INSURANCE MARKET EQUILIBRIUM: CONTRACT FORMATION, HETEROGENEITY, AND OPERATIONAL EFFICIENCY

A Dissertation
Submitted
to the Temple University Graduate Board

In Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy

By
Shuo Qiu
August, 2008

ABSTRACT

The three essays of this dissertation investigate the insurance equilibrium from various perspectives. The first essay uses Cournot game-theoretic model to study the insurance contract formation and provides theoretical justification for policy limit. The second essay introduces buyers' heterogeneous risk aversion into Wilson's equilibrium, derives new equilbria, and provides the conditions under which those new equilibria will hold. The third essay studies the operational efficiency of life insurers in China. Through comparing the efficiency of domestic and foreign life insurers, decomposing their efficiency scores, figuring out the directions and potential they could improve, and analyzing the change and driver of productivity, the essay gives insights of the fast-developing life insurance industry in China.

ACKNOWLEDGMENTS

I sincerely appreciate all the guidance, support, and encouragement from my advisor, Professor Michael Powers. The knowledge and attitude I learned from him will be invaluable treasure throughout the rest of my life. I am also grateful to the other committee members who taught and directed me throughout my Ph.D. study at Temple. They are Professor Mary Weiss, Professor David Cummins, Professor Connie Mao, and Professor Sanat Sarkar. Professor Bingzheng Chen helped develop my initial interest in insurance research and continued to support me generously. I am indebted to him.

To Mum, Dad, and April.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INSURANCE AND REINSURANCE CONTRACT FORMATION: INSIGHTS FROM THE COURNOT PARADIGM

Introduction

Optimal Insurance

It is well known that in an insurance transaction between a single risk-averse buyer and a single risk-neutral seller, the buyer will purchase full coverage unless the transaction involves frictional costs (see Mossin, 1968). When transaction costs are introduced, the buyer will prefer to purchase insurance with a deductible (see Arrow, 1971). More general results were given by Raviv (1979), who considered a single-buyer, single-seller scenario in which both parties are risk averse.

Raviv's (1979) basic theorem states that there exists a class of Pareto-optimal solutions in which the buyer and seller agree on policies in which either: (1) there is a deductible (i.e., no coverage) up to some specified dollar amount x, above which the seller provides coinsurance; or (2) there is an interval of full coverage up to a specified point y, above which the seller provides only coinsurance. Raviv (1979) also showed that the optimal insurance contract is characterized by a deductible if and only if the seller's settlement-cost function has a positive first derivative everywhere.

Interestingly, Raviv's (1979) core results are not entirely consistent with the behavior of real-world markets, as recognized by the author himself. Whereas the results provide a theoretical justification for the existence of the insurance deductible, they fail to show why the insurance policy limit – a contract provision at least as common in practice as the deductible –

should exist. Raviv explained this discrepancy by arguing that risk-averse sellers operate in markets subject to substantial price regulation in which premiums may be insufficient to cover the sellers' full economic costs. In such cases, the Pareto-optimal insurance contract possesses a policy limit without any deductible. One problem with this explanation is that not all insurance markets are closely regulated; and even in lines with little or no price regulation (e.g., commercial general liability insurance), contracts invariably are characterized by policy limits.

Subsequent research on optimal insurance has been extensive, with notable contributions by Schlesinger (1981), Huberman et al. (1983), Briys & Louberg´e (1985), Gollier & Schlesinger (1996), Meyer & Ormiston (1999), and Cummins & Mahul (2004). This literature has explored the optimality of deductibles in a variety of contexts, with only Huberman et al. (1983) arguing for the optimality of policy limits (as an artifact of bankruptcy laws). However, Huberman et al. (1983), like most researchers following Raviv (1979), focused on the demand side, treating insurance sellers as risk-neutral decision makers.

In the present article, we employ a Cournot market-game model to solve for insurance contract forms in market equilibrium. Taking insurers to be risk averse, we find that the equilibrium policies typically involve full insurance up to a given policy limit, with no deductible or coinsurance characteristics.

Cournot Market Games

The Cournot equilibrium represents the earliest application of a non-cooperative game-theoretic solution concept to the analysis of economic markets. Originally proposed by Cournot (1838) to model the behavior of duopolies making competitive quantity offers, the equilibrium requires that each player select the strategy that is the "best response" to the equilibrium strategies of the other player. In this sense, the Cournot equilibrium forms an important special case of the strategic equilibrium concept developed formally by Nash (1951) over a century later.

The Cournot approach may be extended to markets with arbitrary numbers of both buyers and sellers, in which the buyers' strategies consist of price bids, and the sellers' strategies consist of quantity offers. (See Dubey, 1982; Dubey & Shubik, 1980.) These models were introduced into the insurance literature by Powers & Shubik (1998) and Powers et al. (1998), who used the Cournot paradigm to study issues of oligopoly power and market configuration. The present work is the first application of the Cournot model to the problem of insurance contract forms.

Insurance Market with Premium Bids, Indemnity Offers

Consider an insurance market with m homogeneous risk-averse buyers, $i = 1, 2, \ldots, m$, and n homogeneous risk-averse sellers, $j = 1, 2, \ldots, n$. Initially, each buyer i possesses net wealth W_B, and each seller j possesses net wealth W_S. During a specified policy period, each buyer i's assets are subject to a nonnegative loss amount, given by a continuous random variable $X_i \sim F_X(x)$, independent of the other buyers' losses. At the beginning of the policy period: each seller j simultaneously makes a strategic insurance-indemnity offer, $I_j(X_i) \in (0, X_i]$, that represents the loss payment that j is willing to remit to each buyer that chooses to purchase insurance from j;[1] and each buyer i makes a strategic insurance-premium bid, π_i, that represents the amount that i is willing to pay for insurance.

---

[1] The indemnity offer is bounded above by the loss amount, X_i, to reduce problems of moral hazard.

The price-formation aspects of our model admit of two interpretations: a global formulation, consistent with the models of Powers & Shubik (1998) and Powers et al. (1998), that posits an invisible market mechanism or "clearinghouse" to assign buyers to sellers and allocate premiums and losses; and a local formulation that avoids the clearinghouse fiction by describing how premiums and losses are allocated on the margin in a neighborhood of a type-symmetric market equilibrium. Because of its less-restrictive assumptions, we take the latter approach.

Local Formulation

In a small neighborhood of any type-symmetric equilibrium, the m homogeneous buyers will have divided themselves among the n homogeneous sellers in such a way that $\mu \approx m/n$ buyers are associated with each of the n sellers (where it is assumed, for simplicity, that m/n is an integer). In this setting, let $j(i)$ denote the seller associated with buyer i, and let M_j denote the set of buyers associated with seller j. We then claim that the following two conditions must hold in the neighborhood of interest: (a) if seller j changes its indemnity offer $I_j(X_i)$ so that $E_{X_i}[I_j(X_i)]$ increases by a small amount, then j's share of total premiums will increase proportionately; and (b) if buyer i increases its premium bid π_i by a small amount, then i's share of its designated indemnity payment also will increase proportionately. Mathematically, condition (a) implies that seller j receives a total premium amount of

$$\left(\frac{\sum_{i \in M_j} E_{X_i}[I_j(X_i)]}{\sum_{i'=1}^{m} E_{X_{i'}}[I_{j'(i')}(X_{i'})]} \right) \sum_{i'=1}^{m} \pi_{i'} \tag{1}$$

whereas condition (b) implies that buyer i receives a total indemnity amount of

$$\frac{\pi_i}{\sum_{i'=1}^{m} \pi_{i'}} m I_{j(i)}\left(X_i\right) \tag{2}$$

Intuitively, expression (1) captures the idea that when seller j improves its indemnity offer, more and/or higher-bidding buyers will migrate to j, thereby increasing j's share of total premiums. Similarly, expression (2) indicates that when buyer i increases its premium bid, i will be able to purchase more coverage, either from its current company or by switching to a different company. Note that, outside of equilibrium, the leading ratios appearing in expressions (1) and (2) are always well defined because all premium bids, π_i, and quantity offers, $I_j(X_i)$, are positive real numbers.[2] Although these ratios are based upon a notion of direct proportionality, this assumption can be relaxed considerably without affecting our basic results.[3]

Finally, we assume that each seller j incurs transaction costs, c, as a linear function of its total losses (i.e., $c = \alpha \sum_{i \in M_j} I_j\left(X_i\right) + \beta$, where α and β are nonnegative constants), and that

---

[2] Note further that, outside of equilibrium, it is possible for buyer i's indemnity amount to be larger than i's actual loss amount (if buyer i's premium bid is larger than the average of all the buyers' premium bids). While such situations clearly aggravate problems of moral hazard, they also are truly reflective of market disequilibrium in the real world; e.g., insurers sometimes settle claims from "favored" policyholders more generously than other claims.

[3] Specifically, the directly proportional ratios (of the form $q_h \Big/ \sum_{h'=1}^{k} q_{h'}$, for the positive vector $\underline{q} = [q_1, q_2, \ldots, q_k]$) may be replaced by quasi-proportional functions $g_h(\underline{q})$ satisfying the following conditions: $\sum_{h'=1}^{k} g_{h'}(\underline{q}) = 1$; $g_h(\underline{q}) = g_{h'}(\underline{\tilde{q}})$ if $q_h = \tilde{q}_{h'}$ and $\underline{q}_{-h}$ permutes $\underline{\tilde{q}}_{-h'}$; and $g_h(\underline{q}) > 0$, $\frac{\partial g_h(\underline{q})}{\partial q_h} > 0$, and $\frac{\partial^2 g_h(\underline{q})}{\partial q_h^2} < 0$ for all h. For simplicity of exposition, we will work with only the directly proportional case.

$W_S > \sum_{i \in M_j} I_j(X_i) + c$, so that j remains able to pay all obligations regardless of the magnitude of its total loss experience.

Equilibrium

Let $u_B(\cdot)$ and $u_S(\cdot)$ denote, respectively, the utility functions of the risk-averse buyers and sellers. We seek the Cournot-Nash equilibrium solution $(\pi^*, I^*(x))$ such that $\pi_i = \pi^*$ maximizes

$$E_{X_i}\left[u_B\left(W_B - \pi_i + \frac{\pi_i}{\sum_{i'=1}^{m} \pi_{i'}} m I_{j(i)}(X_i) - X_i \right) \right]\Bigg|_{\substack{\mu = m/n \\ \pi_{i'} = \pi^*, i' \neq i \\ I_{j(i)} = I^*}} \tag{3}$$

over $\pi_i > 0$ and $I_j(x) = I^*(x)$ maximizes

$$E_{X_i; i \in M_j}\left[u_S\left(W_S + \left(\frac{\sum_{i \in M_j} E_{X_i}[I_j(X_i)]}{\sum_{i'=1}^{m} E_{X_{i'}}[I_{j'(i')}(X_{i'})]} \right) \sum_{i'=1}^{m} \pi_{i'} - \sum_{i \in M_j} I_j(X_i) - c \right) \right]\Bigg|_{\substack{\mu = m/n \\ \pi_{i'} = \pi^*, \\ I_{j'} = I^*, j' \neq j}} \tag{4}$$

over $I_j(x) \in (0, x]$.

From (3) we find

$$\frac{\partial}{\partial \pi_i} E_{X_i}\left[u_B\left(W_B - \pi_i + \frac{\pi_i}{\pi_i + \sum_{\substack{i'=1\\ i'\neq i}}^{m} \pi^*} mI^*(X_i) - X_i\right)\right]\Bigg|_{\pi_i=\pi^*}$$

$$= E_{X_i}\left[u'_B\left(W_B - \pi_i + \frac{\pi_i}{\pi_i + \sum_{\substack{i'=1\\ i'\neq i}}^{m} \pi^*} mI^*(X_i) - X_i\right)\left[-1 + \frac{\sum_{\substack{i'=1\\ i'\neq i}}^{m} \pi^*}{\left(\pi_i + \sum_{\substack{i'=1\\ i'\neq i}}^{m} \pi^*\right)^2} mI^*(X_i)\right]\right]\Bigg|_{\pi_i=\pi^*}$$

$$= E_{X_i}\left[u'_B\left(W_B - \pi^* + I^*(X_i) - X_i\right)\left[-1 + \frac{m-1}{m\pi^*} I^*(X_i)\right]\right]$$

$$= \frac{m-1}{m\pi^*} E_{X_i}\left[u'_B\left(W_B - \pi^* + I^*(X_i) - X_i\right) I^*(X_i)\right] - E_{X_i}\left[u'_B\left(W_B - \pi^* + I^*(X_i) - X_i\right)\right] = 0 \qquad (5)$$

In (4) we maximize

$$\int_{(R^+)^\mu} u_S\left(W_S + \left(\frac{\sum_{i=1}^{\mu} E_{X_i}[I_j(X_i)]}{\sum_{i=1}^{\mu} E_{X_i}[I_j(X_i)] + \mu \sum_{\substack{j'=1\\ j'\neq j}}^{n} \bar{I}^*}\right) \sum_{i'=1}^{m} \pi^* - \sum_{i=1}^{\mu} I_j(x_i) - c\right) \prod_{i=1}^{\mu} f_X(x_i) d\underline{x}$$

over $I_j(x) \in (0, x]$ subject to the constraint

$$\int_{(R^+)^\mu} \left[\sum_{i=1}^{\mu} I_j(x_i)\right] \prod_{i\in M_j} f_X(x_i) d\underline{x} - \sum_{i=1}^{\mu} E_{X_i}[I_j(X_i)] = 0$$

where $\bar{I}^* = E_X[I^*(X)]$. To solve this optimization problem using the calculus of variations, let

$Y = \sum_{i=1}^{\mu} I_j(x_i)$ and $Y'_i = I'_j(x_i)$ for all i, and also

$$\varphi\left(Y;Y'_1,\ldots,Y'_\mu;x_1,\ldots,x_\mu\right)=u_S\left(W_S+\frac{\sum_{i=1}^{\mu}E_{X_i}[I_j(X_i)]}{\sum_{i=1}^{\mu}E_{X_i}[I_j(X_i)]+\mu\sum_{\substack{j'=1\\ j'\neq j}}^{n}\bar{I}\,*}\sum_{i'=1}^{m}\pi*-\sum_{i=1}^{\mu}I_j(x_i)-c\right)\prod_{i=1}^{\mu}f_X(x_i)$$

and

$$\gamma\left(Y;Y'_1,\ldots,Y'_\mu;x_1,\ldots,x_\mu\right)=\left[\sum_{i=1}^{\mu}I_j(x_i)\right]\prod_{i\in M_j}f_X(x_i)$$

We then want

$$=\left[\frac{\partial\varphi}{\partial Y}-\sum_{i=1}^{\mu}\frac{d}{dx_i}\left(\frac{\partial\varphi}{\partial Y'(x_i)}\right)-\lambda\frac{\partial\gamma}{\partial Y}+\lambda\sum_{i=1}^{\mu}\frac{d}{dx_i}\left(\frac{\partial\gamma}{\partial Y'(x_i)}\right)\right]\Bigg|_{I_j=I*}=0$$

which implies

$$\left.\frac{\partial\varphi}{\partial Y}\right|_{I_j=I*}=\lambda\left.\frac{\partial\gamma}{\partial Y}\right|_{I_j=I*}$$

(Since $\frac{\partial\varphi}{\partial Y'(x_i)}\equiv 0$ and $\frac{\partial\gamma}{\partial Y'(x_i)}\equiv 0$). Thus,

$$u'_S\left(W_S+\frac{\sum_{i=1}^{\mu}E_{X_i}[I_j(X_i)]}{\sum_{i=1}^{\mu}E_{X_i}[I_j(X_i)]+\mu\sum_{\substack{j'=1\\ j'\neq j}}^{n}\bar{I}\,*}\sum_{i'=1}^{m}\pi*-\sum_{i=1}^{\mu}I_j(x_i)-c\right)\left[\frac{\mu\sum_{\substack{j'=1\\ j'\neq j}}^{n}\bar{I}\,*}{\left(\sum_{i=1}^{\mu}E_{X_i}[I_j(X_i)]+\mu\sum_{\substack{j'=1\\ j'\neq j}}^{n}\bar{I}\,*\right)^2}\sum_{i'=1}^{m}\pi*-1-c'\right]\Bigg|_{I_j=I*}$$

$$=u'_S\left(W_S+\mu\pi*-(1+\alpha)\sum_{i=1}^{\mu}I*(x_i)-\beta\right)\left[\frac{(n-1)\pi*}{n\bar{I}\,*}-(1+\alpha)\right]=\lambda$$

which yields

$$\sum_{i=1}^{\mu} I^*(x_i) = \frac{1}{1+\alpha}\left[W_S + \mu\pi^* - \beta - u'^{-1}_S\left(\frac{\lambda}{\left[\frac{(n-1)\pi^*}{n\bar{I}^*} - (1+\alpha)\right]}\right)\right] \tag{6}$$

For pure internal solutions $I^*(x_i)$ (i.e., solutions such that $I^*(x_i) \le x_i$ for $i = 1,2,\ldots,\mu$), condition (6) implies

$$I^*(x_i) = I^{\#} = \frac{1}{(1+\alpha)\mu}\left[W_S + \mu\pi^* - \beta - u'^{-1}_S\left(\frac{\lambda}{\left[\frac{(n-1)\pi^*}{n\bar{I}^*} - (1+\alpha)\right]}\right)\right] \tag{7}$$

for some constant λ. However, boundary solutions $I^*(x_i) = x_i$ will occur if

$$u'_S\left(W_S + \mu\pi^* - (1+\alpha)\sum_{i=1}^{\mu} I^*(x_i) - \beta\right)\left[\frac{(n-1)\pi^*}{n\bar{I}^*} - (1+\alpha)\right] > \lambda$$

and this inequality must hold for any particular $x_h < I^{\#}$ because

$$\frac{\partial}{\partial x_h}\left\{u'_S\left(W_S + \mu\pi^* - (1+\alpha)\sum_{i=1}^{\mu} I^*(x_i) - \beta\right)\left[\frac{(n-1)\pi^*}{n\bar{I}^*} - (1+\alpha)\right]\right\}\Bigg|_{x_h < I^{\#}}$$

$$= -(1+\alpha)\left[\frac{(n-1)\pi^*}{n\bar{I}^*} - (1+\alpha)\right]u''_S\left(W_S + \mu\pi^* - (1+\alpha)\sum_{i=1}^{\mu} I^*(x_i) - \beta\right)\frac{\partial I^*(x_h)}{\partial x_h}\Bigg|_{x_h < I^{\#}} < 0$$

Therefore,

$$I^*(x) = \begin{cases} x, & \text{for} \quad x < I^{\#} \\ I^{\#}, & \text{otherwise} \end{cases} \tag{8}$$

and the unknowns π^*, λ, and $I^{\#}$ may be found by solving equations (5), (7), and (8) simultaneously.

Clearly, the form taken by the equilibrium contract in (8) is that of full insurance subject to a policy limit.

Conclusion

The principal contribution of the present research is to offer a simple economic explanation for the proliferation of policy limits in various lines of insurance. Without disputing the existence of alternative explanations – such as Raviv's (1979) regulatory rationale, Huberman et al.'s (1983) bankruptcy argument, or simply the need of sellers to truncate losses characterized by unbounded first or second moments – we would argue that our analysis covers a substantially broader range of cases, including many in which other explanations fail.

We hasten to note that the policy-limit contract, while optimal in market equilibrium, remains dominated by the Pareto-optimal – but possibly unachievable – deductible contract. Naturally, we are confident that insurance contracts will continue to be written with deductibles (as well as prorate arrangements) for very sound economic and institutional reasons, especially the desire of sellers to reduce moral hazard through loss sharing.

References

Arrow, K. J. (1971). *Essays in the theory of risk bearing*. North-Holland, Amsterdam.

Briys, E. P., & Louberg´e, H. (1985). On the theory of rational insurance purchasing: a note. *Journal of Finance*, *40*(20), 577-581.

Cournot, A. A. (1838). *Recherches sur les principes math´ematiques de la th´eorie des richesses*. Hachette, Paris.

Cummins, J. D., & Mahul, O. (2004). The demand for insurance with an upper limit on coverage. *Journal of Risk and Insurance*, *71*(2), 253-264.

Dubey, P. (1982). Price-quantity strategic market games. *Econometrica*, *50*(1), 111-126.

Dubey, P., & Shubik, M. (1980). A strategic market game with price and quantity strategies. *Journal of Economics, 40*(12), 25-34.

Gollier, C., & Schlesinger, H. (1996). Arrow's theorem on the optimality of deductibles: a stochastic dominance approach. *Economic Theory, 7*(2), 359-363.

Huberman, G., Mayers, D., & Smith, Jr., C. W. (1983). Optimal insurance policy indemnity schedules. *Bell Journal of Economics, 14*(2), 415-426.

Meyer, J., & Ormiston, M. B. (1999). Analyzing the demand for deductible insurance. *Journal of Risk and Uncertainty, 18*(3), 223-230.

Mossin, J. (1968). Aspects of rational insurance purchasing. *Journal of Political Economy, 76*, 553-568.

Nash, J. F., Jr. (1951). Noncooperative games. *Annals of Mathematics, 54*, 286-295.

Powers, M. R., & Shubik, M. (1998). On the tradeoff between the law of large numbers and oligopoly in insurance. *Insurance: Mathematics and Economics, 23*(2), 141-156.

Powers, M. R., Shubik, M., & Yao, S. T. (1998). Insurance market games: scale effects and public policy. *Journal of Economics, 67*(2), 109-134.

Raviv, A. (1979). The design of an optimal insurance policy. *American Economic Review, 69*(1), 84-96.

Schlesinger, H. (1981). The optimal level of deductibility in insurance contracts. *Journal of Risk and Insurance, 48*(3), 465-481.

CHAPTER 2

INSURANCE MARKET EQUILIBRIUM WITH HETEROGENEOUS RISK AVERSION

Introduction

Studies of insurance market equilibrium often address problems of incomplete information. Rothschild & Stiglitz (1976) considered a basic model with buyers of different risk levels (loss probabilities), and identified a well-known separating equilibrium. Since this equilibrium does not always exist, researchers have considered alternative models. Arguing that Rothschild & Stiglitz' (1976) separating equilibrium may not exist because insurers do not respond to their competitors' decisions, Wilson introduced a new equilibrium concept, hereafter termed the "Wilson equilibrium."

Previous research on insurance market equilibrium generally has made an assumption of complete markets. Mayers & Smith (1983) proved that sufficient conditions for insurance demand to be independent of other financial assets are: (1) there exists neither moral hazard nor adverse selection; and (2) insurance-policy indemnities must be independent of the buyer's financial assets, human resources, and other policy indemnities. However, all these assumptions are not realistic in the real world. For a typical buyer, considerations of age, family, financial assets, and expectation of future income all affect the demand for insurance.

This raises an important question: How can we take all such factors bet taken into account simultaneously? One approach to this problem is through an assumption of heterogeneous risk aversion. Since different buyers have different risk exposures, different endowments, and different substitutions of insurance, their attitudes toward risk must be different, and their utility functions cannot be the same. With these differences, it is reasonable to argue that buyers with the same loss probability may make different insurance-purchasing decisions.

We note that the literature on heterogeneous risk aversion is rather limited. Our paper aims to improve knowledge in this area.

Arrow and Pratt provided the most commonly used measure of risk aversion, $\rho = -\frac{u''(\cdot)}{u'(\cdot)}$, in which $u(\cdot)$ denotes the utility function of a buyer. For risk-averse buyers, $\rho > 0$. The larger ρ is, the more risk averse the buyer is. In this paper, we assume that buyers have constant absolute risk aversion (CARA); that is to say, absolute risk aversion does not change with a buyer's wealth.

Section 2 introduces the assumptions and lemmas of Wilson's (1977) model, and we briefly discuss the existence and uniqueness of the associated equilibrium. Section 3 provides the principal results of the paper. We investigate insurance market equilibrium when buyers have heterogeneous risk aversion. We first challenge the lemmas of Wilson (1977), and then provide several modified lemmas. Consequently, we are able to discuss the existence of new equilibria under these modified lemmas. We also compare the equilibria before and after the modifications. In Section 4, various practical implications and areas of further research are discussed.

Wilson's Model

Assumptions

Wilson (1977) assumed that buyers are heterogeneous only in their loss probabilities, that price is the unique factor determining insurance demand, and that equilibrium relies on the supply and demand of insurance. He also assumed that insurers and buyers make the same estimates of loss probabilities and loss severities, but that insurers do not know how much risk each individual buyer represents.

Assume that buyers have two possible future incomes: x if a loss happens, and y if the loss does not happen $(y > x > 0)$. The vector (x, y) is called a consumption vector. It is further assumed that buyers are indexed by the superscript $\{1, \cdots, I\} = I^*$ according to their loss probabilities. We let P^i denote the loss probability of buyer i. When $i > j$, we have $0 < P^j < P^i < 1$. All the buyers have the same utility function $u(\cdot)$, with $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Therefore, we have constant risk aversion across buyers; i.e., $\rho = -\dfrac{u''(\cdot)}{u'(\cdot)}$, $i \in I$.

> **Definition 1:** Any insurance policy can be denoted by a vector $s = (s_1, s_2)$, in which s_1 denotes premium and s_2 denotes indemnity. If a buyer purchases the policy s, then his/her consumption vector is $(x - s_1 + s_2, y - s_1)$.
>
> **Definition 2:** The space of the insurance policy is
>
> $\overline{S} = \{s \in R^2 : x - s_1 + s_2 \geq 0, y - s_1 \geq 0\}$.

Given the utility function $u(\cdot)$ and loss probability P^i, we can use v^i to denote the expected utility if a buyer purchases policy s; i.e.,

$$v^i(s) = P^i u(x - s_1 + s_2) + (1 - P^i) u(y - s_1).$$

According to utility theory, a buyer always chooses the insurance policy that maximizes his or her expected utility. Letting be a non-empty set $S \subset \overline{S}$, and we define the demand function as $d^i(s, S) \in R_+^I$, where $d^i(s, S)$ denotes the number of buyers of type i who purchase policy s according to their expected utilities.

It is assumed that transaction costs are zero. When buyers of type i purchase policy s, the profit of an insurer is equal to its premiums minus expected indemnity payments. Therefore, the profit function of an insurer, $R(\cdot)$, is $R(s,b)=\sum_{i\in I} b^i(s_1 - P^i s_2)$, where b denotes the number of buyers who purchase the policy s. The market profit function of S is defined as $R^*(\cdot;S)$, and we have $R^*(s,S)=R^*(s,d(s,S))$. If S is the set of all the insurance policies that the market can provide, then $R^*(s,S)$ can be regarded as the total profit of the market.

Preliminary Results

Wilson equilibrium is based on a series of lemmas. To understand Wilson equilibrium, it's necessary to have good knowledge of these lemmas. Here, we introduce Wilson's lemmas without proof. We will then check whether these lemmas could still hold after heterogeneous risk aversions are introduced in the next section. Based on the modified lemmas, new kinds of equilibria will be reached.

Lemma 1: For each $i \in I$, the expected utility function $v^i(\cdot)$ is a twice-differentiable concave function defined on $\bar{S}$.

Lemma 2: For each $i \in I$, if $s_2 < [=,>](y-x)$, then $\left.\frac{ds_1}{ds_2}\right|_{v^i(s)} < [=,<] P^i$.

Lemma 2 implies that within the same level of expected utility, buyers always prefer policies with higher indemnities, and full insurance is optimal for all buyers.

Lemma 3: If $i > j$, then $\left.\frac{ds_1}{ds_2}\right|_{v^i(s)} > \left.\frac{ds_1}{ds_2}\right|_{v^j(s)} > 0$.

Lemma 3 implies that the slope of the indifference curve of higher-risk buyers (buyers with a higher loss probability) is always greater (steeper) than that of lower-risk buyers (buyers with a lower loss probability).

Lemma 4: Let $S_i^*(S)$ and $S_j^*(S)$ denote the insurance policies that buyers i

and j choose to purchase according to their expected utilities. If $s^i \in S_i^*(S)$, $s^j \in S_j^*(S)$, and $i > j$, we then have $s^i \geq s^j$.

Lemma 4 implies that buyers with a higher loss probability will purchase at least as much insurance as buyers with a lower loss probability. Figure 1 shows the meaning of this lemma. The notation $v^i(s^1)$ denotes the indifference curve of buyer i when he or she buys the policy s^1. When $i > j$, the indifference curves $v^i(s^1)$ and $v^j(s^1)$ intersect at s^1. For the lower-risk buyer j, policy s^1 and policy s^2 result in the same utilities. However, for the higher-risk buyer i, policy s^2 provides a greater expected utility higher than does policy s^1, because the expected utility curve that crosses s^2 is below $v^i(s^1)$.

Lemma 4 is an extension of Lemma 3. Lemma 3 shows that indifference curves of higher-risk buyers must be steeper than indifference curves of lower-risk buyers, and the two indifference curves have a unique intersection s^1. Lemma 4 points out that on the indifference curve $v^j(\mathbf{s})$, the policy s^2, which is to the right of s^1, is preferred by higher-risk buyers.

Lemma 5: Let e_i denote a vector of a Euclidean space with *i* dimensions. If $i > j$, then for any insurance policy $s = (s_1, s_2) \in \overline{S}$, $R(s, e^i) > [=,<] R(s, e^j)$ iff $s_2 < [=,>] 0$.

Lemma 5 is necessary for the existence of equilibrium. It implies that buyers are able to rank the policies according to the profits these policies produce for the insurers. This ranking is consistent with the preferences obtained in Lemma 3 and Lemma 4.

Lemma 6: For $s \in S$, if $s_2 < 0$, then $R(s, d^i(s)) \leq 0$.

Lemma 6 implies that the insurance market will not provide policies with negative indemnities.

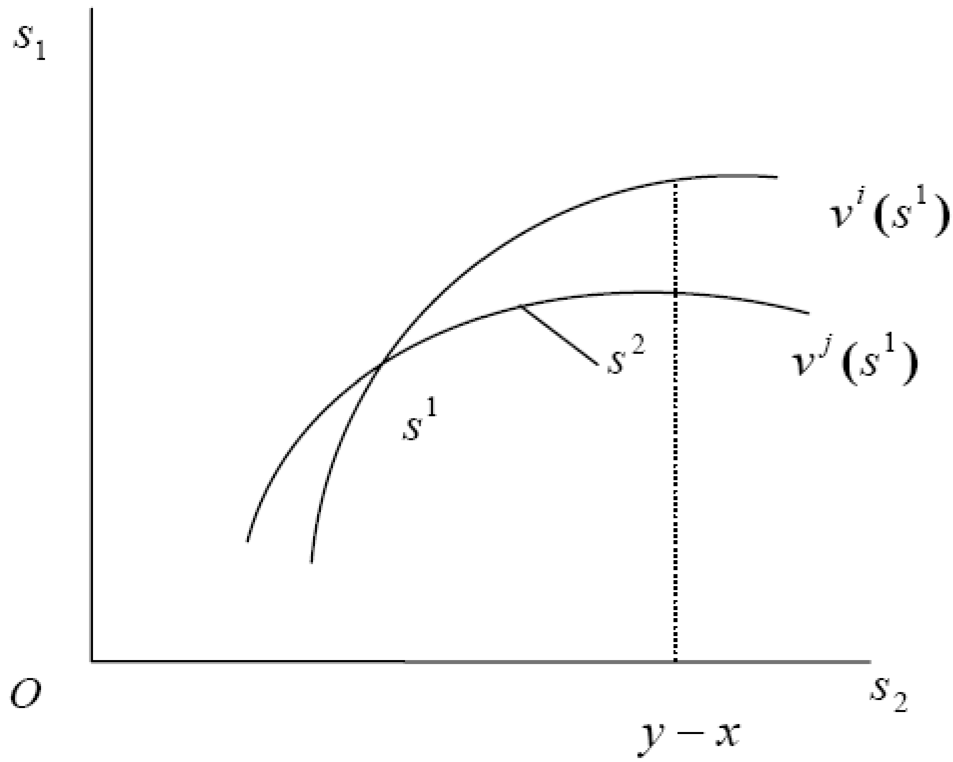

Figure 1. Utilities of Insurance Consumers with Different Risks

Equilibrium Results

Wilson equilibrium is one of demand and supply, in which insurers respond to the decisions of their competitors. This type of equilibrium is based upon the assumption that buyers are heterogeneous only in their loss probabilities.

Definition 3: We call S^* a Wilson equilibrium if:

(i) For each $s \in \bar{S}$, $R^*(s;S^*) \geq 0$; and

(ii) There does not exist an $S \subset \bar{S}$ such that for every $s \in S$,

$R^*(s;Q^*(S^*,S) \cup S) \geq 0$, with strict inequality for at least some $s \in S - S^*$.

In the above expression, $Q^*(S^*, S)$ denotes the set of policies obtained by providing a new policy S and deleting all the policies originally yielding negative profits. Condition (i) says that in equilibrium, each policy must have nonnegative profit. Condition (ii) says that there cannot be a new insurance policy such that if we add the new policy into the policy set and delete all the policies that originally generated negative profit, then each policy in the new policy set can have nonnegative profit, with at least some policies having positive profit.

There are two types of Wilson equilibria – separating and pooling. Each type of equilibrium exists under certain conditions.

Separating Equilibrium

Consider two insurance buyers $i, j \in I$, and $i > j$. In Figure 2, $\overline{OP^i}$ denotes the policy set sold to buyer i that generates zero profit for the insurers, and $\overline{OP^j}$ denotes the policy set sold to buyer j that generate zero profit for the insurers. We use $\overline{OP^{ij}}$ to denote the policy set sold to both buyers i and j that generate zero profit for the insurers. Under complete information, the insurance market equilibrium is $\{s^1, s^2\}$, and both buyers can reach full insurance. However, under incomplete information, insurers cannot recognize the risk of buyers. If insurers provide $\{s^1, s^2\}$, then higher-risk buyers would purchase s^1, because s^1 can bring them higher expected utility. The insurers will lose money in such circumstances. Therefore, under incomplete information, the market equilibrium should be $\{s^2, s^3\}$, in which s^3 is the intersection of $v^i(s^2)$ and $\overline{OP^j}$. At this point, higher-risk buyers would purchase s^2, and lower-risk buyers would purchase s^3. Insurers would gain zero profit, and the market reaches a separating equilibrium.

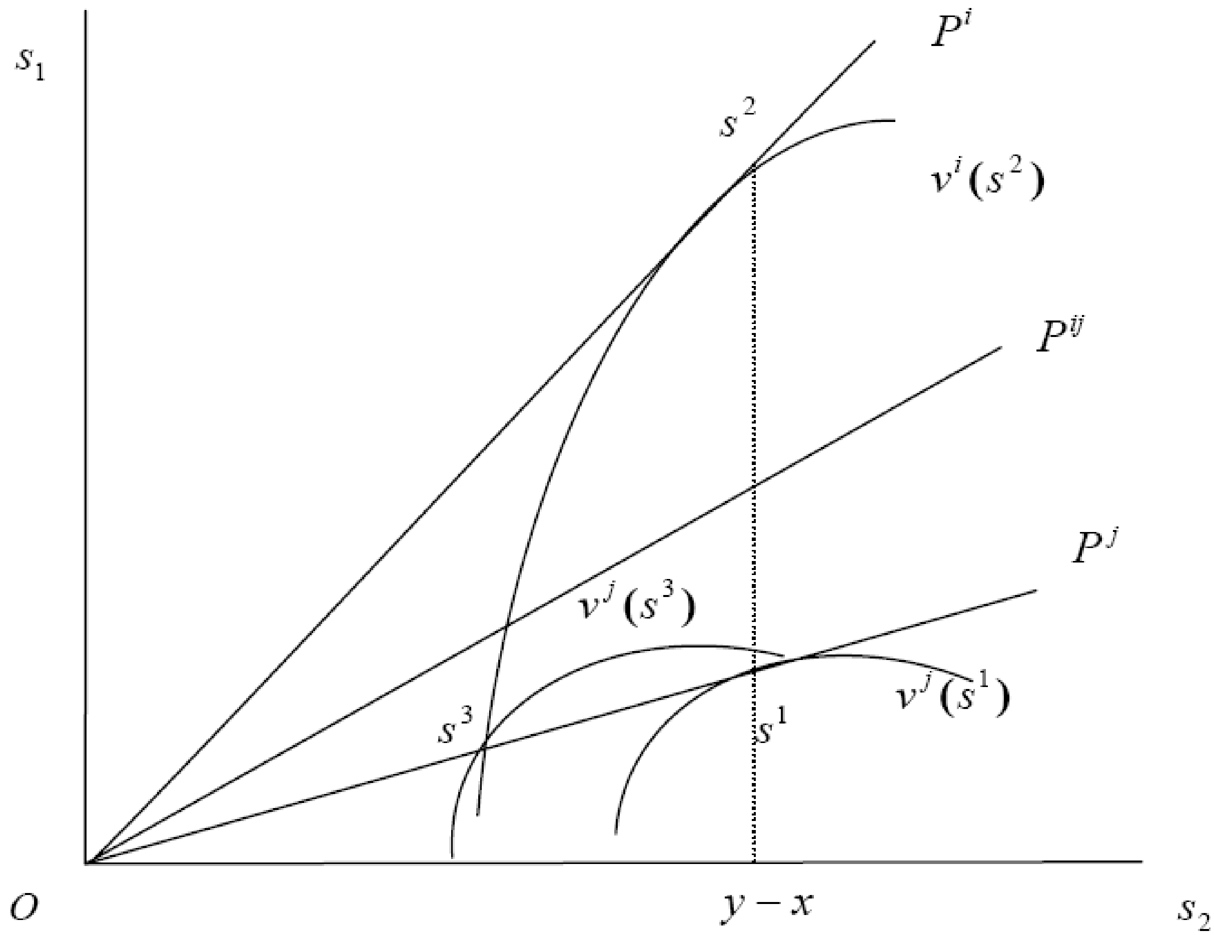

Figure 2. Separating Equilibrium with Two Different Risk Aversions

Pooling Equilibrium

If there is a relatively high proportion of low-risk buyers, then $\overline{OP^{ij}}$ will approach $\overline{OP^{j}}$. The expected utility curve $v^j(s^3)$, for the low-risk buyer *j* purchasing policy s^3, will cross $\overline{OP^{ij}}$. As is shown in Figure 3, if another insurer provides a policy below $v^j(s^3)$ but above $\overline{OP^{ij}}$, then both buyer i and buyer j will choose to purchase this new policy, and at the same time, insurers would generate nonnegative profit. Therefore, the market equilibrium should be the point $\{s^4\}$, where $\overline{OP^{ij}}$ is tangent to the expected utility curve of the low-risk buyer. At this pooling equilibrium, the insurers would generate zero profit.

Wilson Equilibrium under Heterogeneous Risk Aversion

Next we investigate market equilibrium under heterogeneous risk aversion; i.e., when $\rho_k = -\dfrac{u_k''}{u_k'}$ is not constant. According to the level of risk aversion, we divide the buyers into K groups, $\{1,\ldots,K\} = K^*$, $K > 1$. When $k_1 > k_2$, we have $\rho_{k_1} > \rho_{k_2}$. For convenience, we use T_k^i to denote a buyer with loss probability i and risk aversion k.

Modified Lemmas

We first modify several important lemmas related to Wilson equilibrium.

Lemma 1*: For buyer T_k^i, the expected utility function $v_k^i(\cdot)$ is a twice-differentiable concave function defined on $\bar{S}$.

Lemma 2*: For buyer T_k^i, if $s_2 < [=,>](y-x)$, then $\left.\dfrac{ds_1}{ds_2}\right|_{v_k^i(s)} > [=,<] P^i$.

Proof: The slope of T_k^i's indifference curve that crosses the policy *s* is

$$\left.\frac{ds_1}{ds_2}\right|_{v_k^i(s)} = \frac{P^i u_k^{i\,\prime}(x - s_1 + s_2)}{P^i u_k^{i\,\prime}(x - s_1 + s_2) + (1 - P^i) u_k^{i\,\prime}(y - s_1)},$$

in which $u_k^{i\,\prime}(\cdot)$ is the first derivative of $u_k^i(\cdot)$. Lemma 2* then holds by inspection.

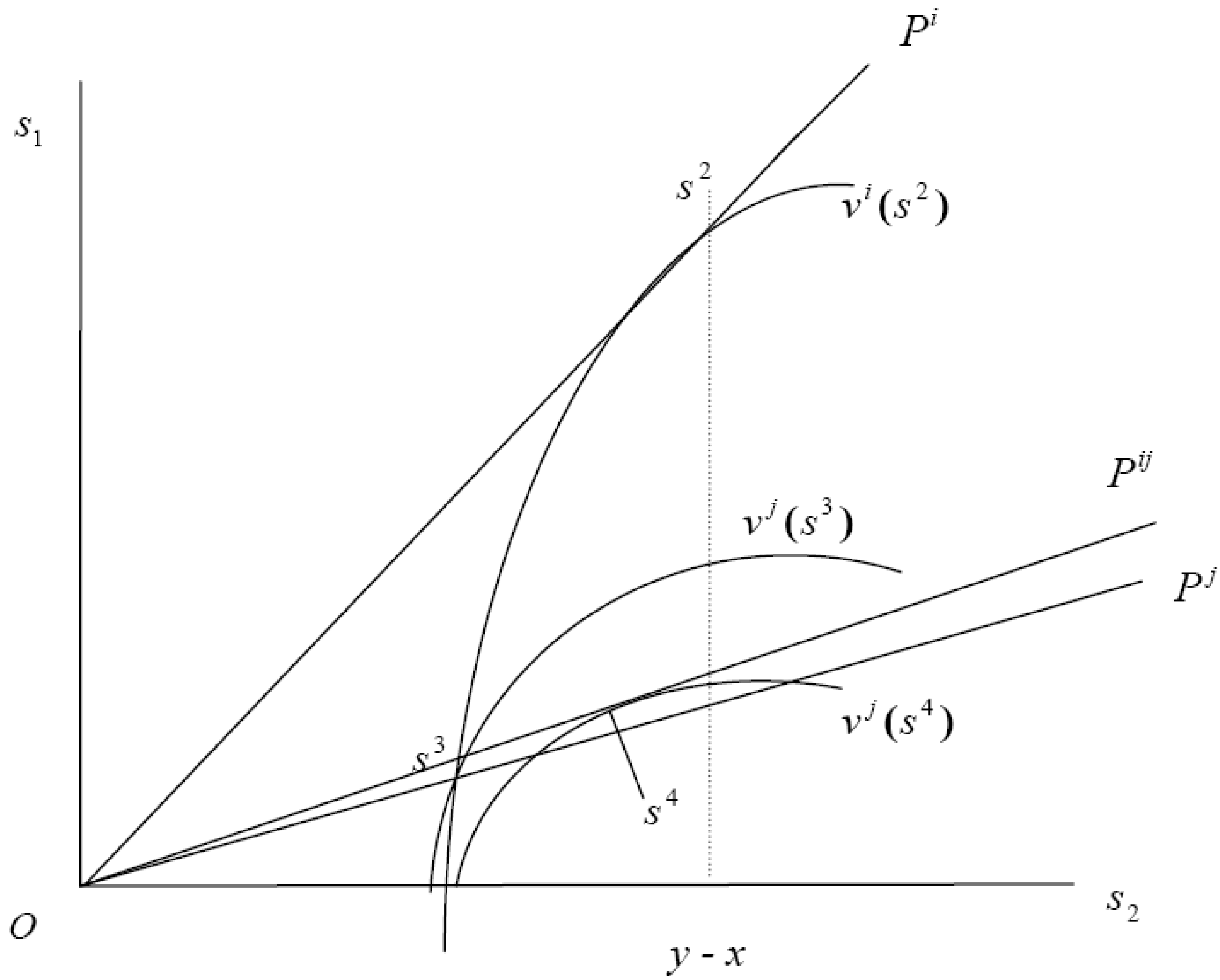

Figure 3. Pooling Equilibrium with Two Different Risk Aversions

Lemma 2* states that at $s_2 = y - x$, the slope of T^i_j's indifference curve is equal to P^i; for $s_2 < y - x$, the slope of the indifference curve is greater than P^i; and for $s_2 > y - x$, the slope is less than P^i. In other words, the slope of T^i_k's indifference curve is independent of his or her level of risk aversion.

Lemma 3*: For buyers $T_{k_1}^i$ and $T_{k_2}^j$, when $P^i > P^j$ there is no guarantee that $\left.\frac{ds_1}{ds_2}\right|_{v_{k_1}^i(s)} > \left.\frac{ds_1}{ds_2}\right|_{v_{k_2}^j(s)}$.

Lemma 3* says that when we consider different levels of risk aversion, Lemma 3 may not hold, as in the following counterexample.

Counterexample

Assume that the utility function of a buyer k is $u_k(w) = -e^{-a_k w}$, where w denotes the buyer's wealth and $a_k > 0$.

Here we consider two types of buyers, T_k^i and T_h^j. We assume that $P^i = 0.5 > P^j = 0.4$ and $a_k = 1 << a_h = 10$ (i.e., the risk aversion of the high-risk buyer is much lower than the risk aversion of low-risk buyer). In addition, we assume that $x = 0, y = 1$, and get the slope of the indifference curve:

$$\left.\frac{ds_1}{ds_2}\right|_{v_k^i(s)} = \frac{P^i a_k e^{-a_k(x-s_1+s_2)}}{P^i a_k e^{-a_k(x-s_1+s_2)} + (1-P^i) a_k e^{-a_k(y-s_1)}} = \frac{1}{1+\left(\frac{1-P^i}{P^i}\right) e^{-a_k(y-x-s_2)}}. \quad (1)$$

For buyer T_k^i, we have

$$\left.\frac{ds_1}{ds_2}\right|_{v_k^i(s)} = \frac{1}{1+\left(\frac{1-0.5}{0.5}\right) e^{-(1-s_2)}} = \frac{1}{1+e^{s_2-1}},$$

and for buyer T_h^j, we have

$$\left.\frac{ds_1}{ds_2}\right|_{v_h^j(s)} = \frac{1}{1+\left(\frac{1-0.4}{0.4}\right) e^{-10(1-s_2)}} = \frac{1}{1+(3/2)e^{10s_2-10}}.$$

Putting them together, we obtain

$$m = \frac{ds_1}{ds_2}\bigg|_{v_k^i(s)} \bigg/ \frac{ds_1}{ds_2}\bigg|_{v_h^j(s)} = \frac{1+(3/2)e^{10s_2-10}}{1+e^{s_2-1}}.$$

When $s_2 < 1 + \frac{\ln(2/3)}{9} < 1 = y - x$, we have $m < 1$; i.e., $\frac{ds_1}{ds_2}\Big|_{v_k^i(s)} < \frac{ds_1}{ds_2}\Big|_{v_h^j(s)}$.

From Figure 4, buyer T_h^j 's indifference curve $v_h^j(s)$ and T_k^i 's indifference curve $v_k^i(s)$ are tangent at $s_2 = 1 + \frac{\ln(2/3)}{9} = 0.955$. We can easily see that after taking the different levels of risk aversion into account, the expected utility curve of the low-risk buyers can be steeper than the expected utility curve of the high-risk buyers in certain areas.

Next, we consider how the buyers would choose insurance when $\frac{ds_1}{ds_2}\Big|_{v_k^i(s)} < \frac{ds_1}{ds_2}\Big|_{v_h^j(s)}$.

For any buyer T_k^i, we always have

$$\frac{ds_1}{ds_2}\bigg|_{v_k^i(s)} = \frac{P^i u_k^{i\,\prime}(x-s_1+s_2)}{P^i u_k^{i\,\prime}(x-s_1+s_2)+(1-P^i)u_k^{i\,\prime}(y-s_1)} = \frac{1}{1+\left(\frac{1-P^i}{P^i}\right)\frac{u_k^{i\,\prime}(y-s_1)}{u_k^{i\,\prime}(x-s_1+s_2)}}. \tag{2}$$

Then we can apply the mean-value theorem to $u_k^{i\,\prime}(y-s_1)$ and $u_k^{i\,\prime}(x-s_1+s_2)$ to obtain

$$\begin{aligned} u_k^{i\,\prime}(y-s_1) &= u_k^{i\,\prime}(x-s_1+s_2) + u_k^{i\,\prime\prime}(\varepsilon)(y-x-s_2) \\ &= u_k^{i\,\prime}(x-s_1+s_2) - \rho_k u_k^{i\,\prime}(\varepsilon)(y-x-s_2), \end{aligned} \tag{3}$$

in which ε lies between $(x-s_1+s_2)$ and $(y-s_1)$.

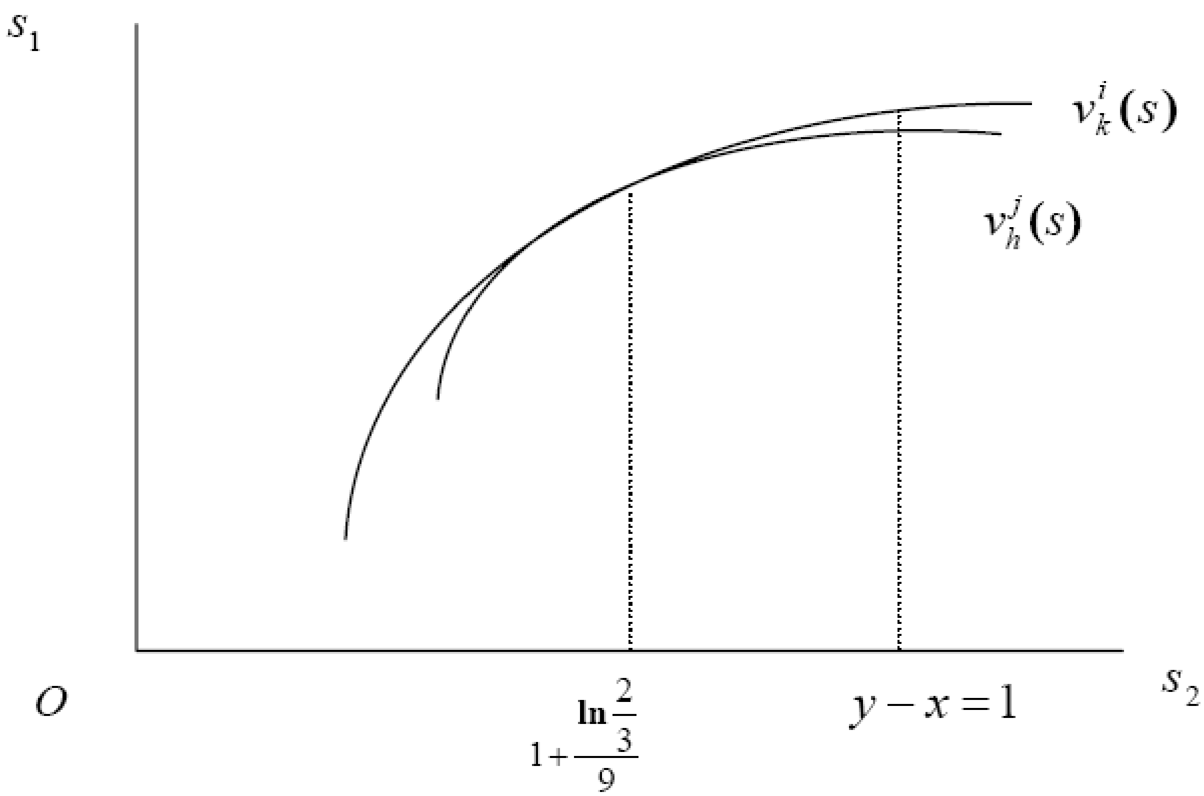

Figure 4. Example of Lemma 3*

Next, focus on the situation in which $\left.\frac{ds_1}{ds_2}\right|_{v_h^j(s)} > \left.\frac{ds_1}{ds_2}\right|_{v_k^i(s)}$. First, we have the equivalent expression

$$\frac{1}{1+\left(\frac{1-P^j}{P^j}\right)\frac{u_h^{j\,\prime}(y-s_1)}{u_h^{j\,\prime}(x-s_1+s_2)}} > \frac{1}{1+\left(\frac{1-P^i}{P^i}\right)\frac{u_k^{i\,\prime}(y-s_1)}{u_k^{i\,\prime}(x-s_1+s_2)}},$$

which implies

$$\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right) < \frac{u_k^{i\,\prime}(y-s_1)}{u_k^{i\,\prime}(x-s_1+s_2)}\cdot\frac{u_h^{j\,\prime}(x-s_1+s_2)}{u_h^{j\,\prime}(y-s_1)}. \tag{4}$$

We define q as

$$q = \frac{u_k^{i\ \prime}(y - s_1)}{u_k^{i\ \prime}(x - s_1 + s_2)} \cdot \frac{u_h^{j\ \prime}(x - s_1 + s_2)}{u_h^{j\ \prime}(y - s_1)},$$

and apply the mean-value theorem to $u_k^{i\ \prime}(y - s_1)$ and $u_h^{j\ \prime}(x - s_1 + s_2)$ to obtain

$$q = \frac{u_k^{i\ \prime}(x - s_1 + s_2) - \rho_k(y - x - s_2)u_k^{i\ \prime}(\varepsilon_1)}{u_k^{i\ \prime}(x - s_1 + s_2)} \cdot \frac{u_h^{j\ \prime}(y - s_1) + \rho_h(y - x - s_2)u_h^{j\ \prime}(\varepsilon_2)}{u_h^{j\ \prime}(y - s_1)}$$

$$= [1 - \rho_k(y - x - s_2)\frac{u_k^{i\ \prime}(\varepsilon_1)}{u_k^{i\ \prime}(x - s_1 + s_2)}][1 + \rho_h(y - x - s_2)\frac{u_h^{j\ \prime}(\varepsilon_2)}{u_h^{j\ \prime}(y - s_1)}].$$

Since $u_k^{i\ \prime}(s) > 0, u_k^{i\ \prime\prime}(s) < 0$, we know that $y - x - s_2 > 0$ and $\varepsilon_1, \varepsilon_2 \in (x - s_1 + s_2, y - s_2)$ imply

$$0 < \frac{u_k^{i\ \prime}(\varepsilon_1)}{u_k^{i\ \prime}(x - s_1 + s_2)} < 1 \text{ and } \frac{u_h^{j\ \prime}(\varepsilon_2)}{u_h^{j\ \prime}(y - s_1)} > 1,$$

and that $y - x - s_2 < 0$ and $\varepsilon_1, \varepsilon_2 \in (y - s_1, x - s_1 + s_2)$ imply

$$\frac{u_k^{i\ \prime}(\varepsilon_1)}{u_k^{i\ \prime}(x - s_1 + s_2)} > 1 \text{ and } 0 < \frac{u_h^{j\ \prime}(\varepsilon_2)}{u_h^{j\ \prime}(y - s_1)} < 1.$$

In both situations we easily have

$$q > [1 - \rho_k(y - x - s_2)][1 + \rho_h(y - x - s_2)],$$

and the equality

$$q = [1 - \rho_k(y - x - s_2)][1 + \rho_h(y - x - s_2)] = 1$$

can be reached only when $y - x - s_2 = 0$.

Using φ to denote the solution set of (4), the inequality

$$[1 - \rho_k(y - x - s_2)][1 + \rho_h(y - x - s_2)] > \left(\frac{1 - P^j}{P^j}\right)\left(\frac{P^i}{1 - P^i}\right) \qquad (5)$$

has the solution set $\phi(s_2)$, where $\phi(s_2) \subset \varphi$.

We then solve for $\phi(s_2)$. Since

$$[1-\rho_k(y-x-s_2)][1+\rho_h(y-x-s_2)]-\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right)>0,$$

it follows that

$$\rho_k\rho_h(y-x-s_2)^2+(\rho_k-\rho_h)(y-x-s_2)+\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right)-1<0.$$

We treat $(y-x-s_2)$ as a variable and try to solve the above inequality in terms of $(y-x-s_2)$. In other words, we study what values $(y-x-s_2)$ will make the inequality hold. In the following discussion, we use the notation $(y-x-s_2)_{1,2}$ to denote the two solutions that make the expressions on the left-hand side equal to zero.

Therefore, when $\Delta=(\rho_k-\rho_h)^2-4\rho_k\rho_h\left[\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right)-1\right]>0$, we have

$$(y-x-s_2)_{1,2}=\frac{(\rho_h-\rho_k)\pm\sqrt{(\rho_k-\rho_h)^2-4\rho_k\rho_h\left[\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right)-1\right]}}{2\rho_k\rho_h}. \qquad (6)$$

When $\min[(y-x-s_2)_{1,2}]<(y-x-s_2)<\max[(y-x-s_2)_{1,2}]$, $\rho_k\rho_h(y-x-s_2)^2+(\rho_k-\rho_h)(y-x-s_2)+\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right)-1<0$ will hold and we have $\left.\frac{ds_1}{ds_2}\right|_{v_h^j(s)}>\left.\frac{ds_1}{ds_2}\right|_{v_k^i(s)}$. When $s_2=y-x$, we always have $\left.\frac{ds_1}{ds_2}\right|_{v_h^j(s)}=P^j<P^i=\left.\frac{ds_1}{ds_2}\right|_{v_k^i(s)}$. Thus, $(y-x-s_2=0)$ cannot make $\rho_k\rho_h(y-x-s_2)^2+(\rho_k-\rho_h)(y-x-s_2)+\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right)-1<0$ hold and $(y-x-s_2=0)\notin\phi(s_2)$. Since $\phi(s_2)\subset\varphi$, we have $(y-x-s_2=0)\notin\varphi$. That is to say, the condition for $\left.\frac{ds_1}{ds_2}\right|_{v_h^j(s)}>\left.\frac{ds_1}{ds_2}\right|_{v_k^i(s)}$ must be either $(y-x-s_2>0)$ or $(y-x-s_2<0)$. The distribution of φ is similar to the distribution of $\phi(s_2)$. From here on, we focus on the solution set of $\phi(s_2)$, which should be similar to the solution set of φ. However, we need to make sure that $(y - x - s_2 < 0)$ cannot be the solution because the insurance coverage cannot exceed the loss amount.

We thus obtain the following important conclusion by solving for $\phi(s_2)$: For buyers who have different levels of risk aversion, when the risk aversion of low-risk buyers is higher than the risk aversion of high-risk buyers, there can be three possible situations, as discussed below.

Situation 1

$\Delta = (\rho_k - \rho_h)^2 - 4\rho_k\rho_h\left[\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right) - 1\right] \leq 0$. There is no root for inequality (5). Lemma 3 still holds and we always have $\frac{ds_1}{ds_2}\Big|_{v_k^i(s)} > \frac{ds_1}{ds_2}\Big|_{v_h^j(s)} > 0$, as is shown in Figure 5.

Situation 2

$\Delta = (\rho_k - \rho_h)^2 - 4\rho_k\rho_h\left[\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right) - 1\right] > 0$, and $a = \max[(y - x - s_2)_{1,2}] < 0$. The points that satisfy $\frac{ds_1}{ds_2}\Big|_{v_k^i(s)} < \frac{ds_1}{ds_2}\Big|_{v_h^j(s)}$ will lie in the area of $s_2 > y - x$, as is show in Figure 6. At $s_2 > y - x$, two indifference curves $v_k^i(\mathbf{s})$ and $v_h^j(\mathbf{s})$ intersect at s^1.

However, since insurers cannot provide indemnity which is higher than the actual loss, we only need to consider the area $s_2 \leq y - x$ here. That is consequently to say that in insurance market under situation 2, indifference curve of high-risk buyer is steeper than the indifference curve of low-risk buyer. Mathematically, when $s_2 \leq y - x$, $\frac{ds_1}{ds_2}\Big|_{v_k^i(s)} > \frac{ds_1}{ds_2}\Big|_{v_h^j(s)}$ always holds.

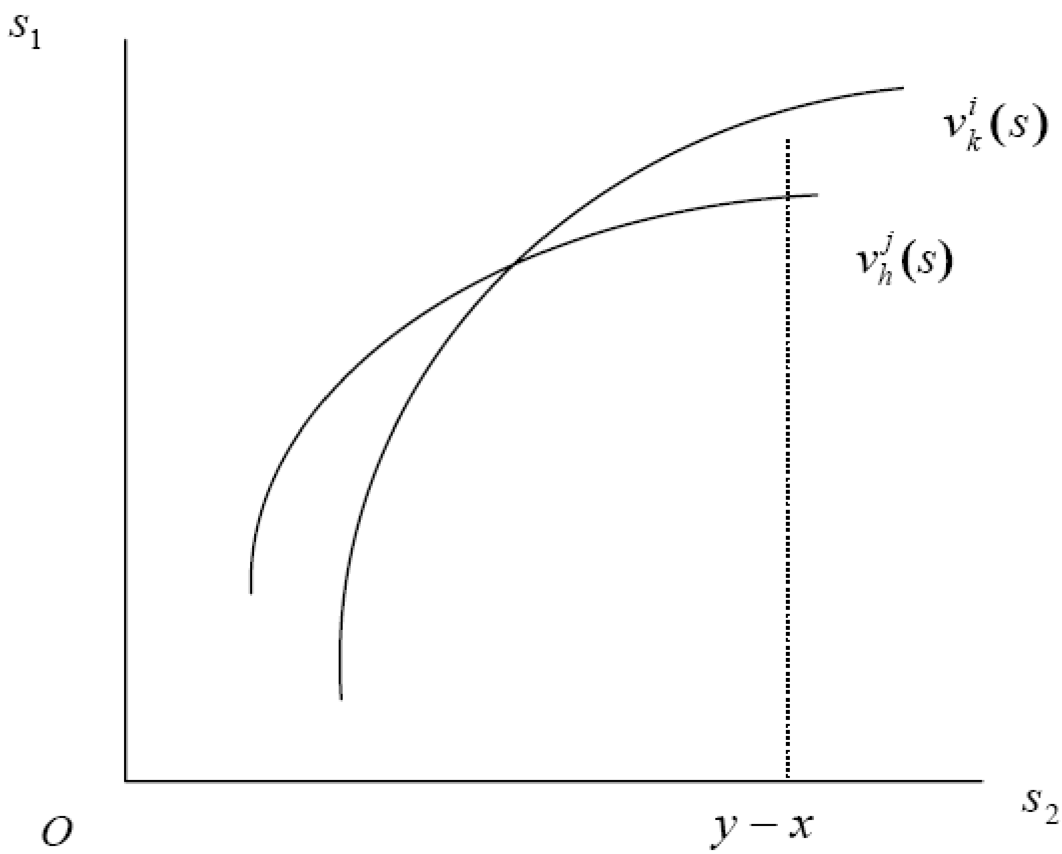

Figure 5. The Consumer's Utility Curve under Situation 1

Situation 3

$$\Delta = (\rho_k - \rho_h)^2 - 4\rho_k\rho_h\left[\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right) - 1\right] > 0$$ and $b = \min[(y - x - s_2)_{1,2}] > 0$. Here the

points that satisfy $\left.\frac{ds_1}{ds_2}\right|_{v_k^i(s)} < \left.\frac{ds_1}{ds_2}\right|_{v_h^j(s)}$ lie in the area of $s_2 < y - x$, as is shown in Figure 7.

When $s_2 < y - x$, two indifference curves $v_h^j(\mathbf{s})$ and $v_k^i(\mathbf{s})$ are tangent to each other at s^1.

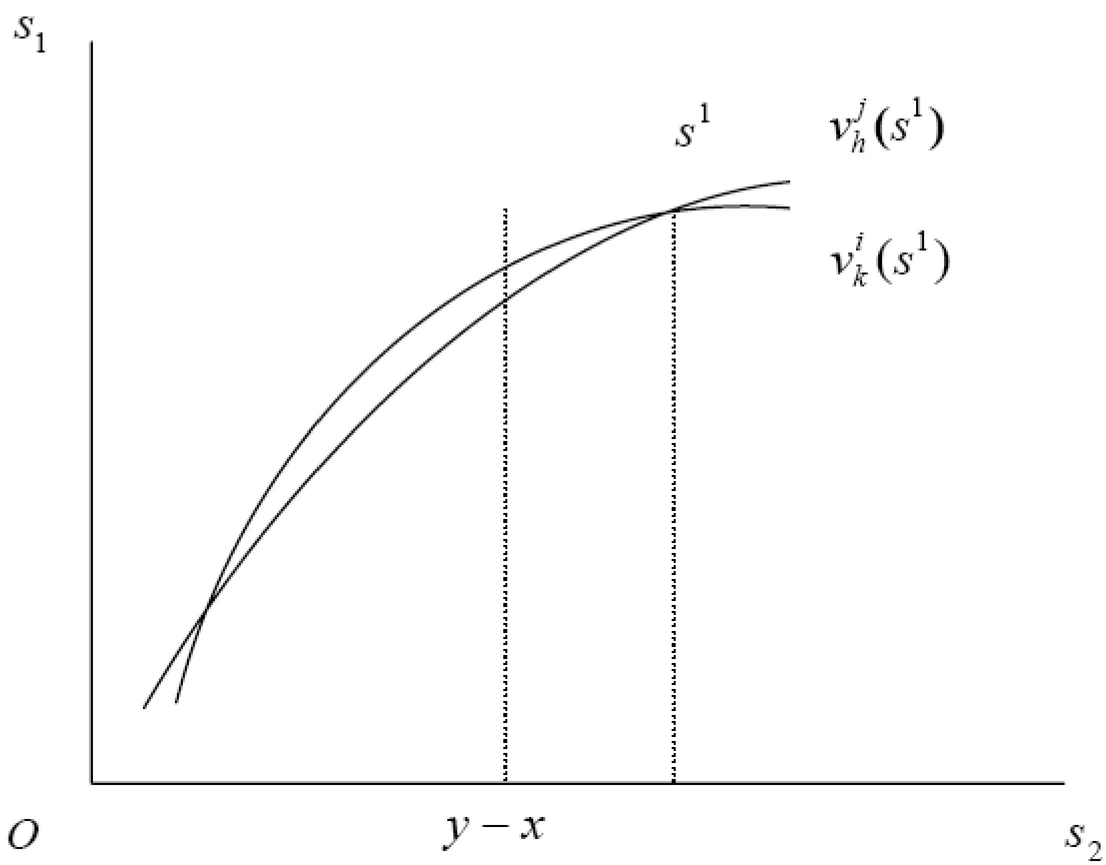

Figure 6. The Consumer's Utility Curve under Situation 2

According to the above counterexample and discussion, when we consider the different levels of risk aversion, there is no guarantee that Lemma 3 holds. Here, we discuss only the situation associated with high-risk buyers with low risk aversion and low-risk buyers with high risk aversion. Other types of buyers will be discussed in the following lemmas.

Lemma 4*: For buyer $T_{k_1}^i$ and $T_{k_2}^j$, when $i > j$, if $s^i \in S_i^*(s)$, $s^j \in S_j^*(s)$, there is no guarantee that $s^i \geq s^j$.

Here, we use $S_i^*(s)$, $S_j^*(s)$ to denote the policy set that buyers $T_{k_1}^i$ and $T_{k_2}^j$ would purchase at the equilibrium. We have a further interpretation for Lemma 4*. According to the results of Lemma 3*, we know there can be three possible situations associated with the indifference curves of buyers with different levels of risk aversion.

Situation 1: Lemma 4 still applies in this situation, and we have $s^i \geq s^j$.

Situation 2**:** According to Lemma 3^*, Lemma 3 still applies in the real market in this situation 2. That is to say when $s_2 \leq y - x$, high-risk buyer will purchase at least the same amount of insurance as low-risk buyer, and we have $s^i \geq s^j$ as is shown in Figure 8.

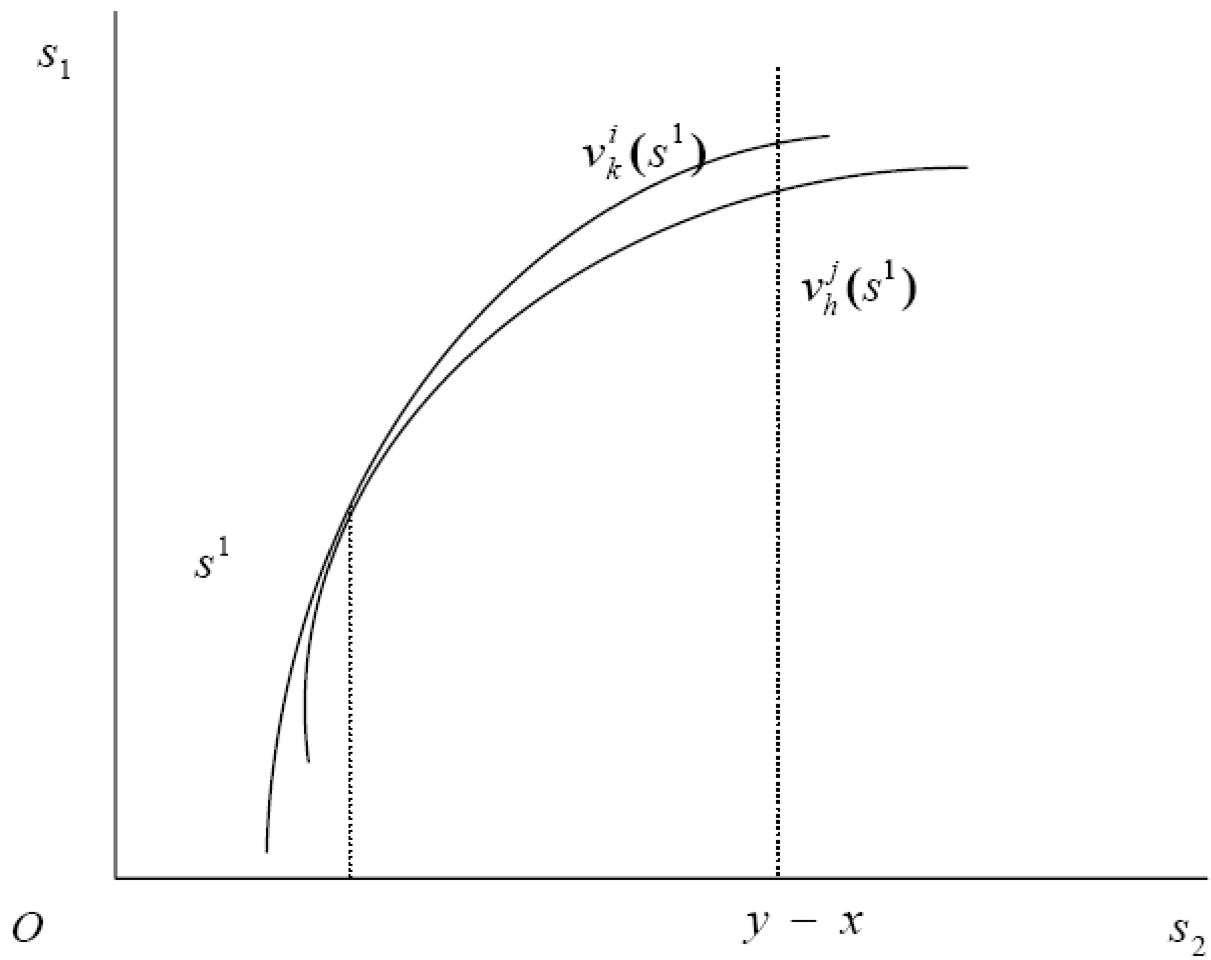

Figure 7. The Consumer's Utility Curve under Situation 3

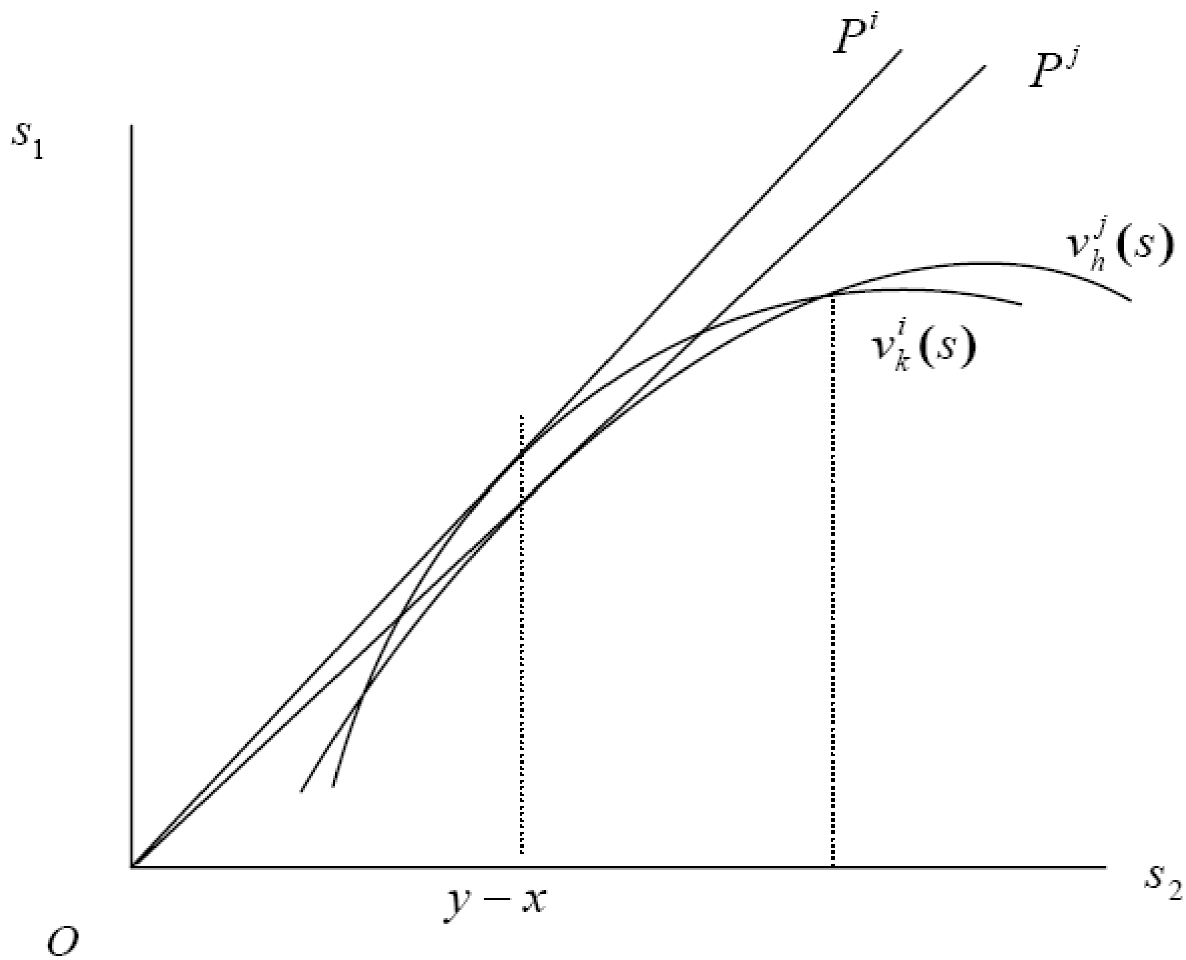

Figure 8. Different Risk Aversions of Lemma 4* in Situation 2

Situation 3**:** Indifference curve $v_h^j(s)$ and indifference curve $v_k^i(s)$ are tangent at s^1 as is shown in Figure 9. T_k^i's expected utility function $v_k^i(s^2)$ that crosses the insurance policy s^2 is higher than $v_k^i(s^1)$. That is to say, in this circumstance, high-risk buyer may prefer the insurance policy that has lower premium and lower indemnity than what low-risk buyer buys. Lemma 4 does not hold here.

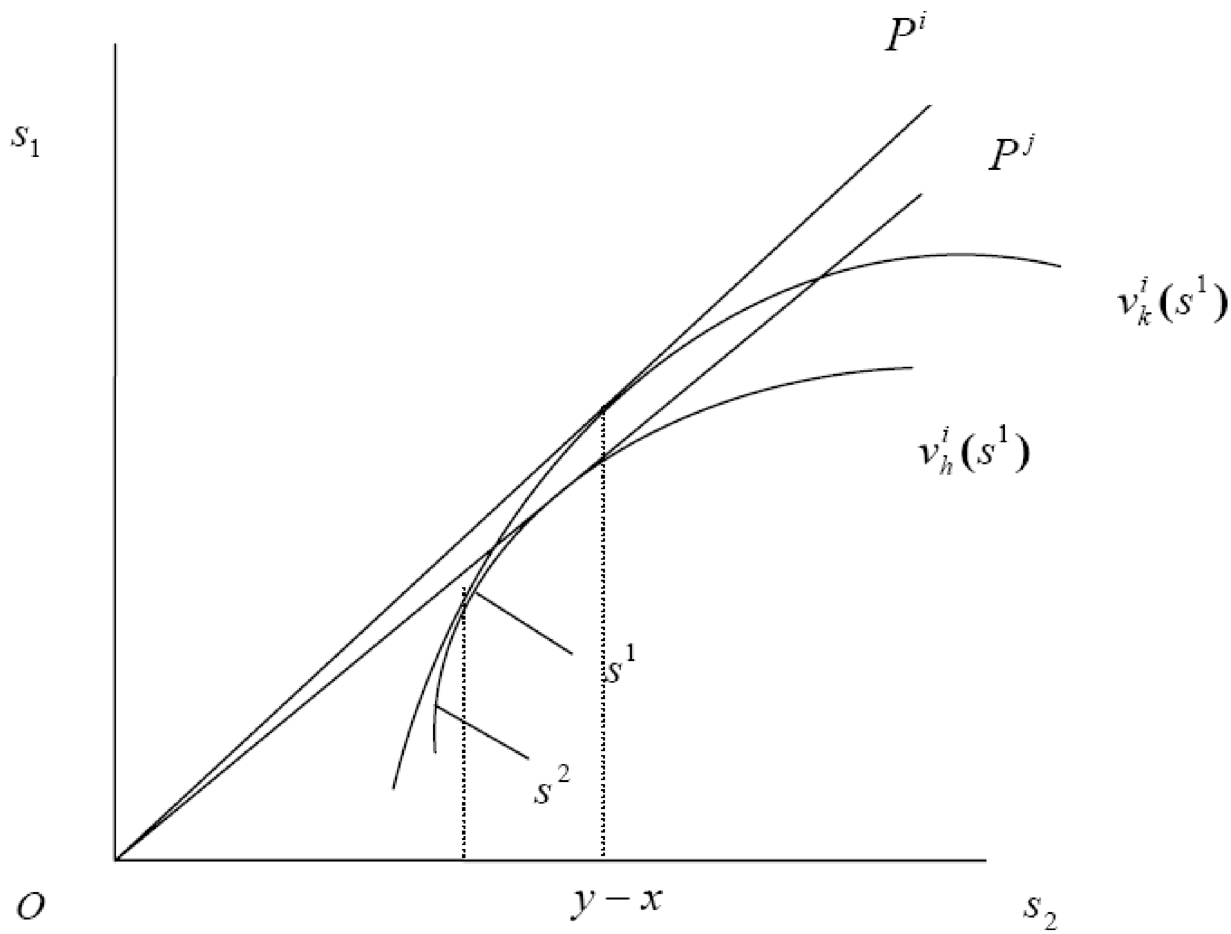

Figure 9. Lemma 4* under Situation 3

Lemma 5*: If $i > j$, for any insurance policy $s = (s_1, s_2) \in \overline{S}$ and any k_1, k_2, when $s_2 < [=,>] 0$, we always have $R(s, e^i_{k_1}) > [=,<] R(s, e^j_{k_2})$.

Proof: After taking the different levels of risk aversion into account, when $i > j$ which is the same as $P^i > P^j$. If $s_2 > 0$, we have

$$R(s, e^i_{k_1}) = s_1 - P^i s_2 < s_1 - P^j s_2 = R(s, e^j_{k_2}).$$ ν

Lemma 6*: Assume $i > j$, for any two buyers T_h^i and T_k^j, if $s_2 \leq y - x$, then we have $\left.\frac{ds_1}{ds_2}\right|_{v_h^i(s)} > \left.\frac{ds_1}{ds_2}\right|_{v_k^j(s)}$.

Proof: We change (6) a little bit and get:

$$(y - x - s_2)_{1,2} = \frac{(\rho_k - \rho_h) \pm \sqrt{(\rho_k - \rho_h)^2 - 4\rho_k\rho_h\left[\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right) - 1\right]}}{2\rho_k\rho_h}$$

. When $\Delta > 0$, since $\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right) > 1$, therefore

$$\sqrt{(\rho_k - \rho_h)^2 - 4\rho_k\rho_h\left[\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right) - 1\right]} < |\rho_k - \rho_h| = \rho_h - \rho_k$$

, and we consequently get $a < 0, b < 0$; that is to say the points that satisfy $\left.\frac{ds_1}{ds_2}\right|_{v_h^i(s)} < \left.\frac{ds_1}{ds_2}\right|_{v_k^j(s)}$ also satisfy $y - x - s_2 < 0$. When $s_2 \leq y - x$, we have $\left.\frac{ds_1}{ds_2}\right|_{v_h^i(s)} > \left.\frac{ds_1}{ds_2}\right|_{v_k^j(s)}$. When $\Delta \leq 0$, $\left.\frac{ds_1}{ds_2}\right|_{v_h^i(s)} > \left.\frac{ds_1}{ds_2}\right|_{v_k^j(s)}$ always holds.

Lemma 6* explains, for the buyer with higher risk and higher risk aversion, that when $s_2 \leq y - x$, his or her indifference curve is always steeper than that of buyer with lower risk and lower risk aversion. For example, when $\rho_h = 10, \rho_k = 1$, and $s_2 = 0.3 < 1 = y - x$, we have

$$\left.\frac{ds_1}{ds_2}\right|_{v_h^i(s)} = \frac{1}{1 + e^{-7}} > \frac{1}{1 + (3/2)e^{-0.7}} = \left.\frac{ds_1}{ds_2}\right|_{v_k^j(s)}.$$

Lemma 7*: As for the buyers T_k^i and T_h^i who have the same loss probability, when $s_2 < [=, >]\, y - x$, we have $\left.\frac{ds_1}{ds_2}\right|_{v_k^i(s)} < [=, >] \left.\frac{ds_1}{ds_2}\right|_{v_h^i(s)}$.

Proof: When the loss probabilities are the same, we can modify (6) as

$$(y-x-s_2)_{1,2}=\frac{(\rho_k-\rho_h)\pm\sqrt{(\rho_k-\rho_h)^2-4\rho_k\rho_h\left[\left(\frac{1-P^j}{P^j}\right)\left(\frac{P^i}{1-P^i}\right)-1\right]}}{2\rho_k\rho_h}$$

$=\dfrac{(\rho_k-\rho_h)\pm|\rho_k-\rho_h|}{2\rho_k\rho_h}$. Therefore, it is easy to obtain that $a=0, b=\dfrac{\rho_k-\rho_h}{\rho_k\rho_h}$, and when $s_2<y-x$ or $s_2>y-x-\dfrac{\rho_2-\rho_1}{\rho_1\rho_2}$, we have $\dfrac{ds_1}{ds_2}\Big|_{v_k^i(s)}<\dfrac{ds_1}{ds_2}\Big|_{v_h^i(s)}$. According to Lemma 2*, when $s_2=y-x$, we have $\dfrac{ds_1}{ds_2}\Big|_{v_k^i(s)}=\dfrac{ds_1}{ds_2}\Big|_{v_h^i(s)}$. Given the continuity of the function, the lemma follows.

We can continue using the above counterexample to illustrate the Lemma 7*. Since the loss probabilities of the two consumers should be the same, we assume $P_i=P_j=0.5$. Then

$$\frac{ds_1}{ds_2}\Big|_{v_k^i(s)}=\frac{1}{1+\left(\frac{1-0.5}{0.5}\right)e^{-(1-s_2)}}=\frac{1}{1+e^{s_2-1}},$$

$$\frac{ds_1}{ds_2}\Big|_{v_h^j(s)}=\frac{1}{1+\left(\frac{1-0.5}{0.5}\right)e^{-10(1-s_2)}}=\frac{1}{1+e^{10s_2-10}}.$$

When $s_2=y-x=1$, we have $\dfrac{ds_1}{ds_2}\Big|_{v_k^i(s)}=\dfrac{ds_1}{ds_2}\Big|_{v_h^j(s)}=0.5$; when $s_2<y-x=1$, for example $s_2=0.3$, we have $\dfrac{ds_1}{ds_2}\Big|_{v_k^i(s)}=\dfrac{1}{1+e^{-0.7}}<\dfrac{1}{1+e^{-7}}=\dfrac{ds_1}{ds_2}\Big|_{v_h^j(s)}$; when $s_2>y-x=1$, for example $s_2=3$, we have $\dfrac{ds_1}{ds_2}\Big|_{v_k^i(s)}=\dfrac{1}{1+e^{2}}>\dfrac{1}{1+e^{20}}=\dfrac{ds_1}{ds_2}\Big|_{v_h^j(s)}$.

Lemma 7* says, for the buyers who face the same loss probability, when $s_2\le y-x$, the indifference curve of buyers with higher risk aversion is steeper than that of buyers with lower risk aversion; when $s_2 > y - x$, the indifference curve of buyers with lower risk aversion is steeper than that of buyers with higher risk aversion.

Lemma 8*: As for the buyers of the same loss probabilities, buyer who has lower risk aversion would purchase less insurance; buyer who has higher risk aversion would purchase more insurance. As is shown in Figure 10, $v_k^i(s^0)$ and $v_h^i(s^0)$ are tangent at s^0, and $v_h^i(s^0)$ is under $v_k^i(s^0)$.

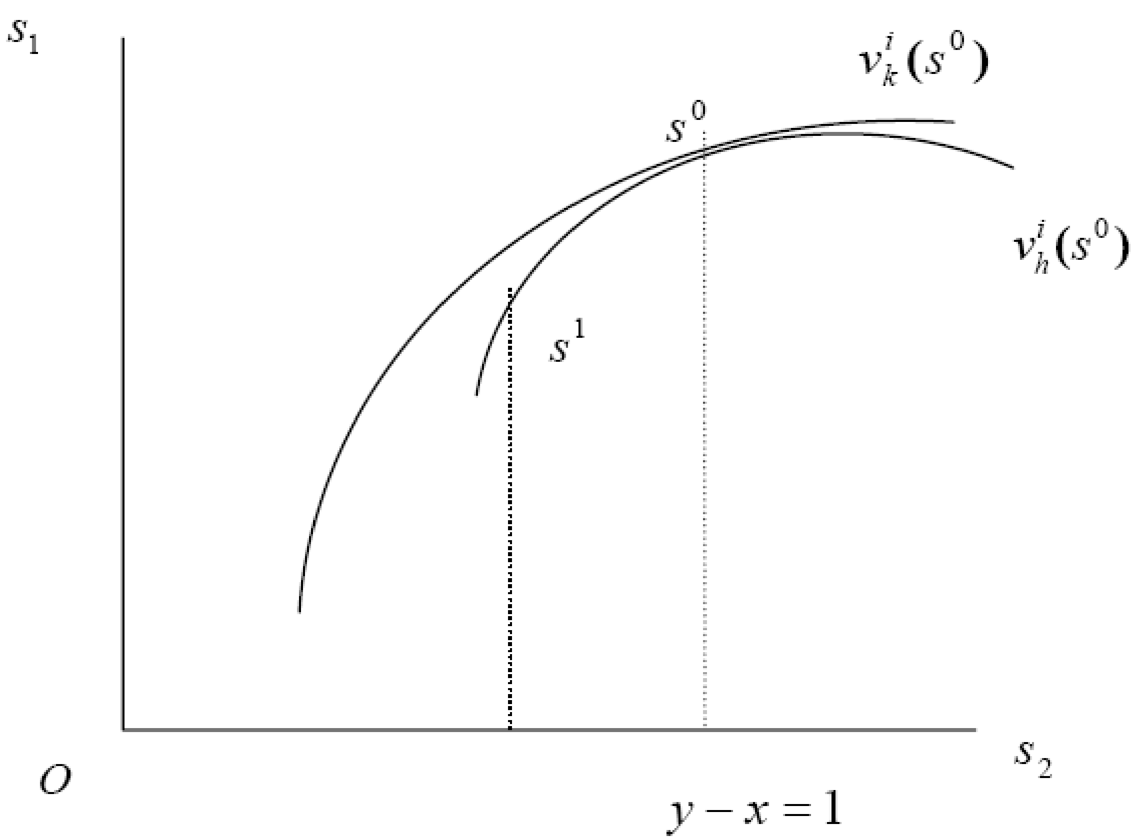

Figure 10. Utility Curves of Consumers with the Same Risk but Different Risk Aversion

As a summary of the above discussion, we note:

(1) For buyers with different risks and different risk aversion, the relationship between their expected utility curves has three possibilities, as has been discussed in Lemma 3^*.

(2) For buyers with the same risk but different risk aversion, the relationship between their expected utility functions is discussed in Lemma 8^*. They will be tangent to $s_2 = y - x$, and the expected utility functions of buyers with higher risk aversion lie under the expected utility functions of buyers with lower risk aversion.

(3) As for buyers with different risks but the same level of risk aversion, their situation is shown in the Situation 1 of Lemma 3^*. The expected utility curve of the high-risk buyer is steeper than that of the low-risk buyer.

We now discuss the market equilibrium after we introduce heterogeneous risk aversion. For convenience, we consider four types of buyers with two loss probabilities (risks) and two different types of risk aversion. That is to say, we divide all the customers into four groups: T_k^j, T_h^j, T_k^i, and T_h^i. Our discussion is based upon the results of Lemma 3^*.

Modified Market Equilibrium, Situation 1

As is shown in Figure 11, according to Lemma 2^*, buyers with the same risk but different risk aversion have expected utility functions whose slopes are equal to loss probabilities P^i at $s_2 = y - x$. The expected utility curve will tangent to $\overline{OP^i}$. In case that the market has complete information, then the Pareto-optimal equilibrium should be $\{s^1, s^2\}$. All the buyers are able to receive full insurance and insurers will gain zero profit.

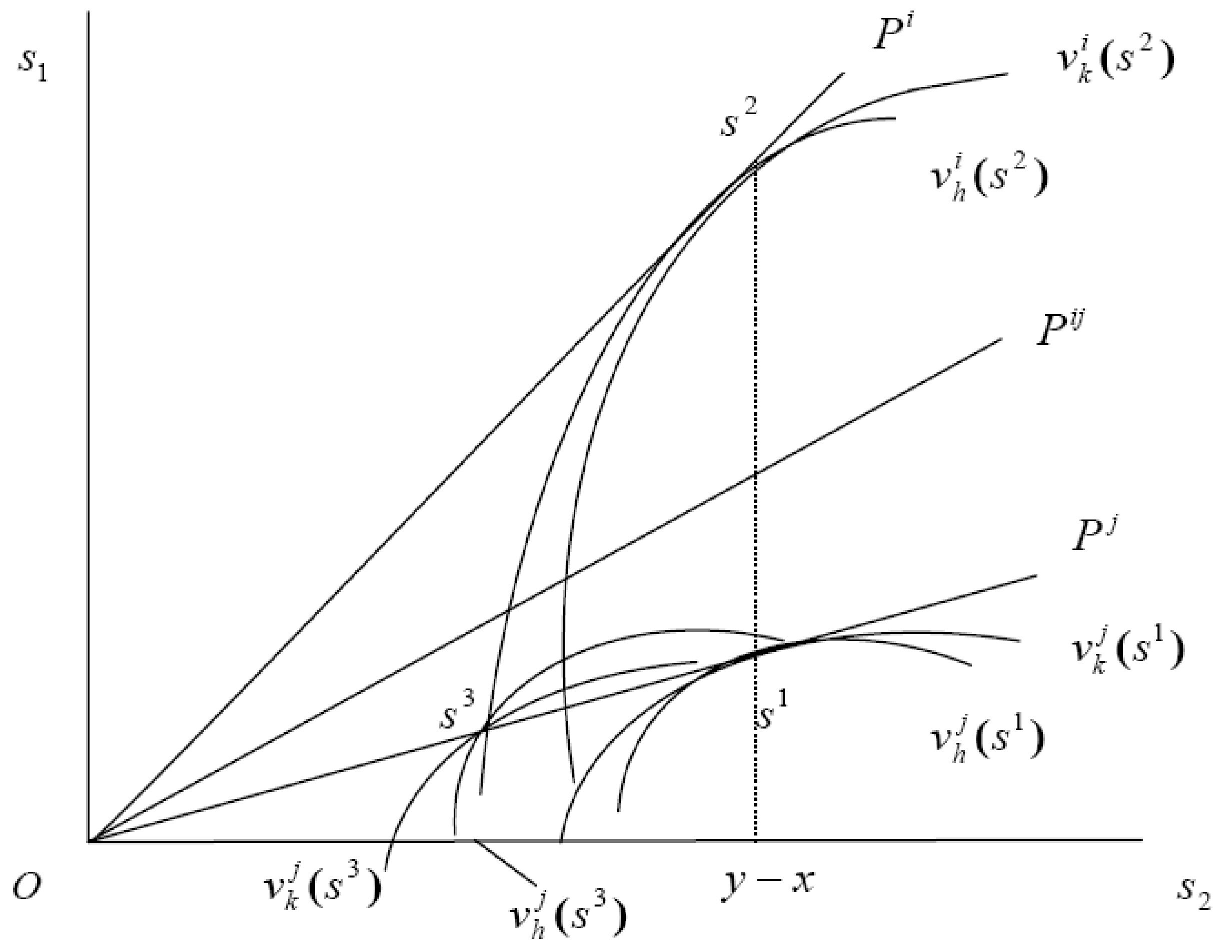

Figure 11. Separating Equilibriums with Two Risks and Two Risk Aversions

In case that market information is asymmetric, insurers cannot recognize different buyers. If both s^1 and s^2 are provided, high-risk buyers T_k^i and T_h^i will purchase s^1 too, because their expected utility at s^1 is higher than the expected utility at s^2. If all the buyers choose to purchase insurance policy s^1, insurers will lose money. Therefore, $\{s^1, s^2\}$ is not the Wilson equilibrium under asymmetric information.

Separating Equilibrium

To reach separating equilibrium, insurers need to provide different policies to different buyers. In order to avoid loosing money, the insurance policies for T_k^j and T_h^j cannot lie under $\overline{OP^j}$. If we want to keep T_k^i and T_h^i from buying the policies designed for T_k^j and T_h^j, these policies cannot lie under $v_k^i(s^2)$ and $v_h^i(s^2)$. Obviously, among all the available policies, T_k^j and T_h^j would prefer the insurance policy s^3 which is the intersection of $v_k^i(s^2)$ (indifference curve of T_k^i that crosses s^2) and $\overline{OP^j}$.

When the indifference curves $v_k^j(s^3)$ and $v_h^j(s^3)$ of the low-risk buyers T_k^j and T_h^j both lie under $\overline{OP^{ij}}$, policies $\{s^2, s^3\}$ constitutes a Wilson equilibrium, because there does not exist another insurance policy that could attract all the four kinds of buyers and at the same time does not generate negative profit for insurers. Here, buyers T_k^j and T_h^j would purchase s^3, while T_k^i and T_h^i would purchase s^2.

Pooling Equilibrium

If there is a large proportion of low-risk buyers among all the buyers, line $\overline{OP^{ij}}$ will approach to line $\overline{OP^j}$, and the indifference curves $v_k^j(s^3)$ and $v_h^j(s^3)$ may go through $\overline{OP^{ij}}$, as is shown in Figure 12. In such condition, if insurer provides a policy which lies above $\overline{OP^{ij}}$ but under $v_k^j(s^3)$ and $v_h^j(s^3)$, then all the buyers who purchase this policy will gain higher expected utility, and profit company will have positive profit. Due to the competition, the insurance policy provided will eventually lies on $\overline{OP^{ij}}$, and insurers will have zero profits.

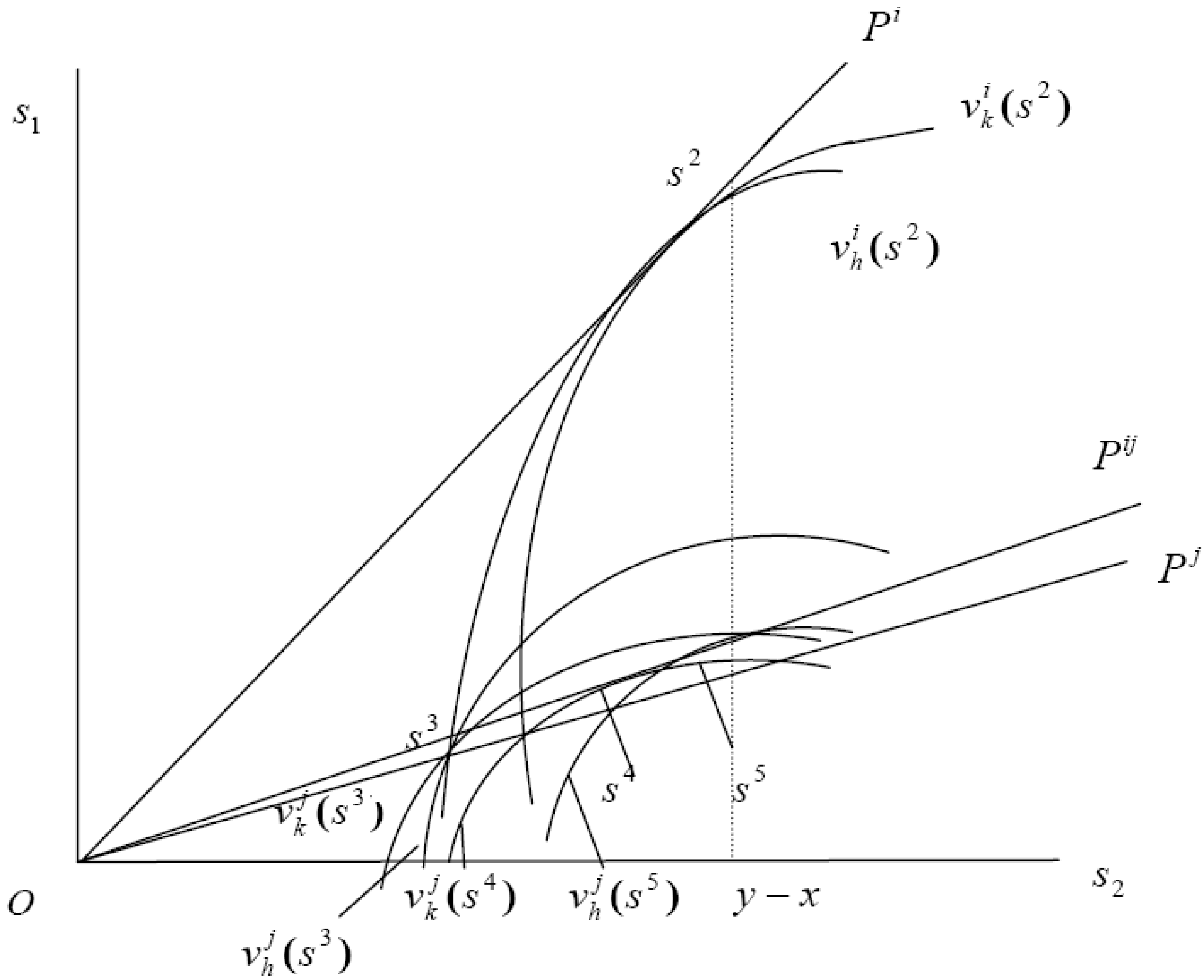

Figure 12. Pooling Equilibrium with Two Risks and Two Risk Aversions

We assume that the expected utility curve of buyer T_k^j is tangent to $\overline{OP^{ij}}$ at s^4, and the utility curve of buyer T_h^j is tangent to $\overline{OP^{ij}}$ at s^5. According to Lemma 7*, when $s_2 < y - x$, we have $\frac{ds_1}{ds_2}\Big|_{v_h^j(s)} > \frac{ds_1}{ds_2}\Big|_{v_k^j(s)}$. However, since $\frac{ds_1}{ds_2}\Big|_{v_h^j(s^5)} = \frac{ds_1}{ds_2}\Big|_{v_k^j(s^4)}$, we easily obtain that $s^4 < s^5$. That is to say buyer T_k^j who has lower absolute risk aversion would prefer policy with less coverage.

We now discuss the market equilibrium when $s^4 < s^5$. First, we test whether $\{s^5\}$ is the Wilson pooling equilibrium. Assume that an insurer provides only one policy s^5, if no other companies provide any insurance policies, all the buyers would purchase s^5, and that company would gain zero profit. However, if another company provides s^4, buyer T_h^j would stick to his choice, but buyer T_k^j would move to s^4. According to the Situation 1 in Lemma 4*, buyers T_k^i and T_h^i would choose s^5 too. Consequently, the company that provides s^5 would lose money, because only when all the insurers come to purchase s^5, this company could keep nonnegative profit. If some low-risk buyers leave, the insurer will not collect enough premium to cover its expected loss. Therefore, policy s^5 cannot exist in the market, leaving only s^4. All buyers would choose to purchase s^4, and insurers would gain zero profit. Therefore, policy $\{s^4\}$, rather than $\{s^5\}$, is the modified Wilson equilibrium.

We also note that the more that T_h^j and T_k^j differ from each other in terms of risk aversion, the further that s^4 is from s^5, and the lower expected utility that buyers T_h^i, T_k^i and T_h^j would gain at the equilibrium of $\{s^4\}$.

Modified Market Equilibrium, Situation 2

In this circumstance, we only need to talk about the equilibrium when $s_2 \le y - x$ is satisfied. It is very similar to Situation 1.

Modified Market Equilibrium, Situation 3

According to the analysis in the above, equilibrium is mainly related to the high-risk buyers with low risk aversion and low-risk buyer with low risk aversion. Therefore, for convenience, we assume that the indifference curves of T_k^i and T_k^j are tangent. We will discuss two different situations.

Assume that point s^1 is the full insurance that T_h^i and T_k^i get; point s^2 is the intersection of $v_k^i(s^1)$ and $\overline{OP^j}$; point s_2^* is where T_k^i and T_k^j are tangent to each other.

The Case of $s_2^* \le s_2(s^2)$*: Separating Equilibrium*

The indifference curve $v_k^j(s^3)$ for buyer T_k^j that crosses s^3 and the indifference curve $v_k^i(s^1)$ are tangent at s_2^*. Point s^2 is the intersection of $v_k^i(s^1)$ and $\overline{OP^j}$. Insurers would provide different policies to different buyers in order to avoid loosing money. Under asymmetric information, insurers could not provide policies $\{s^1, s^3\}$ at the same time, because high-risk buyers would also purchase s^3, which will cause the insurers to lose money. We note that $\{s^1, s^2\}$ is the real Wilson equilibrium. Because according to Lemma 2^*, high-risk buyer would purchase policy s^1 which provides higher indemnity, rather than the policy s^2 which give the same level of expected utility; low-risk buyers would purchase s^2, and insurers would gain zero profit. Here, indifference curves $v_h^j(s^2)$ and $v_k^j(s^2)$ do not go through $\overline{OP^{ij}}$, and separating equilibrium exists.

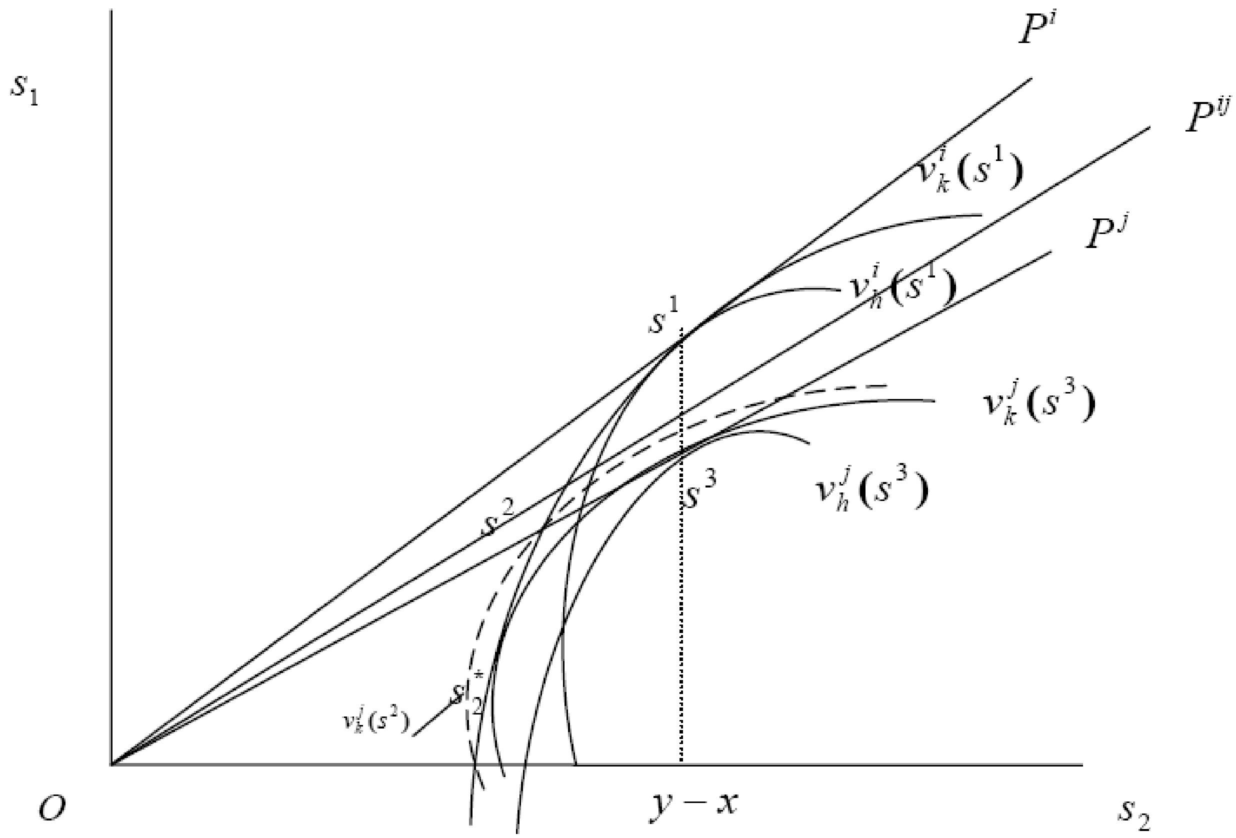

Figure 13. Separating Equilibrium_1 in Situation 3

The Case of $s_2^* \leq s_2(s^2)$*: Pooling Equilibrium*

When indifference curves $v_h^j(s^2)$ and $v_k^j(s^2)$ go through $\overline{OP^{ij}}$, according to the result of Situation 1, assume that the indifference curves of T_k^j and T_h^j are tangent to $\overline{OP^{ij}}$ at s^4 and s^5. Through similar analysis, we have the conclusion that $\{s^4\}$ is the Wilson Pooling Equilibrium.

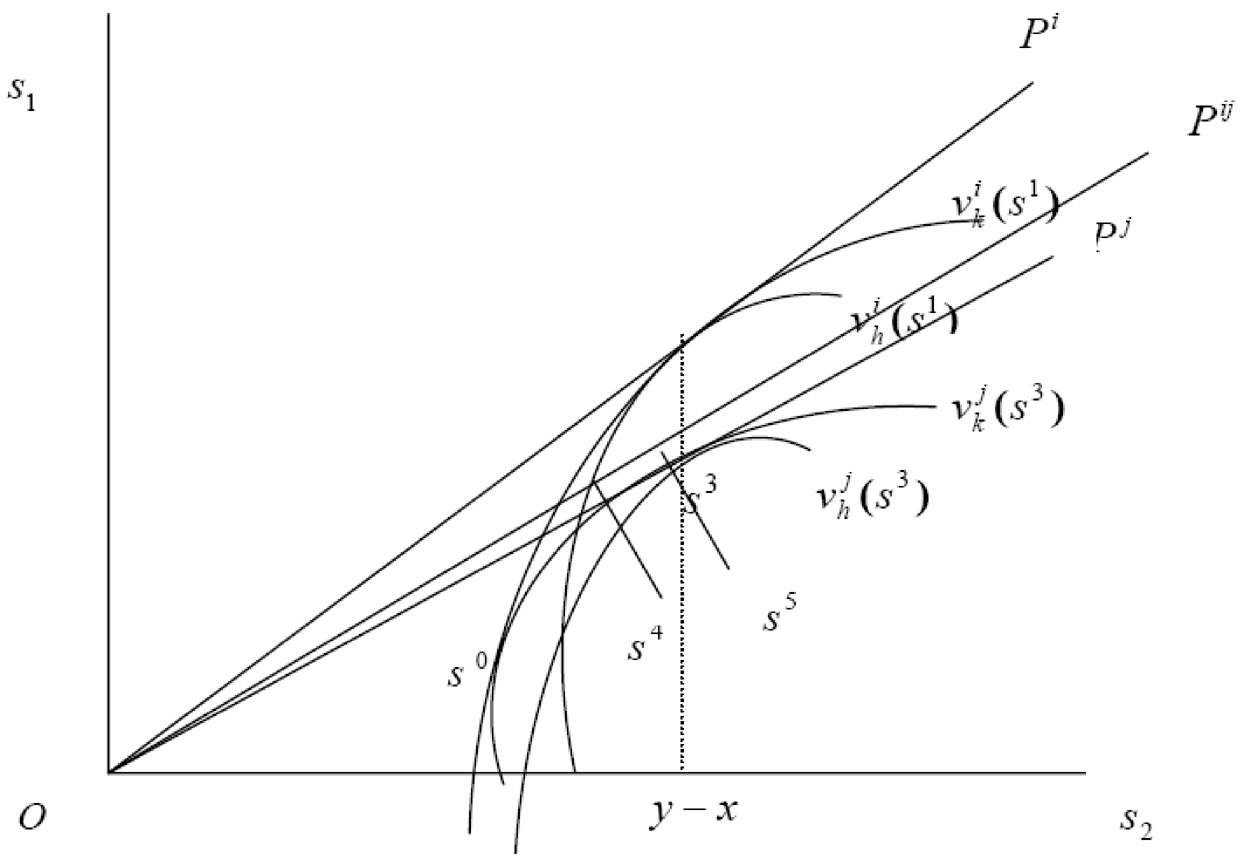

Figure 14. Pooling Equilibrium_1 in Situation 3

The Case of $s_2^* > s_2(s^2)$*: Separating Equilibrium*

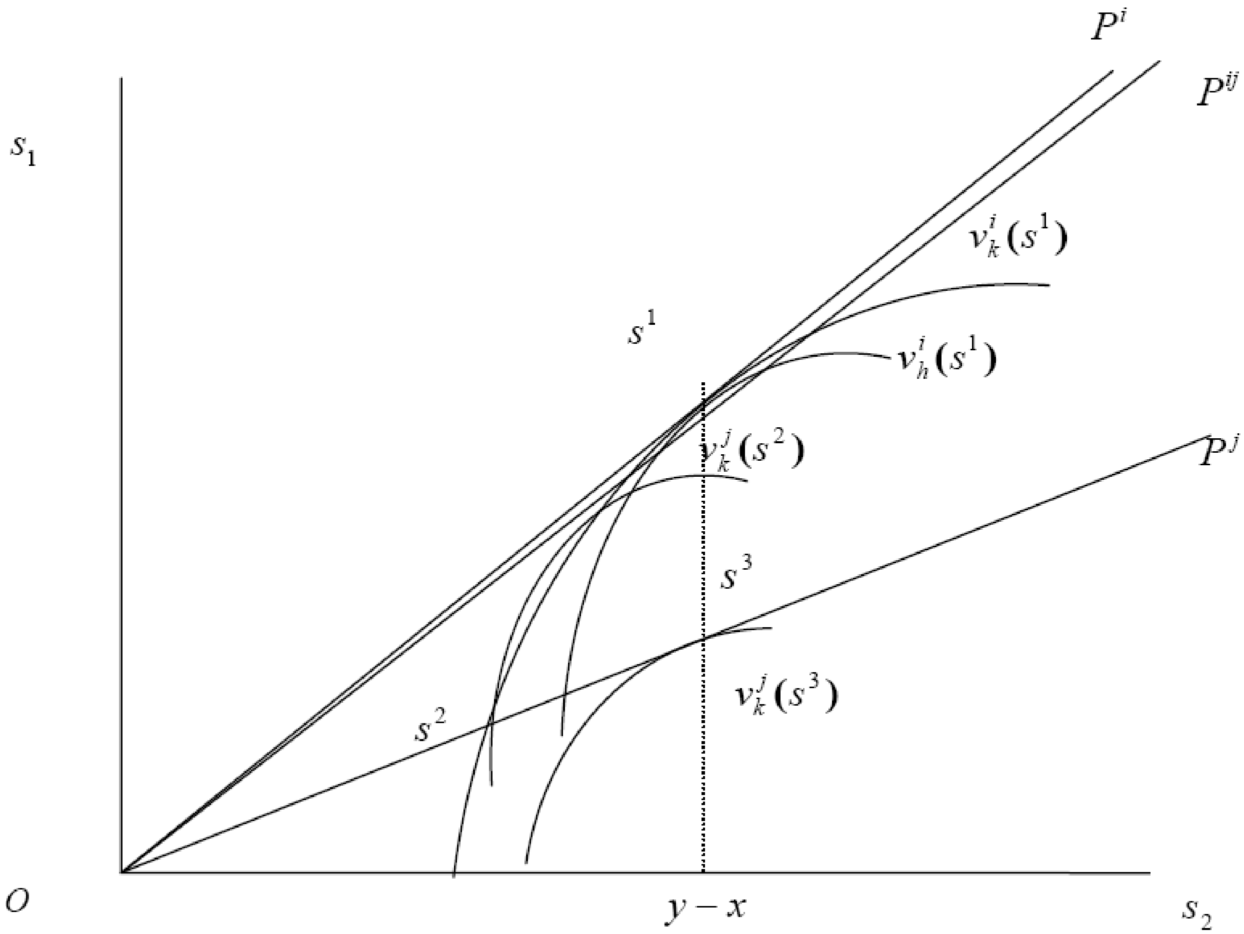

Figure 15. Separating Equilibrium_2 in Situation 3

Note that $v_k^j(s^2)$ is the indifference curve of T_k^j that goes across s^2. As is shown in Figure 15, $\{s^1, s^2\}$ is not Wilson equilibrium, because if another insurer provides a policy under $v_k^j(s^2)$ but above $v_k^i(s^1)$, then low-risk buyers T_k^j and T_h^j would move to purchase this policy, while high-risk buyers T_h^i and T_k^i would stick to s^1, and insurers would gain positive profit.

Assume the indifference curve of T_k^j is tangent to $v_k^i(s^1)$ at s^4, the indifference curve of T_h^j is tangent to $v_k^i(s^1)$ at s^5. According to Lemma 7^*, it is easy to prove that $s^4 < s^5$.

When both $v_h^j(s^5)$ and $v_k^j(s^4)$ lie under $\overline{OP^{ij}}$, we need to investigate whether $\{s^1, s^4, s^5\}$ is a Wilson equilibrium. As is shown in Figure 15, buyer T_k^j buys insurance policy s^4, buyer T_h^j buys insurance policy s^5, while T_h^i and T_k^i purchase insurance policy s^1. Since policies s^4 and s^5 lie above $\overline{OP^j}$, insurers would gain positive profit. If another insurer provides a policy under $v_h^j(s^5)$ and $v_k^j(s^4)$, then buyers T_k^j and T_h^j would choose to purchase this policy. However, T_h^i and T_k^i will come to purchase this policy too, which causes this company to lose money. Therefore, $\{s^1, s^4, s^5\}$ is Wilson equilibrium. We note that the insurer which provides three kinds of polices will gain positive profit. This differs from Wilson's conclusion that all the insurers would gain zero profit.

If $v_h^j(s^5)$ goes through $\overline{OP^{ij}}$, indifference curve $v_k^j(s^4)$ will lie under $\overline{OP^{ij}}$, as is shown in Figure 17. We know when only buyers T_h^j, T_h^i, and T_k^i purchase the policies on $\overline{OP^i}$, insurers would gain positive profit. However, if only these buyers purchase insurance policies on $\overline{OP^{ij}}$, insurers would gain negative profit. Therefore, we could find a point between $v_k^i(s^1)$ and $\overline{OP^{ij}}$ at which when only these three buyers purchase this policy, the insurer would gain zero profit. We denote this point as s^*. When point s^* is above $v_h^j(s^5)$, if another insurer provides a policy which under $v_h^j(s^5)$ but above $\overline{OP^{ij}}$, buyers T_h^j, T_h^i, and T_k^i would all choose to purchase this policy. Buyer T_k^j will stick to s^4, the second insurer would lose money. Therefore, policies $\{s^1, s^4, s^5\}$ are still Wilson equilibrium.

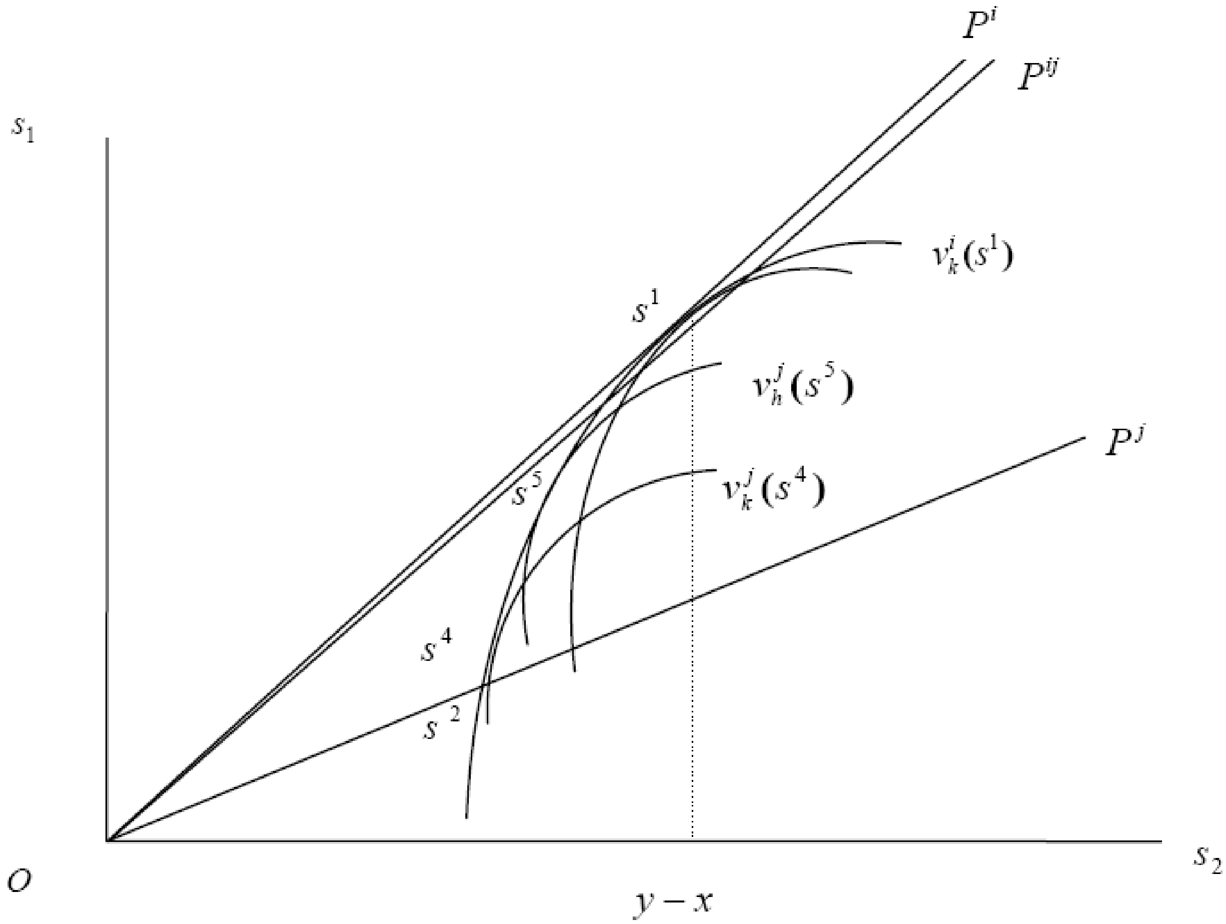

Figure 16. Separating Equilibrium in Situation 3

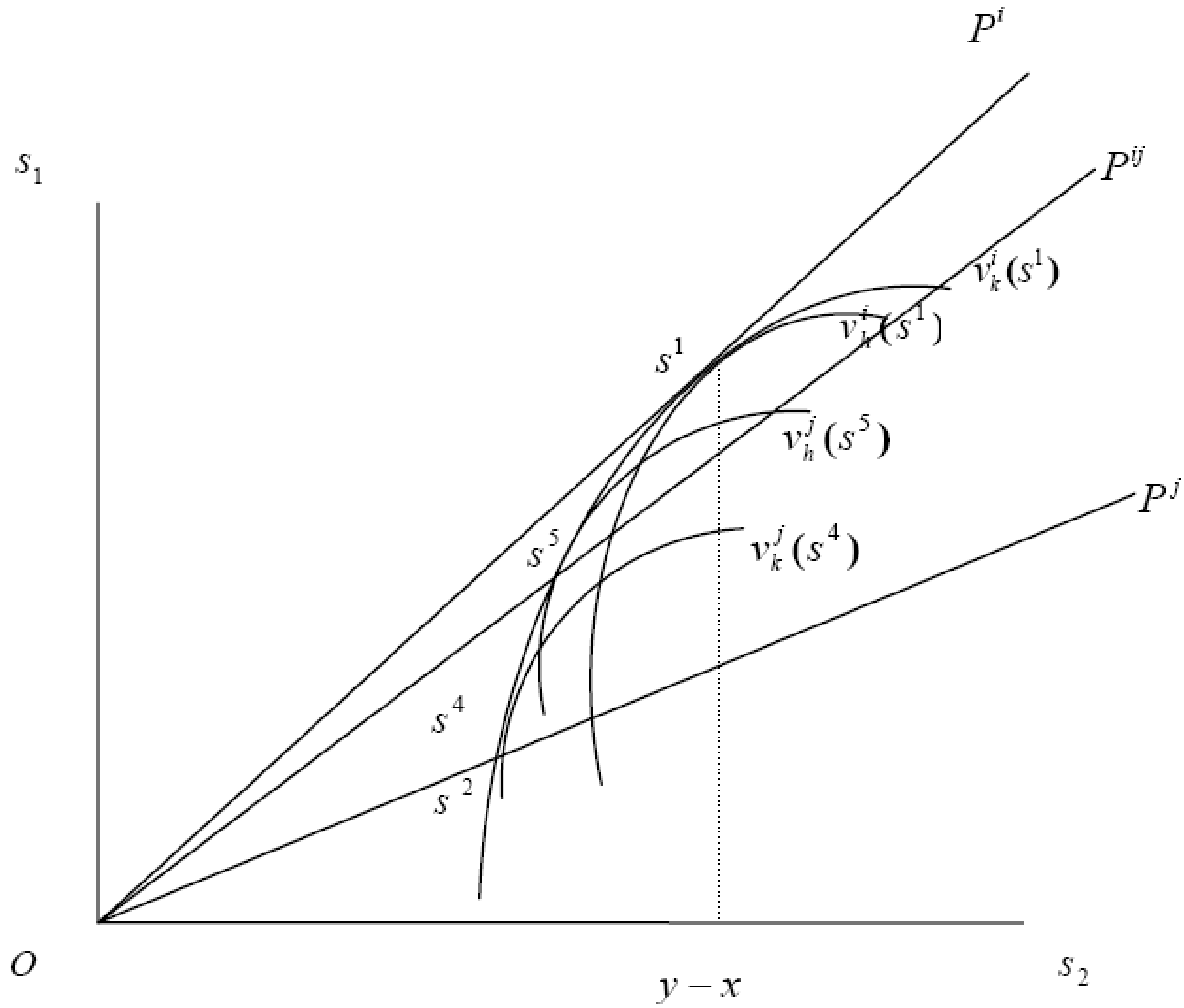

Figure 17. Analysis of Separating Equilibrium in Situation 3

When point s^* is under $v_h^j(\mathbf{s^5})$, if an insurer provides s^*, then buyers T_h^j, T_h^i and T_k^i all would choose to purchase policy s^*. Assume that the indifference curve of T_k^j is tangent to $v_k^i(\mathbf{s^*})$ at s^6. If point s^6 lies under $\overline{OP^j}$, we can prove that $\{s^*, s^6\}$ is the Wilson equilibrium, and the insurer that provides these policies would gain positive profit. We note this is not a strict concept of separating equilibrium, because some low-risk buyers are not separated. If point s^6 lies under $\overline{OP^j}$, the market equilibrium is just like what we have discussed in i).

The Case of $s_2^* > s_2(s^2)$*: Pooling Equilibrium*

If indifference curves $v_h^j(\mathbf{s^5})$ and $v_k^j(\mathbf{s^4})$ both go through $\overline{OP^{ij}}$, this is similar to the pooling equilibrium in Situation 2. Assume that the indifference curve of T_k^j is tangent to $\overline{OP^{ij}}$ at s^7, then $\{s^7\}$ is the pooling equilibrium, and insurers gain zero profit.

Uniqueness and Pareto Optimality

Next, we discuss the uniqueness of the modified Wilson equilibrium. For convenience, we only consider two different risk levels and two different risk aversion. They are denoted as T_k^j, T_h^j, T_k^i, and T_h^i.

As is shown in Figure 19, buyer T_k^j and line $\overline{OP^{ij}}$ are tangent at s^0 which is the pooling equilibrium mentioned above; point s^1 is the intersection of $v_h^i(\mathbf{s^2})$ and $\overline{OP^j}$. When $v_k^j(\mathbf{s^1})$ crosses s^1, policies $\{s^1, s^2\}$ and $\{s^0\}$ are both Wilson equilibria. The utilities of buyers T_h^j, T_k^i, and T_h^i are higher at $\{s^0\}$ than at $\{s^1, s^2\}$. Therefore, we argue that $\{s^0\}$ is better than $\{s^1, s^2\}$ in terms of Pareto optimality.

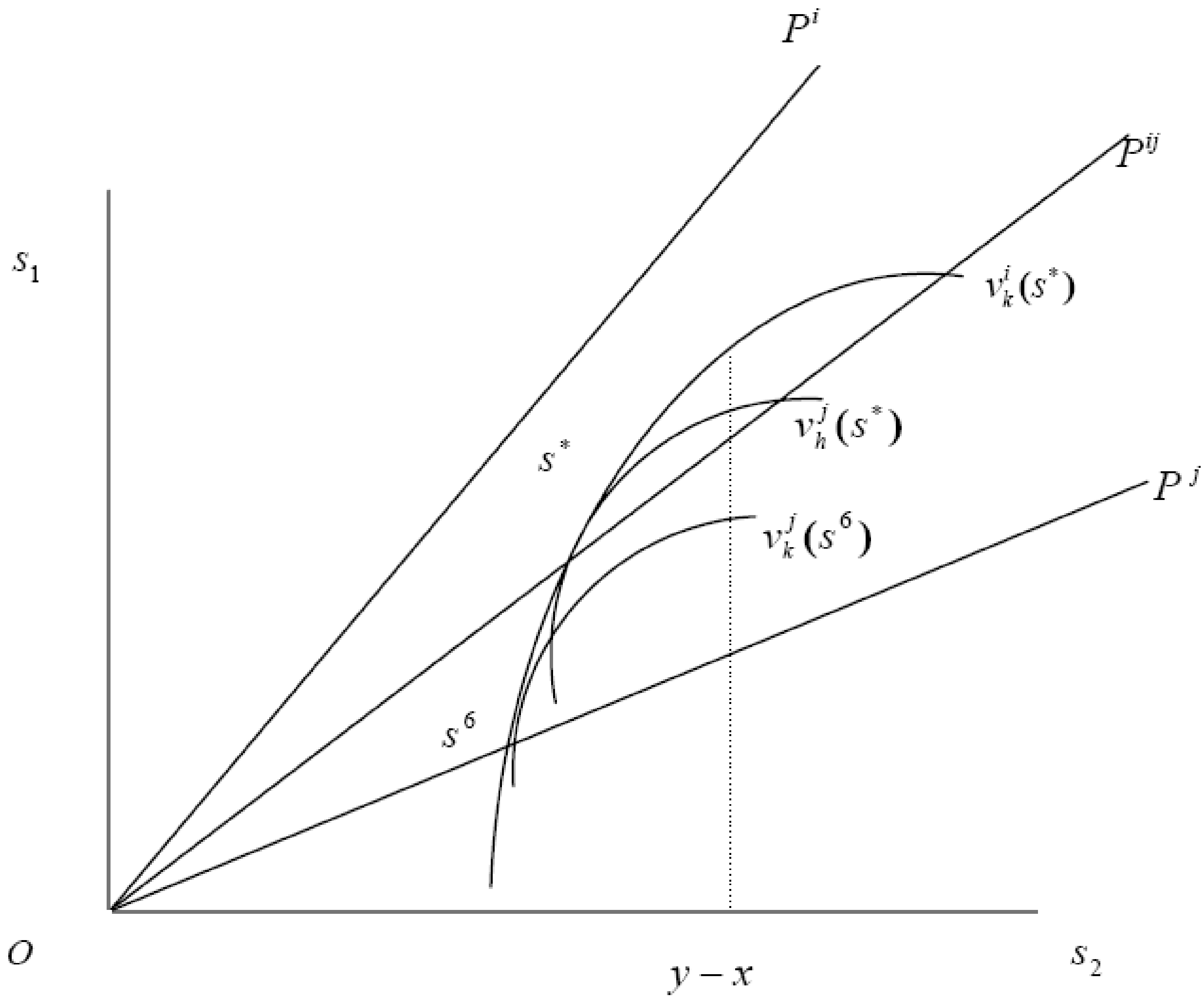

Figure18. Separating Equilibrium_3 in Situation 3

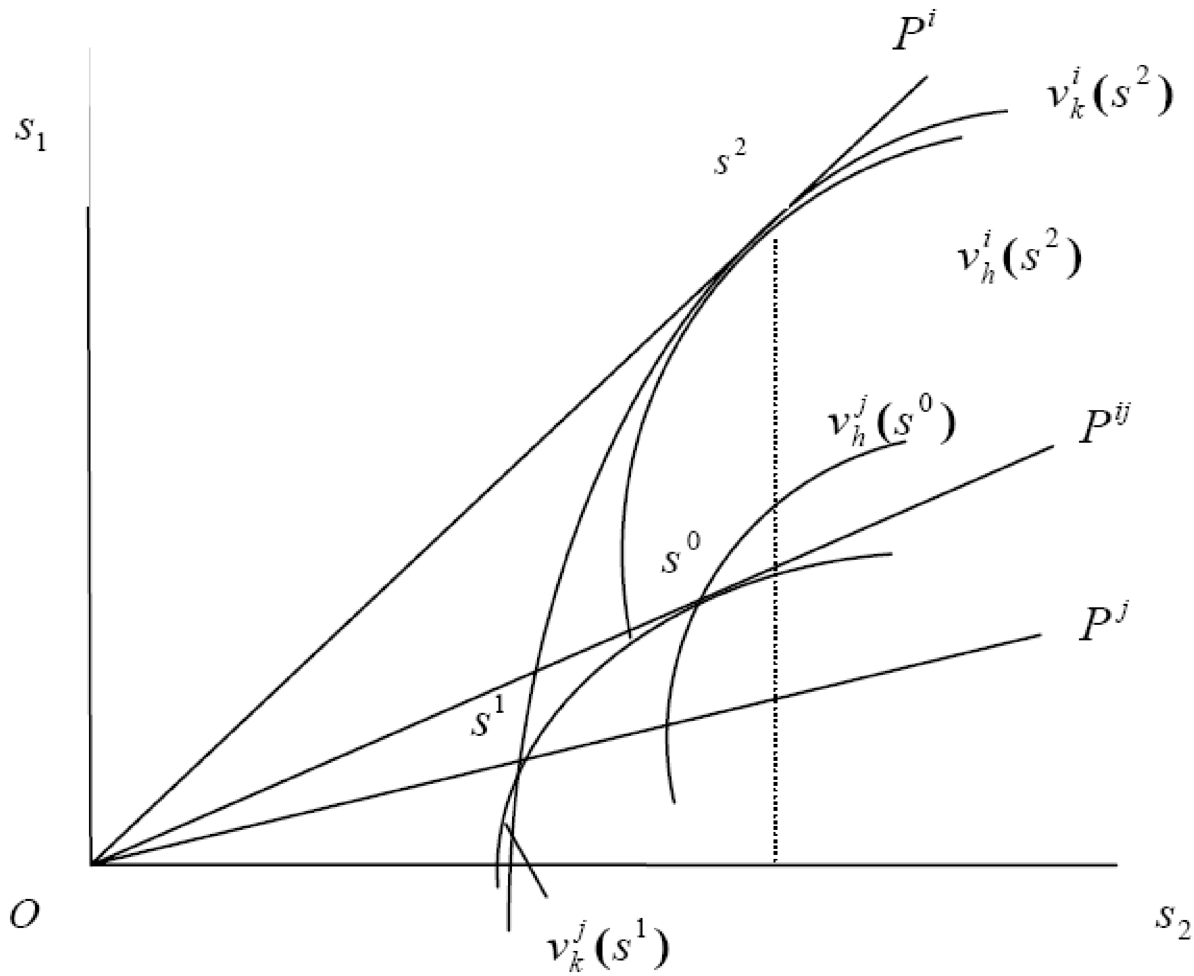

Figure 19. The Uniqueness and Pareto Optimality of the Modified Wilson Equilibrium

According to the analysis of uniqueness and Pareto optimality, modified Wilson equilibria are not unique. However, there must be a Wilson equilibrium is Pareto optimal.

Characteristics of Equilibrium

According to the above analysis, when we take the different risk aversion into account, modified Wilson equilibrium has the following characteristics:

(1) Wilson separating equilibrium exists when there is relatively a small proportion of low-risk buyers. While buyers with different risk aversion exist, the policies that low-risk buyers would purchase depend on the group of high-risk buyers who have the lowest risk aversion. The lower their risk aversion are, the less coverage that the low-risk buyers could receive. That is to say that the low risk aversion of high-risk buyers has significant negative externality.

(2) Wilson pooling equilibrium exists when there is a high proportion of low-risk buyers. When there are buyers of different risk aversion, the Wilson pooling equilibrium depends on the low-risk buyers who also have low risk aversion. The less risk averse they are, the more negative externality they would bring.

(3) At the modified Wilson equilibrium, insurers might gain positive profit. At the separating equilibrium, some low-risk buyers still cannot be totally separated.

(4) The modified Wilson equilibrium might not be unique. However, there is always a Pareto-optimal one.

Conclusion

In real-world insurance markets, it is important to recognize differences in risk aversion, because buyers actually have different perceptions of risk. This paper addresses market equilibrium with heterogeneous risk aversion. Our results show that new equilibria can be reached under this assumption.

For buyers, we use expected-utility theory to model the self-selection process. For insurers, we assume that they respond to their competitors' decisions, and therefore design policies with nonnegative profit. The insurers thus rely on the buyers' self-selection process to separate different types of buyers.

In practice, it would be difficult for insurers to achieve such equilibria for several reasons. First, insurers make different estimates of loss frequencies and loss severities from buyers. Second, the buyers' risk-aversion levels may change across time and wealth (e.g., they may have increasing absolute risk aversion and/or decreasing absolute risk aversion). Finally, the assumption of no transaction costs is unrealistic, since the costs associated with entry and withdrawal are very important. Nevertheless, from certain perspectives, the present research provides several insights into the market. In analyzing equilibrium, we observe the effect of different levels of risk aversion on market efficiency. Using the results of this paper, buyers can be guided to alter their behavior in order to make markets more efficient.

Our model concludes that at some equilibria, insurers may gain positive profit. This contradicts results of the prior literature. In future research, we plan to provide more thorough explanations for this unusual behavior.

References

Doherty, N. A., & Schlesinger, H. (1983). Optimal insurance in incomplete markets. *Journal of Political Economy*, *91*(6), 1045-1054.

Mayers, D., & Smith, C. W. (1983). The interdependence of individual portfolio decisions and the demand for insurance. *Journal of Political Economy, 91*(6), 304-311.

Rothschild, M., & Stiglitz, J. (1976). Equilibrium in competitive insurance market: an essay on economics of imperfect information. *Quarterly Journal of Economics, 90*(4), 629-649.

Smart, M. (2000). Competitive insurance market with two observables. *International Economics Review*, *41*(1), 153-169.

Wilson, C., (1977). A model of insurance markets with incomplete information. *Journal of Economic Theory*, *16*(2), 167-207.

CHAPTER 3

DEVELOPMENT OF CHIENESE LIFE INSURANCE INDUSTRY: AN EFFICIENCY ANALYSIS

Introduction

The Chinese life insurance industry has experienced rapid expansion in recent years, with annual premiums growing from 10 billion U.S. dollars in 1999 to 46 billion U.S. dollars in 2006. In addition to steadily increasing demand, two major supply-side trends have encouraged the development of the industry. First, under the World Trade Organization (WTO) framework, the Chinese government lowered entry barriers to foreign insurers, allowing them to establish joint-venture life insurance firms in China. Second, domestic life insurers strengthened themselves through IPOs and other market developments. (For example, China Life has become the second largest insurance company in the world in terms of market capitalization.) These changes in regulatory policy and market structure present an important question for investors and regulators: How efficient is this development?

In the present article, we use Data Envelopment Analysis (DEA) to study the efficiency of life insurers in China. A comparison of domestic insurers with foreign insurers shows that these two groups have different development patterns, and further analysis of scale economy provides some evidence regarding the potential growth that can be achieved through increasing returns to scale. To investigate firm characteristics, we calculate the direction and potential for improvement that each insurer could realize in various input/output dimensions. We employ the Malmquist-Index approach to distinguish between changes in efficiency and technical progress.

Data and Methodology

Our database derives primarily from volumes of the *China Insurance Year Book* (2001to 2006), supplemented by data provided by the Chinese Insurance Regulatory Commission. All monetary data are stated in local currency (millions RMB) and indexed to a base year of 2001 using the annual CPI of China. Since the American Insurance Group (AIG) maintains different branches in China, each of which files a separate financial statement to the regulator, we use the aggregate data from all AIG branches so that this company is comparable to other insurers with nationwide business.

With regard to methodology, we note that DEA has been used widely in efficiency analysis, especially in economic research. We use the BCC[4] version of the DEA model because it is able to distinguish pure technical efficiency from scale efficiency. Another reason for using this model is that it allows us to identify the improvement directions and potentials for each decision-making unit (DMU; i.e., individual life insurer).

The choice of input and output indicators has been controversial in the literature. We believe that the value-added approach is the most appropriate, and that output indicators need to reflect the three major services provided by life insurers: financial intermediation, risk-pooling and risk-bearing, and "real" financial services related to insured losses.[5] Therefore, we mainly follow Cummins & Weiss (1998), and Cummins, Tennyson, & Weiss (1999) in defining our input and output indicators. The three input indicators for life insurers are equity capital (X1), number of employees (X2), and agent, materials, and other related costs (X3). The first three

---

[4] See Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science, 30*, 1078-1092.

output indicators are formed by taking the sums of incurred losses and additions to reserves for three major lines of business: annuity (Y1), savings related (Y2), and life and health (Y3). We combine the group-insurance and personal-insurance figures within each of the above lines to avoid an excessively large number of output indicators. A fourth indicator is a life insurer's invested assets (Y4).

Empirical Results and Analysis

The summary statistics from our efficiency analyses are presented in Table 1. In this table, results for the categories of all life insurers, domestic life insurers, and foreign life insurers are listed separately. Technical efficiency (TE) reflects a firm's overall efficiency performance, and is the product of pure technical efficiency (PTE) and scale efficiency (SE). Pure technical efficiency reflects the efficiency of resource allocation and internal management, whereas scale efficiency reflects whether or not the firm is operating at the optimal scale. Either increasing returns to scale or decreasing returns to scale will result in scale inefficiency.

The mean technical efficiency of all life insurers decreases from 0.650 in 2001 to 0.505 in 2005. The inefficiency (i.e., deviation from 1.0) is caused by both pure technical inefficiency and scale inefficiency, with scale inefficiency being relatively more severe. This is observed throughout the sample period (2001 to 2005) for the aggregate of all insurers, and implies that the Chinese life nsurance market as a whole has potential to improve in terms of both pure technical efficiency and firm scale. Domestic insurers generally show better efficiency performance than foreign insurers. The medians and means of domestic insurers' technical efficiencies are much higher

[5] See Cummins, J. D., Tennyson, S., & Weiss, M. A. (1999). Consolidation and efficiency in the u.s. life insurance industry. *Journal of Banking and Finance, 23,* 325-357.

than those of foreign insurers in all the sample years. This is mainly caused by the poor efficiency performance of the new joint-venture foreign insurers. These observations are consistent with the current oligopolistic market structure. Traditional domestic life insurers have advantages in terms of market share, distribution systems, client relationships, and brand. New foreign insurers must invest heavily and possibly even experience losses before they truly can become efficient and compete with the domestic giants.

The decomposition of technical efficiency shown in Table 1 implies that foreign insurers have relatively better performance in pure technical efficiency than in scale efficiency. Consequently, scale economy represents a genuine problem for foreign insurers, and our results suggest that their scales are far from optimal. Even though the Chinese Insurance Regulatory Commission allows foreign insurers to enter the Chinese market via joint-venture insurance companies, the intensive regulation of rates, new products, and investments are serious obstacles to further development for these new insurers, and so their relative advantages in actuarial science, production innovation, investment, and international experience cannot be realized. These considerations limit the foreign insurers' growth, and result in low scale efficiency. AIG appears as an exception among foreign insurers, having maintained relatively greater efficiency throughout the sample years, primarily because of its early establishment. In contrast, domestic life insurers have much better performance in scale efficiency, which can be observed from the minimum scale efficiency scores of those insurers. However, this trend was affected in 2005 because of the entry of three new domestic life insurers.

In Table 2, a further analysis of scale economy shows that about 72 percent of insurers exhibit increasing returns to scale and 22 percent show constant returns to scale, with only two insurers in 2005 showing decreasing returns to scale. This further confirms that the Chinese life

Table 1. Summary Statistics of Efficiency Results

2005	All Insurers (N=36)			Domestic Insurers (N=14)			Foreign Insurers (N=22)		
	TE	PTE	SE	TE	PTE	SE	TE	PTE	SE
Median	0.417	0.869	0.580	0.841	0.989	0.979	0.362	0.768	0.553
Mean	0.505	0.783	0.601	0.614	0.844	0.679	0.436	0.744	0.551
StD	0.368	0.243	0.347	0.416	0.247	0.391	0.325	0.238	0.316
Min	0.037	0.234	0.041	0.047	0.234	0.073	0.037	0.366	0.041
Max	1.000	1.000	1.001	1.000	1.000	1.001	1.000	1.000	1.000
2004	All Insurers (N=28)			Domestic Insurers (N=8)			Foreign Insurers (N=20)		
	TE	PTE	SE	TE	PTE	SE	TE	PTE	SE
Median	0.435	0.989	0.672	0.997	1.000	0.997	0.287	0.797	0.410
Mean	0.558	0.840	0.626	0.952	0.977	0.974	0.400	0.785	0.486
StD	0.383	0.199	0.359	0.075	0.057	0.037	0.339	0.210	0.334
Min	0.048	0.397	0.048	0.793	0.837	0.901	0.048	0.397	0.048
Max	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2003	All Insurers (N=24)			Domestic Insurers (N=8)			Foreign Insurers (N=16)		
	TE	PTE	SE	TE	PTE	SE	TE	PTE	SE
Median	0.401	1.000	0.616	1.000	1.000	1.000	0.357	0.922	0.537
Mean	0.584	0.855	0.673	0.896	0.914	0.964	0.429	0.826	0.528
StD	0.332	0.204	0.299	0.228	0.183	0.096	0.260	0.213	0.254
Min	0.112	0.397	0.125	0.355	0.489	0.726	0.112	0.397	0.125
Max	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2002	All Insurers (N=17)			Domestic Insurers (N=6)			Foreign Insurers (N=11)		
	TE	PTE	SE	TE	PTE	SE	TE	PTE	SE
Median	0.471	1.000	0.769	1.000	1.000	1.000	0.417	0.948	0.524
Mean	0.639	0.871	0.724	0.972	0.973	1.000	0.458	0.816	0.573
StD	0.342	0.201	0.306	0.067	0.067	0.001	0.286	0.229	0.282
Min	0.158	0.436	0.167	0.835	0.837	0.998	0.158	0.436	0.167
Max	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2001	All Insurers (N=13)			Domestic Insurers (N=5)			Foreign Insurers (N=8)		
	TE	PTE	SE	TE	PTE	SE	TE	PTE	SE
Median	0.471	1.000	0.631	1.000	1.000	1.000	0.430	0.965	0.430
Mean	0.650	0.951	0.676	0.967	1.000	0.967	0.451	0.920	0.494
StD	0.321	0.092	0.307	0.074	0.000	0.074	0.238	0.108	0.245
Min	0.253	0.746	0.257	0.835	1.000	0.835	0.253	0.746	0.257
Max	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

insurance industry possesses substantial growth potential. Domestic insurers are more likely to possess constant returns to scale and decreasing return to scale. The percentage of foreign life insurers that have constant returns to scale ranges from 12 percent to 20 percent across the sample years, and no foreign insurer exhibits decreasing returns to scale in any year. In addition, different foreign insurers have constant returns to scale in different sample years, implying that no foreign insurers could remain scale efficient across years. The number of domestic insurers with decreasing returns to scale is noteworthy. In 2004, there were 4 out of 8 domestic life insurers exhibiting decreasing returns to scale. This suggests that unbridled growth eventually may result in scale inefficiency. Although the entire industry is growing rapidly, individual insurers must try to avoid careless and inefficient development.

For technically inefficient firms, DEA can identify the input or output directions in which those firms can improve, as well as how much they can improve. This is realized by calculating the shadow inputs/outputs for each DMU. The potential improvements for 36 life insurers in 2005 are shown in Table 3. The first 14 insurers in the table are domestic insurers and the last 22 are foreign insurers.

From Table 3, we observe that life insurers in the Chinese market can improve relatively more in terms of input than output. All of the inefficient insurers can improve efficiency by better controlling their inputs. In addition, the table shows that those insurers can improve with regard to all three input indicators. This suggests that insurers should recognize that they may have excessive inputs and unnecessary costs associated with development, even if development appears rapid and healthy on the surface.

All the life insurers can improve only in terms of certain output indicators. This reflects the fact that inefficient insurers specialize in a few lines of business and the overall efficiency is

Table 2. Distribution of Scale Economies

2005	All Life Insurers		Domestic Life Insurers		Foreign Life Insurers	
	Number	Percentage	Number	Percentage	Number	Percentage
IRS	26	72.22%	7	50.00%	19	86.36%
CRS	8	22.22%	5	35.71%	3	13.64%
DRS	2	5.56%	2	14.29%	0	0.00%
Total	36	100.00%	14	100.00%	22	100.00%
2004	All Life Insurers		Domestic Life Insurers		Foreign Life Insurers	
	Number	Percentage	Number	Percentage	Number	Percentage
IRS	16	57.14%	0	0.00%	16	80.00%
CRS	8	28.57%	4	50.00%	4	20.00%
DRS	4	14.29%	4	50.00%	0	0.00%
Total	28	100.00%	8	100.00%	20	100.00%
2003	All Life Insurers		Domestic Life Insurers		Foreign Life Insurers	
	Number	Percentage	Number	Percentage	Number	Percentage
IRS	19	79.17%	5	62.50%	14	87.50%
CRS	5	20.83%	3	37.50%	2	12.50%
DRS	0	0.00%	0	0.00%	0	0.00%
Total	24	100.00%	8	100.00%	16	100.00%
2002	All Life Insurers		Domestic Life Insurers		Foreign Life Insurers	
	Number	Percentage	Number	Percentage	Number	Percentage
IRS	9	52.94%	0	0.00%	9	81.82%
CRS	7	41.18%	5	83.33%	2	18.18%
DRS	1	5.88%	1	16.67%	0	0.00%
Total	17	100.00%	6	100.00%	11	100.00%
2001	All Life Insurers		Domestic Life Insurers		Foreign Life Insurers	
	Number	Percentage	Number	Percentage	Number	Percentage
IRS	7	53.85%	0	0.00%	7	87.50%
CRS	5	38.46%	4	80.00%	1	12.50%
DRS	1	7.69%	1	20.00%	0	0.00%
Total	13	100.00%	5	100.00%	8	100.00%

Table 3. Potential of Improvement in 2005

Life Insurers	X1-X^1	X2-X^2	X3-X^3	Y^1-Y1	Y^2-Y2	Y^3-Y3	Y^4-Y4
PICC Life	811	142	15	1653	7	13	0
PICC Health	793	343	64	1272	5	8	0
China Life	0	0	0	0	0	0	0
Taiping Life	419	2166	199	2034	465	0	92
Minsheng Life	397	1278	405	1460	85	151	0
China Pacific Life	0	0	0	0	0	0	0
Ping An Life	560	586	521	206	0	0	706
Hua Tai Life	164	122	30	105	3	0	0
New China Life	0	0	0	0	0	0	0
Tai Kang Life	295	3078	558	3005	0	0	0
Sino Life	0	0	0	0	0	0	0
Union Life	0	0	0	0	0	0	0
Greatwall Life	212	372	46	405	2	3	0
National-Life	405	74	19	664	3	5	0
Manulife-Sinochem	75	213	184	589	37	0	0
Pacific-Antai Life	32	259	204	0	196	0	0
Allianz China	24	114	86	0	0	0	62
AXA-Minmetals	111	272	112	98	18	0	0
China Life-CMG	0	0	0	0	0	0	0
CITIC-Prudential	130	631	364	288	95	0	1405
John Hancock Tianan	68	106	38	0	7	0	40
Generali China Life	0	0	0	0	0	0	0
Sun Life Everbright	109	148	93	60	91	0	256
Haier New York Life	81	252	85	84	21	0	399
ING Capital Life	310	288	89	64	13	0	0
AEGON-CNOOC Life	298	366	106	0	19	0	210
CIGNA & CMC	184	119	52	377	2	8	0
Aviva-Cofco Life	182	372	108	0	51	0	1042
Nissay-SVA Life	195	115	41	155	3	0	0
Heng An Standard Life	1111	321	15	840	30	0	0
Skandia-BSAM Life	132	72	40	194	1	3	0
Sino-US MetLife	258	215	159	136	36	0	0
Cathey Life	650	39	14	84	21	0	13
Samsung Air China Life	0	0	0	0	0	0	0
CitiInsurance Life	342	145	99	524	2	4	0
AIG	123	414	273	0	0	0	0

affected by weaknesses in other lines. More specifically, most inefficient insurers can improve in annuity products and savings-related products, whereas few can improve in life and health products. This is because most new insurers give priority to developing the business of life and health insurance. Another reason is that annuities and cash-value insurance are still at an early stage in the Chinese market, and these products are dominated by several traditional life insurers. Furthermore, some insurers have great potential to improve in certain input or output directions, which suggests that they are rather immature in those dimensions. This is a notable characteristic of emerging markets.

Finally, DEA enables us to study the productivity of each life insurer. In the analysis below, we use the Malmquist Index (MI) approach in which an increase in the MI means improvement in productivity. The improvement in productivity can be decomposed into two factors: (1) change in technical efficiency and (2) technical progress. The identification of the two MI drivers allows us to judge the pattern of an individual insurer's development. More specifically, we can tell whether the insurer realizes technical progress or relies only on changes in technical efficiency to improve.

Table 4. Summary of Efficiency Changes and Technical Progress

	Change of Efficiency				Technical Progress			
	Increase	Constant	Decrease	Total	Increase	Constant	Decrease	Total
2001-2002	4	3	6	13	10	0	3	13
2002-2003	5	4	8	17	10	0	7	17
2003-2004	7	4	13	24	14	0	10	24
2004-2005	12	4	12	28	16	0	12	28

Change in technical efficiency can be calculated by the DEA approach discussed above. However, we need to use panel data so that the efficiency scores in different years are comparable. To meet this requirement, when the MI analysis is conducted for any two consecutive sample years, we delete the new insurers that entered the market in the second year.

The signs of technical-efficiency changes vary considerably. In all time periods, several large domestic insurers maintained technical efficiency scores of 1. At the same time, many insurers had deceasing technical efficiency. With the entry of new insurers, competition became more intense. Some life insurers succeeded in improving technical efficiency, whereas others failed. However, technical progress shows a different trend. Many insurers made technical progress across the years, and fewer lost ground. We note further that few insurers were able to make continual technical progress. This is because it is relatively easy for new insurers to achieve technical progress. Older insurers have to switch to a reliance on changes in efficiency to improve productivity.

The MI was more likely to increase than to decrease in the sample years. For example, 17 insurers realized an MI increase in 2004-2005, whereas only 11 realized a decrease. This is a positive sign for the market. Although competition became more intense, insurers still could find ways to improve productivity. Unfortunately, it is not clear what specific types of insurers experienced decreases in productivity. In our sample, both domestic insurers and foreign insurers had such results.

The decomposition of the MI is also informative. We identify the factors substantially affecting the MI as those for which the absolute change in efficiency or technical progress is greater than 0.1. Table 5 reveals that technical progress is an important driver for insurers' productivity. Sometimes, technical progress alone affects the MI, and sometimes there is a mixed

Table 5. Summary of Malmquist Index Results

	Malmquist Index			Drivers of Malmquist Index		
	Increase	Decrease	Total	Change of Efficiency	Technical Progress	Both
2001-2002	9	4	13	1	4	8
2002-2003	10	7	17	2	5	10
2003-2004	13	11	24	1	9	14
2004-2005	17	11	28	6	6	16

effect of technical efficiency and technical progress. It is unlikely that a change in technical efficiency, by itself, is the sole factor driving productivity. In the first three sample periods, there were only 4 cases in which a change in technical efficiency was the only driver. In the period 2004 - 2005, the productivity of 6 insurers was affected solely by a change in technical efficiency. The economic explanation of this change is an interesting topic for further research.

Conclusion

With continually increasing demand, the Chinese life insurance industry has grown rapidly in terms of both premium income and the number of insurers. Change in the regulatory environment and the entry of foreign insurers are restructuring the market. In this article, we assessed the market's development from the perspective of efficiency. Using DEA, we measured the efficiencies of life insurers and identified factors leading to inefficiency. We concluded that domestic life insurers generally have better efficiency performance, and foreign joint-venture insurers should focus on scale economy for their future development. Maintaining growth is an appropriate strategy and should be a particularly high priority for foreign insurers. At the same time, domestic insurers should try to avoid unmanaged growth caused by potential costs associated with decreasing returns to scale.

In calculating the shadow inputs/outputs, we investigated the directions along which an individual firm can improve, and by how much it can improve. We concluded that controlling inputs, as opposed to outputs, is more important for inefficient insurers. In addition, we noted that inefficiency is also caused by the fact that some insurers focus on only a few lines of business, with little output in other lines.

Finally, we studied productivity, which is the product of efficiency change and technical progress. We found that life insurers in the Chinese market do not maintain a consistent pattern of increases or decreases in either efficiency or technical progress. However, it appears to be easier for the new insurers both to realize technical progress and to improve technical efficiency. With regard to productivity, the MI increases across years for more than one-half of the life insurers. We noted that technical progress is a frequent driver for changing the MI, and that an efficiency change, by itself, is rarely the sole driver affecting the MI.

References

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science, 30*, 1078-1092.

Berger, A. N., & Humphrey, D. B. (1992). Measurement and efficiency issues in commercial banking. In Z. Griliches (Ed.), *Output measurement in the service sectors*. Chicago, IL: University of Chicago Press.

Berger, A. N., Hunter, W. C., & Timme, S. G. (1993). The efficiency of financial institutions: a review and preview of research past, present and future. *Journal of Banking and Finance, 17*, 221-249.

Charnes, A., Cooper, W. W., Lewin, A., & Seiford, L. (1993). *Data envelopment analysis: theory, methodology and applications*. Norwell, MA: Kluwer Academic Publishers.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*, 429-444.

Cummins, J. D. (1999). Efficiency in the u.s. life insurance industry: are insurers minimizing costs and maximizing revenues? In J. D. Cummins & A. M. Santomero (Ed.), C*hanges in the life insurance industry: efficiency, technology and risk management*. Boston, MA: Kluwer Academic Publishers.

Cummins, J. D., Tennyson, S., & Weiss, M. A. (1999). Consolidation and efficiency in the U.S. life insurance industry. *Journal of Banking and Finance, 23*, 325-357.

Cummins, J. D., & Weiss, M. A. (1993). Measuring cost efficiency in the property and liability insurance industry. *Journal of Banking and Finance, 17*, 463-481.

Cummins, J. D., & Weiss, M. A. (2000). Analyzing firm performance in the insurance industry. In G. Dionne (Ed.), *Handbook of insurance*. Boston, MA: Kluwer Academic Publishers.

Cummins, J. D., & Zi, H. (1998). Measuring economic efficiency of the U.S. life insurance industry: econometric and mathematical programming methods. *Journal of Productivity Analysis, 10*, 131-152.

Houston, D., & Simon, R. (1970). Economies of scale in financial institution: a study in life assurance. *Econometrica, 38*, 856-864.

Wei, Q. (1998). DEA for the measurement of relative efficiency. Beijing, China: Renmin University Press.

Weiss, M. A. (1986). Analysis of productivity at the firm level: An application to the life insurers. *Journal of Risk and Insurance, 53*, 49-84.

Yuengert, A. M. (1993). The measurement of efficiency in life insurance: estimates of a mixed normal-gamma error model. *Journal of Banking and Finance, 17*, 483-496.

CONCLUSION

Investigating insurance equilibrium is meaningful since it reveals how the insurance market is formed and what role an underlying factor will play in the market formation. This dissertation studies three separate topics of insurance equilibrium: contract formation, heterogeneity, and operational efficiency. It extends literature of insurance equilibrium by developing game-theoretic model, including buyer's risk versions in the insurance equilibrium, and utilizing new data to evaluate insurers' operational efficiency.

The first chapter of this dissertation investigates the insurance contract formation under the framework of Cournot game-theoretic model. By designing a bid-offer market mechanism, we provide an economic explanation for insurance policy limit, which commonly exists in the real market but is hardly justified in the insurance literature. We prove that policy limit has sound reasons to exist even in the absence of solvency constraint.

The second chapter extends Wilson's insurance equilibrium by introducing heterogeneity of buyers' risk aversion. Under this new circumstance, we first raise a counterexample of Wilson's underlying lemmas and then discuss how these lemmas can be modified. With the modified lemmas, we develop a series of new equilibria and figure out the conditions of the new equilibria. This extension of Wilson's equilibrium reveals that separating equilibrium sometimes may just partially separates high-risk buyers and low-risk buyers; and that insurers may be able to keep positive profit under competition.

The third chapter uses Data Envelopment Analysis (DEA) to study the operational efficiency of life insurers in China. Even though the whole industry is developing rapidly, efficiency analysis provides insights about whether the development is at the cost of large investment. We compare the efficiency of domestic and foreign life insurers, decompose their efficiency scores, investigate the directions and potential they could improve, and analyze the change and drivers of productivity. We point out that domestic life insurers are generally more efficient than foreign life insurers, and foreign life insurers should set improving scale with high priority. Most life insurers' inefficiency is caused by the fact that they focus on a few lines of business, and technical progress is a frequent driver of the change of productivity.

REFERENCES

Arrow, K. J. (1971). *Essays in the theory of risk bearing*. North-Holland, Amsterdam.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science, 30*, 1078-1092.

Berger, A. N., & Humphrey, D. B. (1992). Measurement and efficiency issues in commercial banking. In Z. Griliches (Ed.), *Output measurement in the service sectors*. Chicago, IL: University of Chicago Press.

Berger, A. N., Hunter, W. C., & Timme, S. G. (1993). The efficiency of financial institutions: a review and preview of research past, present and future. *Journal of Banking and Finance, 17*, 221-249.

Briys, E. P., & Louberg´e, H. (1985). On the theory of rational insurance purchasing: a note. *Journal of Finance*, *40*(20), 577-581.

Charnes, A., Cooper, W. W., Lewin, A., & Seiford, L. (1993). *Data envelopment analysis: theory, methodology and applications*. Norwell, MA: Kluwer Academic Publishers.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*, 429-444.

Cournot, A. A. (1838). *Recherches sur les principes math´ematiques de la th´eorie des richesses*. Hachette, Paris.

Cummins, J. D. (1999). Efficiency in the u.s. life insurance industry: are insurers minimizing costs and maximizing revenues? In J. D. Cummins & A. M. Santomero (Ed.), C*hanges in the life insurance industry: efficiency, technology and risk management*. Boston, MA:

Kluwer Academic Publishers.

Cummins, J. D., & Mahul, O. (2004). The demand for insurance with an upper limit on coverage. *Journal of Risk and Insurance, 71*(2), 253-264.

Cummins, J. D., Tennyson, S., & Weiss, M. A. (1999). Consolidation and efficiency in the U.S. life insurance industry. *Journal of Banking and Finance, 23*, 325-357.

Cummins, J. D., & Weiss, M. A. (1993). Measuring cost efficiency in the property and liability insurance industry. *Journal of Banking and Finance, 17*, 463-481.

Cummins, J. D., & Weiss, M. A. (2000). Analyzing firm performance in the insurance industry. In G. Dionne (Ed.), *Handbook of insurance*. Boston, MA: Kluwer Academic Publishers.

Cummins, J. D., & Zi, H. (1998). Measuring economic efficiency of the U.S. life insurance industry: econometric and mathematical programming methods. *Journal of Productivity Analysis, 10*, 131-152.

Doherty, N. A., & Schlesinger, H. (1983). Optimal insurance in incomplete markets. *Journal of Political Economy*, *91*(6), 1045-1054.

Dubey, P. (1982). Price-quantity strategic market games. *Econometrica*, *50*(1), 111-126.

Dubey, P., & Shubik, M. (1980). A strategic market game with price and quantity strategies. *Journal of Economics, 40*(12), 25-34.

Gollier, C., & Schlesinger, H. (1996). Arrow's theorem on the optimality of deductibles: a stochastic dominance approach. *Economic Theory*, *7*(2), 359-363.

Houston, D., & Simon, R. (1970). Economies of scale in financial institution: a study in life assurance. *Econometrica, 38*, 856-864.

Huberman, G., Mayers, D., & Smith, Jr., C. W. (1983). Optimal insurance policy indemnity schedules. *Bell Journal of Economics, 14*(2), 415-426.

Mayers, D., & Smith, C. W. (1983). The interdependence of individual portfolio decisions and the demand for insurance. *Journal of Political Economy, 91*(6), 304-311.

Meyer, J., & Ormiston, M. B. (1999). Analyzing the demand for deductible insurance. *Journal of Risk and Uncertainty, 18*(3), 223-230.

Mossin, J. (1968). Aspects of rational insurance purchasing. *Journal of Political Economy, 76*, 553-568.

Nash, J. F., Jr. (1951). Noncooperative games. *Annals of Mathematics, 54*, 286-295.

Powers, M. R., & Shubik, M. (1998). On the tradeoff between the law of large numbers and oligopoly in insurance. *Insurance: Mathematics and Economics, 23*(2), 141-156.

Powers, M. R., Shubik, M., & Yao, S. T. (1998). Insurance market games: scale effects and public policy. *Journal of Economics, 67*(2), 109-134.

Raviv, A. (1979). The design of an optimal insurance policy. *American Economic Review, 69*(1), 84-96.

Rothschild, M., & Stiglitz, J. (1976). Equilibrium in competitive insurance market: an essay on economics of imperfect information. *Quarterly Journal of Economics, 90*(4), 629-649.

Schlesinger, H. (1981). The optimal level of deductibility in insurance contracts. *Journal of Risk and Insurance, 48*(3), 465-481.

Smart, M. (2000). Competitive insurance market with two observables. *International Economics Review, 41*(1), 153-169.

Wei, Q. (1998). DEA for the measurement of relative efficiency. Beijing, China: Renmin University Press.

Weiss, M. A. (1986). Analysis of productivity at the firm level: An application to the life insurers. *Journal of Risk and Insurance, 53,* 49-84.

Wilson, C., (1977). A model of insurance markets with incomplete information. *Journal of Economic Theory, 16*(2), 167-207.

Yuengert, A. M. (1993). The measurement of efficiency in life insurance: estimates of a mixed normal-gamma error model. *Journal of Banking and Finance, 17*, 483-496.