new steady state is reached. The results of the simulation are shown in Figure 4.11. The first thing to note is the immediate and permanent change to the *capital type*. In the face of a permanent higher cost of energy it is optimal to begin investing in higher efficiency capital right from the start. The level of *investment* reacts quickly too, increasing 1.0% in the initial period. It continues to rise and has jumped 1.5% after five periods, and then slows as it approaches its new steady state approximately 2.0% higher than the previous steady state.

The immediate reaction of investment in new capital comes at the expense of *consumption*, which declines 0.25% in order to give up resources for investment. Consumption continues its decline in ensuing years, reaching a trough of 0.31% four periods after policy implementation. Thereafter consumption increases and returns to its pre-policy level after 20 years on its way to a new steady state of 0.2% above the initial level. As indicated by the steady state comparison, *GDP* will eventually be slightly higher (0.7%) under the new energy tax compared to the capital income tax. But the transition to the new steady state is lengthy, requiring in excess of forty years to approach the new level.

The structure of the model is such that although the choice variables react sharply to the new tax regime, the state variables evolve only slowly. This can be seen in *energy use* and the *capital stock*. The goal of the new energy tax is to reduce energy use, which is accomplished, but over a lengthy period. Use is reduced by 12% in the first ten years, but progress is slower as more years pass. After twenty years energy use has declined by 16.5%. After fifty years the economy is nearly converged to its

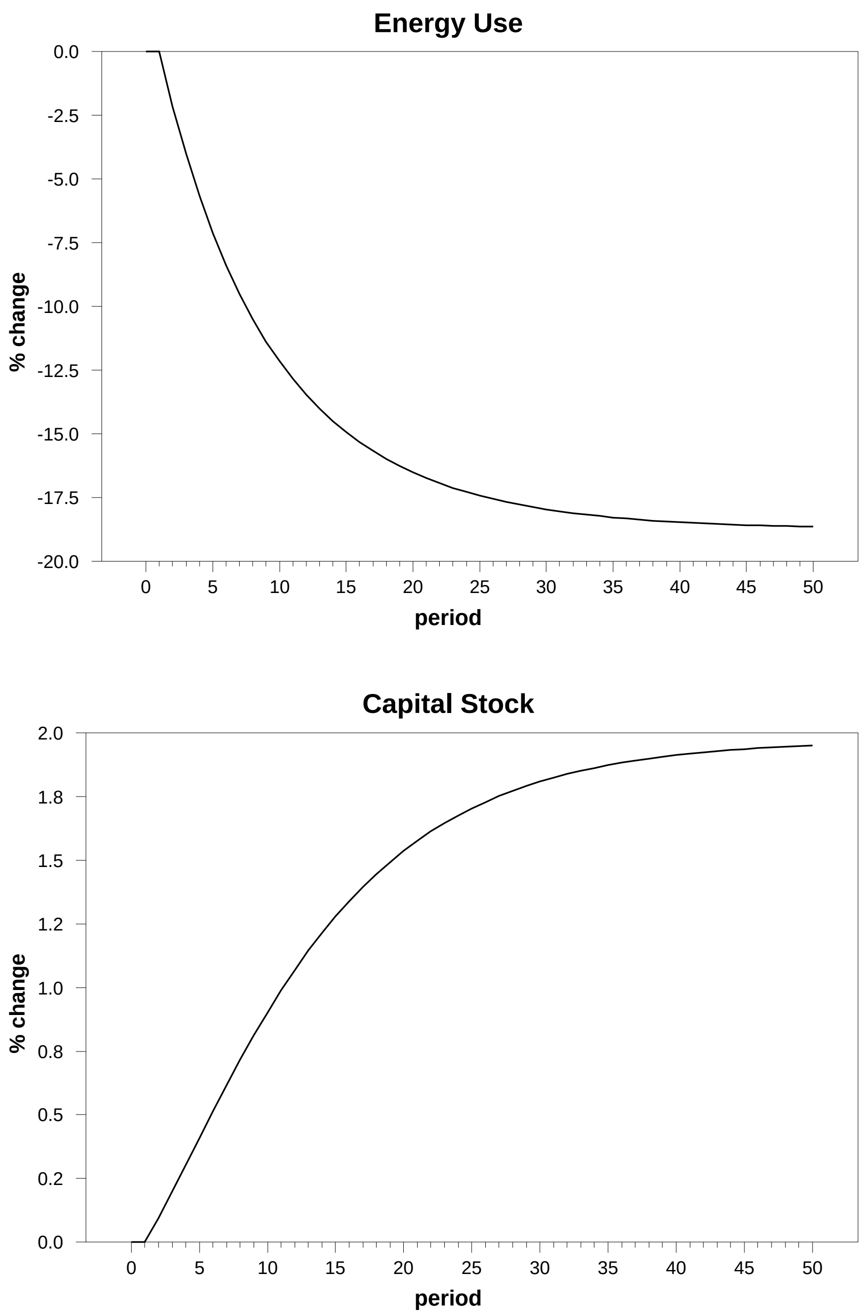

Figure 4.10: Shift from Capital Income Tax to Energy Tax

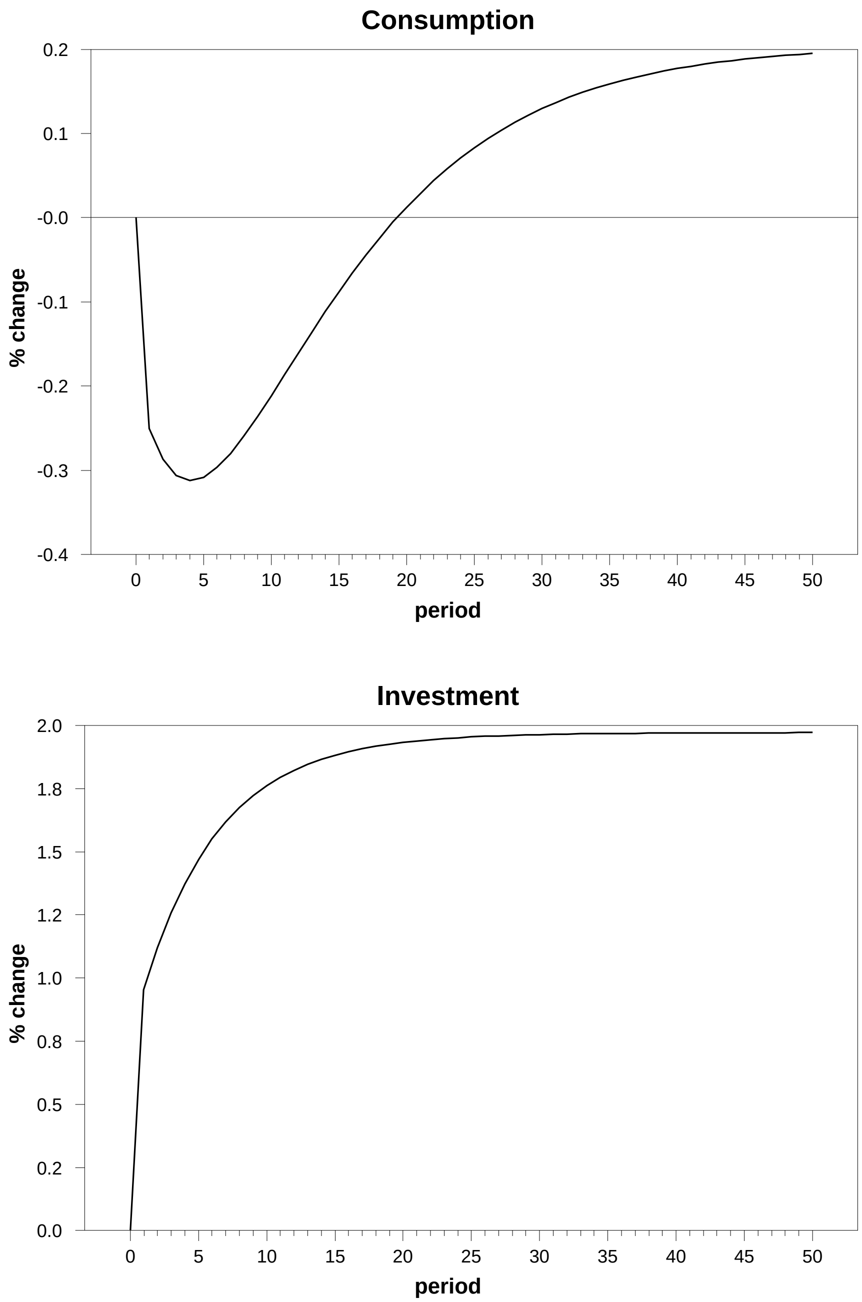

Figure 4.11: Shift from Capital Income Tax to Energy Tax

new equilibrium for energy use. The capital stock evolves slowly as well, converging to its new steady state of 2.0% higher after fifty years.

Lastly, total government *tax revenues* are increased when the new tax regime is implemented. Recall that the design of the simulation is to implement a revenue-neutral change in tax rates. It is clear from Figure 4.11, though, the tax change is not revenue neutral during the transition. At implementation, the new energy tax and elimination of the capital income tax yields a 23% increase in total revenue in the first period. As the economy transitions its capital stock and energy usage, revenue falls and returns to its steady state level after fifty years. Over the transition period the total excess revenue generated from the change in tax structure is the area under the curve. That area has a value of 2.13, so a cumulative increase of 213% above the annual, steady state value of tax revenue. The cumulative increased tax burden has a numerical value of .026, approximately 1.7% of annual GDP.

Transition from Investment Tax to Energy Tax

Figures 4.12 and 4.13 show the transition from an investment tax to an energy tax. As would be expected, the transition paths are very similar. The exception is in the choice variables of *investment* and *consumption*. In this simulation, investment reacts more swiftly than when the transition comes from a capital income tax regime. This is because an investment tax is being lifted here, leading to the immediate reaction. The impact on consumption is similar, but that variable resumes growth immediately afterwards whereas it spends four years declining in the previous simulation. Generally the impacts on the state variables of GDP, energy use, and the capital stock are the same, as is the dynamics and magnitude of tax revenues.

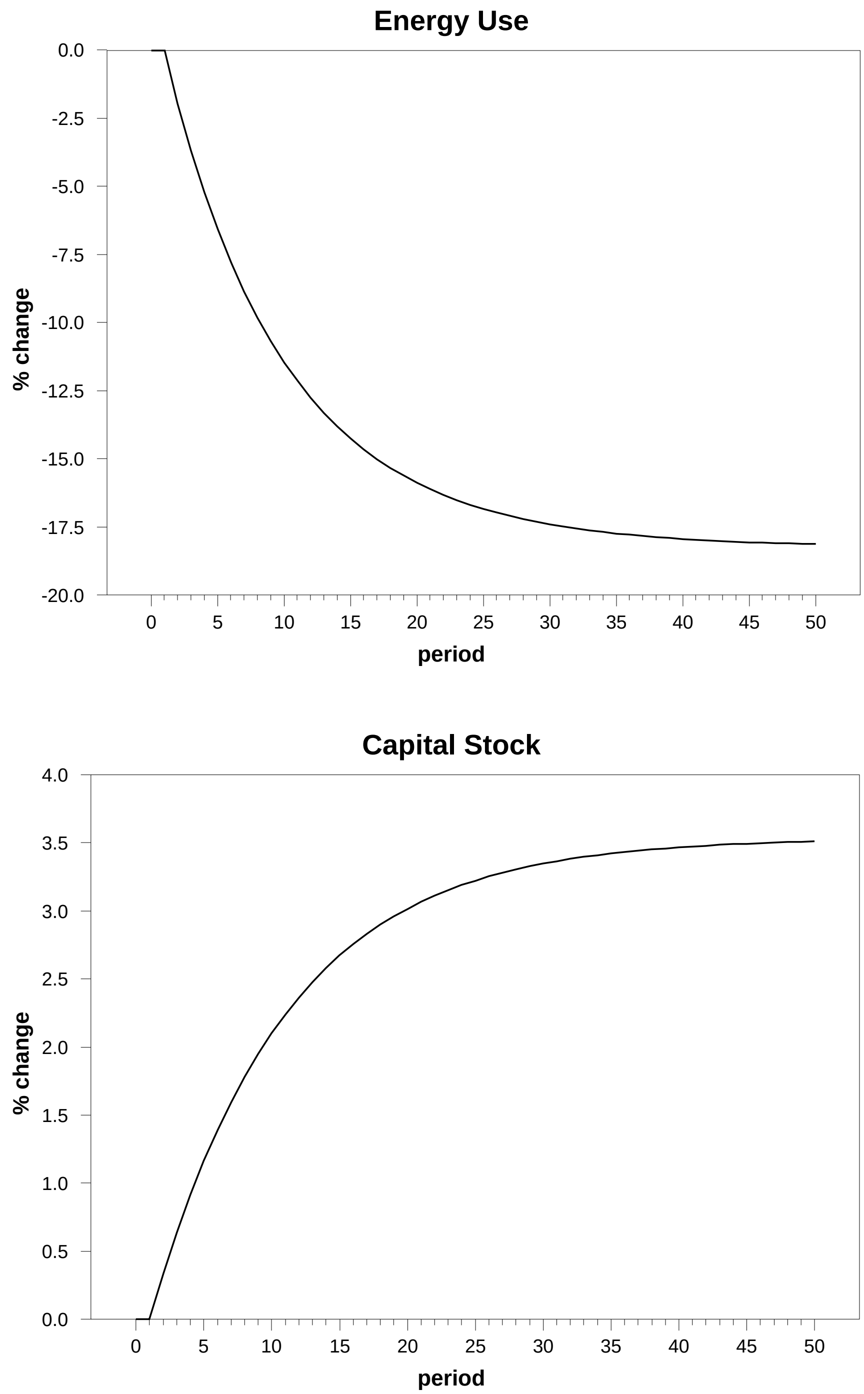

Figure 4.12: Shift from Investment Tax to Energy Tax

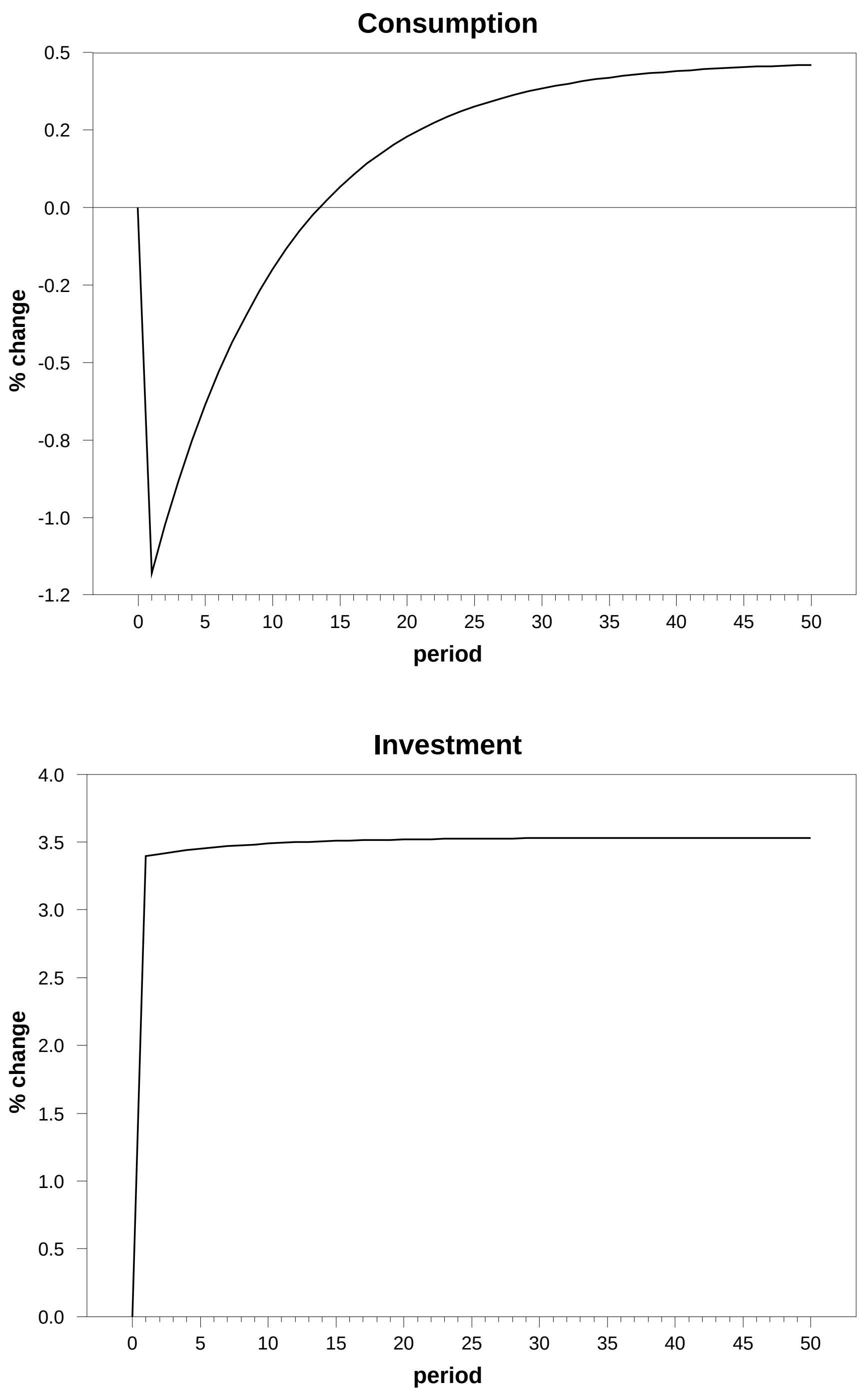

Figure 4.13: Shift from Investment Tax to Energy Tax

CHAPTER 5
CONCLUSION

The issues of energy use, energy taxation, and the ensuing impacts on the macroeconomy are at the forefront of public policy debate. Over the past decade the scientific community has increasingly endorsed the conclusion that human actions, mostly the use of fossil fuels, is causing climate change. As a result, there has been growing support both in the United States and abroad for adopting policies to reduce greenhouse gas emissions. Although there has been increasing accord on the causes of climate change, there remains a wide dispersion on the projected macroeconomic effects of climate change policy.

The academic and professional literature on the topic project impacts that vary greatly. While some of the disagreement can be attributed to researchers modeling different scenarios, one would expect some convergence when modeling a common scenario such as implementation of the Kyoto Protocol which would have required the United States to reduce total greenhouse gas emissions to 7% below those emitted in 1990. But a survey of published reports finds projected impacts on GDP vary by a factor of more than eight.

One reason for the wide disagreement is the variety of ways in which researchers model the interaction of energy and capital in the production function. At one extreme is complete rigidity of the capital-energy input ratio. At the other is complete flexibility of the ratio in response to relative price changes. One class of models strikes a balance between the two, with short-run rigidity and long-run flexibility. This approach makes use of heterogeneous capital types.

In this dissertation I develop a model with capital types to examine the impact of carbon mitigation policies on a model economy that imports all of its energy. There is a single type of energy which is used in conjunction with an array of capital types differentiated by their energy intensity. High capital types require less energy but are more costly. Once investment in capital has occurred, the energy requirement is fixed. This modeling structure well replicates the empirically observed high complementarity of energy and capital in time series data and their substitutability in cross section data.

The use of capital types leads to the curse of dimensionality, which is handled by simplifying the model. I develop a social planner version of the model with an energy tax, capital income tax, and investment tax which mimics the behavior of a competitive equilibrium of households and firms.

The model shows that significant energy reductions can be achieved with relatively low impacts on welfare measurements such as GDP and consumption. An energy tax of 22.9% reduces aggregate energy use by 20% with reductions of 0.7% of GDP, and 0.4% of consumption. Importantly, though, the economy's rigid capital stock leads to a very long transition period, in which the change in consumption is negative for as much as twenty years. If the energy tax is compensated for by removing a capital income tax of 2.068% or an investment tax of 3.03%, the net effect on GDP and consumption is positive.

It is also possible to achieve the 20% reduction in energy use by taxing either investment or capital. A capital tax of 28% or an investment tax of 39% achieves the reduction, but with disastrous and identical effects on GDP (-20%) and consumption (-11%). These taxes work only indirectly on energy use, so the only way to achieve the reduction is to push down overall output so low that far less energy is needed.

The model's implications for carbon mitigation policy is that an energy tax is an effective method for achieving significant reductions in energy use, and the pairing with a capital income tax reduction may be an attractive option. However, the long transition period is necessary and is certainly longer than political election cycles, making commitment to such a policy difficult. Energy reduction policies can be successful in reducing greenhouse gas emissions, but they should not be oversold, as there will need to be sacrifices made in terms of consumption to accommodate higher investment.

Lastly, the model presented and this entire dissertation is silent on the distributional impacts of carbon mitigation policies. When viewing the model variables in aggregate, it is fairly straightforward that an energy tax combined with a reduction in the capital income tax is an effective tool for reducing energy with net benefits for the economy GDP and consumption. But the model says nothing about the distribution of those benefits. In an economy such as the U.S., the imposition of an energy tax undoubtedly affects all agents while the ameliorating effects of the lightened capital income tax burden might be enjoyed by a smaller share of the population. If policy-makers wish to manage the impacts on different segments of society, this model does not help address those issues.

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Appendices

APPENDIX A
DATA

In this appendix I describe my sources for raw data and the procedure for developing my full dataset. Nearly all data comes from two sources: the Bureau of Economic Analysis[1] (BEA) and the Energy Information Administration[2] (EIA). To create the energy price, use, and expenditure series, I follow the method of Atkeson and Kehoe(1999). For the national accounts data procedures (output, capital stock, consumption, etc.) I follow the method developed by Cooley
(1995).

Energy Data

Energy price and usage data comes from the Annual Energy Review 2008 (AER) published by the EIA. The usage data covers consumption of coal, petroleum, natural gas, and electricity by the residential, commercial, industrial, transportation, and power generation sectors. It is published in terms of heat rate, or British Thermal Units (BTUs). The power generation sector both consumes and produces energy. It consumes coal, natural gas, and petroleum and produces electricity to be consumed by the other sectors. Simply adding together all consumption would then result in some double counting. In order not to double count the energy consumed and produced by the electrical power sector, I remove the consumption of fossil fuels by that sector.

The price data is more straightforward. I use nominal producer price data as published in the AER. These prices are published in cents per kilowatt hour (KwH). A conversion is necessary to conform with the usage data, which is in billions of BTUs. I convert the price data from cents per KwH to dollars per million BTU using the standard heat rate conversion.[3]

I create nominal total energy expenditure series for each fuel using the usage and price series. That is, they are the simple products of usage in terms of BTUs, and

[1]U.S. Department of Commerce.
[2]U.S. Department of Energy.
[3]The conversion is 3,412 BTUs per KwH.

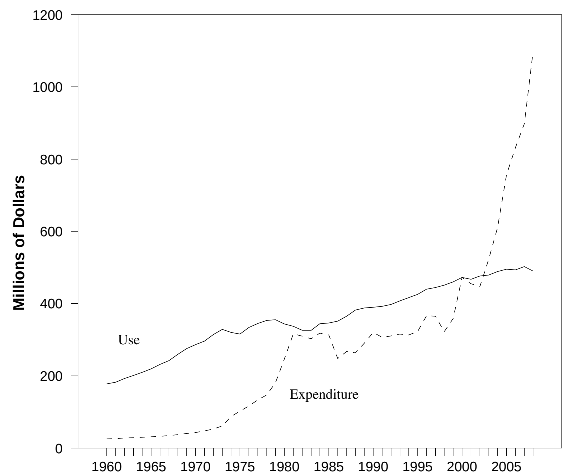

Figure A.1: Energy Use and Expenditure 1960-2008

Notes: The Expenditure and Use series are shown here with millions of dollars on the vertical axis. The Expenditure series is nominal and the Use is in terms of year 2000 dollars.

nominal prices in terms of BTUs. The sum of expenditures by fuel yields a *total energy expenditure* series.

I create a single, total energy usage series in constant dollar terms using the total usage series and the nominal price in the year 2000 for each type of fuel. I use 2000 as the base year, and sum each of the series to create the *total energy usage* series. Finally, a single price series for energy is needed. To do this I take the ratio of the *total energy expenditure* series and the *total energy usage* series. This is, then, an energy price deflator with 2000 as the base year and a measure of the nominal energy price. I deflate this nominal energy price series using the Implicit Price Deflator from the BEA to generate the real energy price, p.

Energy Estimation

The energy price is determined exogenously, and follows an ARMA(1,1) process

$$log(p_{t+1}) = (1-\rho)log(\overline{p}) + \rho_t log(p_t) + \eta\epsilon_{p,t-1} + \epsilon_{p,t}. \tag{A.1}$$

I estimate Equation (A.1) and get the following parameters

$$\rho = 0.93,$$

$$\eta = 0.42.$$

Macroeconomic Data

Most of the necessary macroeconomic data on capital stock, output, investment, and consumption is provided by the BEA National Income and Product Accounts (NIPA) program. There are two necessary adjustments, though. The first is to augment the output data with the flow of services that consumers receive from energy using durable goods. The second is the simple removal of the energy producing sector from the aggregate figures.

The augmentation of consumer output is necessary because households are significant energy users in the economy. Consumers respond to energy prices in the same way firms do, by investing accordingly from the menu of energy efficient products. High energy prices lead consumers to invest more heavily in energy efficient homes and appliances, for example. NIPA does publish investment and the total stock of consumer durables, but there is no reflection of the services that consumers yield from these goods. In short: the data to this point show the costs of energy investment and energy consumption, but the BEA's measure of GDP does not include the benefits.

I follow Cooley(1995) in estimating the output from consumer durables. The necessary steps are to calculate the implied, economy-wide interest and depreciation rates from known variables. I derive the interest rate series using the NIPA data for output, capital stock and investment. The depreciation rate for consumer durables is derived from the corresponding series for investment and stock. I then calculate the flow of services from consumer durables using those interest and depreciation rates. This imputed flow of services from consumer durables is then added to the published GDP figures to create a total output series.[4]

The second data adjustment is to remove the output and capital stock figures of the energy producing sector. Including them would amount to double-counting, as

[4]This procedure is explained in more detail in the next section.

my interest is to investigate how the rest of the economy responds to changes in prices paid to that sector. I simply subtract the output and capital stock of the sector from the economy-wide totals.

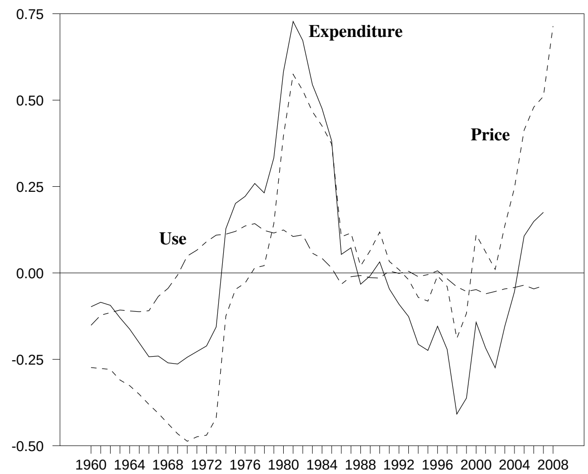

Figure A.2: Energy Price, Use, and Expenditure 1960-2008

Notes: All series are displayed as logarithms and normalized to their mean value. The Use series is shown as a ratio to real GDP. The Expenditure series is shown as a ratio to nominal GDP. The Price series is the implicit energy price deflator in 2000 dollars divided by the GDP implicit price deflator.

Figure 4.3 exposes the most interesting features of the data. Each of the series are logarithms and normalized to their mean value. The price series is the implicit energy price deflator in 2000 dollars divided by the GDP deflator. As would be expected, the price series displays massive spikes in 1973 and again in 1979. These spikes correspond to the OPEC oil embargo and the Iranian Islamic Revolution, respectively. In the following two decades, energy prices fell precipitously. More recently, energy prices have once again started to rise, and is reflected in the graph.

Energy expenditures in Figure 4.3 are shown as nominal expenditures on energy as a share of nominal GDP. As might be expected, the behavior of energy expenditures tracks closely with energy prices. The economy could not make significant short run changes to the existing capital stock in response to sudden price changes. The economy must purchase enough energy to operate its capital stock, even at high prices.

Energy use is shown as the ratio of total usage to GDP. The most striking feature of the series is the low volatility compared with prices and expenditures. It is clear, though, that energy use changed over time. During the period from 1960 to 1973, energy use as a share of GDP is increasing. The energy price spikes of the 1970's, though, led to investment in capital with better energy efficiency, and the series flattened before starting a downward trend. This reveals the feature of interest here: high energy prices do indeed prompt investment in energy-efficient capital at the margin, but this moves the economy's total energy efficiency slowly. The difference in volatility can also be seen in Table A. The standard deviations reveal the higher volatility of the price and expenditure series, compared with energy usage.

Variable	*Mean*	σ
Price	1.063	.354
Expenditure	5.78	1.78
Use	5.47	1.02

Table A.1: Descriptive Statistics

Notes: Calculations are performed on the original series, not the demeaned logarithms shown in the Figures. The mean of the Price series is somewhat meaningless, as it is the average ratio of two price indices. The standard deviation shows volatility. Both Expenditure and Use show shares. Nominal energy expenditures in the data set are on average 5.78% of nominal GDP, and real energy usage is on average 5.47% of real GDP.

The underlying cause behind the behavior of the Use series is the capital-energy ratio of the economy, which is presented in Figure A.3. This figure shows the ratio declining from 1960 to 1973 as the economy was investing in less efficient capital. That trend reverses after the price spikes of the 1970s. Also worth noting is the stabilization of the ratio in the 1980 to 1995 period. This marked a period when energy prices were falling as a result of decreased tensions in the Middle East and production from major, new fossil fuel deposits found outside that volatile region. Prices began to rise again in 1995-2008, and the capital energy ratio followed suit.

Flow of Output from Consumer Durables

Generally, the task is to impute the value of services derived from the stock of consumer durables. To do this, I assume that these services are generated in the same way that services are derived from the privately held capital stock in the economy.

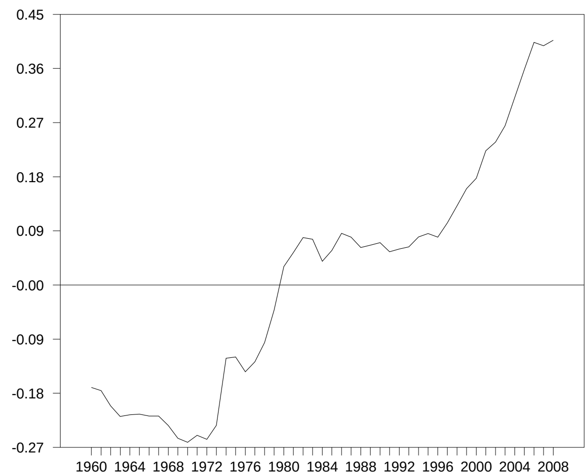

Figure A.3: Capital Energy Ratio 1960-2008

Notes: The series is displayed as a logarithm and normalized to its mean value.

The privately held capital stock K generates income

$$Y_P = (i + \delta_P)K_P, \tag{A.2}$$

where P denotes *Private*, i is the interest rate, and δ is the depreciation rate. Each term in Equation A.2 is calculable from the NIPA data. For consumer durables, the capital stock, K_C is reported in the NIPA data, and the depreciation rate, δ_C, is implied by the K_C and investment in consumer durables. An interest rate is not available. I assume that the interest rate for consumer durables is the same as the private sector, and I derive Y_C. That calculated output is then added to Gross Domestic Product as reported in NIPA.

The first task is to calculate the interest rate implied by the data. The income data, though, needs a slight adjustment because of the ambiguity of two sources of income: proprietor's income and the small measurement discrepancy between *Net National Product* and *National Income*. Proprietor's income is certainly earned by capital to some degree, but some might be earned by the very consumer durables that I am attempting to treat here. A similar argument applies to the unknown income that is the difference between national product and income.[5] I assume that the share

[5]The measurement error between National Product and National Income is very small. It's

of these two income categories that is earned by private capital is the same as the economy-wide share earned by private capital. Let θ_P represent the share of income earned by private capital

$$\begin{aligned} Y_P =& \theta_P GNP \\ =& (YRENT + YPROF + YNINT) + \\ & \theta_P(YPROP + NNI - NP) + CFC, \end{aligned} \tag{A.3}$$

where the first three terms are rental income, corporate profits, and interest income and are all unambiguously earned from the private capital stock. The last term is *consumption of fixed capital* or depreciation. It is straightforward to solve the above for θ_P

$$\theta_K = \frac{(YRENT + YPROF + YNINT) + CFC}{GNP - (YPROP + NNI - NP)},$$

and apply this equation to the data. My results show a reasonable return for capital over the 1960 to 2008 period. Capital earns 28.5% of total income, and the relationship is fairly stable with a standard deviation of 1.3%. I then calculate the total income earned by capital (Y_P), by multiplying its share by total income

$$Y_P = \theta_K \cdot GNP,$$

and the interest rate is calculated using Equation A.2. My calculations yield reasonable results for the interest rate, which averages 6.1% over 1960 to 2008 with a standard deviation of 0.7%.

The last item needed is a depreciation rate for the consumer durables sector, δ_C. The NIPA reports the stock of consumer durables as well as purchases of new consumer durables, so the depreciation rate is implied by the familiar capital accumulation law of motion

treatment here does not affect results, but I make the calculations for completeness.

$$K_{C,t+1} = (1 - \delta_C)K_{C,t} + X_{C,t}.$$

My estimates of the consumer durables depreciation rate for 1960 to 2008 yield an average of 17.6% and a standard deviation of 2.8%. The depreciation rate is higher than we typically see for capital in macroeconomic data. But consumer durables as reported in the NIPA include any consumer goods intended to last more than three years. A relatively higher depreciation rate is expected. Lastly, we apply the data to Equation A.2 to calculate the flow of services from consumer durables, and add this calculation to GDP as reported in the NIPA.

The augmentation of GDP with the flow of services from consumer durables has a noticeable but not overwhelming effect. Total GDP for the period is increased by an average of 5.5% with a standard deviation of 0.86%.

APPENDIX B
LOG LINEARIZATION

In this appendix I show the log linearization carried out to approximate the decision functions around the steady state. The method is originally developed by King, Plosser, and Rebelo 1988 and used extensively in the literature. For more discussion of the method see Uhlig 1995. The three equations are (4.8), (4.9), and (3.16). For each variable I replace the time subscripts t and $t+1$ with 1 and 2, respectively

$$c_1 \equiv c_t,$$
$$c_2 \equiv c_{t+1}.$$

Let a tilde symbol represent the log deviation of a variable from its steady state value

$$\tilde{c_i} \equiv log(c_i) - log(\bar{c}) \quad for \quad i = 1, 2.$$

The log deviation is approximately equal to the percent deviation.

$$c_i = \bar{c}e^{\tilde{c_i}} \approx \bar{c}(1 + \tilde{c_i}).$$

The First Euler Equation

The first Euler equation is

$$u_c(t) = \beta E_t \left[u_c(t+1) \left[g_z(t+1)(1 - \tau^k_{t+1}) + \frac{1 + \tau^x_{t+1}}{f_v(t+1)}(1 - \delta) \right] \right] \frac{f_v(t)}{1 + \tau^x_t}.$$

Using the functional forms and parameters from Chapter 4 as well as the substitutions above, the Euler becomes

$$\frac{1}{c_1} = \beta \left[\frac{1}{c_2} \left[\alpha z_2^{\alpha-1}(1 - \tau^k_2) + \frac{1 + \tau^x_2}{\gamma v_2^{\gamma-1}}(1 - \delta) \right] \right] \frac{\gamma v_1^{\gamma-1}}{1 + \tau^x_1}.$$

Rearranging, applying the approximation method above and simplifying gives

$$(1+\tau_1^x)[c_2][\gamma v_2^{\gamma-1}] = (1-\tau_2^k)\beta[c_1][\alpha z_2^{\alpha-1}][\gamma v_2^{\gamma-1}][\gamma v_1^{\gamma-1}] + (1+\tau_2^x)(1-\delta)\beta[c_1][\gamma v_1^{\gamma-1}],$$

$$(1+\tau_1^x)[\bar{c}e^{\tilde{c_2}}][\gamma(\bar{v}e^{\tilde{v_2}})^{\gamma-1}] = (1-\tau_2^k)\beta[\bar{c}e^{\tilde{c_1}}][\alpha(\bar{z}e^{\tilde{z_2}})^{\alpha-1}][\gamma(\bar{v}e^{\tilde{v_2}})^{\gamma-1}][\gamma(\bar{v}e^{\tilde{v_1}})^{\gamma-1}] + (1+\tau_2^x)(1-\delta)\beta[\bar{c}e^{\tilde{c_1}}][\gamma(\bar{v}e^{\tilde{v_1}})^{\gamma-1}],$$

$$\begin{aligned}&(1+\tau_1^x)[\bar{c}(1+\tilde{c_2})][\gamma\bar{v}^{\gamma-1}(1+\tilde{v_2}(\gamma-1))]\\ &= (1-\tau_2^k)\beta[\bar{c}(1+\tilde{c_1})][\alpha\bar{z}^{\alpha-1}(1+\tilde{z_2}(\alpha-1))][\gamma\bar{v}^{\gamma-1}(1+\tilde{v_2}(\gamma-1))][\gamma\bar{v}^{\gamma-1}(1+\tilde{v_1}(\gamma-1))]\\ &+ (1+\tau_2^x)(1-\delta)\beta[\bar{c}(1+\tilde{c_1})][\gamma\bar{v}^{\gamma-1}(1+\tilde{v_1}(\gamma-1))].\end{aligned} \tag{B.1}$$

The Second Euler Equation

The second Euler equation is

$$\begin{aligned}&u_c(t)(1+\tau_t^x)\left[v_t - \frac{f(v_t)}{f_v(t)}\right]\\ &= \beta E_t\left[u_c(t+1)\left[\left[v_{t+1} - \frac{f(v_{t+1})}{f_v(t+1)}\right](1-\delta)(1+\tau_{t+1}^x) - p_{t+1}(1+\tau_{t+1}^m)(1-\tau_{t+1}^k)\right]\right].\end{aligned}$$

Implementing the functional forms and substitutions yields

$$(1+\tau_1^x)\frac{1}{c_1}v_1\left(1-\frac{1}{\gamma}\right) = \beta\left[\frac{1}{c_2}\left[v_2\left(1-\frac{1}{\gamma}\right)(1-\delta)(1+\tau_2^x) - p_2(1+\tau_2^m)(1-\tau_2^k)\right]\right].$$

Rearrange, log linearize and simplify

$$(1+\tau_1^x)c_2v_1\left(1-\frac{1}{\gamma}\right) = \beta c_1 v_2\left(1-\frac{1}{\gamma}\right)(1-\delta)(1+\tau_2^x) - \beta c_1 p_2(1+\tau_2^m)(1-\tau_2^k),$$

$$(1+\tau_1^x)[\bar{c}e^{\tilde{c_2}}][\bar{v}e^{\tilde{v_1}}]\left(1-\frac{1}{\gamma}\right) = \beta[\bar{c}e^{\tilde{c_1}}][\bar{v}e^{\tilde{v_2}}]\left(1-\frac{1}{\gamma}\right)(1-\delta)(1+\tau_2^x) - \beta[\bar{c}e^{\tilde{c_1}}][\bar{p}e^{\tilde{p_2}}](1+\tau_2^m)(1-\tau_2^k),$$

$$(1+\tau_1^x)[\bar{c}(1+\tilde{c_2})][\bar{v}(1+\tilde{v_1})]\left(1-\frac{1}{\gamma}\right) = \beta[\bar{c}(1+\tilde{c_1})][\bar{v}(1+\tilde{v_2})]\left(1-\frac{1}{\gamma}\right)(1-\delta)(1+\tau_2^x) - \beta[\bar{c}(1+\tilde{c_1})][\bar{p}(1+\tilde{p_2})](1+\tau_2^m)(1-\tau_2^k). \tag{B.2}$$

The Feasibility Constraint

The feasibility constraint is

$$g(z_t)(1-\tau_t^k) - m_t p_t(1+\tau_t^m)(1-\tau_t^k) = c_t + x_t(1+\tau_t^x).$$

Implementing the functional forms and substitutions yields

$$z_1^\alpha(1-\tau_1^k) - m_1 p_1(1+\tau_1^m)(1-\tau_1^k) = c_1 + x_1(1+\tau_1^x).$$

Log linearize and simplify

$$\left[(\bar{z}e^{\tilde{z1}})^\alpha\right](1-\tau_1^k) - [\bar{m}e^{\tilde{m1}}][\bar{p}e^{\tilde{p_1}}](1+\tau_1^m)(1-\tau_1^k) = [\bar{c}e^{\tilde{c_1}}] + [\bar{x}e^{\tilde{x1}}](1+\tau_1^x),$$

$$\begin{aligned}\left[\bar{z}^{\alpha}(1+\tilde{z1}\alpha)\right](1-\tau_1^k)-[\bar{m}(1+\tilde{m1})][\bar{p}(1+\tilde{p_1})](1+\tau_1^m)(1-\tau_1^k)&\\ =[\bar{c}(1+\tilde{c_1})]+[\bar{x}(1+\tilde{x1})](1+\tau_1^x).&\end{aligned}\tag{B.3}$$

Equations (B.1) and (B.2) are the linearized versions of optimal behavior about steady state values. I use them with Equation (B.3) to estimate linear decision functions in Chapter 4.

APPENDIX C

IMPULSE RESPONSE

This appendix shows the responses of choice and state variables to price shock of one standard deviation (33%). In each graph, the horizontal axis shows the number of periods post-shock and the vertical axis shows percent deviation from the steady state.

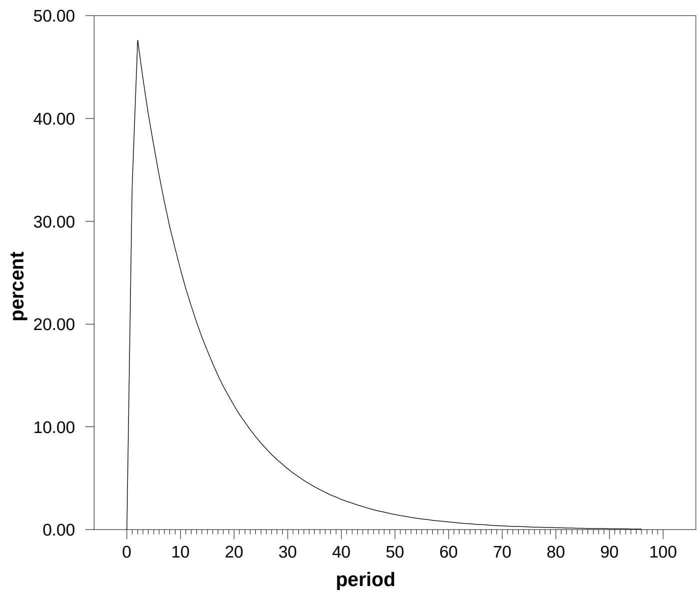

Figure C.1: Energy Price

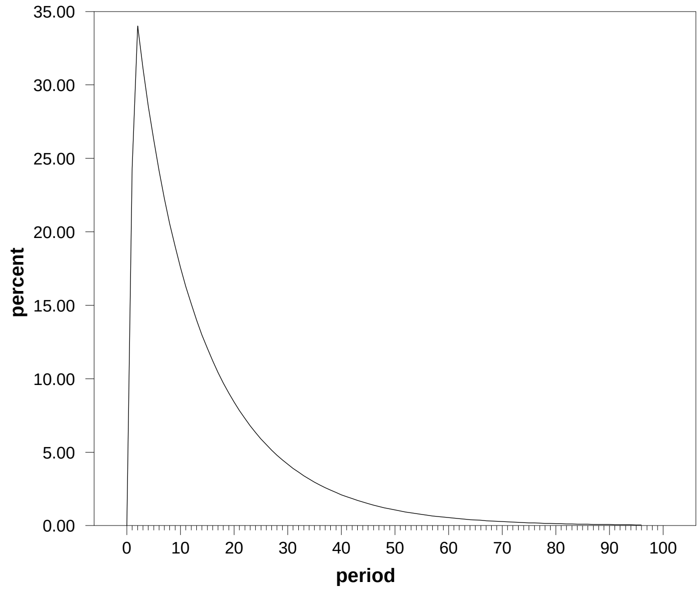

Figure C.2: Capital Type

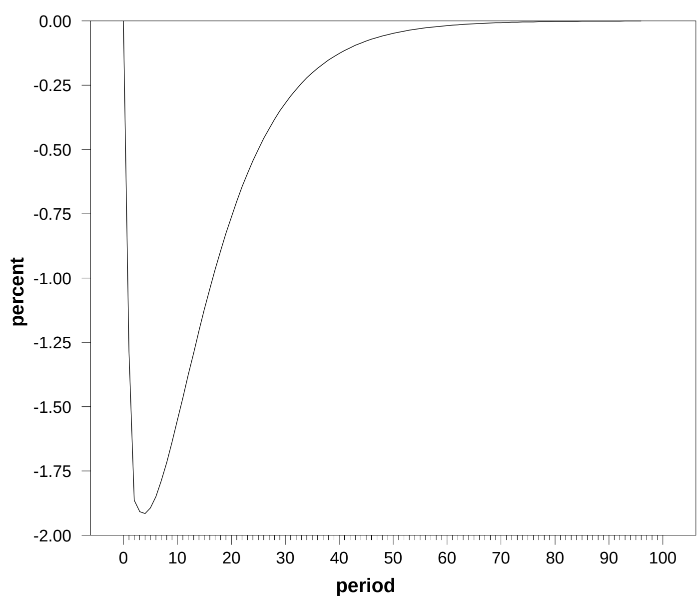

Figure C.3: GDP

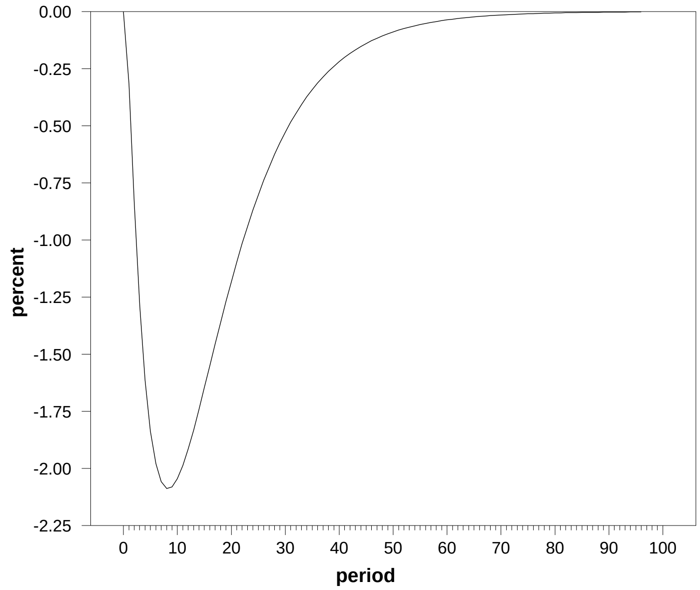

Figure C.4: Consumption

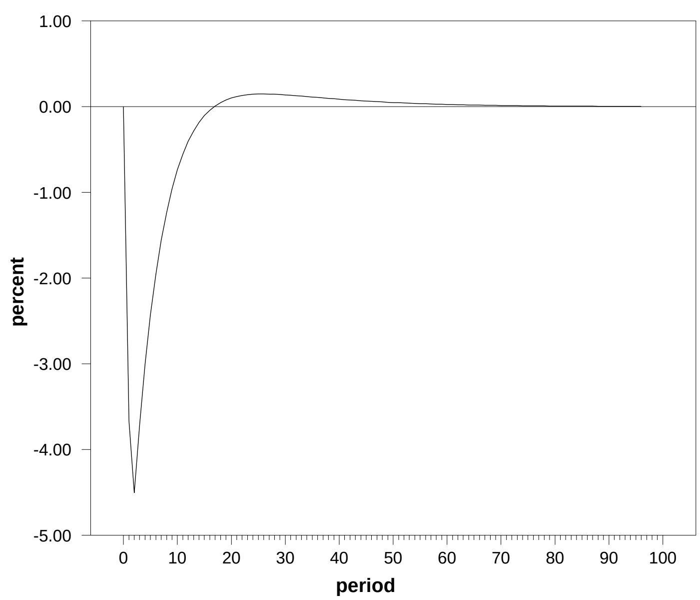

Figure C.5: Investment

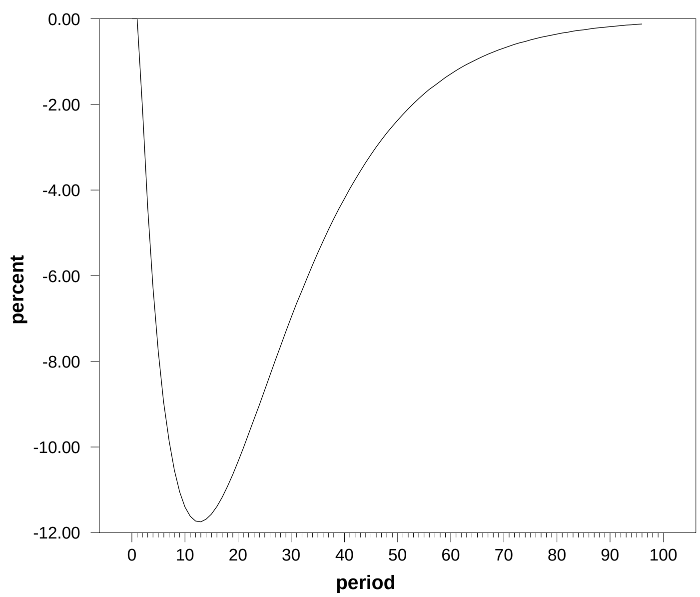

Figure C.6: Energy Use

APPENDIX D

TAX SIMULATIONS

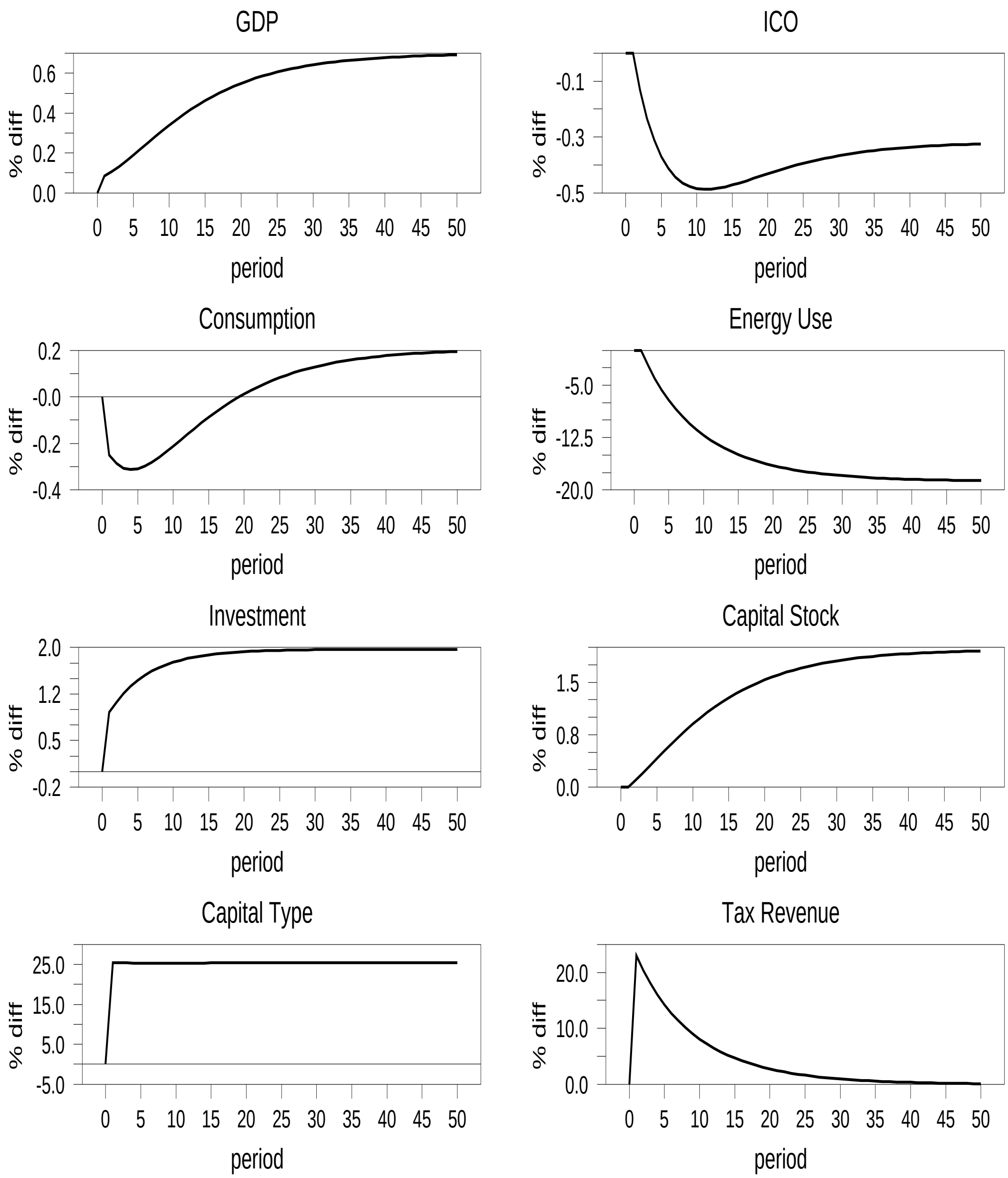

Figure D.1: Shift from Capital Income Tax to Energy Tax

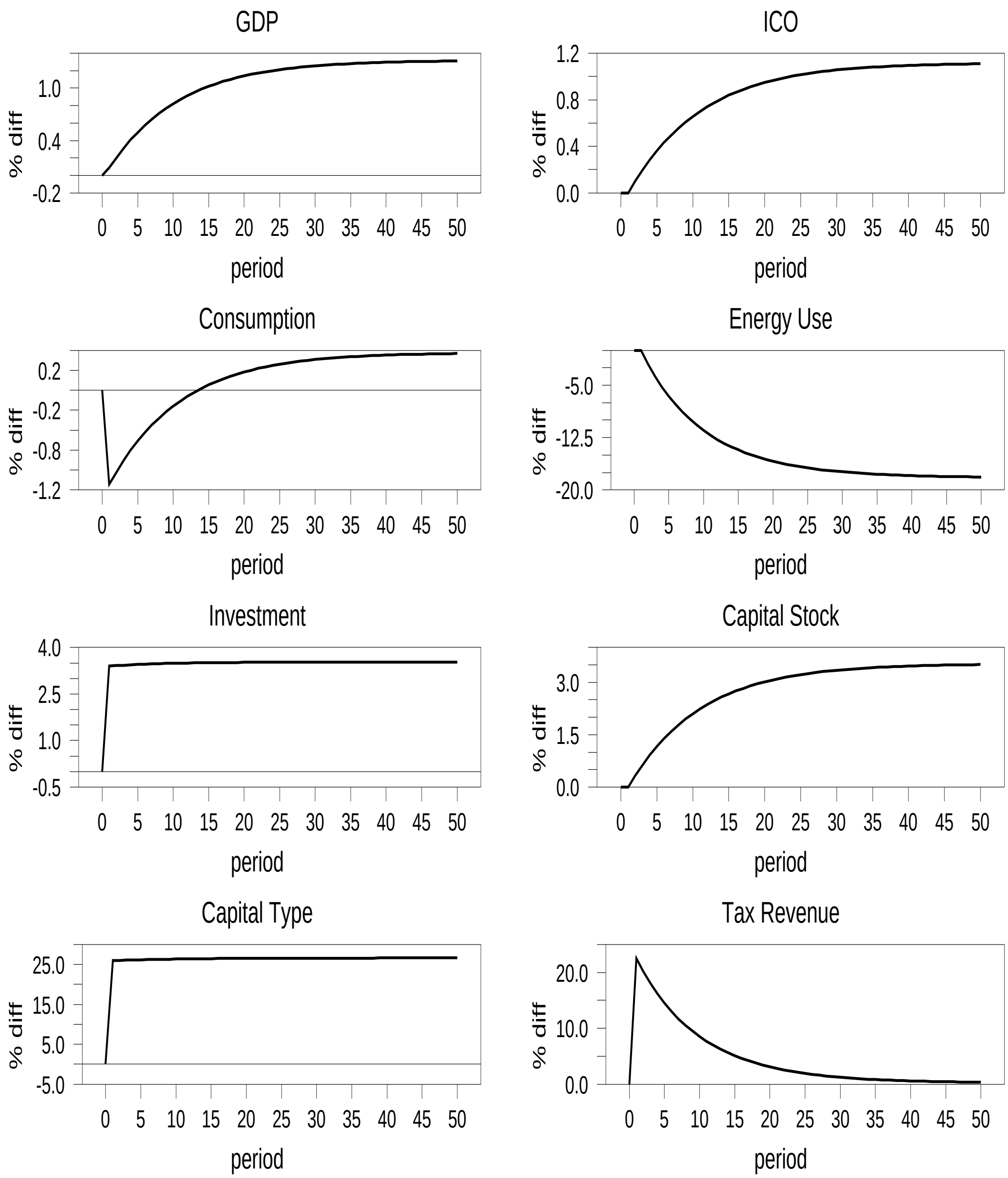

Figure D.2: Shift from Investment Tax to Energy Tax

Dynamic Energy Models and Carbon Mitigation Policies

Mathematica Code

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Create Model and Steady State Equations

- **Clear existing functions and turn off spelling alerts**

```
Clear["Global`*"]
Off[General::spell1]
Off[General::spell]
```

- **Set parameter values and steady state price (from data)**

```
param = {β → 0.96, δ → 0.10, γ → .90, α → .43, ρ → 0.934, η → .42};
exog = {p → 1.0627};
```

- **Steady State Equations and some Identities**

$$Lv = Vbar == \frac{\beta\, p\, (1 - \tau r)\, (1 + \tau m)}{\left(1 - \frac{1}{\gamma}\right) (\beta\, (1 - \delta) - 1)\, (1 + \tau x)};$$

$$Lz = Zbar == \left(\frac{\alpha\, \gamma\, Vbar^{(\gamma-1)}\, (1 - \tau r)}{\left(\frac{1}{\beta} - (1 - \delta)\right) (1 + \tau x)}\right)^{\frac{1}{1-\alpha}};$$

$$Le = Ebar == \frac{Zbar}{Vbar^{\gamma}};$$

$$Lk = Kbar == Ebar * Vbar;$$

$$Lx = Xbar == \delta\, Kbar;$$

$$Lm = Mbar == \frac{Xbar}{\delta\, Vbar};$$

$$Ly = Ybar == Zbar^{\alpha} - Mbar\, p\, ;$$

$$Lc = Cbar == Ybar - Xbar;$$

$$VCutoff = \left(\frac{Pbar\, (1 + \tau m)}{\alpha\, Zbar^{\alpha-1}}\right)^{\frac{1}{\gamma}};$$

Solve for four separate steady states

- **eq0 is the zero tax steady state**

```
eq0 = Join[{Lv, Lz, Le, Lk, Lx, Lm, Ly, Lc}] /. param /.
    {p → 1.0627, τr → 0.0, τx → 0.0, τm → 0.0};
steady = FindRoot[eq0, {Vbar, 39}, {Zbar, 3.1}, {Ebar, .1},
    {Kbar, 4.1}, {Xbar, .4}, {Mbar, .1}, {Ybar, 1.5}, {Cbar, 1.1}];
steady = Flatten[{steady, Pbar -> p /. exog}];
steady = Flatten[{steady, VSbar -> VCutoff /. τm → 0 /. param /. steady}]
```

```
{Vbar → 67.5127, Zbar → 2.78449, Ebar → 0.062849, Kbar → 4.2431, Xbar → 0.42431,
 Mbar → 0.062849, Ybar → 1.48646, Cbar → 1.06215, Pbar → 1.0627, VSbar → 5.22726}
```

- **eq1 is the steady state associated with a 2.068% tax on capital income**

```
eq1 = Join[{Lv, Lz, Le, Lk, Lx, Lm, Ly, Lc}] /. param /.
    {p → 1.0627, τr → 0.02068, τx → 0.0, τm → 0.0};
steadyr = FindRoot[eq1, {Vbar, 39}, {Zbar, 3.1}, {Ebar, .1},
    {Kbar, 4.1}, {Xbar, .4}, {Mbar, .1}, {Ybar, 1.5}, {Cbar, 1.1}];
steadyr = Flatten[{steadyr, Pbar -> p /. exog}];
steadyr = Flatten[{steadyr, VSbar -> VCutoff /. τm → 0 /. param /. steadyr}]
```

```
{Vbar → 66.1165, Zbar → 2.69411, Ebar → 0.0619636, Kbar → 4.09682, Xbar → 0.409682,
 Mbar → 0.0619636, Ybar → 1.46552, Cbar → 1.05583, Pbar → 1.0627, VSbar → 5.11916}
```

- **eq2 is the steady state associated with a 3.03% tax on investment**

```
eq2 = Join[{Lv, Lz, Le, Lk, Lx, Lm, Ly, Lc}] /. param /.
    {p → 1.0627, τr → 0.0, τx → 0.0303, τm → 0.0};
steadyx = FindRoot[eq2, {Vbar, 39}, {Zbar, 3.1}, {Ebar, .1},
    {Kbar, 4.1}, {Xbar, .4}, {Mbar, .1}, {Ybar, 1.5}, {Cbar, 1.1}];
steadyx = Flatten[{steadyx, Pbar -> p /. exog}];
steadyx = Flatten[{steadyx, VSbar -> VCutoff /. τm → 0 /. param /. steadyx}]
```

```
{Vbar → 65.5272, Zbar → 2.65629, Ebar → 0.0615881, Kbar → 4.03569, Xbar → 0.403569,
 Mbar → 0.0615881, Ybar → 1.45663, Cbar → 1.05307, Pbar → 1.0627, VSbar → 5.07354}
```

- **eq3 is the steady state associated with a 22.9% tax on energy use**

```
eq3 = Join[{Lv, Lz, Le, Lk, Lx, Lm, Ly, Lc}] /. param /.
    {p → 1.0627, τr → 0.0, τx → 0.0, τm → 0.229};
steadym = FindRoot[eq3, {Vbar, 39}, {Zbar, 3.1}, {Ebar, .1},
    {Kbar, 4.1}, {Xbar, .4}, {Mbar, .1}, {Ybar, 1.5}, {Cbar, 1.1}];
steadym = Flatten[{steadym, Pbar -> p /. exog}];
steadym = Flatten[{steadym, VSbar -> VCutoff /. τm → 0.229 /. param /. steadym}]
```

```
{Vbar → 82.9731, Zbar → 2.68556, Ebar → 0.050349, Kbar → 4.17761, Xbar → 0.417761,
 Mbar → 0.050349, Ybar → 1.47577, Cbar → 1.05801, Pbar → 1.0627, VSbar → 6.42431}
```

- **Store some of the solution values to be used later**

```
Vr = Vbar /. steadyr; Zr = Zbar /. steadyr; Er = Ebar /. steadyr; Kr = Kbar /. steadyr;
Xr = Xbar /. steadyr; Mr = Mbar /. steadyr; Yr = Ybar /. steadyr;
Cr = Cbar /. steadyr; Pr = Pbar /. steadyr; VSr = VSbar /. steadyr;
Vx = Vbar /. steadyx; Zx = Zbar /. steadyx; Ex = Ebar /. steadyx; Kx = Kbar /. steadyx;
Xx = Xbar /. steadyx; Mx = Mbar /. steadyx; Yx = Ybar /. steadyx;
Cx = Cbar /. steadyx; Px = Pbar /. steadyx; VSx = VSbar /. steadyx;
Vm = Vbar /. steadym; Zm = Zbar /. steadym; Em = Ebar /. steadym; Km = Kbar /. steadym;
Xm = Xbar /. steadym; Mm = Mbar /. steadym; Ym = Ybar /. steadym;
Cm = Cbar /. steadym; Pm = Pbar /. steadym; VSm = VSbar /. steadym;
```

- **Checking equivalence of tax revenues from the three solutions above**

$$\left\{\left(\alpha\,\gamma\,\mathrm{Kr}^{\alpha\,\gamma-1}\,\mathrm{Er}^{(1-\gamma)\,\alpha}\right)*\mathrm{Kr}*\tau\mathrm{r}\ /.\ \mathrm{param}\ /.\ \tau\mathrm{r}\to .02068,\ \mathrm{Pm}*\mathrm{Mm}*\tau\mathrm{m}\ /.\ \{\tau\mathrm{m}\to 0.229\},\ \mathrm{Xx}*\tau\mathrm{x}\ /.\ \{\tau\mathrm{x}\to 0.0303\}\right\}$$

```
{0.0122558, 0.0122528, 0.0122282}
```

Log Linearization and Decision Functions

- **Log Linearized Euler Equations and Constraint**

$$\mathrm{euler1} = (1+\tau\mathrm{x})\ \mathrm{Cbar}\ (1+\mathrm{c2})\ \gamma\ \mathrm{Vbar}^{\gamma-1}\ (1+\mathrm{v2}\ (\gamma-1)) == \beta\ \mathrm{Cbar}\ (1+\mathrm{c1})\ \gamma\ \mathrm{Vbar}^{\gamma-1}\ (1+\mathrm{v1}\ (\gamma-1))\ \alpha\ \mathrm{Zbar}^{\alpha-1}\ (1+\mathrm{z2}\ (\alpha-1))\ \gamma\ \mathrm{Vbar}^{\gamma-1}\ (1+\mathrm{v2}\ (\gamma-1))\ (1-\tau\mathrm{r}) + \beta\ \mathrm{Cbar}\ (1+\mathrm{c1})\ \gamma\ \mathrm{Vbar}^{\gamma-1}\ (1+\mathrm{v1}\ (\gamma-1))\ (1-\delta)\ (1+\tau\mathrm{x})\ ;$$

$$\mathrm{euler2} = (1+\tau\mathrm{x})\ \mathrm{Cbar}\ (1+\mathrm{c2})\ \mathrm{Vbar}\ (1+\mathrm{v1})\ \left(1-\frac{1}{\gamma}\right) == \beta\ \mathrm{Cbar}\ (1+\mathrm{c1})\ \mathrm{Vbar}\ (1+\mathrm{v2})\ \left(1-\frac{1}{\gamma}\right)\ (1-\delta)\ (1+\tau\mathrm{x}) - \beta\ \mathrm{Cbar}\ (1+\mathrm{c1})\ \mathrm{Pbar}\ (1+\mathrm{p2})\ (1+\tau\mathrm{m})\ (1-\tau\mathrm{r});$$

(* constraint =0==Zbar^{α}(1+ z1 α)(1-τr)-
Mbar Pbar(1+m1)(1+p1)(1+τm)(1-τr) - Xbar(1+x1)(1+τx)- Cbar(1+c1); *)

$$\mathrm{constraint} = 0 == \mathrm{Zbar}^{\alpha}\ (1+\ \mathrm{z1}\ \alpha) - \mathrm{Mbar}\ \mathrm{Pbar}\ (1+\mathrm{m1})\ (1+\mathrm{p1}) - \mathrm{Xbar}\ (1+\mathrm{x1}) - \mathrm{Cbar}\ (1+\mathrm{c1});$$

```
L0 = {euler1, euler2, constraint};
```

- **Linear Decision Functions for control variables (c, x, v) and transition equations for state variables (z, m, p)**

```
cFunc = {c1 → acp p1 + acz z1 + acm m1, c2 → acp p2 + acz z2 + acm m2};
xFunc = {x1 → axp p1 + axz z1 + axm m1, x2 → axp p2 + axz z2 + axm m2};
vFunc = {v1 → avp p1 + avz z1 + avm m1, v2 → avp p2 + avz z2 + avm m2};
allFunc = Flatten[{cFunc, xFunc, vFunc}];
```

$$\text{zTrans} = \text{z2} \to (1+\text{z1})\ (1-\delta) + \frac{\text{Xbar Vbar}^{\gamma-1}}{\text{Zbar}}\ (1+\text{x1})\ (1+\text{v1}\ (\gamma-1)) - 1;$$

$$\text{mTrans} = \text{m2} \to (1+\text{m1})\ (1-\delta) + \frac{\text{Xbar}\ (1+\text{x1})}{\text{Mbar Vbar}\ (1+\text{v1})} - 1;$$

```
pTrans = {p2 → ρ p1};
allTrans = Flatten[{zTrans, mTrans, pTrans}];
L1 = L0 /. allFunc /. allTrans /. allFunc;
```

- **There are four different sets of substitution rules. One for the zero tax steady state, and one for each of the tax revenue neutral steady states.**

```
allSub = Flatten[{steady, param, {p → 1.0627, τr → 0.0, τx → 0.0, τm → 0.0}}];
allSubr = Flatten[{steadyr, param, {p → 1.0627, τr → 0.02068, τx → 0.0, τm → 0.0}}];
allSubx = Flatten[{steadyx, param, {p → 1.0627, τr → 0.0, τx → 0.0304, τm → 0.0}}];
allSubm = Flatten[{steadym, param, {p → 1.0627, τr → 0.0, τx → 0.0, τm → 0.229}}];
```

- **Create 9 equations for 9 unknowns by setting each state variable to 1 and others to zero**

```
eq1 = L1 /. {p1 → 1, z1 → 0, m1 → 0};
eq2 = L1 /. {p1 → 0, z1 → 1, m1 → 0};
eq3 = L1 /. {p1 → 0, z1 → 0, m1 → 1};
```

- **Set up four different sets of equations to be used below, one for each tax regime**

```
eq = Flatten[Join[{eq1, eq2, eq3}]] /. allSub;
eqr = Flatten[Join[{eq1, eq2, eq3}]] /. allSubr;
eqx = Flatten[Join[{eq1, eq2, eq3}]] /. allSubx;
eqm = Flatten[Join[{eq1, eq2, eq3}]] /. allSubm;
```

- **For each tax regime, find the corresponding coefficients for the linear decision functions**

```
sol = FindRoot[eq, {acp, -1.5}, {acz, 1.5}, {acm, -0.5}, {avp, 0.5}, {avz, -0.5},
  {avm, 0.5}, {axp, 0.5}, {axz, 0.5}, {axm, -0.5}, MaxIterations → 10 000 000]

{acp → -0.0110117, acz → 0.864586, acm → -0.0434776, avp → 0.757027, avz → 0.363102,
 avm → -0.0084404, axp → -0.129843, axz → -0.590184, axm → -0.048573}

solr = FindRoot[eqr, {acp, -1.5}, {acz, 1.5}, {acm, -0.5}, {avp, 0.5}, {avz, -0.5},
  {avm, 0.5}, {axp, 0.5}, {axz, 0.5}, {axm, -0.5}, MaxIterations → 10 000 000]

{acp → -0.0111731, acz → 0.855798, acm → -0.0433806, avp → 0.75669, avz → 0.361472,
 avm → -0.00847197, axp → -0.131936, axz → -0.598254, axm → -0.0489305}

solx = FindRoot[eqx, {acp, -1.5}, {acz, 1.5}, {acm, -0.5}, {avp, 0.5}, {avz, -0.5},
  {avm, 0.5}, {axp, 0.5}, {axz, 0.5}, {axm, -0.5}, MaxIterations → 10 000 000]

{acp → -0.0112112, acz → 0.85213, acm → -0.0433035, avp → 0.756505, avz → 0.360749,
 avm → -0.00850188, axp → -0.132923, axz → -0.60176, axm → -0.0491817}
```

```
solm = FindRoot[eqm, {acp, -1.5}, {acz, 1.5}, {acm, -0.5}, {avp, 0.5}, {avz, -0.5},
  {avm, 0.5}, {axp, 0.5}, {axz, 0.5}, {axm, -0.5}, MaxIterations → 10 000 000]
```

```
{acp → -0.00252326, acz → 0.855458, acm → -0.0350755,
 avp → 0.758669, avz → 0.362161, avm → -0.00682748,
 axp → -0.121687, axz → -0.592426, axm → -0.0392468}
```

Create decision functions and laws of motion for use in the simulations below

- **The control variable decision functions make use of the coefficients derived above. The transition equations for state variables are log linear versions of the original transition equations. There are also identities to track the values of the cutoff capital type (vs) and the lowest capital type invested in at any time (vmin). The "dp" function is used when there is a simulated, stochastic price series. The "de" function creates a lagged version of the error term for implementing the ARMA(1,1) process.**

```
dc := c[[i]] = acp * p[[i]] + acz * z[[i]] + acm * m[[i]] ;
dv := v[[i]] = avp * p[[i]] + avz * z[[i]] + avm * m[[i]] ;
dx := x[[i]] = axp * p[[i]] + axz * z[[i]] + axm * m[[i]] ;
dz :=
```

$$z[[i+1]] = (1 + z[[i]])\ (1 - \delta) + \text{Xbar}\ (1 + x[[i]])\ \frac{\text{Vbar}^{\gamma-1}}{\text{Zbar}}\ (1 + v[[i]]\ (\gamma - 1)) - 1;$$

$$\text{dm} := m[[i+1]] = (1 + m[[i]])\ (1 - \delta) + \text{Xbar}\ (1 + x[[i]])\ \frac{1}{\text{Mbar Vbar}\ (1 + v[[i]])} - 1;$$

$$(*\ \text{dy}:=y[[i]]=\frac{\text{Zbar}^{\alpha}}{\text{Ybar}}(1+z[[i]]\ \alpha)(1-\tau r[[i]])-\frac{\text{Pbar Mbar}}{\text{Ybar}}\ (1+p[[i]])(1+m[[i]])(1+\tau m[[i]])(1-\tau r[[i]])-1;\ *)$$

$$\text{dy} := y[[i]] = \frac{\text{Zbar}^{\alpha}}{\text{Ybar}}\ (1 + z[[i]]\ \alpha) - \frac{\text{Pbar Mbar}}{\text{Ybar}}\ (1 + p[[i]])\ (1 + m[[i]]) - 1;$$

$$\text{dk} := k[[i+1]] = (1 + k[[i]])\ (1 - \delta) + \frac{\text{Xbar}}{\text{Kbar}}\ (1 + x[[i]]) - 1;$$

$$\text{dvs} := \text{vs}[[i]] = \frac{\text{Pbar}}{\alpha\,\gamma\ \text{Zbar}^{\alpha-1}\ \text{Vbar}^{\gamma}}\ (1 + \tau m[[i]])\ p[[i]] - z[[i]]\ (\alpha - 1)\ \frac{1}{\gamma};$$

```
dvmin := vmin[[i]] = Min[v];
dp := p[[i + 1]] = ρ p[[i]] + η e0[[i]] + e1[[i]];
de := e0[[i]] = e1[[i - 1]];
```

- **Some functions for cosmetic reasons when graphing. The simulations can only run to the second-to-last location in each array, because of the state variables which have a lag structure. These functions simply trim the array by one value after the simulation is complete.**

```
funcTrim := {c = Delete[c, -1]; v = Delete[v, -1];
  x = Delete[x, -1]; m = Delete[m, -1];
  z = Delete[z, -1]; k = Delete[k, -1];
  y = Delete[y, -1]; p = Delete[p, -1]; vs = Delete[vs, -1];
  vmin = Delete[vmin, -1]; TaxRev = Delete[TaxRev, -1];}
```

Simulation using historical US data

- **Set the decision function coefficients (stored in "sol") and steady state values (stored in "steady") to the zero tax values**

```
acp = acp /. sol; acz  =  acz /. sol; acm = acm /. sol;
avp = avp /. sol; avz  =  avz /. sol; avm = avm /. sol;
axp = axp /. sol; axz  =  axz /. sol; axm = axm /. sol;
Cbar =  Cbar /. steady;  Ybar = Ybar /. steady;  Kbar = Kbar /. steady;
Xbar = Xbar /. steady;  Zbar = Zbar /. steady;
Mbar = Mbar /. steady;  Pbar = Pbar /. steady;  Vbar = Vbar /. steady;
VSbar = VSbar /. steady;
ρ = ρ /. param; β = β /. param; δ = δ /. param;
γ = γ /. param; α = α /. param;  η = η /. param;
```

- **Import the historical data from a .csv file. The start of the simulation has a price well above the steady state, so I created a "burn in" time to get the state variables reasonably adjusted to the energy price at that time.**

```
Clear[p];
(* SetDirectory["C:\\Documents and Settings\\Luke
    Tilley\\My Documents\\Econ\\Dissertation\\Mathematica"]; *)
SetDirectory["C:\\Documents and Settings\\c1lat01\\my documents\\PuttyClay"];
pIMPORT = Flatten[Import["PRICE2.csv", "CSV"]];
Pbar  = Mean[pIMPORT];
p  = Log[pIMPORT / Pbar];
obs = Length[p];
```

- **Clear out any existing variable arrays and create new, blank arrays for each**

```
Clear[v, c, z, x, m, y, k, vs, vmin];

v = Table[0, {obs}]; c = Table[0, {obs}]; z = Table[0, {obs}];
y = Table[0, {obs}]; x = Table[0, {obs}]; m = Table[0, {obs}];
k = Table[0, {obs}]; vs = Table[0, {obs}]; vmin = Table[0, {obs}];
τr = Table[0, {obs}];  τx = Table[0, {obs}];  τm = Table[0, {obs}];
```

- **Run the simulation and trim off the last period of each array**

```
Do[{dc, dv, dx, dz, dm, dy, dk, dvs, dvmin}, {i, 1, obs - 1}];
Clear[TaxRev];
TaxRev  =  ((E^(m+Log[Mm]) * E^(p+Log[Pm])) * τm) +
   ((α γ (E^(k+Log[Km]))^(α γ-1) (E^(m+Log[Mm]))^((1-γ) α)) * (E^(k+Log[Km])) * τr) + ((E^(x+Log[Xm])) * τx);
funcTrim;
```

- **Create arrays with levels for each variable. The simulation is run using log deviations.**

```
CLvl = E^(c+Log[Cbar]); XLvl = E^(x+Log[Xbar]);
VLvl = E^(v+Log[Vbar]); VSLvl = E^(vs+Log[VSbar]); VMinLvl = E^(vmin+Log[Vbar]); ZLvl = E^(z+Log[Zbar]);
MLvl = E^(m+Log[Mbar]); YLvl =  E^(y+Log[Ybar]);  KLvl = E^(k+Log[Kbar]); PLvl = E^(p+Log[Pbar]);
```

- **Export the data with a datestamp included in the filename**

```
Export[StringJoin[{"USSim_",
    DateString[{"MonthName", "_", "Day", "_", "Year", "_", "Second", ".xls"}]}],
  {CLvl, XLvl, VLvl, ZLvl, MLvl, YLvl, KLvl, PLvl, TaxRev}, "Table"];
```

Impulse Response

- **Set the decision function coefficients (stored in "sol") and steady state values (stored in "steady") to the zero tax values**

```
acp = acp /. sol; acz = acz /. sol; acm = acm /. sol;
avp = avp /. sol; avz = avz /. sol; avm = avm /. sol;
axp = axp /. sol; axz = axz /. sol; axm = axm /. sol;
Cbar = Cbar /. steady; Ybar = Ybar /. steady; Kbar = Kbar /. steady;
Xbar = Xbar /. steady; Zbar = Zbar /. steady;
Mbar = Mbar /. steady; Pbar = Pbar /. steady; Vbar = Vbar /. steady;
VSbar = VSbar /. steady;
ρ = ρ /. param; β = β /. param; δ = δ /. param;
γ = γ /. param; α = α /. param; η = η /. param;
```

- **Set up the price series with zero deviations and no errors**

```
p = Table[0, {101}];
obs = Length[p];
e1 = Table[0, {obs}];
e0 = Table[0, {obs}];
```

- **Insert one positive standard deviation in the first period. Then run "de" and "dp" to carry that initial error through the ARMA(1,1) process**

```
e1[[1]] = Log[1.33326];
Do[{de}, {i, obs}];
e0[[1]] = 0;
Do[{dp}, {i, obs - 1}];
```

- **Clear out any existing variable arrays and create new, blank arrays for each**

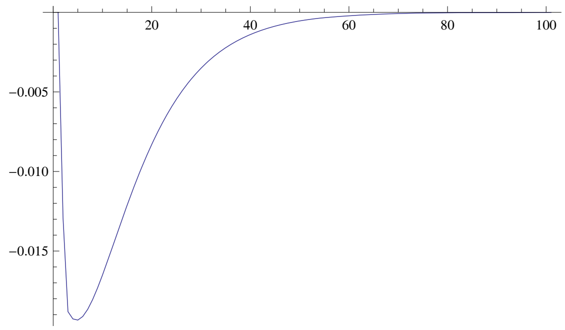
```
Clear[v, c, z, x, m, y, k, vs, vmin]; obs = Length[p];
v = Table[0, {obs}]; c = Table[0, {obs}]; z = Table[0, {obs}];
y = Table[0, {obs}]; x = Table[0, {obs}]; m = Table[0, {obs}];
k = Table[0, {obs}]; vs = Table[0, {obs}]; vmin = Table[0, {obs}];
τr = Table[0, {obs}]; τx = Table[0, {obs}]; τm = Table[0, {obs}];
```

- **Run the simulation**

```
Do[{dc, dv, dx, dz, dm, dy, dk, dvs, dvmin}, {i, 1, obs - 1}];
Clear[TaxRev];
```

- **Visually inspect a variable**

```
ListLinePlot[y]
```



Capital Tax to Energy Tax

- **Set the decision function coefficients (stored in "solm") and steady state values (stored in "steadym") to those associated with a 22.9% tax on energy**

```
acp = acp /. solm; acz  = acz /. solm; acm = acm /. solm;
avp = avp /. solm; avz  = avz /. solm; avm = avm /. solm;
axp = axp /. solm; axz  = axz /. solm; axm = axm /. solm;
Cbar =  Cbar /. steadym;  Ybar = Ybar /. steadym;  Kbar = Kbar /. steadym;
Xbar = Xbar /. steadym;  Zbar = Zbar /. steadym;  Mbar = Mbar /. steadym;
Pbar = Pbar /. steadym;  Vbar = Vbar /. steadym; VSbar = VSbar /. steadym;
CSbar = CSbar /. steadym; XSbar = XSbar /. steadym;
ρ = ρ /. param; β = β /. param; δ = δ /. param;
γ = γ /. param; α = α /. param;  η = η /. param;
p = Table[0, {101}];
obs = Length[p];
```

- **Set the starting point as the steady state when there is a tax on capital. Because the simulation are done in log difference form, this means setting each variable as being the log difference between the capital tax steady state (with a "r" subscript for each variable) and the new target steady state with a tax on energy (with a "m" subscript for each variable). The simulation starts in period 6, so the taxes are set with a tax on capital in the first five periods, and zero thereafter. And there is a zero tax on energy in the first five periods, with the tax implemented in period 6.**

```
v = Table[Log[Vr / Vm], {obs}]; c = Table[Log[Cr / Cm], {obs}];
z = Table[Log[Zr / Zm], {obs}];
y = Table[Log[Yr / Ym], {obs}]; x = Table[Log[Xr / Xm], {obs}];
m = Table[Log[Mr / Mm], {obs}];
k = Table[Log[Kr / Km], {obs}];  vs = Table[0, {obs}]; vmin = Table[0, {obs}];
τr = Table[0.0, {obs}];  τx = Table[0.0, {obs}];  τm = Table[0.229, {obs}];
Do[τr[[i]] = 0.02068, {i, 1, 5}];  Do[τm[[i]] = 0.0, {i, 1, 5}];
```

- **Run the simulation**

```
Do[{dc, dv, dx, dz, dm, dy, dk, dvs, dvmin}, {i, 6, obs - 1}];
Clear[TaxRevLvl];
TaxRevLvl = ((E^(m+Log[Mm]) * E^(p+Log[Pm])) * τm) +
    ((α γ (E^(k+Log[Km]))^(α γ-1) (E^(m+Log[Mm]))^((1-γ) α)) * (E^(k+Log[Km])) * τr) + ((E^(x+Log[Xm])) * τx);
TaxRevSS = Table[TaxRevLvl[[1]], {Length[TaxRevLvl]}];
TaxRev = 100 * (TaxRevLvl / TaxRevSS - 1);
funcTrim;
```

- **Convert from log differences to percent deviation from steady state for graphs**

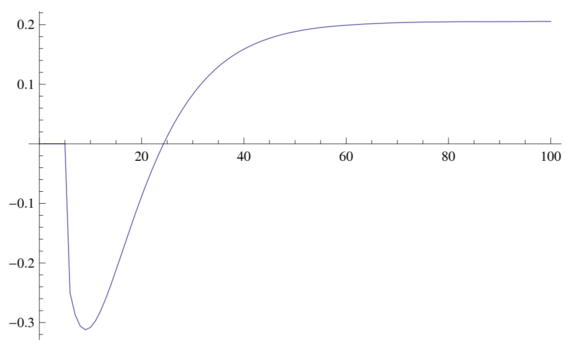
```
CPctDev = 100 * ((E^(c+Log[Cm]) / Cr) - 1); XPctDev = 100 * ((E^(x+Log[Xm]) / Xr) - 1);
VPctDev = 100 * ((E^(v+Log[Vm]) / Vr) - 1); VSPctDev = 100 * ((E^(vs+Log[VSm]) / VSr) - 1);
ZPctDev = 100 * ((E^(z+Log[Zm]) / Zr) - 1); MPctDev = 100 * ((E^(m+Log[Mm]) / Mr) - 1);
YPctDev =  100 * ((E^(y+Log[Ym]) / Yr) - 1);
KPctDev = 100 * ((E^(k+Log[Km]) / Kr) - 1); PPctDev = 100 * ((E^(p+Log[Pm]) / Pr) - 1);

SetDirectory["E:\\Dissertation\\RATS"];

Export[StringJoin[{"SimKtoM_",
     DateString[{"MonthName", "_", "Day", "_", "Year", "_", "Second", ".xls"}]}],
   {CPctDev, XPctDev, VPctDev, ZPctDev, MPctDev, YPctDev,
    KPctDev, PPctDev, TaxRev}, "Table"];
```

- **Visually inspect a variable**

```
ListLinePlot[CPctDev, PlotRange → All]
```



Investment Tax to Energy Tax

- **Set the decision function coefficients (stored in "solm") and steady state values (stored in "steadym") to those associated with a 22.9% tax on energy**

```
acp = acp /. solm; acz  = acz /. solm; acm = acm /. solm;
avp = avp /. solm; avz  = avz /. solm; avm = avm /. solm;
axp = axp /. solm; axz  = axz /. solm; axm = axm /. solm;
Cbar =  Cbar /. steadym;  Ybar = Ybar /. steadym;  Kbar = Kbar /. steadym;
Xbar = Xbar /. steadym;  Zbar = Zbar /. steadym;  Mbar = Mbar /. steadym;
Pbar = Pbar /. steadym;  Vbar = Vbar /. steadym; VSbar = VSbar /. steadym;
CSbar = CSbar /. steadym; XSbar = XSbar /. steadym;
ρ = ρ /. param; β = β /. param; δ = δ /. param;
γ = γ /. param; α = α /. param;  η = η /. param;
p = Table[0, {101}];
obs = Length[p];
```

- **Set the starting point as the steady state when there is a tax on investment. Because the simulation are done in log difference form, this means setting each variable as being the log difference between the investment tax steady state (with an "x" subscript for each variable) and the new target steady state with a tax on energy (with a "m" subscript for each variable). The simulation starts in period 6, so the taxes are set with a tax on investment in the first five periods, and zero thereafter. And there is a zero tax on energy in the first five periods, with the tax implemented in period 6.**

```
v = Table[Log[Vx / Vm], {obs}]; c = Table[Log[Cx / Cm], {obs}];
z = Table[Log[Zx / Zm], {obs}];
y = Table[Log[Yx / Ym], {obs}]; x = Table[Log[Xx / Xm], {obs}];
m = Table[Log[Mx / Mm], {obs}];
k = Table[Log[Kx / Km], {obs}];  vs = Table[0, {obs}]; vmin = Table[0, {obs}];
τr = Table[0.0, {obs}];  τx = Table[0.0, {obs}];  τm = Table[0.229, {obs}];
Do[τx[[i]] = 0.0303, {i, 1, 5}];  Do[τm[[i]] = 0.0, {i, 1, 5}];
```

- **Run the simulation**

```
Do[{dc, dv, dx, dz, dm, dy, dk, dvs, dvmin}, {i, 6, obs - 1}];
Clear[TaxRevLvl];
TaxRevLvl  =  ((E^(m+Log[Mm]) * E^(p+Log[Pm])) * τm) +
    (((α γ (E^(k+Log[Km]))^(α γ-1) (E^(m+Log[Mm]))^((1-γ) α)) * (E^(k+Log[Km])) * τr) + ((E^(x+Log[Xm])) * τx);
TaxRevSS  =  Table[TaxRevLvl[[1]], {Length[TaxRevLvl]}];
TaxRev  =  100 * (TaxRevLvl / TaxRevSS - 1);
funcTrim;
```

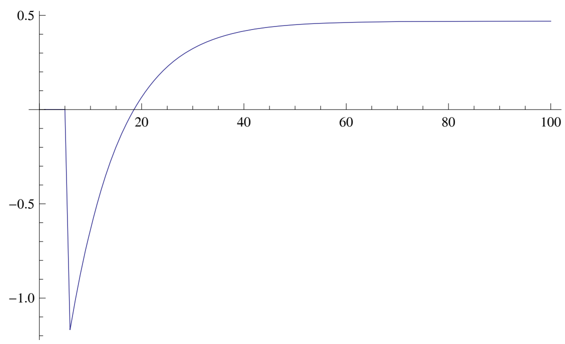
- **Convert from log differences to percent deviation from steady state for graphs**

```
CPctDev = 100 * ((E^(c+Log[Cm]) / Cx) - 1); XPctDev = 100 * ((E^(x+Log[Xm]) / Xx) - 1);
VPctDev = 100 * ((E^(v+Log[Vm]) / Vx) - 1); VSPctDev = 100 * ((E^(vs+Log[VSm]) / VSx) - 1);
ZPctDev = 100 * ((E^(z+Log[Zm]) / Zx) - 1); MPctDev = 100 * ((E^(m+Log[Mm]) / Mx) - 1);
YPctDev =  100 * ((E^(y+Log[Ym]) / Yx) - 1);
KPctDev = 100 * ((E^(k+Log[Km]) / Kx) - 1); PPctDev = 100 * ((E^(p+Log[Pm]) / Px) - 1);

Export[StringJoin[{"SimXtoM_",
     DateString[{"MonthName", "_", "Day", "_", "Year", "_", "Second", ".xls"}]}],
   {CPctDev, XPctDev, VPctDev, ZPctDev, MPctDev, YPctDev,
    KPctDev, PPctDev, TaxRev}, "Table"];
```

- **Visually inspect a variable**

```
ListLinePlot[CPctDev, PlotRange → All]
```



Stochastic Simulation

- **Set the decision function coefficients (stored in "sol") and steady state values (stored in "steady") to the zero tax values**

```
acp = acp /. solm; acz  =  acz /. solm; acm = acm /. solm;
avp = avp /. solm; avz  =  avz /. solm; avm = avm /. solm;
axp = axp /. solm; axz  =  axz /. solm; axm = axm /. solm;
Cbar =  Cbar /. steadym;  Ybar = Ybar /. steadym;  Kbar = Kbar /. steadym;
Xbar = Xbar /. steadym;  Zbar = Zbar /. steadym;  Mbar = Mbar /. steadym;
Pbar = Pbar /. steadym;   Vbar = Vbar /. steadym; VSbar = VSbar /. steadym;
CSbar = CSbar /. steadym; XSbar = XSbar /. steadym;
ρ = ρ /. param; β = β /. param; δ = δ /. param;
γ = γ /. param; α = α /. param;  η = η /. param;
```

- **Create empty arrays for the price series and the error terms**

```
p = Table[0, {1001}];
obs = Length[p];
e1  =  Table[0, {obs}];
e0  =  Table[0, {obs}];
```

- **Generate stochastic disturbances in e1, and then create the one period lagged version for the ARMA(1.1) process, and create the price series**

```
e1 = Table[RandomReal[NormalDistribution[0, .108]], {obs}];
Do[{de}, {i, obs}];
e0[[1]] = 0;
Do[{dp}, {i, obs - 1}];
```

- **Clear out any existing variable arrays and create new, blank arrays for each**

```
Clear[v, c, z, x, m, y, k, vs, vmin];
v = Table[0, {obs}]; c = Table[0, {obs}]; z = Table[0, {obs}];
y = Table[0, {obs}]; x = Table[0, {obs}]; m = Table[0, {obs}];
k = Table[0, {obs}]; vs = Table[0, {obs}]; vmin = Table[0, {obs}];
τr = Table[0, {obs}];  τx = Table[0, {obs}];  τm = Table[0.0, {obs}];
```

- **Run the simulation**

```
Do[{dc, dv, dx, dz, dm, dy, dk, dvs, dvmin}, {i, 1, obs - 1}];
Clear[TaxRev];
TaxRev = ((E^(m+Log[Mm]) * E^(p+Log[Pm])) * τm) +
    ((α γ (E^(k+Log[Km]))^(α γ-1) (E^(m+Log[Mm]))^((1-γ) α)) * (E^(k+Log[Km])) * τr) + ((E^(x+Log[Xm])) * τx);
funcTrim;
```

- **Create arrays with levels for each variable. The simulation is run using log deviations.**

```
CLvl = E^(c+Log[Cbar]); XLvl = E^(x+Log[Xbar]);
VLvl = E^(v+Log[Vbar]); VSLvl = E^(vs+Log[VSbar]); VMinLvl = E^(vmin+Log[Vbar]); ZLvl = E^(z+Log[Zbar]);
MLvl = E^(m+Log[Mbar]); YLvl = E^(y+Log[Ybar]); KLvl = E^(k+Log[Kbar]); PLvl = E^(p+Log[Pbar]);
```

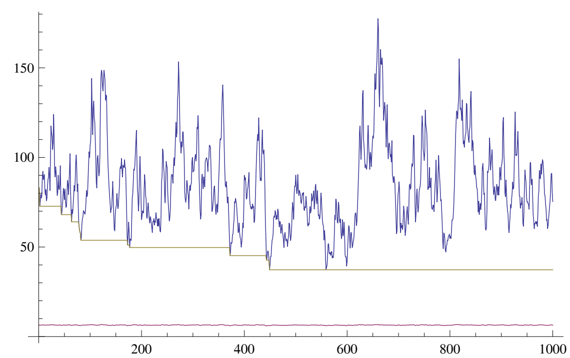
```
(* SetDirectory["C:\\Documents and Settings\\Luke
    Tilley\\My Documents\\Econ\\Dissertation\\RATS"]; *)
SetDirectory["C:\\Documents and Settings\\c1lat01\\my documents\\PuttyClay"];
```

```
Export[StringJoin[{"StochasticSim_",
    DateString[{"MonthName", "_", "Day", "_", "Year", "_", "Second", ".txt"}]}],
  {CLvl, XLvl, VLvl, VSLvl, VMinLvl, ZLvl, MLvl, YLvl, KLvl, PLvl, TaxRev},
  "Table"];
```

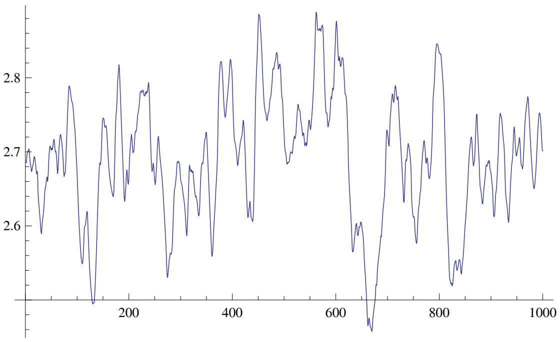
```
Min[VLvl] / Max[VSLvl]
```

```
5.51262
```

```
ListLinePlot[{VLvl, VSLvl, VMinLvl}]
```



```
ListLinePlot[ZLvl]
```




```
Max[ZLvl]
```

2.88888

```
Min[ZLvl]
```

2.45782

q(v) Simulation

- **Run a simulation to support Proposition 3 that there would only be investment in one capital type. The simulation below shows the shape of the q(v) function from Equation 3.3 as a function of v**
- **The simulation is run for capital types up to v=200 with 100 different realized price series with 100 observations each. It is also done over a range of values for z**

```
vlow = 1;
vhigh = 200;
vincr = 1;
zlow = 2.2;
zhigh = 3.4;
zincr = .1;
pcount = 100;
pobs = 100;
v = Table[i, {i, vlow, vhigh, vincr}];
vcount = Length[v];
qv = Table[0, {vcount}];
zrange = Table[i, {i, zlow, zhigh, zincr}];
ptable = Table[0, {i, pcount}, {j, pobs}];
```

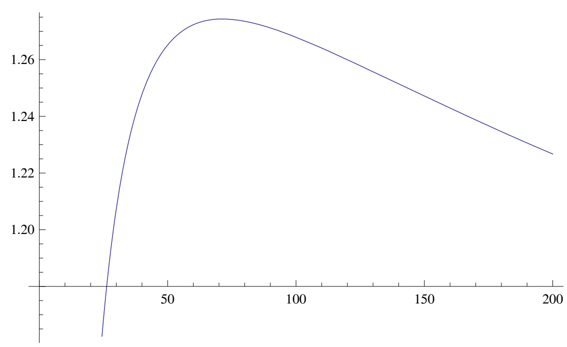
```
Timing[For[i = 1, i < pcount + 1, i ++,
  Clear [p, e1, e0]; p = Table[0, {pobs}];
  e1 = Table[0, {pobs}]; e0 = Table[0, {pobs}];
  e1 = Table[RandomReal[NormalDistribution[0, .108]], {pobs}]; Do[{de}, {i, pobs}];
  e0[[1]] = 0; Do[{dp}, {i, pobs - 1}]; PLvl = E^(p+Log[Pbar]); ptable[[i]] = PLvl;
  Do[
   For[j = 1, j < pobs + 1, j ++,
    For[w = 1, w < (Length[zrange] + 1),
     w ++, qv[[a]] = qv[[a]] + 1/(pcount * Length[zrange])
        (1 - δ)^j (α zrange[[w]]^(α-1) v[[a]]^γ/v[[a]] - 1/v[[a]] PLvl[[j]])]
   ]
   , {a, 1, vcount, 1}]]]
```

```
{427.859, Null}
```

```
Max[qv]
Count[qv, Max[qv]]
ListLinePlot[qv]
```

```
1.27435
```

```
1
```




```
Export[StringJoin[{"qvSim_", DateString[{"MonthName", "_",
      "Day", "_", "Year", "_", "Second", ".txt"}]}], {qv}, "Table"];
```