AN EXPLORATORY STUDY OF THE FACTORS RELATED TO SUCCESSFUL MATHEMATICAL PROBLEM SOLVING ON NON-ROUTINE UNCONSTRAINED TASKS

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ABSTRACT

A main goal of mathematics educators is to guide students in becoming better problem solvers; however, the recipe for successful problem solving is complex due to the varying factors that play a role in the problem solving process (Schoenfeld, 1992). There is a limited amount of research that examines problem solving when students work on non-routine problems outside of the classroom; therefore, the goal of this study is to use secondary data analysis to discover what factors (Schoenfeld, 1992) relate to problem solving on non-routine unconstrained tasks of students in the middle grades. Identifying the factors that relate to successful unconstrained non-routine problem solving can help mathematics teachers and policy makers make more informed decisions about curriculum and instruction in order to enhance problem solving aptitude. Using Schoenfeld’s (1992) theoretical framework for mathematical behavior, the following question set the groundwork for the current study: What resource (computational skills and heuristics), control (self-regulation), and belief/affect factors (demographics, motivation, and anxiety) both individually and collectively relate to unconstrained non-routine mathematical problem solving? The research question is answered in a series of three stages that examines how the factors relate to a) problem correctness, b) correct problem set-up, and c) problem completion. Results suggest that higher levels of self-regulation, and SES status predict problem completion; higher self-regulation, ability beliefs, and SES predict correctly setting-up the problem; and higher levels of anxiety and stronger computational skills predict solving the problem correctly. Reasons for the patterns of results are discussed, as well as suggestions for future research to extend on the current findings.
I dedicate this dissertation to my loving parents, Tom and Donna. Without you, my journey through graduate school would not be possible.

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CHAPTER 1
INTRODUCTION

Background

People of all backgrounds face problems they need to solve on a day-to-day basis. These problems vary from elementary school homework problems to problems in the work place. On that premise, the concept of problem solving varies depending on the amount of steps necessary to solve the problem, the strategies to solve the problem, and the context in which the problem takes place. Due to the wide range of problems people face and the variety of problem solving environments, problem solving is a complex phenomenon that occurs in all fields of study. More specifically, with relation to mathematics, mathematical problem solving ability and performance are major factors that influence students’ mathematical achievement (NCTM, 2000; Schoenfeld, 2007), especially during middle school, a critical time for math learning given the status of Algebra as a gatekeeper course (Adelman, 2006). Problem solving tends to be a very complex mathematical task, which involves the application of content knowledge, appropriate problem solving strategies, and non-cognitive variables such as beliefs and affective factors (Frenshe & Funke, 1995, Schoenfeld, 1992).

In the field of mathematics education, the definition of problem solving is the “mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development” (NCTM, 2010). A vast amount of literature exists on the teaching and learning of problem solving in mathematics education (Schoenfeld, 2007). With an ample amount of research showing the importance of problem solving aptitude on mathematics achievement (Schoenfeld,
The National Council for Teachers of Mathematics’ (NCTM) Agenda for Action, emphasized that problem solving should be a main focus in curricula across the nation (NCTM, 1980). With many changes being proposed to mathematics curricula, the U.S. National Research Council formed the Mathematical Sciences Education Board (MSEB), whose main job was to pay close attention to the critical issues in mathematics education. In 1989, the MSEB emphasized a change in mathematics curricula in its report, Everybody Counts; NCTM followed with the Curriculum and Evaluation Standards for School Mathematics (1989), which had five general goals for all K-12 students, one of which was for students to become mathematical problem solvers (Schoenfeld, 2007). NCTM then published the Principles and Standards for School Mathematics (2000), which reflected on the research and practice throughout the past decade and proposed critical standards for all K-12 curricula (Schoenfeld, 2007). One of the pivotal standards focused on problem solving, emphasizing that all curricula should allow students to: (a) develop new mathematical knowledge through problem solving, (b) solve problems that arise in varying contexts, (c) apply and adapt useful strategies to solve problems, and (d) monitor and reflect on the process. From these reports, it is clear that problem solving has been a central focus in mathematics education research and practice for decades (Schoenfeld, 2007).

As history shows, a main goal of mathematics educators is to guide students to become better problem solvers; however, the recipe for successful problem solving is complex due to the varying factors that play a role in the problem solving process (Schoenfeld, 1992). In addition to the complex nature of the problem solving process,
the success of mathematical problem solving varies globally with students in the United States generally being out-performed by those from Asian countries (Cai 2000a, 2000b; Cai & Hwang, 2002). Research shows that students in Asian countries (i.e., China and Japan) tend to use more appropriate and advanced problem solving strategies than their U.S counterparts, thus out-performing U.S students on problem solving tasks. Chinese and Japanese students also tend to use abstract, algebraic strategies significantly more than U.S students, which leads to more successful problem solving outcomes (Cai 2000a, 2000b; Cai & Hwang, 2002). In order for the U.S to stay globally competitive and to improve students’ mathematical problem solving performance, more research is needed to understand and identify the specific factors that relate to successful problem solving strategies and achievement.

To improve the problem solving ability of U.S students, more effective interventions must be implemented. Research shows that problem solving ability and success can be developed in classroom settings with appropriate interventions (Carpenter, Fennama, Peterson, Chiang & Loef, 1989; Englard, 2010, Jitendra & Star, 2012; Montague, 1992; Varcaretu, 2008); however more research on the underlying factors that contribute to problem solving success is necessary. Appropriate interventions can help guide changes to curriculum and instruction to foster a deep understanding of content knowledge, prepare students to apply their knowledge to novel situations, and promote self-regulated learning and positive mathematical beliefs. If researchers can better identify the imperative factors that lead to successful problem solving, mathematics teachers and policy makers can aid students to become better problem solvers by making the appropriate changes to curriculum and instruction.
Statement of Purpose

Problem solving research has been conducted in an array of different contexts ranging from research on young children to individuals in the work place. Problem solving is a complex phenomenon that is affected by numerous factors such as mathematical ability, strategy selection, self-regulation, and motivational factors (Schoenfeld, 1992). Much of the research on mathematical problem solving, in particular, examines one particular factor in depth, but there are seldom studies that explore several factors collectively. For example, some studies take a close look at how content knowledge affects problem solving (Canobi, Reeve, Pattison, 2003; Hecht & Vagi, 2012), while others concentrate on specific problem solving strategies and how the strategies play a role in problem solving success (Cai, 2000a, 2000b; Huntly & Davis, 2008; Lannin, Barker, & Townsend, 2006; Montague, 1992). Moreover, other studies focus on belief and affective factors such as motivation (Areepattamannil & Freeman, 2008; Gottfriend, 1990; Halawah, 2006; Lepola, Niemi, Kuikka, & Hannula, 2005) and mathematical anxiety (Kyttälä, M., & Björn, 2014; Vukovic et al., 2012) on the problem solving process.

However, the majority of studies on mathematical problem solving do not examine how students perform on non-routine, unconstrained tasks. To understand the definition of non-routine problems, it is important to first understand what it means to be a routine problem. Gilfeather and Regato (1999) describe routine problems as tasks which use steps to facilitate the use of already known procedures in order to find a solution. Routine problems are generally defined as being well-structured because the tasks are clearly formulated and typically require the use of standard algorithms. For
instance, consider the one-step equation $x + 3 = 12$. Generally, students will remember the specific procedure to subtract three from both sides of the equation to find the answer of nine because they have practiced the procedure over and over again. In contrast, there are other mathematical problems that are considered *non-routine*. Non-routine problems are tasks that do not require the use of a known algorithm, and for which there is no clear, prescribed approach to find the solution. Instead, non-routine tasks require the use of a different type of strategy such as looking for a pattern, guessing and checking, making and solving a similar problem, or drawing a diagram (Gilfeather & Regato, 1999; Pólya; 1945, Schoenfeld; 1992). Another characteristic of non-routine tasks is that they may be solved in more than one way. For example, two students solving a problem may arrive at the same answer two different ways.

Schoenfeld (1992) makes it very clear that the definitions of what scholars and educators mean by the term *problem solving* can be conflicting. For the purpose of this paper, it can be argued that non-routine problems allow students to solve a problem in such a way that the process gives them “… the potential to provide intellectual challenges for enhancing students' mathematical understanding and development” (NCTM, 2010). Often times, problem solving is used to describe the process when individuals solve routine problems, which are often called exercises. In contrast to problem solving with routine exercises, this study explores what relates to student success when they engage in a non-routine problem that makes them use a heuristic strategy to explore and engage in a problem for an extended period of time.

While an ample amount of research exists on non-routine problem solving in various domains (Becker & Shimada, 1997; Cai & Cifarelli, 2005; Cifarelli & Cai, 2005),
seldom does research explore how various factors relate to \textit{unconstrained}, non-routine problem solving. Unconstrained problem solving refers to solving a problem without time constraints that students often face both in the classroom and on standardized tests. Often times, time constraints prohibit individuals from retrieving the appropriate resources to solve a problem (Bowden, 1985; Hopkins & Egeberg, 2009). The purpose of the present study is to look at how various factors (Schoenfeld, 1992) both individually and collectively affect middle school students’ unconstrained, non-routine problem solving ability when given the opportunity to work on a problem at length. The various factors in this study draw on the Mathematical Problem Solving Framework of Schoenfeld (1992), which include: (a) Resources (computational skills and heuristics), (b) Control (self-regulation), and (c) Belief and Affective Factors (motivation, mathematical anxiety, and demographics). A more detailed description of the theoretical framework can be found in Chapter 2.

Research Question and Research Design

Every year, the Academy, a private K-12 college preparatory school in a Northeastern part of the United States, has its middle school students complete a problem solving project as part of its curriculum. The problem solving project requires the students to select one non-routine problem and work on the problem for 14 days outside of the classroom. The Academy saves the students’ projects to compare student work from year-to-year. The school’s math department also gathers additional information on their students’ content knowledge, self-regulation, beliefs, and feelings towards mathematics for departmental purposes. Since the Academy conducts a problem solving project using non-routine problems and gathers additional information on student
characteristics, it provided researchers the unique opportunity to explore the factors that relate to unconstrained, non-routine problem solving through a secondary data analysis.

The primary research question in the proposed study is: What resource (computational skills and heuristics), control (self-regulation), and belief/affect factors individually and collectively relate to unconstrained non-routine mathematical problem solving success of middle school students? The research question is answered in a series of three stages that examines how the factors relate to a) problem correctness, b) correct problem set-up, and c) problem completion.

Significance of Study

The proposed study will make a significant contribution to the field of education in several ways. First and foremost, since problem solving is a major topic in U.S mathematics curricula and a crucial factor in predicting mathematical success (NCTM, 2000; Schoenfeld, 2007), it is essential to determine the underlying factors that relate to successful problem solving. This study will not only examine how more observable factors such as computational skills and problem solving strategies play a role in determining non-routine problem solving competency, but also how more non-observable factors such as mathematical beliefs and affects contribute. Secondly, identifying the factors that relate to successful problem solving can help mathematics teachers and policy makers make more informed decisions about curriculum and instruction in order to enhance problem solving aptitude.

Lastly, though affective factors such as mathematical anxiety have been shown to have a negative effect on mathematical achievement when students are put under time constraints (Andrews & Brown, 2015; Ashcraft & Krause, 2007; Hoffman, 2010;
Hopkins & Egeberg, 2009), it is necessary to determine whether they influence problem solving success when participants are able to work on a mathematical problem at length. If affective factors do not contribute in an unconstrained problem solving situation, this would suggest that even if individuals (e.g., with mathematical anxiety) are not successful problem solvers when given a short amount of time to solve a non-routine problem, they may be successful in an outside of the classroom environment such as the work place.

Definition of Terms

This study addresses many terms used in mathematics education, some of which can be used in more than one way depending on the context. The following is a comprehensive list of terms used throughout the paper along with their definitions in order to avoid misinterpretations:

- **Affective factors** is an umbrella term which includes mathematics anxiety and motivational factors such as self-efficacy, interest, value, and goal (McLeod and Adams’, 1989; Schoenfeld, 1992).

- **Beliefs** refer to an individual’s mathematical perspective or worldview (Schoenfeld, 1985, 1992).

- **Gender** is defined as the traits, behaviors, and expectations that culture trains individuals to practice and hold (Gilbert, 2001).

- **Heuristics** are the rules and techniques for effective problem solving, which for the purpose of this study is synonymous to problem solving strategies and problem set-up (Pólya, 1945; Schoenfeld, 1992).
○ **Mathematical anxiety** is an affective factor that is an individual’s belief that they will not be able to solve a mathematical problem that he or she encounters (Hoffman, 2010). To be more specific, it can be a feeling of worry, fear, or nervousness that negatively affects mathematical performance (Hembree, 1990; Lewis, 1970).

○ **Mathematical problem solving** are mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development (NCTM, 2010).

○ **Motivation** is an affective factor and explains why an individual behaves and thinks in a certain way (Graham & Weiner, 1996; Weiner, 1992); however, it is important to note that the term motivation has several components, which include self-efficacy, interest, value, goal, and self-regulation.

○ **Non-routine problems** are mathematical tasks that do not require the use of a known algorithm. There is no clear, prescribed approach at finding the solution (Gilfeather & Regato, 1999; Pólya; 1945, Schoenfeld; 1992).

○ **Resources** are the critical tools necessary to solve a given problem. For the purpose of this study, the resources include computational skills and heuristics (problem solving strategies) (Schoenfeld, 1985, 1992).

○ **Routine problems** are tasks that facilitate the use of already known procedures in order to find a solution (Gilfeather & Regato, 1999).

○ **Self-efficacy** refers to an individual’s beliefs about their capability to perform a specific task (Bandura, 1986, 1997; Galla & Wood, 2012).
- **Self-regulation** regulation is a control factor (Schoenfeld, 1992) that is defined as the self-generated control and consistent adaptation of actions, emotions, thoughts, and attention for the purpose of reaching a goal (Cleary & Chen, 2009; Zimmerman 1989, 2000).

- **Socioeconomic status** (SES) is defined as a hierarchical ranking of a person or family unit based on the control of power, wealth, and/or social status (Mueller & Parcel, 1981).

- **Unconstrained** problem solving refers to solving a problem without time constraints that students often face both in the classroom and on standardized tests. The “unconstrained” amount of time for this study is 14 days, which differs from the typical time constraints that students face in the classroom.

**Organization of Paper**

The first chapter of this dissertation discussed the background of mathematical problem solving, its importance, the problem at hand, the purpose of the study, and operational definitions to avoid misinterpretations throughout the paper. The second chapter explains the relevance of Schoenfeld’s (1992) mathematical problem solving framework for the purpose of the study along with a comprehensive literature review on the factors related to mathematical problem solving. Chapter 2 also uses the literature to explain the importance of individually and collectively examining the factors that relate to problem solving. Chapter 3 explains the quantitative methodology and statistical analyses that are used and Chapter 4 discusses the results by examining how the individual factors correlate with each other and with unconstrained non-routine problem solving. Chapter 5 concludes the paper by discussing the results, limitations,
implications, and suggestions for further research. Measures and additional information discussed in Chapter 3 and 4 can be found in the Appendix listed after the references.
CHAPTER 2

LITERATURE REVIEW

Historical Background on Problem Solving

A vast amount of literature exists on the teaching and learning of problem solving throughout the last four decades in numerous domains (Schoenfeld, 2007). During the 1970’s, there was a strong research emphasis to identify characteristics of successful problem solvers by conducting various teaching experiments. Early research started with exploring simple tasks, such as the Tower of Hanoi problem (Newell & Simon, 1972), which could be solved in a reasonable amount of time, required specific problem solving strategies, and demonstrated the process of solving “real” problems (Frensch & Funke, 1995, p. 2); however, researchers began to argue that findings from simple tasks that were conducted in laboratories did not accurately depict real-life problems that tended to be more complex (Frensch & Funke, 1995). It was also found that problem solving research findings regarding predictors, strategies, and outcomes varied based on the domain. In examining problem solving, North American and European researchers had slightly different theoretical perspectives (Frensch & Funke, 1995).

Since the process of problem solving appeared to vary based on context, North American researchers in the 1980’s started investigating problem solving in an array of fields, such as domains in the sciences, humanities and athletics. The focus of research in North America was to compare successful and unsuccessful problem solvers, which they referred to as experts and novices (Anderson, Boyle, & Riser, 1985; Frensch & Funke, 1995; Sternberg & Frensch, 1991). For example, research on novice verse expert problem solving was examined with electronics and computer skills (Kay, 1991; Lesgold
& Lajoie, 1991), strategy game playing (Sternberg & Frensch, 1991), and reading and writing (Bryson, Bereiter, Scardamalia & Joram, 1991; Stanovich & Cunningham, 1992). It was also during this time period that the importance and influence of metacognition in problem solving had been established, especially self-regulation and monitoring (Schoenfeld, 1992).

On the European front, scholars took a slightly different approach to problem solving research. In contrast to examining simple problem solving tasks in specific domains, European researchers (e.g., Broadbent, 1977; Dörner, 1975, 1985; Dörner, Kreuzig, Reither, & Stäudel's, 1983; Ringelband, Misiak, & Kluwe, 1990) studied complex, laboratory tasks that took a real-life approach to problem solving. For example, Broadbent (1977) emphasized the difference between cognitive problem solving processes that function under what he called awareness versus outside of awareness. In his research, he typically utilized mathematically specific computerized systems. Dörner (1975; 1985) was interested in how the various components of problem solving intertwined. He was interested in the motivational, cognitive, and social pieces of problem solving and how these components worked together. As the historical research indicates, both North American and European researchers concluded that problem solving differs based on the process, context, and domain. They also determined that results found in laboratory experiments were more difficult to generalize to real-world problem scenarios than originally thought. More research was needed to explore problem solving outside of the laboratory setting to gain a better understanding of how problem solving happens in the real world.
Theoretical Framework

As the historical context of problem solving shows, mathematical problem solving is a complex process that involves multiple components including content knowledge, problem solving strategies, metacognition, motivation, anxiety, and sociocultural factors. Since education research is often broken down and segregated into different concentrations such as motivation, metacognition, or achievement, to name a few, educational researchers tend to focus on one aspect or another based on their expertise. Focusing on one particular aspect (i.e., motivation or metacognition) allows researchers to gain a deeper understanding of how that factor relates to a specific task; however, measuring only one or two components at a time does not allow researchers to identify which factors are most significant. If researchers want to gain a more well-rounded understanding of what relates to problem solving, they must consider all of the contributing factors. In order to account for all of the factors, Schoenfeld (1992) created a framework to examine multiple aspects that relate to problem solving by drawing on the historical context of mathematical behavior.

Many of the theoretical frameworks for mathematical behavior and problem solving are derived from Information Processing Theory (Newell & Simon, 1972; Simon, 1979). This is mainly because Information Processing Theory focuses on the numerous cognitive and metacognitive processes and strategies that people use as they solve problems in various domains (Montague & Bos, 1990). Drawing from Information Processing Theory and on the work of George Pólya (1945), Alan Schoenfeld (1985, 1992) created a framework for the analysis of mathematical behavior in order to determine what people know and what people do as they work on mathematical
problems. Schoenfeld (1985, 1992) designed his framework to best predict what takes place as a person attempts to find the solution to a mathematical problem. He argued that there are three categories of behavior and knowledge in order to describe the problem solving process, which include: (a) Resources (the knowledge base), (b) Control/Metacognition, and (c) Beliefs and Affects.

Schoenfeld (1985, 1992) defined resources to be the critical tools necessary to solve a given problem. In order to solve a problem, the student must have the necessary content knowledge and use the proper problem solving strategies for the task at hand. He calls this the problem solver’s “initial search space” (Schoenfeld, 1985, p. 17). The initial search space provides the problem solver with the specific pathways that are open for exploration. Within this category of the framework, it is important for an individual to determine what he or she knows, believes, or suspects to be true about a problem. The specific types of knowledge that relate to successful problem solving performance include: (a) the intuitive and informal knowledge about the topics (b) definitions, facts, and algorithmic procedures, (c) routine procedures and relevant competencies, and (d) knowledge about the rules of discourse. To elaborate, informal knowledge may include a person’s perceptions, predictions, and preliminary ideas about a problem. The facts and definitions include understanding the words and meanings of the problem. For example, if a problem asks a person to solve for the missing variable, one must understand what it means to be a variable and know the necessary algorithmic steps to solve an equation. Schoenfeld uses a geometric example to explain both routine procedures and knowledge of discourse. He states that routine procedures differ from the facts and algorithmic procedures because they are not always right and wrong. For instance, he says certain
proof techniques are not algorithmic, but differ depending on the problem and the particular strategy the problem solver chooses to take. The knowledge of discourse is described as understanding the rules in the particular domain by knowing what is permitted and what is not. For example, Schoenfeld (1992) suggests that one cannot determine the diameter of a circle by simply guessing where the middle of the circle may be. Mathematics, in particular, tends to be a more formal domain where rules must be followed.

In addition to the necessary mathematical knowledge, Schoenfeld (1985, 1992) discusses the importance of *heuristics*. Using Pólya’s definition from *How to Solve It* (1945), he states:

> Heuristic, or heuretic, or “ars inveniendi” was the name of a certain branch of study, not very clearly circumscribed, belonging to logic, or to philosophy, or to psychology, often outlined, seldom presented in detail, and as good as forgotten today. The aim of heuristic is to study the methods and rules of discovery and invention…Heuristic, as an adjective, means, “serving to discover” (p. 112-113).

Today, the term heuristics has become practically synonymous with problem solving strategies (Schoenfeld, 1985). Heuristics are the rules and techniques for effective problem solving, which have been a major focus of problem solving research in mathematics education for decades. During the problem solving process, Schoenfeld argues the importance of exploration. Successful problem solvers must successfully analyze, design, explore, implement, and verify their work during the problem solving process. The analysis piece requires either understanding, simplifying, or reformulating the problem. The design of the problem involves constructing an argument. For example, a problem solver must have a plan with reasoning to explain his or her exploration. Once the problem is analyzed, designed, and explored, it must be
successfully implemented. Successful implementation requires the avoidance of procedural errors or misconceptions. Lastly, verification involves checking work to catch mistakes. Schoenfeld (2012) suggests that expert mathematical problem solvers (i.e., mathematicians) in comparison to novice problem solvers (i.e., students) do a better job at successfully implementing problem solving strategies. In fact, Pólya (1945) would argue that mathematical problem solving does not focus on the application of routine algorithms, but instead focuses on solving unfamiliar tasks when the solutions are not obvious. When individuals are drilled with exercises and routine practice, Pólya would argue that they are not developing the broad set of problem solving skills and strategies necessary to develop an expert mathematical mind (i.e., a mathematician’s thinking).

Furthermore, in order to successfully implement the resources, Schoenfeld (1985, 1992) argues the importance of control or metacognition when problem solving. Under the umbrella of metacognition, Schoenfeld focused on self-regulation. He argues that as research on child development and mathematics education converged, the concept of self-regulation became more important. Self-regulation is important in the problem solving framework because it is a category of behavior which describes how students use the information available to them. To be an expert problem solver, Schoenfeld (1992, 2012) argues that students must be able to effectively use their resources and strategies through self-regulation. The individual must know how to accurately plan, assess, and make conscious decisions to be successful (Schoenfeld, 1992).

Moreover, Schoenfeld’s (1992) framework suggests that an individual’s beliefs and affects are also influential in the problem solving process (McLeod and Adams’, 1989; Schoenfeld, 1985, 1992). Schoenfeld (1985, 1992) describes beliefs and affects to
be a student’s mathematical perspective or worldview. He argues students’ beliefs about themselves, their mathematical experiences, and their environment can mold the way their resources, heuristics, and controls function together. For example, a student who is not confident in mathematics because he or she was unsuccessful in the past may give up after a short period of time, even though he or she possesses the content knowledge to solve the problem. Belief and affect factors include feelings towards mathematics, which include mathematics anxiety and motivational factors such as self-efficacy, self-regulation, interest, value, and goal. It is important to note that sociocultural factors such as one’s gender and socioeconomic status may also have an influence on one’s beliefs and affects, thus, it cannot be ignored when examining problem solving (Boaler, 2002; Brotman & Moore, 2007). On this premise, it is important to examine how these factors influence an individual’s content knowledge, problem solving strategies, and control during the problem solving process. Since this study is taking into account multiple factors that relate to mathematical problem solving, it is best to use Schoenfeld’s (1992) framework as a guide for exploration.

Resources

Schoenfeld’s argues (1992) that a problem solver’s resources consist of the necessary tools to solve the problem at hand. These important tools include an individual’s content knowledge. The content knowledge necessary to solve both routine and non-routine problems involves the use of computational skills and appropriate problem solving strategies. In order to successfully solve the task at hand, the problem solver needs to have the relevant mathematical knowledge along with the ability to access and use the knowledge appropriately. This section discusses the literature on the
relationship of both computational skill and heuristics on mathematics achievement and mathematical problem solving.

*Computational Skills and Mathematics Achievement*

In order to solve mathematics problems, both routine and non-routine, individuals must have the appropriate computational skills at hand (NCTM, 2000). Computational skills are both the “selection and application of arithmetic operations to calculate solutions to mathematical problems” (Millians, 2011, p. 396). To elaborate, computational skills involve the ability to calculate basic operation problems accurately using mental methods, by showing work, and by using other tools such as a calculator. Computational skills also require the appropriate selection of arithmetic operations. Often times in the literature, this is referred to as procedural skills because it involves knowing the skills, steps, or actions in order to appropriately solve a problem (Canobi, 2009; Rittle-Johnson et al., 2001). For example, the appropriate computational skills for simplifying an expression involves knowing how to execute the order of operations without making arithmetic errors.

Research shows that an individual’s computational knowledge has an impact on their overall mathematics achievement (Canobi, 2004; 2005; Canobi, Reeve, & Pattison; 2003; Dowker, 1998; Hallet, Nunes & Bryant, 2010; Hecht 1998; LeFevre et al., 2006; Star & Rittle-Johnson 2009). The effect of computational skills on mathematics achievement has been found in a wide range of ages and ability levels. The various domains where computational skills have been explored include studies involving addition and subtraction (Hallet, Nunes & Bryant, 2010; Hecht 1998), estimation (Dowker, 1998; Star & Rittle-Johnson 2009), counting (LeFevre et al., 2006), and
equation solving (Durkin, Rittle-Johnson, & Star, 2011). Falkner, Levi, and Carpenter (1999) also showed that computational skills in young children may even have an impact on algebraic achievement. They found that many students in grades one through six were unable to recognize the equal sign as a relational symbol, but instead understood it as an operational symbol. For example, students may fill in the blank, $2 + 10 = \underline{} + 6$, with a 12 showing a misunderstanding of the equal sign. This is significant because computational errors can have a major impact on algebraic computation and problem solving. This particular study is consistent with other findings that also highlight the importance of computational skills and their relationship to mathematics achievement. In fact, the inability to appropriately apply computational skills has shown to be detrimental for students moving from learning arithmetic in elementary school to algebraic topics in the middle grades (Alibali, Knuth, Hattikudur, McNeil, & Stephens, 2007; Baroody & Ginsburg, 1983; Stacey & MacGregor, 1990).

*Computational Skills with Routine Problems*

In addition to computational skills affecting mathematics achievement in general, it also affects individuals’ ability to solve routine problems. Research shows that there is a correlation between children’s computational skills and ability to solve routine problems and exercises (Canobi, 2004, 2005; Canobi & Pattison, 2003; Hecht 1998). Canobi (2004, 2005) and Canobi and Pattison (2003) examined both computational skills and conceptual understanding with relation to elementary arithmetic topics. The purpose of Canobi’s (2004, 2005) studies was to determine if patterns occur in children’s computational skill and conceptual understanding and if these patterns relate to the efficiency and flexibility of their problem solving characteristics. The procedure
consisted of a problem solving task to assess routine problem solving ability and the use of part-whole relations, as well as a judgment task, which allowed students to justify their answers. The students were also asked to solve sets of addition and subtraction problems. Results showed various profiles of problem solving strategies, speed, and problem solving accuracy. The children’s problem solving profiles of computational skill and conceptual knowledge were related to individual differences in grade level and part-whole knowledge. Canobi’s results suggest that identifying children’s computational skill and conceptual understanding on arithmetic topics, such as adding and subtracting, is important for understanding children’s problem solving development. Canobi and Pattison (2003) also showed that students had different computational profiles, which was determined from their problem solving strategy selection and problem accuracy. These results showed that routine problem solving success and computational understanding are correlated. The authors concluded that determining individual computational profiles gives insight into the development of children’s mathematical knowledge of early arithmetic problem solving skills. Distinct profiles of computational and conceptual understanding were identified; however, the author only compared these factors to routine tasks. More research is needed to determine how computational skills affect problem solving on more complex, non-routine problems.

Additionally, research studies have focused around computational skills with relation to routine fraction problem solving tasks. For example, Hecht (1998) examined computational fluency and conceptual understanding of fractions with seventh and eighth grade students. The purpose of Hecht’s (1998) study was to gain more knowledge on the individual differences of students’ understanding of fractions. Results of this study
showed computational and conceptual knowledge explained the variability in the computational fraction problem solving, as well as how accurately the students set up word problems involving fractions. The author concluded that this study shows how computational and conceptual knowledge are associated with individual differences of fraction problem solving skills. This study also provides additional evidence on the individual computational skill differences that arise in seventh and eighth grade students. This implies that computational skills affect students’ problem solving success in various ways with regards to routine arithmetic problem solving on constrained tasks. More research is needed to determine if other factors, such as one’s self-regulation or feelings towards mathematics, have an impact with this particular age group on unconstrained non-routine tasks.

**Heuristics**

In addition to computational skills, Schoenfeld (1992, 2012) classifies the use of appropriate problem solving strategies as a necessary resource in mathematical problem solving. He would argue that, in addition to the appropriate computational skills, one must know the appropriate problem solving strategies in order to set up and solve the problem. According to him and many researchers, problem solving strategy selection and implementation are critical steps on the path to successful problem solving (Huntly & Davis, 2008; Lannin, Barker, & Townsend, 2006; Montague, 1992; Pólya, 1945; Scheonfeld 1985, 2012). Numerous studies have been conducted to determine what types of strategies students of varying backgrounds, ability, and age level use as they solve problems. While this may seem obvious, it is important to note that appropriate problem solving strategies and set up vary based on problem type. For example, routine problems
require students to pay close attention to the details and often apply algorithms to solve the problem, while non-routine problems may require students to explore related problems, identify patterns, and grapple with unfamiliar tasks (Schoenfeld, 1992). In fact, Pólya (1945) would argue that mathematical problem solving does not have students apply routine algorithms, but instead work on unfamiliar tasks when the solutions are not obvious. When students are drilled with exercises and routine practice, Pólya would argue that they are not developing the broad set of problem solving skills necessary to solve problems as mathematicians. Due to the importance of strategy selection and implementation when problem solving, this section reviews the empirical research that has been conducted with how problem solving strategies relate to mathematics achievement and problem solving success.

**Importance of Heuristics for Mathematics Achievement**

Research shows that using appropriate mathematical strategies and being able to set up a strategy appropriately when solving problems relates to general mathematics achievement (Chamot et al., 1992; Perveen, 2010; Sebrechts et al., 1996). For example, Sebrechts, Enright, Bennett, and Martin (1996) were interested in identifying different problem solving strategies and how these particular algebraic strategies are indicators of overall quantitative reasoning ability on a standardized achievement test. To answer these questions, they analyzed the solutions of undergraduate students, all from the same fairly prestigious university, to word problems from the Graduate Record Exam (GRE). When solving algebraic word problems, results showed that students used four major strategies, which included equation formulation, ratios, simulation, and other unsystematic approaches, which included diagrams and verbal descriptions. They found
that high achieving students typically used more equation strategies and fewer unsystematic approaches than the low achieving students. It can be seen that the majority of correct answers have strategy formulations with thought-out planning. Sebrechts et al. also identified where errors occurred during the problem solving process. The six errors happened during: (a) situation comprehension otherwise known as understanding the problem, (b) strategy formulation, (c) plan construction, (d) general plan development, (e) specific plan development, and (f) procedural implementation. These results showed that appropriate problem solving strategies require an understanding of the problem at hand along with precise computational skills. Since problem solving strategy selection and implementation plays an important role in mathematics achievement on standardized tests, it is important to consider this as a variable when analyzing factors that relate to successful problem solving.

*Heuristics in Routine Problem Solving*

In addition to problem solving strategies affecting general mathematics achievement on standardized tests, research shows that varying problem solving strategy approaches of students can affect their routine problem solving achievement. For example, a longitudinal study by Ross, McNaught, Reys, Grouws, and Chávez (2011) took place over the course of three years to examine changes in problem solving strategies with relation to problem solving success on a routine algebra problem with high school students. Ross et al. examined over a thousand students’ strategy choices on an algebraic problem that could be solved arithmetically or algebraically. They were specifically interested in determining what strategies the majority of students would use, which strategies would be most likely to produce a correct answer, and if students would
continue to use the same strategy to solve the same problem over the course of three years. Using a system of equations exercise as their problem to track over three years, they anticipated several different successful approaches to solving the problem. They found that only about 20% of the students successfully solved the problem. Students did not improve on this problem over the course of three years and continued to use ineffective strategies or no strategy at all. This study illuminates the importance of having an appropriate problem strategy and set-up in order to have the chance to correctly apply the computational skills. It is important to examine what factors relate to both successfully setting up and successfully solving a problem. Since this study only focused on one system of equations problem, additional studies are needed to determine if these results are consistent with other types of non-routine problems.

There have also been studies to show that interventions that help improve strategy selection can improve problem solving success (Englard, 2010; Jitendra & Star, 2012). Englard wanted to determine if using a model-based approach, as used by Singapore students (Jiang, Hwang, & Cai, 2014), to show third grade students how to problem solve would improve their performance on fraction-based tasks. The model-based approach had students draw figures as a strategy to help them solve word problems (e.g., creating part and whole fractions bars, p.158). Using a pretest-posttest design, Englard found that students in the model-based treatment group out-performed those students in the control group. The results imply that showing students the model-based strategy can help improve problem solving performance and, more importantly, it illuminates the fact that improvement in problem strategy implementation can improve problem solving performance. Jitendra and Star (2012) also conducted a study that shows how
improvements in problem solving selection and implementation can improve problem solving performance. The purpose of their study was to determine if schema-based instruction (SBI) would help improve seventh grade students’ problem solving performance on routine percent problems. Results show that SBI improved the performance on percent problems of both high and low-achieving students, although it had a greater impact on high-achieving students. However, contrary to their hypothesis, the high achieving students were not able to transfer the SBI. Results also showed that low achieving students may need more time, as well as support, to set up and implement different problem solving strategies. More research is needed to determine if permitting more time for students to retrieve appropriate strategies will improve overall problem solving performance. It is unclear if students of varying ability levels may be able to appropriately set-up, complete, and correctly solve a non-routine problem when permitted an unconstrained amount of time to do so.

From the literature, it is evident that problem solving retrieval and implementation play a crucial role for routine problem solving success. It is also clear that certain problems may require more planning and execution time than others. The next section reviews the specific studies that not only explore resource factors (computational skills and problem solving strategies) during the problem solving process, but with non-routine problems in particular.

Resource Factors in Non-routine Tasks

In addition to computational knowledge and problem solving strategies playing an important role in routine problem solving success, they also affect problem solving success during non-routine tasks. Lee and Chen (2009) investigated the effects of how
question prompts, prior knowledge, including computational skills, and the combination of question prompts in combination with prior knowledge affected problem-solving performance on non-routine mathematical tasks. The questions were considered non-routine because they prompted students to focus on aspects of the problem through the course of multiple phases and required the use of a non-algorithmic strategy to solve the problem. The prior knowledge instrument measured for the understanding of algebraic expressions and arithmetic sequences, which both measure an individual’s computational skills. Results of this study showed that the students who have higher prior computational knowledge perform better than those who have lower computational knowledge on both simple and complex, non-routine problems. This implies that computational skills are necessary in the process of solving non-routine problems. Individuals must also be able to appropriately select and execute their computational knowledge to have success on non-routine problems.

The literature on problem solving strategies with non-routine tasks varies because some studies examine how students use a particular strategy when solving a non-routine problem, while others let students explore in order to identify the differences in strategies. To be more specific, Capraro, An, Ma, Rangel-Chavez, and Harbaugh (2012) took a close look at how math and science pre-service teachers implemented the guess-and-check method when solving a non-routine problem. Eight pre-service teachers were asked to solve a non-routine, open-ended triangle task problem that had four unique solutions. After the participants worked on the problems, they were interviewed and asked about their problem solving process. The qualitative results showed that the primary strategy used was guess and check; however, it was often used ineffectively.
These results show that individuals may have trouble determining the correct strategy to use and implement when given non-routine tasks because non-routine tasks are not as straight-forward as routine problems that require specific algorithms to solve.

Non-routine problem solving research has also been done with younger participants. Montague and Bos’ (1990) examined the problem solving characteristics of eighth grade students as they solved non-routine word problems. More specifically, they took a close look at the cognitive knowledge, self-regulation, and use of problem solving strategies and how these factors related to problem solving success of high-achieving, average-achieving, low-achieving, and learning disabled eighth grade students. After various tests were administered to the participants in each group and statistical analyses were conducted, several significant differences were evident between groups. They found that the learning disabled students did not perform poorly on non-routine word problems due to computational errors, but instead because of incorrect strategy selection. They also concluded that learning disabled and low-achieving students had a significantly more difficult time determining the appropriate problem solving strategies to use than the other two groups. High-achieving students were significantly better at predicting, selecting, and executing appropriate problem solving strategies to multi-step word problems than the low-achieving and learning disabled students. The results imply that low-achieving and learning disabled students may not have a more difficult time solving non-routine word problems because of poor basic arithmetic skills, but because they may have a more difficult time retrieving the appropriate problem solving strategies to solve the problem. While Montague and Bos (1990) identify differences between the varying
ability groups, more research is needed to identify if other factors, such as motivation and mathematical anxiety, also played a role during the problem solving process.

It is important to note that students’ problem solving strategies on non-routine problems may be different depending on how much time they have to solve the problem. Cifarelli and Cai (2005) conducted an exploratory study that examined how two college students formulated and solved non-routine, open-ended mathematical tasks using a computer program. The participants were interviewed and video-taped as they solved a problem called the Billiard Ball task and the Number Array task and were given as much time as they needed to solve the problems. Given that there were no time constraints on the non-routine tasks, the researchers could observe how the participants’ problem solving strategies changed and evolved over time into more advanced strategies. This implies that students may retrieve more appropriate problem solving strategies when given more time, thus perform better on problem solving tasks when not put under the pressure of time constraints.

*Resources Summary*

From the literature, it is clear that resource factors, both computational knowledge and problem solving selection and implementation, have a strong relationship to mathematical achievement and problem solving success. In the past two decades, the main focus of research on computational knowledge in mathematical learning has centered on elementary topics (i.e., operations with integers and fractions) using routine problem solving tasks. From the majority of these studies, researchers can group individuals into clusters depending on their strengths and weaknesses of their computational understanding to see how it relates to their problem solving; however,
more research is needed with non-routine tasks. Also, the majority of research on computational knowledge with relation to problem solving does not examine the impact of affective factors such as motivation and anxiety. More research is needed to determine how, not only computational knowledge, but various factors influence non-routine problem solving.

It is also clear that problem solving strategy selection and implementation affect overall mathematical achievement, general routine problem solving, and non-routine problem solving. The bulk of research in this area focuses on the types of problem solving strategies students of varying age and ability level select as they solve different types of mathematical problems, both routine and non-routine. With regards to non-routine problem solving in particular, the research shows that students may have a more difficult time retrieving and implementing the appropriate strategy due to other affective factors (Montague & Bos, 1990). Since the retrieval and implementation of problem solving strategies may vary based on other factors and time constraints, more research is needed to determine if self-regulation, motivation, and mathematical anxiety along with resource factors relate to non-routine problem solving.

Control

To successfully implement the resource factors (computational skills and problem solving strategies), Schoenfeld (1985, 1992) argues the importance of control—often called metacognition in the psychological literature. In the general problem solving process, control involves one’s planning, monitoring, decision-making, and metacognitive acts. Being able to control one’s thoughts and actions during the problem solving process is impacted by an individual’s ability to self-regulate. Self-regulation is
defined as the self-generated control and consistent adaptation of actions, emotions, thoughts, and attention for the purpose of reaching a goal (Cleary & Chen, 2009; Zimmerman 1989, 2000). Self-regulation is imperative in order to properly use the necessary resources and to retrieve the appropriate problem solving strategies to solve a problem. The literature indicates that self-regulation begins affecting children’s academic success at early ages and persists throughout their education. It not only affects their mathematics achievement, but relates to problem solving on both routine and non-routine problems.

Self-regulation and Mathematics Achievement

As mentioned, self-regulation begins affecting individual’s academic achievement at early ages. Blair, Ursache, Greenberg, Vernon-Feagans, and The Family Life Project Investigators (2015) at Penn State University and the University of North Carolina investigated how self-regulation relates to mathematics achievement and reading ability of low-income children in the early elementary grades (K-2). Math achievement was measured using the Woodcock-Johnson III Tests of Achievement Applied Problems subtest and reading ability was assessed using the letter-word identification subtest. Executive function tasks were used to measure the participants’ self-regulation. Significant effects of self-regulation measures on mathematics achievement was consistent as children developed over kindergarten through second grade, but did not consistently affect reading ability while controlling for family demographic factors. Since the majority of the sample included low-income children, more research is needed with an economically diverse sample. Self-regulation is also only investigated with general mathematical ability on a standardized test and more research is needed to
examine how self-regulation effects children’s ability on more specific mathematical tasks.

It is important to discuss that the impact of self-regulation on mathematics achievement may vary based on age group. For instance, Cleary and Chen (2009) explored the relationships between student grade level, ability group, and math course with self-regulation and motivational factors of sixth and seventh grade students. More specifically, they wanted to determine if trends in student motivation and self-regulation varied based on grade and math course. Results indicated that seventh graders use more effective self-regulation strategies than sixth graders and seventh grade ability levels are more clearly differentiated based on self-regulation and motivation factors. In addition, self-regulation and motivational aspects tended to increase with more advanced courses (i.e., honors vs. regular courses). The effect of self-regulation on achievement even persists through college. Briley, Thompson, and Iran-Nejad (2009) investigated how self-regulation affects the achievement of college level students in remedial math courses. The participants in this study included undergraduate students enrolled in nine sections of Intermediate Algebra. To gather information about students’ self-regulation, a survey was administered and compared to standardized multiple-choice midterm and final exams. Results show that self-regulation is correlated with general academic achievement, as well as mathematics achievement.

In addition to self-regulation varying by age, the effect of self-regulation on mathematics achievement can vary by gender. Matthews, Ponitz, and Morrison (2009) investigated how both early gender differences in self-regulation affected academic achievement in kindergarten students. Matthews et al. conducted a five-year longitudinal
study to collect an array of data on previous knowledge, child behavior, self-regulation characteristics, and achievement. Results indicated that teachers rated girls to be significantly more self-regulated in the classroom than boys. This implies that early gender differences have an impact on self-regulation and, in turn, self-regulation has an impact on mathematics achievement. When examining academic achievement, it is important to examine both gender and self-regulation together because they tend to be highly correlated; however, this may differ as children get older. While this study looked at general mathematical achievement, it is also important to examine how self-regulation and gender impact more specific mathematical tasks such as problem solving.

Research also shows that improvements in self-regulation can help improve mathematics achievement. Perels, Dignath, and Schmitz (2009) conducted an intervention with hopes of enhancing mathematical achievement by improving self-regulation strategies of sixth grade students. The participants were split into two groups. Both groups were covering a unit on divisors and multipliers, while one group was provided with in-class training on self-regulated learning strategies and the other group was not. After the intervention, the students were given a self-regulation questionnaire. The experimental group displayed significantly more self-regulated behavior than the control group. More importantly, the experimental group scored significantly higher on their post-test on divisors and multipliers. These results indicate that self-regulation is a predictor of mathematics achievement and can improved through interventions; however, this study was limited to only divisors and multipliers. More research is needed to determine if self-regulation has a direct relation to other types of non-routine problems.
Self-regulation in Routine Problem Solving

Self-regulation does not only affect general mathematical achievement, but also how students solve routine problems. Throndsen (2010) conducted a longitudinal study on self-regulated learning of early elementary school children. His primary goal was to discover the relationship between children’s basic mathematical skills and strategy use on arithmetic problems with their self-regulation and motivational beliefs. Secondly, he wanted to determine if students at different mathematical ability levels differ with regards to their self-regulation characteristics. Throndsen collected data during three different time periods. Children’s basic math skills were measured through standardized achievement tests, while data on children’s strategy use, metacognitive competence, and motivational beliefs was gathered through interviews. Results showed that high-performing (very good) students used significantly more advanced mathematical problem solving strategies than the other two groups. Analyses also showed that children in varying ability groups differed in self-regulated learning. For example, lower ability students displayed lower declarative and situational metacognitive knowledge. These results imply that children’s self-regulation while solving basic arithmetic problems affects low-performing students.

An additional study also shows that improving self-regulated problem solving strategies can help improve overall problem solving performance. An intervention study by Montague (1992) explored the effectiveness of metacognitive strategies on problem solving success. The study examined the cognitive and metacognitive problem solving processes of eighth grade students with learning disabilities. She was particularly interested in determining if cognitive and metacognitive instruction could help improve
problem solving skills. The metacognitive strategies were comprised of self-instruction, self-question, and self-regulation. After two treatments were conducted, she found that students who received complementary instruction of cognitive and metacognitive strategies became more effective problem solvers than students who received either cognitive or metacognitive instruction alone. However, students did not retain the problem solving strategies over time. Additional research is needed with varying groups of students in the general education population. The study at hand only examined six middle school students with learning disabilities. It is not clear if this specific instruction would be effective with students of varying background knowledge or with non-routine mathematics problems.

*Self-regulation in Non-routine Problem Solving*

In addition to examining how self-regulation affects the problem solving process in general, there are studies that take examine how self-regulation affects non-routine problem solving; however, these studies are limited. Shin, Jonassen, and McGee (2002) examined the predictors of both routine and non-routine problem solving success in a multimedia environment with 9th grade astronomy students. Shin and colleagues predicted the factors relating to solving well-structured, routine problems may not be sufficient for solving non-routine problems. Using rubric scoring systems to assess problem solving skills on both routine and non-routine problems and instruments to measure both cognitive and affective variables, their regression analyses showed that conceptual knowledge, justification skills, science attitudes, and self-regulation predicted non-routine problem solving scores; however, only conceptual knowledge and justification skills predicted closed problem scores. These results show that differences
occur between problem solving with routine problems versus non-routine problems with regards to self-regulation. Students who tend to be more self-regulated and have more positive attitudes towards astronomy tend to be better at solving non-routine problems; however more research is needed to determine if these results would be replicated during non-routine mathematical tasks.

Self-regulation Summary

From the literature, it is evident that self-regulation affects both mathematics achievement and problem solving. An individual must know when and how to use their resources and how to retrieve the appropriate problem solving strategies for the task at hand. The effect of self-regulation on mathematics achievement and problem solving differs by age, ability level, gender, and problem type (Blair et al., 2015; Briley et al., 2009; Cleary & Chen, 2009; Matthews et al., 2009; Thondsen, 2010); however it is unclear how self-regulation affects non-routine mathematical problem solving success along with other factors. Since the majority of research on self-regulation and mathematical problem solving focuses on tasks with time constraints, more research is needed to determine how self-regulation affects students’ success on non-routine tasks when time constraints are not in place.

Beliefs and Affects

As defined in the beginning of this chapter, beliefs and affects are a student’s mathematical perspective or worldview (Schoenfeld, 1992). An individual’s beliefs and affects towards themselves, their mathematical experiences, and their environment can mold the way their resources and control factors function together. Research shows that these factors which include mathematics anxiety and motivational factors such as self-
efficacy, interest, value, and goal have shown to make an impact on mathematics
achievement (Andrews and Brown, 2015; McLeod and Adams’, 1989; Panaoura, 2013;
Schoenfeld, 1992; Schommer-Aikins, Duell, and Hutter, 2005). Gender and
socioeconomic status are also important factors to consider because they can influence an
individual’s beliefs and affects towards mathematics (Barnes, 2000; Boaler, 2002). Their
beliefs and affective traits allow individuals to draw on their resources and utilize them
either successfully or unsuccessfully. With this premise, there is a wide range of research
in the field of mathematics education that examines beliefs and affects both individually
and in combination with other factors to examine the impact they have on both
mathematics achievement in general and on more specific tasks such as problem solving.

Schoenfeld (1992) states that students compile the majority of their beliefs and
affects about formal mathematics along with their sense of discipline from the
mathematics classroom and the community surrounding them. He also argues that
students’ beliefs and affects impact their behavior in ways that are extremely powerful
and consequential towards their mathematical success. There have been several recent
studies to determine how a) motivation towards mathematics, b) mathematical anxiety,
and c) gender and socioeconomic status impact achievement in an array of contexts
(Callejo & Vila, 2009; Ishida, 2002; Schommer-Aikins, Duell, and Hutter, 2005).

Motivational Factors

One of the major categories under the beliefs/affects umbrella is motivation and
its components. The concept of motivation explains why an individual behaves and
thinks in a certain way (Graham & Weiner, 1992; Weiner, 1996); however, motivation
has an array of components, which include self-efficacy, interest, value, goal, and self-
regulation (discussed in the previous section). The effects of motivation on achievement tend to differ by subject matter and age (Gottfried, 1990). Research has shown that motivation has a positive correlation with academic achievement in the elementary grades (Gottfriend, 1990; Lepola, Niemi, Kuikka, & Hannula, 2005); however, some studies show less of an effect of motivation on mathematics achievement in higher grades (Areepattamannil & Freeman, 2008; Halawah, 2006). Regardless of the variety of impact in previous studies, the concept of motivation cannot be overlooked. In this section, the components of motivation are investigated both individually and collectively to examine the impact they have on both mathematics achievement and problem solving.

*Self-efficacy and Mathematics Achievement*

One major component under the umbrella of motivation is self-efficacy. Self-efficacy refers to an individual’s beliefs about their capability to perform a specific task (Bandura, 1986, 1997; Galla & Wood, 2012). Research has shown self-efficacy to be a major component of motivation that affects mathematics achievement (Cifarelli, Goodson-Epsy, and Chae, 2010; Panaoura, 2013; Peklaj, Podlesek, & Pečjak, 2014). Peklaj, Podlesek, and Pečjak (2014) investigated how previous knowledge, gender, personality traits, and math motivation predicted students’ math grades. While this study does not directly indicate what factors relate to mathematical problem solving, it shows how specific motivational constructs (i.e., self-efficacy) predict mathematics achievement. The participants included high school students from six different schools in various regions of Slovenia. Instruments included mathematical tasks from the PISA 2000, a standardized assessment, which measured previous knowledge, motivational strategies, self-efficacy, and interest in mathematics, as well as the students’ final grade.
Results showed that self-efficacy, along with other variables such as previous knowledge and personality traits, had a direct effect on students’ final math grades. This study shows the importance of examining self-efficacy variables when researching mathematics achievement; however, more research is needed to determine if these results can be replicated with younger participants during non-routine problem solving tasks.

*Self-efficacy and General Problem Solving*

Cifarelli, Goodson-Epsy, and Chae (2010) also found that self-efficacy beliefs had an effect on problem solving strategies and performance of College Algebra students. Using a triangulation mixed methods research design, they administered The Mathematical Belief Systems Survey (Yackel, 1984) and conducted student interviews. Written mathematical work of differing mathematical topics was also used to examine the participants’ problem solving strategies. From the survey and interviews, two participants who used more complex problem solving strategies tended to have more positive views towards mathematics and higher self-efficacy beliefs. These two students also tended to be more persistent during the problem solving process when they encountered difficulties. On the other hand, students who had more negative beliefs towards mathematics and lower self-efficacy struggled when difficulties arose while problem solving. The results imply that self-efficacy has an effect on problem solving strategies, persistence, and overall performance; however, the sample in this study was small. A larger sample size is needed in order to make generalizations about the relationship of self-efficacy on problem solving.
Self-efficacy in Non-routine Problem Solving

Research shows that self-efficacy can also have an effect on non-routine problem solving. Panaoura (2013) examined how students’ self-efficacy beliefs affect students’ ability to use representations on solving non-routine geometrical tasks. She wanted to determine if self-efficacy differs by grade level and to examine the interrelations between various self-beliefs and geometrical performance. The participants included students at the primary level (grades 5 and 6) and students at the secondary level (grades 7 and 8). A test with 12 geometrical tasks was administered to the students. The test was divided into three parts, which consisted of geometrical figure perceptual and recognition ability, area and perimeter measurement tasks, and verbal problems that relate to discursive figure apprehension. A questionnaire was used to measure students’ self-efficacy beliefs towards the geometrical tasks. Two models were created for both the primary and secondary grade levels. Both models indicated interrelations between self-efficacy beliefs and geometrical figure understanding. Results also indicated that prior experiences with solving geometrical tasks affect self-efficacy beliefs and, in turn, self-efficacy beliefs affect geometrical tasks performance. This study confirms the significance of self-efficacy on non-routine mathematical problem solving in grades 5 through 8.

Interest, Value, and Goals in Math Achievement

In addition to self-efficacy, other components of motivation include interest, value, and goal. Renninger and Hidi (2002) defined interest to be both the feelings, thoughts, and engagement towards a particular activity. Mitchell (1993) argues that interest stems from the interactions people have with the activities and contexts in which
they take part. In addition to interest, value is an important component of motivation because it varies based on the importance or worth the activity at hand has to the individual (Eccles et al., 1983; Higgins, 2007). Lastly, an individual’s goal is considered to be the aim or purpose with respect to the activity or activities at hand (Elliot & Murayama, 2008).

Steinmayr and Spinath (2009) examined various components of motivation including achievement motives, self-concept, values, and goal orientation and how these factors predicted mathematics achievement of eleventh and twelfth grade students in Germany. Using instruments to measure these constructs and comparing it to mathematics achievements and general academic achievement measures, analyses showed that motivational variables accounted for 46% of the variance with value in particular accounting for 32.3%. This study shows the strong impact motivational components have on overall mathematics and academic achievement. Kloosterman and Cougan (1994) also investigated students’ beliefs and confidence about learning mathematics and how mathematical beliefs vary with age and ability level. The participants included students in grades one through six who were part of a larger project to improve mathematics teaching. After conducting student interviews to gather information about student interest and confidence with relation to age and ability level, the researchers found that ability level in the lower grades (i.e., first, and second) did not affect one’s self-efficacy or interest in mathematics; however, changes were apparent in third grade. In grades three through sixth, there were no low-achieving students who expressed a high interest or high self-efficacy towards mathematics. The results from these studies imply that as students get older, their self-efficacy and interest in
mathematics starts to change due to their ability level. Additional research is needed to determine if other factors in combination with self-efficacy and interest affect mathematical problem solving. More research is also needed to determine if self-efficacy affects individual’s ability to appropriately set-up, complete, or correctly solve a mathematical task. It is unclear at what stage self-efficacy affects the problem solving process.

*Interest, Value, and Goals in Problem Solving*

Other aspects of motivation also affect children’s ability to problem solve. Schommer-Aikins, Duell, and Hutter (2005) examined the various mathematical problem solving beliefs of middle school students. They wanted to investigate if there is a relationship about students’ general epistemological beliefs (i.e., beliefs about knowledge, speed of learning, and ability to learn) and their domain-specific beliefs about mathematical problem solving (value and usefulness). Using a large sample of students from two middle schools, students were given questionnaires about their epistemological beliefs and beliefs about problem solving. They used the Kansas State Assessment instrument to measure mathematical problem solving ability. After conducting a path analysis to show the relationships among mathematical problem solving beliefs, general epistemological beliefs, and academic success, results indicated a significant correlation between students finding math to be useful and their mathematical problem solving performance. The epistemological belief that learning is fixed also had a significant correlation with mathematical problem solving. These results imply that student beliefs about the value and usefulness of mathematics along with having a fixed or non-fixed mindset can have a significant impact on problem solving performance.
Motivational components also relate to non-routine problem solving. Eseryel, Law, Ifenthaler, Ge, and Miller (2014) examined the relationships between motivation, engagement, and complex non-routine problem solving in a game-based learning environment with eighth and ninth graders. Using a motivational questionnaire for measures on students’ motivational characteristics and engagement in the complex problem solving scenario as the dependent variable, analyses showed that interest actually had a negative correlation with engagement during the problem solving process. The authors suggest that the reason for the loss in interest is due to the task at hand. They suggest that the students were more interested before playing the game because their expectations were high, but lost interest when playing the game because they were disappointed. These results imply that the relationship between interest and non-routine problem solving engagement and success may vary based on the context.

Mathematical Anxiety

In addition to motivational factors, mathematical anxiety is another affective factor that has an impact on mathematics achievement (Hembree, 1990; Hoffman, 2010; Lewis, 1970; Richardson & Suinn, 1972). It refers to an individual’s belief that they will not be able to solve a mathematical problem that he or she encounters (Hoffman, 2010); this can manifest as a feeling of worry, fear, or nervousness that affects mathematical performance (Hembree, 1990; Lewis, 1970).

Mathematical Anxiety and Mathematics Achievement

Mathematical anxiety does not only relate to mathematics achievement, but is shown to have a negative effect on a person’s working memory and mathematical success (Andrews & Brown, 2015; Ashcraft & Krause, 2007; Hoffman, 2010). A study by
Andrews and Brown (2015) investigated mathematics anxiety and how it affects mathematics performance on standardized tests. Using student surveys from orientation, grades, and SAT scores to compare to scores on the Abbreviated Math Anxiety Scale (AMAS), results showed a negative correlation between SAT math scores and mathematics anxiety along with a negative correlation between math grades and anxiety. An important finding to note is that students who received an “A” in the course reported the same mathematics anxiety of students who failed the course; therefore, prior knowledge along with other affective factors may influence mathematics anxiety. Mathematics anxiety also may affect individuals differently depending on the context. For example, a student may be less anxious if he or she already predicts failure in a course verses if he or she is taking a standardized test for college acceptance. While this study shows that mathematics anxiety is a critical factor in mathematics performance on standardized tests, more research is needed to determine if math anxiety affects individuals in various ways depending on different contexts.

 Mathematical Anxiety in Problem Solving

Mathematical anxiety not only affects overall mathematical achievement, but inhibits students’ problem solving performance. Hoffman (2010) examined how mathematics anxiety affected problem solving efficiency of pre-service teachers. Using the Mathematics Anxiety Scale (MAS) to measure affective dispositions for mathematics and computer-based multiplication problems at two difficulty levels, it was found that mathematics anxiety was related to a low perception of one’s math skills. There was also a significant correlation between anxiety and problem-solving accuracy. While mathematics anxiety is prevalent in both high school and college, it is important to note
that mathematics anxiety also affects younger children (Kyttälä, M., & Björn, 2014; Vukovic et al., 2012). Vukovic, Kieffer, Bailey, and Harari (2013) conducted a study that examined how mathematics anxiety affects mathematics performance on arithmetic word problems both concurrently and longitudinally of second and third grade students. Results show a negative correlation between anxiety and word problem performance. Kyttälä, and Björn (2014) also examined various predictors of mathematical performance on word problems, one of which included anxiety. Their analyses showed that anxiety was negatively correlated with word problem solving success, which is consistent with the findings from other studies (Hembree, 2010; Vukovic et al., 2012).

Guven and Cabakcor (2013) also took a look at the varying factors that influence mathematical problem solving. They examined seventh grade students’ academic success and various affective factors that influence their mathematical problem solving performance. More specifically, they examined the effects of students’ mathematical anxiety, attitudes towards problem solving, problem solving beliefs, and self-efficacy on problem solving achievement test. Questionnaires were used to measure problem solving attitude and beliefs, anxiety, and self-efficacy. A problem solving achievement test was also administered, which consisted of 15 multiple-choice questions. After multiple regression analyses were conducted, results show that various factors affect problem solving achievement. There was a strong significant correlation between academic success and problem solving achievement and a moderately significant correlation between self-efficacy, math anxiety, attitudes, and beliefs on problem solving achievement. This study adds to the literature on the complexity of problem solving and how problem solving is affected by many factors including mathematics anxiety;
however, because the problem solving test was multiple choice, it is difficult to determine the participants’ difficulties during the problem solving process. For example, it is unclear if computational errors or unsuccessful problem solving strategies affected the students’ problem solving success more than the affective factors.

In addition to mathematical anxiety affecting students’ problem solving performance with basic arithmetic problems and on standardized tests, it can also relate to non-routine problem solving. Kaufmann and Vosburg (1997) examined how math anxiety affects what they called creative problem solving of high school students. The creative problems involved two non-routine insight problems that were characterized as unstructured. The participants were required to take a questionnaire asking about their moods before solving the problems, then proceeded to work on the non-routine tasks in a test-like setting during school hours. Results indicated that anxiety had a negative correlation with problem solving performance. As the students’ anxiety increased, their level of performance dropped. This shows that anxiety can have a negative effect on non-routine problem solving performance; however, more research is needed to see how anxiety affects non-routine problem solving when students are not in a test environment. The pressure of the clock may have caused anxiety to have more of an effect on the students’ problem solving performance.

*Gender*

In addition to motivation and anxiety, gender is a sociocultural factor that has shown to have an impact on individual’s beliefs and attitudes towards mathematics. Before examining gender in mathematics and mathematical problem solving, it is important to define the term. Gender is defined as the feelings, behavior, and attitudes
that a particular culture associates with a person’s biological sex (American Psychological Association, 2012). Brotman and Moore (2008) further explain the ideas of femininity and masculinity to be terms that explain the cultural aspects of gender. For example, Gilbert and Cavert (2003) explain that the idea of femininity is not always associated with females and masculinity is not always associated with males. It is culture that develops the femininity and masculinity in males and females. Further, Jo Boaler (2002) explains gender is not only comprised by a set of characteristics that are shared by a group of people, but also gender is developed by different situations and interactions amongst people.

*Gender and Mathematics Achievement*

When it comes to mathematical problem solving, gender is an important factor to consider because it can have an effect on an individual’s beliefs and affects towards mathematics throughout their educational career (Barnes, 2000; Boaler, 2002). The majority of research in the past several decades shows that the gender differences in mathematics do not primarily happen in early education, but typically occur in the middle school years and beyond (Hyde et al., 1990, Maccoby & Jacklin, 1974). Fennema and Sherman (1978) constructed one of the first studies to examine gender differences in mathematics achievement of students in middle school and found that girls have lower confidence levels than boys in mathematics beginning at sixth grade.

Gender differences continue to make an impact as individuals get older. On the mathematics section of the Scholastic Aptitude Test (SAT), females tend to score approximately 0.30 standard deviations below males (College Board, 2007). In addition, according to the U.S Department of Commerce’s Economics and Statistics
Administration of 2009, approximately half of the work force consists of women, but women only hold approximately 25% of science, technology, engineering, and math (STEM) jobs. Throughout the last decade, there are more women who are college educated than in the past, but the underrepresentation in STEM still exists. According to the Census Bureau’s 2009 American Community Survey (ACS), approximately 2.5 million college-educated women had STEM degrees in contrast to 6.7 million men. Specifically, since 1994, the representation of women in mathematics jobs has declined (Beede et al., 2009).

**Gender and Routine Problem Solving**

In addition to differences in mathematics achievement, males and females tend to differ on problem solving tasks. Fennema, Carpenter, and Jacobs (1998) argue that gender differences in achievement at the elementary school level are minimal, but they suggest boys and girls take different approaches to learning. They explain that in grades one through three, there are no gender differences in computational skills such as addition, subtraction, multiplication, and division, but, typically, boys tend to use abstract strategies to solve problems while girls use methods such as modeling and counting; therefore, in the elementary school years, boys and girls develop a different understanding of mathematics as a result of differing problem solving strategies. They argue that different learning approaches in the early years affect learning in the future because the lack of understanding becomes more critical as one develops.

Fennema and Sherman (1977) also conducted one of the earliest studies examining problem solving gender differences at the high school level. They observed 589 females and 644 males from four predominately Caucasian schools to study
mathematics achievement and the differences related to spatial visualization. They argue that females have as much mathematical ability and potential, but cultural belief factors caused gender differences. Battista (1990) also orchestrated a study to examine gender differences at the high school level. He investigated the ways spatial thinking affects learning and problem solving in high school geometry. Results suggested that males and females differ in spatial visualization and performance in geometry, but have similar logical reasoning skills.

There are various factors that affect the gender gap (Jungwirth, 1991; Battista, 1990; Becker, 1991; & Boaler, 1994; 1997) in differing contexts; however gender differences during the activity of problem solving outside the classroom is seldom explored. More research on problem solving success outside of the classroom is needed since increased gender differences can occur inside the classroom. Barnes (2000) examined dominant characteristics of males in the classroom. Barnes looked at the ways gender and power roles affected interactions within the classroom environment. This particular study observed a 10th grade mathematics class in a private coeducational Australian school. One element, in particular, Barnes examined is the discourse between female and male students. She argued that masculinity forms two male sub-groups: The “Mates” and the “Technophiles”. The Mates are characterized as athletic and popular young males who lead group interactions for the sake of avoiding independent work, while the Technophiles shied away from group interactions and believed their own mathematical knowledge to be superior to others in the class. She suggests the Mates’ masculine characteristics such as behavior, tone of voice, and gestures dominated the classroom environment, as well as their intelligence and quick thinking. Also, in some
cases, the Mates displayed their dominance by questioning the teacher’s authority. In addition to the Mates, the Technophiles showed a different form of masculinity. Instead of dominating the classroom by their personality, they kept to themselves, belittling those who saw them as less intelligent. After examining the studies of Fennema and Sherman (1978) and Barnes (2000), it is clear that classroom environment shapes the differing identities of males and females and has an effect on the gender gap. More research is needed to determine if gender has an effect on mathematical ability outside of the classroom.

*Gender and Non-routine Problem Solving*

In addition to gender differences in mathematics achievement and problem solving on specific tasks, males and females also differ on how they solve non-routine problems. Moreno and Mayer (1999) took a close look at how males and females responded to the following open-ended, non-routine question: “What could you do to decrease the intensity of lightning?” (p. 357). The participants in this study included 810 college students and were all given 2.5 minutes to answer the question. Results showed the answers to the question differed by gender. Analyses indicated that females were eight times more likely than males to explain that nature cannot be altered. While this result is the answer to a very specific question, it implies that females and males may think differently due to other variables when solving non-routine problems. More research is needed in other domains to see if these results are consistent.

*Socioeconomic States (SES), Achievement, and Problem-Solving*

In addition to one’s gender, socioeconomic status (SES) has shown to be a demographic factor that influences one’s beliefs and affects towards mathematics, thus
affecting overall academic achievement. Socioeconomic status (SES) is defined as a hierarchical ranking of a person or family unit based on the control of power, wealth, and/or social status (Mueller & Parcel, 1981). There are numerous studies indicating a positive correlation between SES and academic achievement (e.g., Coe, Peterson, Blair, Schutten, & Peddie, 2013; Haveman & Wolfe, 1995; Klebanov, Brooks-Gunn, & Duncan, 1994; Sirin, 2005; Smith, Brooks-Gunn, & Klebanov, 1997); however, it can vary based on academic domain (e.g., science, social studies, and mathematics). Shaw and Barbuti (2010) found that students of lower SES were underrepresented in STEM majors and were more likely to drop out of STEM fields. Cadigan, Wei, and Clifton (2013) also found that SES had a positive correlation with mathematics achievement of Canadian students enrolled at a private K-12 school. The purpose of their study was to determine if there were correlations between standardized math test scores, motivation, instructional time, student morale, and gender. Their finding show that SES, gender, and student morale were all positively correlated with the students’ mathematics achievement; however, more research is needed on SES along with other factors to see how it affects students’ achievement on more non-routine tasks.

Belief/Affect Factors Summary

From the literature, it is clear that motivation, anxiety, and demographic factors all relate to mathematics achievement, how individuals solve problems, along with how people solve non-routine problems. It is evident that the majority of research on these factors examined the relationship to mathematical success during time constraining tasks (Andrews & Brown, 2015; Hoffman, 2010; Kyttälä, M., & Björn, 2014; Vukovic et al., 2012); however, it is unclear if these factors, along with resource and control factors
would relate to performance on non-routine mathematical tasks. More research is also needed to determine if these factors have an effect on students’ problem solving performance outside of a classroom or test setting where students have the opportunity to take their time and not be under pressure to finish a problem. Furthermore, with the existence of gender differences in mathematics achievement and mathematical problem solving, gender as a contributing factor to mathematical problem solving cannot be ignored. More research is needed to determine how gender both individually relates to non-routine problem solving, as well as collectively with other factors. While Schoenfeld’s (1992) four categories of examining problem solving are essential, the culture and context in which problem solving takes place must be considered.

Global Differences and Interventions

The bulk of this literature review summaries the research on the factors that relate to mathematics achievement and successful problem solving; however, it is important to summarize why exploring factors that relate to problem solving is important. The literature in this section shows that there are major global differences in problem solving competency with the U.S underperforming compared to other countries. Research also shows that interventions can help students improve their problem solving ability. In this section, the literature on global differences and examples of problem solving interventions are summarized to illuminate the importance of problem solving research and to show the need for more research on the factors that relate to problem solving. The following studies show the problem solving strategies of students in various countries and show that critical differences emerged between students. This section reviews the literature on problem solving across the globe and explains why and how these
differences occur. Following the research on global differences are two examples on problem solving interventions. The issue of global differences and the need for more effective interventions are two of the main reasons why researchers need to better identify the contributing factors to problem solving success and determine how the factors relate to problem solving both individually and collectively.

*Global Problem Solving Differences*

Cai (1998) investigated the problem solving differences between students in the U.S, China, and Japan. The results showed that elementary students in China and Japan are more likely to use algebraic strategies and representations when problem solving than students in the U.S. However, in this study, the reasons why students used different strategies for varying tasks were unclear. In a follow-up study, Cai (2000a) examined the different problem solving approaches of U.S and Chinese sixth grade students on *process-constrained* and *process-open* problems. He also investigated how U.S and Chinese students’ solution representations and strategies affect their performance differences on various tasks. Cai defined process-constrained problems as a problem that can “be solved either with a standard algorithm or with a more flexible application of an algorithm” (p. 310) and process-open problems as needing no formal algorithm. Results showed that U.S and Chinese students’ approaches to process-open tasks differed and Chinese students performed significantly better than U.S students on the process-constrained tasks. A qualitative analysis was performed to determine why there were differences on performance between the tasks. The results of the analysis showed that during the problem solving process, Chinese students were more likely to use symbolic representations and routine algorithms, while U.S students typically used concrete visual
representations. More research is needed to determine exactly why the difference in strategy use occurs by examining other factors.

In a separate analysis, Cai (2000b) focused on how sixth grade students in the U.S and China solved a specific type of problem: the arithmetic averaging algorithm, also known as the mean or average formula. After exploring the responses of U.S and Chinese students’ answers to two arithmetic average problems, he found several differences between the problem solving process and solution representations. In general, Chinese students were more successful than U.S students at finding accurate numerical solutions to both of the problems, but similar cognitive difficulties occurred with both groups. Both groups of students had difficulties applying the algorithm not due to their procedural knowledge of the algorithm, but due to their conceptual knowledge. However, even though difficulties with both groups occurred due to lack of conceptual knowledge, the conceptual representations of the two problems differed between groups. As discussed in Cai’s previous studies (1998, 2000a), Chinese students used more advanced algebraic representations, while U.S students used more concrete pictorial representations. Overall, with regards to these specific problem types, students who used more advanced algebraic representations were generally better problem solvers than those who did not. Researching how students solve different problems that assess different topics will provide more insight into what relates to students’ mathematical problem solving ability.

In addition to examining how students problem solve, Cai and Hwang (2002) explored how U.S and Chinese students problem pose, or formulate and present problems. In general, they wanted to identify the relationships between sixth grade
students problem posing and problem solving performance and how it differs between countries. Knowing that Chinese students tend to use more abstract representations when problem solving (Cai 2000a, 2000b), they were curious to see if Chinese students pose problems that require extensions beyond the information provided within the problem. As previous research suggests, Chinese students use more abstract, complex strategies when solving problems, although, when only analyzing students who use concrete problem solving strategies, the performance between the groups was almost the same. As expected, as problems become more abstract, Chinese students out-performed U.S students. The results also show that those who chose concrete strategies were more likely to make errors than those who chose abstract strategies. The types of problems posed between the groups were also significantly different, with U.S students posing vaguer, general algorithm-based problems. The Chinese sample showed a much stronger connection between problem posing and problem solving activities. For example, a U.S student may pose a problem that requires abstract problem solving strategies, but not be able to solve it. Since Chinese students tend to have better abstract problem solving strategies than U.S students, they are more likely to solve problems that require abstract thinking.

Further research was conducted to investigate how Chinese and U.S students’ use of strategies and representations relate to their algebra learning (Cai, 2004). Instead of examining only sixth graders, Cai studied students in grades 4 through 8 to gain a better understanding of how algebra learning affects problem solving strategies and representations. As mentioned, Chinese students typically learn algebraic concepts before U.S students; therefore, he hypothesized that eighth grade students in the U.S
might use similar strategies and representations as Chinese sixth grade students. Results showed that whether U.S students learned algebraic concepts or not, they were still more likely to use pictorial representations as opposed to algebraic. On the other hand, Chinese students were more likely to use algebraic representations even if they had no formal algebra learning. More research is needed to explain why this occurs. It is quite possible that Chinese students are more comfortable using algebraic strategies because they are exposed to these strategies at early ages and have more experience using them as they problem solve.

A later study by Jiang, Hwang, and Cai (2014) examined Chinese and Singapore sixth-grade students to examine their problem solving strategies and performance on speed problems. They distributed problems to all the participants and categorized their strategies as arithmetic, algebraic, model drawing, or no strategy. Overall, they found that the Chinese students outperformed the Singapore students. They also found that the groups used different problem solving strategies. The Chinese students used algebraic strategies more successfully and more frequently than Singapore students, but the Chinese students did not use as many different types of strategies as the Singapore students. It is also interesting to note that the Singapore students’ use of a model drawing to solve a fraction-based problem helped them out-perform the Chinese students on that particular problem. More research is needed to determine what other factors contribute to strategy selection. Researchers need to investigate the specific cognitive and affective factors of students to gain more insight into why differences occur in strategy selection in order to put effective interventions in place.
Interventions

Numerous research studies examined ways to improve students’ problem solving ability by putting interventions in place. For example, a study by Loehr, Fyfe, and Rittle-Johnson (2014) examined different approaches to teaching problem solving strategies to determine what instructional approach is more effective. Some educators and researchers argue the importance of having the student attempt to solve a problem before instruction (Chi, 2009; Piaget, 1973; Schwartz, Lindgren & Lewis, 2009), while others argue the importance of providing instruction first (Hiebert et al., 2003, Kirschner, Sweller & Clark, 2006). However, both approaches to problem solving instruction have shown to be beneficial in differing contexts. More research is needed to determine what factors yield more significance in different contexts and with varying types of problems. More research on the contributing factors can help make interventions, such as this, more effective.

Another example shows how teaching the problem solving process can help students’ problem solving success. Varcaretu (2008) wanted to help ninth grade Romanian students improve their problem solving skills by helping them improve Polya’s (1945) first step to problem solving, “understanding the problem” (p. 5). The purpose of her study was to help students identify the unknown(s), the essential data, and the specific condition by developing a simplified model. She found that having students write, critique, and revise problems in a simpler way had positive effects on students’ overall ability to problem solve. She also found that making students carefully select the important information in a problem helped improve their overall self-monitoring habits. The results suggest that students can be effectively taught problem solving strategies;
however more research is needed to determine the other contributing factors to problem solving success in order to develop better interventions that may help improve self-regulation and motivation or decrease mathematical anxiety.

Gaps in the Literature

There are two major gaps in the literature regarding factors that relate to mathematical problem solving. First, there is a limited amount of research that combines all of the factors of Schoenfeld’s (1992) mathematical problem solving framework. More research is needed to determine how procedural and conceptual content knowledge, appropriate problem solving strategies, self-regulation, and belief/affective factors collectively influence mathematical problem solving. The research shows that each one of the factors individually affects mathematics achievement and problem solving success; however, it is still unclear what factors are more important than others.

Secondly, the majority of research on mathematical problem solving examines students solving closed problems under specific time constraints. More research is needed to determine if the respective factors affect non-routine problem solving differently when individuals are not under time pressure. As mentioned, Jitendra and Star (2008) found that low-achieving students might need more time than high-achieving students to provide appropriate strategies. Research shows that problem solving can be a very complicated and complex process for children (Silver, 1986, Schoenfeld 1985, 2012), and children need varying amounts of time to show their best work. For example, Buschman (2003) quotes a child who says, “I have to think about a problem a long time, and sometimes I need a break to let my brain rest ‘cause it starts to hurt” (p. 542).
shows that students with varying backgrounds might develop different problem solving strategies if not under a time constraint.

Research Questions

The two major gaps in research with relation to mathematical problem solving have led to one over-arching research question. The research question is as follows: What resource (computational skills and heuristics), control (self-regulation), and belief/affect factors (motivation, mathematical anxiety, demographics) individually and collectively relate to unconstrained non-routine mathematical problem solving success of middle school students?
CHAPTER 3

METHODOLOGY

The methodology chapter describes the research design, context, participants, procedures, and secondary data that is used to answer the current research question. After revisiting the purpose of the study and the research question, the reasoning behind the proposed methodology is explained. Following the explanation of the research design, procedures, participants, secondary data, and a description of the statistical analyses is provided. To conclude, the hypotheses for the study are discussed.

Purpose and Research Questions

The purpose of this study is to identify what factors both individually and collectively related to successful non-routine, unconstrained mathematical problem solving for the given sample of students. Drawing from the mathematical problem solving framework of Schoenfeld (1992), the factors being examined include, (a) resource factors (computational skills and heuristics), (b) control (self-regulation), and (c) belief and affective factors (motivational factors and mathematical anxiety). By identifying factors that relate to successful problem solving, mathematics educators and policy makers can make informed decisions on how to improve middle school students’ mathematical problem solving ability.

Research Design
The current study uses a correlation design (e.g., Creswell, 2014) to better measure how independent factors relate to successful problem solving (dependent factor). Additionally, the exploration of predetermined theories (e.g., Schoenfeld, 1992) through the use of surveys, achievement tests, student work, and demographic data to generalize from a sample to a larger population warrant a quantitative design. The goal of the research design is to explore factors that relate to successful problem solving so further research can identify the predictive relationships among factors in order for interventions to be put in place. Thus, the most appropriate and effective method to analyze the factors is through a regression-based analysis (Tabachnick & Fidell, 2007).

Since there are multiple factors being examined in this study, it requires the use of numerous measures. In order to measure non-routine problem solving, a problem solving project showing student work was analyzed and coded. To identify strengths of computational skills and problem solving strategies, student placement exams based on math course were used. Further, a self-reporting questionnaire, homework completion grade, and time to complete the problem solving project were used to measure self-regulation. Additional questions on the self-reporting questionnaire were used to measure motivation and anxiety factors. The data in this study were used for a variety of reasons, which include face-validity, cost, and convenience. Given that the students are required to complete the problem solving project, placement exam, and questionnaire at their school, the data is pre-existing and available for secondary data analysis. The problem solving project and placement exam were developed by the school’s middle school math department; therefore, it was free of charge. In addition, the face-validity of the measures from the questionnaire are supported from a variety of other studies (e.g.,
Middleton & Midgley, 2002; Newton, 2009; Patrick, Ryan, & Kaplan; 2007; Turner, Meyer, Midgley, & Patrick, 2003). Demographic factors such as grade, course, course level, gender, and socioeconomic status were also provided by the school.

Participants

The participants of this study consisted of 242 middle school students from a private independent school in a Northeastern area of the United States. In the 2015-2016 academic year, there were 74 sixth grade students (32 female, 42 male), 89 seventh grade students (45 female, 44 male), and 79 eighth grade students (34 female, 45 male). For the purpose of this study, the school’s name will be The Academy in order to protect the faculty members, staff, and students involved. The Academy is known to be an elite PreK-12 college preparatory school that is located in a suburban area. Since it is a private institution, which is not fully state-funded, the school requires students to pay tuition to attend. For the 2015-2016 year, the middle grades tuition was $30,600. Some families pay for tuition in full or on a 10-month payment plan, while other families are provided financial aid.

Students in The Academy’s middle school have a traditional schedule such that they have math class for 40-45 minutes for six days out of a seven class day rotation for the entire academic year. The school has what are called “skip days” which means that math class skips once every seven school days. The math classes also vary from 40-45 minutes depending on what time of the day the class period falls. The students whose data are being analyzed in the present study were enrolled in either Pre-Algebra, Algebra A, or Algebra B as their math course. It is important to have participants in varying
grade levels and ability levels to determine if the factors related to problem solving differ based on these levels. For example, computational skills, problem solving strategies, control factors, beliefs, and affective factors may have different impacts depending on the grade level and course. To gain a better understanding of the grade level and content associated with each course, the descriptions are listed.

Course Content Descriptions

The majority of sixth grade students were enrolled in the Pre-Algebra (regular) course. The Pre-Algebra course covered the following topics: Algebraic expressions and integers, solving one-step equations and inequalities, decimals and equations, factors, operations with fractions, exponents, ratios, proportions, percents, solving multi-step equations and inequalities, data analysis, and probability. Approximately 20.3% of sixth graders take Pre-Algebra (Honors). The Pre-Algebra (Honors) course covered the same material, but moved at a slightly faster pace. The students were also posed with more challenging problems, which focus more on critical thinking. Since they moved at a slightly faster pace, in addition to the topics covered in the Pre-Algebra (regular course), the honors course covered inductive and deductive reasoning, arithmetic and geometric sequences, the binary sequence, the sequence of squares, the sequence of cubes, and the Fibonacci sequence (Charles, Davison, Landau, McCracken, & Thompson, 2004; Jacobs, 1994).

The majority of seventh grade students were enrolled in the Algebra A course. The Academy splits Algebra 1 across two years. The majority of students took the first part of Algebra 1 in seventh grade and the second part in 8th grade. The major topics covered in Algebra A, in this particular order, were as follows: (a) an introduction to
algebraic expressions, (b) equations and inequalities, (c) solving and graphing linear equations, (d) polynomials, and (e) factoring (Bittinger, Ellenbogen, & Johnson, 2014).

Approximately 27% of the seventh grade students take Algebra A (Honors). The Algebra A (Honors) course covered the same material, but moved at a slightly faster pace. The students were also posed with more challenging problems, which focused more on higher-level thinking. Since they moved at a slightly faster pace, in addition to the topics covered in the Algebra A (regular course), the honors course also covered permutations, combinations, and probability (Jacobs, 1994).

The majority of eighth grade students took the Algebra B course, which is considered the second year of Algebra 1. The major topics covered in Algebra B, in this particular order, were as follows: (a) polynomials and factoring, (b) rational expressions and equations, (c) solving and graphing systems of equations, (d) radical expressions and equations, and (e) quadratic equations (Bittinger, Ellenbogen, & Johnson, 2014).

Approximately 24.1% of the eighth grade students take Algebra B (Honors). The Algebra B (Honors) course covered the same material, but moved at a slightly faster pace. The students were also posed with more challenging problems, which focused more on critical thinking. Since they moved at a slightly faster pace, in addition to the topics covered in the Algebra B (Regular) course, the honors course had lessons that introduced them to topology and graph theory (Jacobs, 1994).

Instruments and Measures

The data analyzed in this study consisted of existing student data on a variety of measures administered by the Academy. These instruments and measures included a grade-wide problem solving project, course placement exams, a questionnaire, and
demographics. In this section, the measures are described in detail and organized by the factor in which it measures. All of the data in this study was considered secondary data because it was pre-existing. All data were de-identified; student names were replaced with code names before they were provided for research analysis. The middle school math department collected the existing data to gather information on how the students in their school perform year-to-year on a problem solving project that was developed by the school. The questionnaire was also used by the middle school mathematics teachers for departmental purposes.

Problem Solving Project (PSP)

It is important to start with the description of The Academy’s Middle School Problem Solving Project (PSP) as it will be used to measure unconstrained, non-routine problem solving success. The Academy’s Middle School PSP was completed by all sixth, seventh, and eighth grade students during January of the 2015-2016 academic year. The overall purpose of the PSP was to have students act as mathematicians as they worked on one (from the option of 3) non-routine problem over the course of 14 days. The Academy’s Mathematics Department used this project to teach students to understand and apply the process of mathematical problem solving. The project also taught students to reason, construct, and evaluate mathematical arguments, and communicate their mathematical thinking by explaining their problem solving strategies. The students were told that the goal is not to solve the problem quickly nor is it necessary to successfully solve the problem, but to immerse themselves in a problem over the course of two weeks. The PSP was introduced to all middle school students in an
assembly format and then explained in more detail by each student’s math teacher. The projects were then distributed, collected, and graded by each student’s math teacher.

In order to introduce the PSP to students, a special assembly was held for all sixth graders, as it was their first year being exposed to the project. The Academy’s Mathematics Department Head conducted a presentation on mathematical problem solving by telling the story of a famous mathematician named Andrew Wiles. The presentation opened with an explanation of what “mathematicians do” and then continued with an explanation of Fermat’s Last Theorem to help the students understand the story of Andrew Wiles. The Department Head told the students that a French mathematician named Pierre de Fermat posed one of the most famous mathematics problems in history. The problem was to prove the equation $a^n + b^n = c^n$ has no whole number solutions if $n$ is a whole number greater than two. For example, $a^3 + b^3 \neq c^3$, $a^4 + b^4 \neq c^4$, $a^5 + b^5 \neq c^5$, and so on. Around 1637, Fermat scribbled in the margin of the book *Arithmetica* what is now known as Fermat’s Last Theorem. Unfortunately, after Fermat died, the proof of Fermat’s Last Theorem was not left behind, which started a three and a half century quest to prove the theorem. The Department Head said that Andrew Wiles read about Fermat’s Last Theorem when he was 10 years old, he then became a professor of mathematics at Princeton University, and devoted seven years of his career on trying to prove the theorem before eventually doing so. A video interview was shown to the students of Andrew Wiles after he solved Fermat’s Last Theorem to illuminate the importance of persistence (Berlinghoff & Gouvêa, 2004). The story of Andrew Wiles was used to introduce, motivate, and excite the students about the PSP and to summarize to the students what is means to problem solve.
The PSP allowed students to work on one non-routine math problem at length over the course of a 14 day period outside of the classroom. Since the students were working on one problem over the course of two weeks, they were not put under the normal pressure of completing a problem within a normal class period. Due to the design of the PSP, students had the option to select one problem from three choices based on their grade level; therefore, in total, there were nine different non-routine questions. (See Appendix A). Since the problems required students to draw on varying types of resources, problem chosen was used as a variable in the data analysis to use as both a control measure as well as to determine if the factors related to non-routine problem solving success differently based on problem selection. The three problems differed by grade level because the project is completed every year; therefore, the problems will be different as students advance to the next grade level. Also, the problems needed to be grade-level appropriate based on the students’ background knowledge. The students used problem solving logs in order to show their work and mathematical reasoning as they solved the problem of their choice. The logs were also used for students to record how long they worked on the problem by recording their start and end time.

Unconstrained, non-routine problem solving success was measured by coding the PSPs to determine if the students a) correctly solved the problem, b) appropriately set-up the problem, but made a computational error, c) appropriately set-up the problem, but did not finish, d) made both a computational error and did not finish or e) inappropriately set-up the problem. It is important to note that all 242 participants showed their work when completing the PSP. For example, no student wrote down an answer without showing the mathematics they used and the reasoning behind it. The coding for unconstrained, non-
routine problem solving was used to differentiate between three student outcomes, which were: a) problem correctness, b) correct problem set-up, and c) problem completion. For the first analysis of problem correctness, all students who correctly solved the problem were coded as 1 and those who did not solve the problem correctly were coded as 0. For the second analysis of correct problem set-up, all of the students who inappropriately set up the problem were coded as 0 and all of the other students were coded as 1; therefore, students who made computational errors and/or did not finish could still be coded as 1 for correctly setting up the problem. For the third analysis of problem completion, all of the students who completed the problem regardless if they set the problem up correctly or made a computational error were coded as 1 and all of the other students who did not finish were coded as 0. To be clear, the participants do not fall into one group or the other. For example, a student can appropriately set-up the problem, complete the problem, and find the correct answer; therefore receive a score of 1 for all three outcomes. In contrast, a student can set up the problem correctly, but fail to complete the problem or find a correct answer; therefore s/he would receive a score of 1 only for correct problem set-up and a 0 for the other two outcomes, and so on. To understand how the problems were coded for problem correctness, correct problem set-up, and problem completion, explanations and examples of student work are presented in the following sections.

Since the design of the project allowed the students to work on the problem for 14 days, in general, there tended to be a vast amount of student work involved; however, the figures below show snap shots of students who arrived at correct answers verses those who did not. If the students did not specifically state their answer in their work, they
stated it somewhere in their reasoning. The figures of student work and explanations of the coding are listed in order by grade level. The explanations discuss how students received a score of 1 or 0 for problem correctness followed by an explanation of how they received a score of 1 or 0 for correct problem set-up. Examples are not provided for problem completion because the codes were simply based on if they arrived at an answer or not. The process of how they did so was irrelevant. PSP problems can be found in Appendix A.

*Sixth Grade Problems*

In order to receive a problem correct score of 1 on the sixth grade Fast Draw problem, the student appropriately created two squares with four, five, and six lines and two equilateral triangles with four and five lines. The lines were also properly labeled to distinguish between the amounts of lines used either in the student’s work or reasoning explanation. Figure 3.1 shows a student who received a score of 1 for problem correctness.
Figure 3.1 Sixth Grade Fast Draw Problem Correctness Score 1

Figure 3.2 Sixth Grade Fast Draw Problem Correctness and Set-up Score 0
If the student did not solve the Fast Draw problem correctly, the student received a score of 0 for problem correctness. Figure 3.2 shows a student who received a score of 0 for problem correctness because s/he did not appropriately count and label the lines of the shapes either in the work or reasoning. This student also received a 0 for correct problem set-up. In order to receive a score of 1 for correct problem set-up, the student must appropriately label the lines to differentiate between drawing a figure with four, five, or six lines. Students who received a 0 for problem correctness, but a 1 for problem set-up either made a computational error by miscounting the lines or did not finish solving the problem.

For the sixth grade Four Four’s problem, in order to receive a score of 1 for problem correctness, the student used the correct order of operations with no computational errors, used four fours, and found all of the numbers between 0 and 9. It can be seen in Figure 3.3 that the student found all of the combinations correctly. This student received a score of 1 for problem correctness. Students did not have to complete the extension in order to correctly solve the problem.

Figure 3.3 Sixth Grade Four Fours Problem Correctness Score 1
Figure 3.4 is a snapshot of a student who received a score of 0 for both problem correctness and correct problem set-up. The student did not use the correct order of operations. It can be seen that s/he added before multiplying or dividing. If s/he wanted to compute the operations in that particular order, s/he would have needed to use parentheses. In addition to not using the correct order of operations, in previous work the student was not always using four fours.

Figure 3.4 Sixth Grade Four Fours Problem Correctness and Set-up Score 0

Figure 3.5 Sixth Grade Goldbach’s Conjecture Problem Correctness Score 1
For the sixth grade Goldbach’s Conjecture problem, students were given a problem correctness score of 1 if s/he found Goldbach pairs for all of the even integers between 4 and 100 without making any errors. As can be seen in Figure 3.5, the student found at least one Goldbach pair for all of the even numbers between 4 and 100.

In contrast, Figure 3.6 shows a student who received a score of 0 for both problem correctness and correct problem set-up. The student received a score of 0 because s/he was using composite numbers, instead of prime numbers, to calculate Goldbach’s pairs. Student’s who received a 1 for correct problem set-up, but a 0 for problem correctness used all prime numbers in their calculations, but would make a computational error by adding the numbers incorrectly or would not finish finding all of the numbers between 4 and 100.
Séventh Grade Problems

In order to receive a problem correctness score of 1 for the seventh grade Diagonal problem, the student showed their work and arrived at an answer of 119 diagonals. Figure 3.7 is a snapshot of a student who used both a pattern and discovered a formula to determine the amount of diagonals in a 17-sided polygon, thus, received a score of 1 for problem correctness.
In contrast, Figure 3.8 is an example of a student who also worked on the Diagonal problem, but received a score of 0 for problem correctness. The student attempted to formulate a pattern by counting the diagonals; however, it can be seen that the student did not arrive at a correct answer due to a computational counting error. S/he tried to draw and count all of the diagonals in a 17-sided polygon, but was unsuccessful and arrived at an answer of 105 diagonals. However, this student received a score of 1 for correct problem set-up. While counting all of the diagonals may not have been the most efficient way to solve the problem, it could be done using this method. Students who received a 0 for correct problem set-up either showed no work, did not understand the concept of diagonals or polygons, or did not draw a diagonal to all of the other vertices.

![Diagram of 17-sided polygon with diagonals]

Figure 3.8 Seventh Grade Diagonal Problem Correctness Score 0 Set-up Score 1

For the Martian Money problem, students who received a score of 1 for problem correctness showed their work and formulated a pattern or formula to determine that 39
reddies was the total amount of money that cannot be made with 5 and 11. Figure 3.9 shows an example of a student who received a score of 1 for problem correctness.

Figure 3.9 Seventh Grade Martin Money Problem Correctness Score 1

Figure 3.10 is an example of a student who received a score of 0 on Martian Money for problem correctness and a 0 for correct problem set-up. He failed to come up with a
pattern or formula to show the correct amount of reddies that cannot be made. Instead the student is using numbers and operations that are not relative to the problem. It is unclear why the student is using the number 7 or multiplying by 1. His strategy is not an effective method to solve the problem. Students who also received a score of 0 for correct problem set up were trying to figure out the maximum numbers of reddies that can (instead of cannot) be made with 5 and 11 reddies without realizing that the answer would obviously be infinite.

For the seventh grade Red Paint problem, students who received a score of 1 for problem correctness showed their work and created a model, devised a pattern, or made a formula to determine that 2 cubes had 0 faces painted, 9 with 1, 12 with 2, 4 with 3, and no cubes had 4, 5 or 6 faces painted. Figure 3.11 is a snap shot of a student who received a score of 1 for problem correctness.

![Figure 3.11 Seventh Grade Red Paint Problem Correctness Score 1](image)

Figure 3.12 is a snap shot of a student who received a score of 0 for both problem correctness and for correct problem set-up for the Red Paint problem. The student failed to figure out the correct answer and did not set-up the problem appropriately. To set up
the problem appropriately, the student must draw a picture or formulate a pattern by envisioning the cube as a 3-dimensional object composed of 27 cubes. The student’s work in Figure 3.12 is unclear and the answer is incomplete. It cannot be determined how the answer of 45 relates to the problem. The student is also not envisioning the cube as a 3-dimensional object.

**Eighth Grade Problems**

In order to receive a problem correctness score of 1 on the eighth grade Cake problem, the student needed to determine that 15 pieces was the total maximum number of slices. While it is not mentioned in the problem, the students were instructed by their teachers that they could not rearrange and stack the cake. To appropriately set-up the problem, the student needed to envision the cake as 3-dimensional object. Figure 3.13 is
a snapshot of a student’s who arrived at an answer of 15 and received a problem correctness score of 1. The most common answers for the Cake problem tended to be 11 and 14. Students who set-up the problem inappropriately and only envisioned the cake as a 2-dimensional object typically got an answer of 11. Students such as the one in the Figure 3.14 set-up the problem appropriately by envisioning the cake as 3-dimensional, but did not realize that the cake can be cut in such a way that produces a piece of cake in the center of the cylinder. Figure 3.14 shows a student who received a problem correctness score of 0, but a correct problem set-up score of 1.

In order to receive a problem correct score of 1 on the eighth grade Tiles problem, the student determined that 60 square units is the maximum area that can be created. By devising a formula, the student’s work in Figure 3.15 shows that s/he arrived at the correct answer.
In contrast, Figure 3.16 shows a student who set-up the Tile problem appropriately and found the answer to be 48 units squared; however s/he did not find the maximum square area; therefore, s/he received a problem correctness score of 0. Students who did not set-up the problem appropriately either showed minimal or no work and/or did not follow the directions to make the same amount of blue and white tiles.
To receive a problem correctness score of 1 on the eighth grade Hiking problem, the student must determine that the maximum distance hiker #1 can go from home to the oasis with the food supply is 15 days. The student’s work in Figure 3.17 shows that s/he arrived at the correct answer.

![Figure 3.17 Eighth Grade Hiking Problem Correctness Score 1](image)

Figure 3.17 Eighth Grade Hiking Problem Correctness Score 1

Figure 3.18 shows a snap shot of a student who set-up the Hiking problem appropriately by making a diagram; however, s/he found the answer to be 14 days due to a computational error in earlier work; therefore, s/he received a problem correctness score of 0 and a correct problem set-up score of 1. Students who received a 0 for correct

![Figure 3.18 Eighth Grade Hiking Problem Correctness Score 0 Set-up Score 1](image)
problem set-up showed minimal or no work and/or misunderstood the problem and thought that hiker #1 also had to come home.

After coding for problem correctness, there were 74 students who correctly solved the non-routine problem and 168 students who did not. When coding for correct problem set-up, there were 216 students who set-up the problem appropriately and 26 students who did not. The last PSP coding mechanism was to differentiate between students who completed the problem from those who did not. Students received a score of 1 for problem completeness if they made a full attempt at solving the problem and concluded their project with an answer, either right or wrong. A complete problem attempt involved showing all of their work during the problem solving process and finishing a minimum of four logs. Completing four logs was the minimum requirement for the PSP. When coding for problem completeness, there were 195 students who completed the problem and 47 students who did not. Table 3.1 shows how many students solved that particular problem along with the percent of students that solved the problem correctly, correctly set-up the problem, and completed the problem.

Table 3.1

*Descriptives Based on Problem Type*

<table>
<thead>
<tr>
<th>Problem Type</th>
<th># of Students</th>
<th>% Correct</th>
<th>% Correct Set-up</th>
<th>% Completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast Draw</td>
<td>37</td>
<td>19%</td>
<td>86%</td>
<td>76%</td>
</tr>
<tr>
<td>Four Four’s</td>
<td>28</td>
<td>39%</td>
<td>89%</td>
<td>64%</td>
</tr>
<tr>
<td>Goldbach’s</td>
<td>9</td>
<td>22%</td>
<td>77%</td>
<td>89%</td>
</tr>
<tr>
<td>Diagonals</td>
<td>38</td>
<td>42%</td>
<td>89%</td>
<td>84%</td>
</tr>
<tr>
<td>Martian Money</td>
<td>21</td>
<td>71%</td>
<td>90%</td>
<td>90%</td>
</tr>
</tbody>
</table>
Table 3.1, continued

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Red Paint</strong></td>
<td>30</td>
<td>53%</td>
<td>93%</td>
<td>87%</td>
</tr>
<tr>
<td><strong>Cake</strong></td>
<td>41</td>
<td>5%</td>
<td>66%</td>
<td>71%</td>
</tr>
<tr>
<td><strong>Tiles</strong></td>
<td>21</td>
<td>10%</td>
<td>95%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Hiking</strong></td>
<td>27</td>
<td>24%</td>
<td>88%</td>
<td>94%</td>
</tr>
</tbody>
</table>

For more details on the PSP directions, grade level problems, and format refer to Appendix A. The rest of this section describes the instruments used to measure the independent variables that were used in the analysis.

**Resource Measures**

A problem solver’s resources consist of both computational skills and being able to appropriately set up a problem (Schoenfeld, 1992). In order to measure the participant’s resource factors, *course placement exams* were used. At the Academy, the placement exams were one piece of information, among others, that was used to place students in the appropriate math course for the next academic year. Placement exams were given to students at the end of April 2016. The exams were taken in the students’ math classes where they had 40 minutes to complete the exam. The placement exams were the same for all sixth grade students entering an Algebra A course, seventh grade students entering an Algebra B course, and eighth grade students entering a Geometry course. The placement exams assessed students on material covered throughout the academic year. The exam included problems that show students’ computational skills and ability to set up an appropriate problem solving strategy. The sixth grade students
took their placement exam with no calculator, while the seventh and eighth grade students were permitted to use a calculator.

The problems on the placement exam were coded as problems that assessed computational skills or strategy competence (heuristics). Computational skills questions measured the students’ ability to calculate basic operation problems accurately using mental methods, by showing work, and/or by using other tools such as a calculator. Computational skills questions also required the appropriate selection of arithmetic operations. The computational skills questions were coded as 1 for a correct answer and 0 for an incorrect answer. Figure 3.19 shows an example of a sixth grader who correctly solved two computational skills questions on the course placement exam. The student appropriately used the order of operations without making any calculation errors.

A. Perform the indicated operations.

1. \( 58 - 193 + 27 \)
   \( \underline{135 + 27} \)
   \( \underline{-108} \)

2. \((-6)(-14)\)

   \( \text{Correct Answer: 84} \)

Figure 3.19 Sixth Grade Computational Skills Correct Example

A. Perform the indicated operations.

1. \( 58 - 193 + 27 \)
   \( \underline{58 - 220} \)
   \( \underline{-162} \)

2. \((-6)(-14)\)

   \( \text{Correct Answer: 84} \)

Figure 3.20 Sixth Grade Computational Skills Incorrect Example

In contrast, Figure 3.20 shows an example of a sixth grade student who solved the computational skills questions incorrectly. On the first problem, the student used the incorrect order of operations by adding before subtracting when the student should have been conducting his or her arithmetic operations from left to right. On the second
problem, the student multiplied two negative numbers together and wrote the product as a negative number instead of a positive number.

Since the course placement exams differed based on grade level, it is important to observe how the computational skills questions became more advanced based on the content covered in the course. For example, Figure 3.21 shows a seventh grade student who solved two computational skills questions correctly. The first computational question in this example required the students to appropriately subtract two polynomials. The second computational question instructed the students to multiply two binomials together and then simplify the terms.

Perform the indicated operation and simplify.

9. \( (12x^2 - 3x + 5) - (3x^3 + 12x^2 - x) \)   
   \[
   = 12x^2 - 3x + 5 - 3x^3 - 12x^2 + x \\
   = -3x^3 + 3x + 5
   \]

10. \( (4n - 7)(n + 6) \)   
   \[
   = 4n^2 - 7n + 24n - 42
   \]

Figure 3.21 Seventh Grade Computational Skills Correct Example

Counter to the figure above, Figure 3.22 shows an example of a seventh grade student who solved two computational questions incorrectly. In the first problem, the student added “-3x” and “x” incorrectly. It can be assumed that the student thought that x

Figure 3.22 Seventh Grade Computational Skills Incorrect Example
represented 0x instead of 1x. In the second problem, the student incorrectly added “24n” and “-7n”. When combining these two like terms, the answer should be positive 17n instead of “-17n”.

The computational skills questions continued to become slightly more advanced on the eighth grade placement exam. The first example problem in Figure 3.23 required the student to divide then simplify an algebraic rational expression. The second problem had the students add together three rational terms then simplify, if possible. The student in Figure 3.23 solved both of the computational problems correctly.

In contrast, Figure 3.24 is an example of a student who made computational errors on both of the problems. On the first problem, the student incorrectly simplified “x” and
“$2x^2$”. Instead of writing $2x$ in the denominator, the student wrote “$x^2$”. The student also failed to keep “$x - 2$” in the denominator. On the second problem, the student correctly finds the common denominator, but fails to multiply the numerators by the appropriate term.

The strategy competence measures differed from the computational skills measures because these questions measured the students’ ability to set-up a problem appropriately. In order to correctly solve these types of problems, it was imperative for the students to use a correct problem solving strategy. For example, the question in Figure 3.25 was a sixth grade question that required the student to either set up an appropriate equation or set up a correct arithmetic operation which shows that he or she knew that $328$ is 82% of the original cost. The strategy competence questions were only coded for correct set-up, not correct computational execution in order to differentiate between the students’ computational skills and strategy competence. The student in Figure 3.25 correctly set up an equation that yielded a correct answer.

21. Mr. Connor bought a dress for his wife on sale for $328.00. If the dress had been marked down 18%, how much did it cost before the sale?

\[
x \cdot 0.82 = 328
\]

\[
x = 400
\]

The dress originally cost $400.

Figure 3.25 Sixth Grade Strategy Competence Correct Example
Figure 3.26 shows the same strategy competence example; however, the student incorrectly set up the problem. The student multiplied the sale price by “1.18”. It can be assumed that the student thought adding 18% of the sale price to the sale price was the same operation as subtracting 18% off of the original price. By setting up this incorrect operation, the student was unable to achieve a correct answer even though his computational skills were correct.

21. Mr. Connor bought a dress for his wife on sale for $328.00. If the dress had been marked down 18%, how much did it cost before the sale?

\[ \begin{align*}
    26 & \quad 24 \\
    \underline{328} & \quad 00 \\
    \underline{2800} & \quad 00 \\
    \hline
    38700 & \quad 00
\end{align*} \]

Figure 3.26 Sixth Grade Strategy Competence Incorrect Example

Similarly to the computational skills questions, the strategy competence questions also became more advanced as the grade level increased to align with the students’

25. The sum of the measures of complementary angles is 90°. If one angle measures 35° less than twice its complement, find the measure of each angle.

\[ \begin{align*}
    n + 2n - 35 &= 90 \\
    +35 &+35 \\
    3n &= 125 \\
    n &= 41 \frac{2}{3} \\
    \frac{145}{3} &\quad \text{and} \quad \frac{125}{3}
\end{align*} \]

Figure 3.27 Seventh Grade Strategy Competence Correct Example
background knowledge. The question in Figure 3.27 shows an example of a seventh grade strategy competence measure. The problem required the student to set up an appropriate equation to determine the missing measures for the complementary angles. The work in Figure 3.27 shows that the student sets up the correct equation that yields a correct answer.

In contrast, the example in Figure 3.28 shows an incorrect set-up. The student ignores the fact that the one angle should be 35° less than twice its complement. Since the student does not appropriately set-up the problem, he or she was unable to properly execute the appropriate computational procedures.

25. The sum of the measures of complementary angles is 90°. If one angle measures 35° less than twice its complement, find the measure of each angle.

![Figure 3.28 Seventh Grade Strategy Competence Incorrect Example]

The eighth grade strategy competence measures are slightly more advanced than the seventh grade measures due to the content covered in the eighth grade course. The
strategy competence question in Figure 3.29 was a mixture problem that required the students to set up a system of equations in order to successfully solve. It can be seen by the student work in this figure that the student appropriately sets up the system of equations. By appropriately setting up the system of equations, the student now has the opportunity to execute the correct computational procedures.

Contrary to the work shown in Figure 3.29, the work in Figure 3.30 shows a student who incorrectly set up the mixture problem. Instead of having the first equation in the system represent the total amount of liters and the second equation represent the mixture of the two types of acid, the student puts the percentages of the acid and total for the liters in the incorrect places.

11. A chemist has a solution that is 30% acid and another that is 60% acid. How much of each is needed to make 120L of a solution that is 40% acid?

Figure 3.30 Eighth Grade Strategy Competence Incorrect Example

The figures presented above for both computational skills and strategy competence (heuristics) only represent a few examples of these measures. On the 6th grade placement exam (Appendix B), there were 17 questions measuring computational skills and 7 assessing strategy competence. On the 7th grade placement exam (Appendix C), there were 14 questions for computational skills and 8 for strategy competence, and
on the 8th grade placement exam (Appendix D), there were 11 questions measuring computational skills and 7 measuring strategy competence. The students were given average scores for both the computational skill and strategy competence measures in order to have an overall computational skill score and overall strategy competence score. Since the placement exams have different questions based on grade level for the purpose of being grade-level appropriate, the overall computational skill score and overall strategy competence score were normed by grade, with each student given a z-score on each measure. To view how the resource factors differed by the demographic data, Table 3.2 shows the descriptive statistics for the resource factors based on gender and financial aid status (SES measure).

Table 3.2
Descriptive Statistics for Resource Factors Based on Z-Score (N = 242)

<table>
<thead>
<tr>
<th></th>
<th>Computational Skills Mean(SD)</th>
<th>Heuristics Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male (N = 131)</td>
<td>.056(1.08)</td>
<td>.108(1.06)</td>
</tr>
<tr>
<td>Female (N = 111)</td>
<td>-.066(.89)</td>
<td>-.128(.91)</td>
</tr>
<tr>
<td>Financial Aid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given (N = 57)</td>
<td>-.233(.89)*</td>
<td>-.238(.92)*</td>
</tr>
<tr>
<td>Not Given (N = 185)</td>
<td>.072(1.02)*</td>
<td>.073(1.01)*</td>
</tr>
</tbody>
</table>

Note: *p < .05. **p < .01.

Control Measures

In order to appropriately measure students’ control (self-regulation), scores from a 34 item survey (See Appendix E) were used for analysis. The self-regulation piece of the survey contained six items measuring to what extent students regulate and plan their cognition. The survey was administered to the students during April of the 2015-2016 academic year approximately two months after the students completed the PSP. This
survey was used by The Academy’s middle school math department to gain knowledge about their students’ feelings and beliefs towards mathematics in general and towards their math class. The survey was administered and completed in the students’ mathematics classes using a paper and pencil instrument using a 5 or a 7-point Likert scale, depending on the measure. An example of one of the self-regulation questions states, “When I run into difficulty doing a math problem, I go back and work out where I went wrong.” The Likert scale ranged from 1 = “Not at all True” to 5 = “Very True.” The self-regulation measure’s face-validity and reliability is supported by a number of other studies (e.g., Middleton & Midgley, 2002; Patrick, Ryan, & Kaplan; 2007; Turner, Meyer, Midgley, & Patrick, 2003).

In order to gather the most accurate results from the motivation, self-regulation, and anxiety survey, a factor analysis was conducted on all 34 items. The purpose of the factor analysis was to determine which questions loaded onto the same factor in order to eliminate questions that were not sufficient. The factor analysis was then used to compute a composite score for each category. The minimum amount of data for a factor analysis was satisfied by using a sample size of 242 participants (Tabachnick & Fidell, 2007). The Kaiser-Meyer-Olkin measure of sampling adequacy was .89, above the commonly recommended value of .6, and Bartlett’s Test of Sphericity was significant ($\chi^2 (465) = 4341.42, p < .01$). The items that loaded onto this factor were all related to self-regulation towards math in general and towards math class, thus, this variable was called *math self-regulation*. The factor analysis showed that the math self-regulation factor accounted for approximately 6% of the variance. Table 3.3 shows the factor loadings for the five math self-regulation items.

Table 3.3
Factor Loadings for Math Self-Regulation ($N = 242$)

<table>
<thead>
<tr>
<th>Self-Regulation</th>
<th>Mean</th>
<th>Standard Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 13</td>
<td>-.62</td>
<td></td>
</tr>
<tr>
<td>Question 16</td>
<td>-.67</td>
<td></td>
</tr>
<tr>
<td>Question 28</td>
<td>-.71</td>
<td></td>
</tr>
<tr>
<td>Question 30</td>
<td>-.48</td>
<td></td>
</tr>
<tr>
<td>Question 31</td>
<td>-.53</td>
<td></td>
</tr>
</tbody>
</table>

Note. Each question can be found in Appendix E. Bold values are sufficient factor loadings above .4

Presented in Table 3.4 are the descriptive statistics for self-regulation based on gender, grade level, and financial aid status (SES measure).

Table 3.4

Descriptive Statistics for Math Self-Regulation (Survey) ($N = 242$)

<table>
<thead>
<tr>
<th>Gender</th>
<th>Mean</th>
<th>Standard Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male ($N = 131$)</td>
<td>0.841</td>
<td>1.00</td>
</tr>
<tr>
<td>Female ($N = 111$)</td>
<td>-0.099</td>
<td>0.77</td>
</tr>
<tr>
<td>Grade Level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th ($N = 74$)</td>
<td>-.108</td>
<td>0.91</td>
</tr>
<tr>
<td>7th ($N = 89$)</td>
<td>0.55</td>
<td>0.87</td>
</tr>
<tr>
<td>8th ($N = 79$)</td>
<td>0.04</td>
<td>0.94</td>
</tr>
<tr>
<td>Financial Aid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given ($N = 57$)</td>
<td>0.22</td>
<td>0.87</td>
</tr>
<tr>
<td>Not Given ($N = 185$)</td>
<td>-0.07</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Note: *$p < .05$. **$p < .01$."

In addition to the self-reported measure of math self-regulation, student’s homework completion throughout the year was used as a control measure. At the Academy, homework was assigned, on average, four days a week during the academic year and was graded solely on completion. Homework completion is an appropriate control measure because homework was often completed at home or during study hall periods. Given that students worked on homework during non-instructional time, it can be implied that homework required independence, planning, and time management
(Ramdass & Zimmerman, 2011). Students also had to remember to bring their homework from their locker to math class to earn credit. This extra step of bringing their homework from their locker required planning, as well. Research suggests that homework enhances the development of self-regulation skills and it also implies that students who complete their homework on a regular basis tend to be more self-regulated than those who do not (Pintrich, 2000; Trautwein & Koller, 2003). Moreover, since the PSP was completed outside of the classroom, it was important to measure how often students completed their other work outside of the classroom (i.e., homework). Presented in Table 3.5 are the descriptive statistics for homework completion based on gender, grade level, and financial aid status (SES measure).

Table 3.5

Descriptive Statistics for Homework Completion (N = 242)

<table>
<thead>
<tr>
<th></th>
<th>Means (100 Total)</th>
<th>Standard Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gender</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male (N = 131)</td>
<td>90.69*</td>
<td>7.09</td>
</tr>
<tr>
<td>Female (N = 111)</td>
<td>92.76*</td>
<td>5.79</td>
</tr>
<tr>
<td><strong>Grade Level</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th (N = 74)</td>
<td>91.29</td>
<td>6.13</td>
</tr>
<tr>
<td>7th (N = 89)</td>
<td>93.33**</td>
<td>6.66</td>
</tr>
<tr>
<td>8th (N = 79)</td>
<td>90.06**</td>
<td>6.58</td>
</tr>
<tr>
<td><strong>Financial Aid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given (N = 57)</td>
<td>90.43</td>
<td>6.20</td>
</tr>
<tr>
<td>Not Given (N = 185)</td>
<td>92.01</td>
<td>6.68</td>
</tr>
</tbody>
</table>

*Note: *p < .05. **p < .01.
Figure 3.31 is a box and whisker plot that shows the distribution of homework grades based on grade level. The homework grades for all of the middle school students range from 66 to 100.

In order for students to self-regulate, they must also be able to control their attention in order to stay focused on the task at hand. This process often requires individuals to clear their minds of distracting thoughts that may prohibit them from completing the task (Winne, 1995). When an individual is able to do this, s/he increases the chances of reaching their goal (i.e, solving a non-routine math problem). Research shows that achievement outcomes increase as focused time spent on-task increases (Kuhl, 1985). As an additional control measure, the students logged how long they were working on the non-routine problem each day. The total amount of time spent working on the non-routine problem was used as an additional continuous control factor to
measure the participants’ *time-on-task*. The descriptive statistics for time-on-task are presented in Table 3.6.

Table 3.6

*Descriptive Statistics for Time-on-task (minutes) (N = 242)*

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gender</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male (N = 131)</td>
<td>64.05</td>
<td>36.50</td>
</tr>
<tr>
<td>Female (N = 111)</td>
<td>66.25</td>
<td>32.63</td>
</tr>
<tr>
<td><strong>Grade Level</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th (N = 74)</td>
<td>63.80</td>
<td>25.91</td>
</tr>
<tr>
<td>7th (N = 89)</td>
<td>70.47</td>
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</tr>
<tr>
<td>8th (N = 79)</td>
<td>60.10</td>
<td>41.35</td>
</tr>
<tr>
<td><strong>Financial Aid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given (N = 57)</td>
<td>63.36</td>
<td>27.62</td>
</tr>
<tr>
<td>Not Given (N = 185)</td>
<td>65.58</td>
<td>36.65</td>
</tr>
</tbody>
</table>

*Note:* *p < .05. **p < .01.*

**Belief and Affect Measures**

In order to appropriately measure students’ belief and affect factors (*motivation* and *mathematical anxiety*), the additional questions from the 34 item survey (See Appendix E) were used for analysis. In addition to measuring self-regulation, this survey measured mathematical anxiety and various aspects of motivation. The survey as a whole was mainly adapted from the work of Eccles, Wigfield, and colleagues (Eccles et al., 1993; Wigfield & Eccles, 2000; Wigfield & Cambria, 2010; Wigfield & Meece, 1988); however, questions measuring mathematical anxiety were drawn from the abbreviated mathematics rating scale (A-MARS) (Richardson & Suinn, 1972). The Richardson and Suinn (1972) measure has shown to have a strong test-retest reliability (Ashcraft, 2002). The reliability and face-validity of Eccles et al. (1993) measures were also supported by an array of other studies (Middleton & Midgley, 2002; Patrick, Ryan, & Kaplan, 2007; Turner, Meyer, Midgley, & Patrick, 2003).
After running the factor analysis, six factors were created. The six factors included math self-regulation that was previously discussed, *ability beliefs*, *interest*, *importance and mastery*, *performance goals*, and *math anxiety*. As mentioned, the minimum amount of data for a factor analysis was satisfied by using a sample size of 242 participants (Tabachnick & Fidell, 2007). Interest accounted for 30.8% of the variance, anxiety accounted for 10.4%, performance goals accounted for 8.9%, importance of mastery accounted for 4.8%, and ability beliefs accounted for 3.9%. Table 3.7 shows the factor loadings.

Table 3.7

*Factor Loadings for Anxiety and Motivation (N = 242)*

<table>
<thead>
<tr>
<th>Question</th>
<th>Interest</th>
<th>Anxiety</th>
<th>Performance Goals</th>
<th>Importance and Mastery</th>
<th>Ability Beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>14</td>
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<td>17</td>
<td></td>
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<td></td>
<td>.76</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.7, continued

Note. Each column represents a factor that each item loaded on. Each question can be found in Appendix E. Bold values are adequate factor loadings above .4

Presented in Table 3.8 are the descriptive statistics for the belief/affect factors based on gender, grade level, and financial aid status (SES measure).

Table 3.8

Descriptive Statistics for Belief/Affect Factors (N = 242)

<table>
<thead>
<tr>
<th></th>
<th>Interest</th>
<th>Anxiety</th>
<th>Performance Goals</th>
<th>Importance and Mastery</th>
<th>Ability Beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male (N = 131)</td>
<td>.13(.94)*</td>
<td>-.22(.92)**</td>
<td>-.01(1.01)</td>
<td>.07(.97)</td>
<td>.17(.94)**</td>
</tr>
<tr>
<td>Female (N = 111)</td>
<td>-.15(.96)*</td>
<td>.26(.80)**</td>
<td>.02(.84)</td>
<td>-.09(.86)</td>
<td>-.20(.92)**</td>
</tr>
<tr>
<td>Grade Level</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th (N = 74)</td>
<td>-.03(.96)</td>
<td>.05(.80)</td>
<td>-.12(.80)</td>
<td>.23(.88)**</td>
<td>.07(.99)</td>
</tr>
<tr>
<td>7th (N = 89)</td>
<td>.20(.92)</td>
<td>.05(.94)</td>
<td>.13(1.03)</td>
<td>.18(.91)**</td>
<td>-.10(.98)</td>
</tr>
<tr>
<td>8th (N = 79)</td>
<td>-.20(.95)</td>
<td>-.10(.94)</td>
<td>-.04(.94)</td>
<td>-.41(.85)**</td>
<td>.04(.87)</td>
</tr>
<tr>
<td>Financial Aid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given (N = 57)</td>
<td>.03(.84)</td>
<td>.00(.91)</td>
<td>.00(.91)</td>
<td>-.09(.90)</td>
<td>-.21(.82)</td>
</tr>
<tr>
<td>Not Given (N = 185)</td>
<td>-.08(.99)</td>
<td>.01(.87)</td>
<td>.01(.86)</td>
<td>.03(.93)</td>
<td>.06(.97)</td>
</tr>
</tbody>
</table>

Note: *p < .05. **p < .01. Means are presented with standard deviations in parentheses

Demographic Measures

The students’ parents are required to list their child’s identified gender at the beginning of the school year; therefore it was reported in the school database. Gender, along with other demographics, were recorded in a program called Faculty Access for the Web (FAWeb). FAWeb also had the students’ grade level, which was used to identify which course level (regular or honors) the student was enrolled in during the 2015-2016 academic school year.

Furthermore, there is an agreement in the literature that the three main indicators of SES tend to be parental income, parental occupation, and parental education (Gottfried, 1985; Mueller & Parcel, 1981). In addition to SES proving to relate to academic performance, it has shown to decrease the effect size of ethnic differences in
mathematics achievement when it is controlled for in the data analysis (Stevenson, Chen, & Uttal, 1990). Since the Academy does not ask parents about their income, occupation, or education, the best measure for SES given the data available was whether the family was provided financial aid. If a family was provided financial aid for their child to attend the Academy, it can be implied that the family did not have the disposable income (~$30,600) to pay for the institution’s tuition. Information on the participants’ financial aid status was provided by the Academy’s Chief Financial Officer (CFO). If students were provided financial aid, they were coded as 0 to indicate they were of lower SES; if they were not provided financial aid, they were coded as 1 to indicate they were of higher SES. Students coded for lower SES can include both low and middle income families because they did not have the disposable income to pay the tuition in full for various reasons. All students coded to be of higher SES are of high income families.

Procedure

Since this study is a secondary data analysis, de-identified, extant data was supplied to the researcher by the Academy at the end of the 2015-2016 academic school year. This timeline in Table 3.9 shows when each piece of data was collected by the Academy throughout the school year. The timeline also shows other necessary pieces of the study’s procedure including approval from the school to use the data and the time period for data analysis.

Table 3.9

<table>
<thead>
<tr>
<th>Date</th>
<th>Activity</th>
</tr>
</thead>
</table>

*Timeline for Data Collection at the Academy*
Table 3.9, continued

<table>
<thead>
<tr>
<th>Date Range</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 26th, 2016</td>
<td>The Academy’s mathematics department head introduced the students to the problem-solving project by presenting the story of Andrew Wiles</td>
</tr>
</tbody>
</table>
| January 26th - February 10th, 2016 | - The students’ mathematics teachers explained the directions of the PSP and the students began working on the project outside of class  
- The students had exactly 14 days to complete the project |
| February 11th-12th, 2016 | The mathematics teachers collected students problem solving projects and began grading |
| April 4th-14th, 2016 | - The students completed the motivation, self-regulation, and math anxiety survey in their respective math classes |
| April 25th-29th, 2016 | - The students took their placement exam in class  
- The placement exam took each student approximately 40 minutes to complete  
- Official approval from Head of School to use the data was obtained |
| June 3rd, 2016 | - IRB proposal for secondary data analysis was submitted |
| June 12th, 2016 | - Homework percentages for the academic year were obtained |
| June 12th-July 12th, 2016 | - De-identified data was supplied to the researcher by The Academy  
- Data analysis was conducted |

Data Analysis

In order to answer the research question, several statistical analyses were conducted to both individually and collectively analyze the impact of the variables. In order to determine how the numerical independent variables (computational skills, heuristics, self-reported self-regulation, homework completion, time on-task, motivation, mathematical anxiety, and demographic factors) individually related to the three outcome
variables of unconstrained, non-routine problem solving, point-biserial correlations were computed.

To analyze how the independent variables collectively relate to unconstrained non-routine problem solving, logistic regressions were conducted with three dichotomous outcomes including a) problem correctness, b) correct problem-setup, and c) problem completion. A logistic regression analysis was appropriate because numerous independent variables were used to predict dichotomous outcome variables. Problem type was controlled for prior to each point-biserial correlation and logistic regression.

Hypotheses

According to the literature, computational skills, appropriate problem solving strategies, self-regulation, motivation, anxiety, gender, and SES all individually related to mathematics achievement and problem solving in various ways. Since all of the factors have shown to individually relate to problem solving, this is most likely going to occur with this sample of students as well. However, it is unclear what factors are more predictive than others with relation to problem solving, especially when the problems are non-routine and the problem solving process does not have time constraints. Shin, Jonassen, and McGee (2002) found that conceptual knowledge and reasoning skills related to closed problem solving of 9th grade astronomy students, but additional factors such as self-regulation and affective factors showed significance during non-routine problem solving. On this premise, it is expected that self-regulation and affective factors may show to be just as significant as computational skills and appropriate strategy use with relation to non-routine problem solving success. Moreover, since the students are permitted to work on the non-routine problem for 14 days, they may not exhibit the same
type of mathematical anxiety that they face in time-constrained settings; therefore, mathematical anxiety may not show as much significance in this particular study.
CHAPTER 4

RESULTS

The results of this study are organized by first discussing the correlations between the independent variables of Schoenfeld’s mathematical problem solving framework to identify how the demographic, resource, control, and belief/affect factors related to each other for the students in this study. The second section addresses the results with regards to the research questions. When examining how the factors related to the process of solving unconstrained non-routine problems, the results first show how the factors individually correlated with the dichotomous outcomes of a) problem correctness, b) correct problem set-up, and c) problem completion by conducting point-biserial correlations. After the independent variables were individually correlated with the three outcome variables, logistic regressions were conducted with all of the independent factors put into one model. The logistic regressions were used to identify what factors show the most significance during the problem solving process.

Independent Variable Correlations

To determine how the demographic, resource, control, and belief/affect factors were inter-related, bivariate correlations were computed. As shown in Table 4.1, the analyses yielded a wide variety of significant inter-correlations. Gender had a negative correlation with both homework grade \((r = -.16, p < .05)\) and mathematical anxiety \((r = - .27, p < .01)\). Since males were dummy coded as 1 and females were coded as 0, this suggests that females tended to have a significantly higher homework grade than males; however, females were more anxious. Gender also showed to be related to both interest \((r = .14, p < .05)\) and ability beliefs \((r = .19, p < .01)\); males were more interested in
mathematics, and males had higher belief that they would succeed on mathematical tasks. These results suggest that control and belief/affect factors differ by gender.

Table 4.1

Correlations Among Independent Variables (N = 242)

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<th>10</th>
<th>11</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Demographics</td>
<td></td>
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<tr>
<td>1. Gender</td>
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<td>3. Course Level</td>
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<td>Resources</td>
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<td>4. Computation</td>
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<td>5. Heuristics</td>
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<td>.13*</td>
<td>.58**</td>
<td>.55**</td>
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<tr>
<td>6. Math Self-regulation</td>
<td>.10</td>
<td>-.13*</td>
<td>-.19**</td>
<td>.30**</td>
<td>-.07</td>
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<tr>
<td>7. Homework</td>
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<td>.24**</td>
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<td>.27**</td>
<td>-.27**</td>
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<td>8. Time</td>
<td>-.03</td>
<td>.03</td>
<td>.07</td>
<td>.14*</td>
<td>.04</td>
<td>-.14*</td>
<td>.28**</td>
<td></td>
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<td>Belief/Affects</td>
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</tr>
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<td>9. Interest</td>
<td>.14*</td>
<td>-.01</td>
<td>.22**</td>
<td>.35**</td>
<td>.19**</td>
<td>-.40**</td>
<td>.30**</td>
<td>.11</td>
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<td>10. Anxiety</td>
<td>-.27*</td>
<td>.04</td>
<td>.10</td>
<td>-.17**</td>
<td>-.13</td>
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<td>-.13*</td>
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<td>-.38**</td>
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<tr>
<td>11. Performance Goals</td>
<td>-.01</td>
<td>.09</td>
<td>.05</td>
<td>.10</td>
<td>-.07</td>
<td>.24**</td>
<td>-.16*</td>
<td>-.03</td>
<td>-.13*</td>
<td>.00</td>
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<td></td>
</tr>
<tr>
<td>12. Importance/Mastery</td>
<td>.09</td>
<td>.08</td>
<td>.17**</td>
<td>.31**</td>
<td>.16*</td>
<td>-.44**</td>
<td>.31**</td>
<td>.19**</td>
<td>.58**</td>
<td>-.17*</td>
<td>-.18*</td>
<td></td>
</tr>
<tr>
<td>13. Ability Beliefs</td>
<td>.19**</td>
<td>.12</td>
<td>.00</td>
<td>.37**</td>
<td>.29**</td>
<td>-.24**</td>
<td>.28**</td>
<td>.07</td>
<td>.39**</td>
<td>-.45**</td>
<td>-.19**</td>
<td>.33**</td>
</tr>
</tbody>
</table>

Note. * p < .05 and ** p < .01

In contrast to gender, socioeconomic status did not correlate with control or belief/affect factors, but instead correlated with the students’ resource factors. Financial aid status, the measure for socioeconomic status, had a positive correlation with both students’ computational skills (r = .13, p < .05) and heuristics or problem set-up accuracy (r = .13, p < .05). Since students who did not receive financial aid were coded as 1 and those who received financial aid were coded as 0, these results indicate that students who were of higher socioeconomic status slightly outperformed those students of lower socio-
economic status on both computational skills and problem set-up accuracy. In addition to gender and socioeconomic status, several variables correlated with course level. Results showed that students who were in the honors math class had stronger computational skills \((r = .57, p < .01)\), problem set-up accuracy \((r = .58, p < .01)\), and homework completion percentages \((r = .24, p < .01)\). Students in the honors class also showed higher levels of motivation such as interest in mathematics \((r = .22, p < .01)\) and seeing mathematics as important \((r = .17, p < .01)\).

In addition to the correlations of the demographic variables and the other factors, the resource factors have significant correlations with the control and belief/affect factors. Problem set-up accuracy (heuristics) positively correlated with homework grade \((r = .27, p < .01)\); however, it did not correlate with the math self-regulation measure nor time to complete the problem solving project. Computational skills had a positive correlation with problem-set up accuracy \((r = .55, p < .01)\), which implies that students who had stronger computational skills also had stronger problem set-up accuracy. Computational skills also positively correlated with all three control measures. Computational skills had a positive correlation with math self-regulation \((r = .30, p < .01)\) and homework completion \((r = .40, p < .01)\), which indicates that students who tended to have stronger computational skills also tended to be significantly more self-regulated. The positive correlation between computational skills and time-to-complete the non-routine, unconstrained tasks also shows that there was a relationship between spending more time on the non-routine problem and computational skill strength.

Additionally, the control factors have significant correlations with the belief/affect factors. Results indicate that homework completion correlated with time to complete the
Homework completion was positively correlated with interest \((r = .30, p < .01)\), importance and mastery \((r = .31, p < .01)\) and ability beliefs \((r = .28, p < .01)\) towards mathematics. These results show that students who tended to have higher homework completion percentages, which indicates that they were more self-regulated students, tended to have higher motivation towards mathematics. Students with higher homework grades tended to have a higher interest in mathematics, value the subject more, and have a stronger belief that they will succeed in the domain. However, performance goals were negatively correlated to homework grades \((r = -.16, p < .05)\); which suggests that students who completed their homework more often were less likely to report wanting to do better or as well as their peers. Moreover, time to complete the PSP also positively correlated with the importance and mastery measure \((r = .19, p < .01)\), which indicates that students who spent more time on the PSP tended to value mathematics more than those who spent less time.

Finally, mathematical anxiety correlated with other belief/affect factors. Anxiety had negative correlations with motivational components. Students with higher levels of mathematical anxiety had lower interest \((r = -.38, p < .01)\), importance and mastery measure \((r = -.17, p < .05)\) and ability beliefs \((r = -.45, p < .01)\) towards mathematics. These results indicate that students who were more anxious about mathematics tended to be less motivated towards the subject due to their interest, importance/mastery value, and ability beliefs. The motivational components also showed to have positive correlations to each other.

In summary, the results show that the factors in Schoenfeld’s (1992) framework have varying correlations to each other. Students of higher SES tend to have slightly
stronger resources (computational skills and problem set-up accuracy) than those of lower SES. Additionally, student resource factors show correlations with their control and belief/affect factors. In general, students with stronger computational skills and problem set-up accuracy tend to be more self-regulated, more motivated, and less anxious than those with weaker computational skills and problem set-up accuracy. Lastly, control factors and motivational factors are also correlated. Students who are more self-regulated tend to be less anxious and show higher motivational characteristics. These results have implications when examining how these factors both individually and collectively relate to the dependent variable (unconstrained, non-routine problem solving). For example, since socioeconomic status and both resource factors were correlated, students of higher socioeconomic status may have made less computational errors when solving non-routine problems. In addition, students who have stronger resource factors also showed higher levels of self-regulation and motivation; however, these factors may have affected the unconstrained non-routine problem solving process differently. For instance, motivation and self-regulation may have only affected problem persistence, while problem correctness was predicted by the resource factors.

Analyses Pertinent to Research Question

To examine how the demographic, resources, control, and belief/affect factors both individually and collectively related to unconstrained, non-routine problem solving, point-biserial correlations and logistic regressions were conducted with three different dependent variables: (1) problem correctness, (2) correct problem set-up, and (3) problem completion. Point-biserial correlations and logistic regressions were suitable for these analyses because the three outcome variables were dichotomous.
Before conducting the logistic regressions, it was important to control for problem chosen. As mentioned in Chapter 3, the design of the project allowed the students to choose one of three problems to work on over the course of 14 days. Since there were three problems per grade level, there was a total of nine possible problem types. Due to the fact that each problem required the students to draw on slightly different resources, the problem chosen will be controlled for during each analysis.

**Problem Correctness**

In order to determine what factors related to problem solving success on unconstrained non-routine tasks, it is important to differentiate between those students who solved the problem correctly versus those who did not. As mentioned in Chapter 3, students received a score of 1 for a correct answer and a score of 0 for an incorrect answer.

Since there were nine different problem choices, three from each grade level, Table 4.2 shows the amount of students who chose that particular problem and the percentage of students who solved the problem correctly. From the percentages of problem correctness based on problem type, students who chose Martian Money had the highest probability of solving the problem correctly and students who chose the Cake problem had the lowest probability. Due to these results, problem chosen was controlled for in the latter analyses in order to provide more accurate results with respect to the independent variables.

Table 4.2

<table>
<thead>
<tr>
<th>Problem Type</th>
<th># of Students</th>
<th>% Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
After controlling for problem chosen, a point-biserial correlation was conducted to determine how each factor individually correlated with problem correctness. These results are displayed in Table 4.3. The results showed several significant correlations. Both resource factors (computational skills and heuristics) were significantly correlated with problem correctness. This indicates that students who solved the problem correctly instead of incorrectly tended to have significantly stronger computational skills \( (r_{pb} = .36, p < .01) \) and were stronger at setting-up mathematics problems \( (r_{pb} = .27, p < .01) \). The only control factor that was significantly correlated with problem correctness was homework completion \( (r_{pb} = .19, p < .01) \). This shows that students who solved the problem correctly instead of incorrectly tended to have higher homework percentages which implies that those students who solved the problem correctly tended to be more self-regulated, as measured by homework. Results also showed a positive correlation between ability beliefs and problem correctness \( (r_{pb} = .20, p < .01) \); which indicates that
students who solved the problem correctly tended to have higher beliefs in their mathematical ability. Lastly, there was a positive correlation between financial aid (measure for SES) and problem correctness ($r_{pb} = .15, p < .05$), which suggests that students who solved the non-routine problems correctly tended to be of higher SES.

Table 4.3

*Point-Biserial Correlations with Independent Variables and Problem Correctness ($N = 242$)*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Problem Correctness</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>.12</td>
</tr>
<tr>
<td>Financial Aid</td>
<td>.15*</td>
</tr>
<tr>
<td><strong>Resources</strong></td>
<td></td>
</tr>
<tr>
<td>Computation</td>
<td>.36**</td>
</tr>
<tr>
<td>Heuristics</td>
<td>.27**</td>
</tr>
<tr>
<td><strong>Controls</strong></td>
<td></td>
</tr>
<tr>
<td>Math Self-Reg</td>
<td>-.07</td>
</tr>
<tr>
<td>Homework</td>
<td>.19**</td>
</tr>
<tr>
<td>Time</td>
<td>.08</td>
</tr>
<tr>
<td><strong>Belief/Affects</strong></td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>.07</td>
</tr>
<tr>
<td>Anxiety</td>
<td>.04</td>
</tr>
<tr>
<td>Performance Goals</td>
<td>.03</td>
</tr>
<tr>
<td>Importance/Mastery</td>
<td>.15*</td>
</tr>
<tr>
<td>Ability Beliefs</td>
<td>.20**</td>
</tr>
</tbody>
</table>

*Note.* *p*.05 and **p*.01

To determine how the factors collectively related to problem correctness, a logistic regression was conducted with all of the factors included in the model while controlling for problem chosen. The logistic regression was conducted with the demographic factors (gender, SES, and course level), resource factors (computational skills and problem-set up accuracy), control factors (math self-regulation, homework
completion, and time to complete), and belief/affect factors (anxiety and motivation components) as the independent variables and problem correctness as the outcome variable. Table 4.4 shows the results.

Table 4.4

<table>
<thead>
<tr>
<th>Variable</th>
<th>B</th>
<th>SE</th>
<th>Wald</th>
<th>OR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>0.99</td>
<td>0.42</td>
<td>5.66</td>
<td>2.76</td>
</tr>
<tr>
<td>Fin. Aid (SES)</td>
<td>0.78</td>
<td>0.49</td>
<td>2.60</td>
<td>2.08</td>
</tr>
<tr>
<td>Course Level</td>
<td>2.16</td>
<td>0.65</td>
<td>11.03**</td>
<td>8.64</td>
</tr>
<tr>
<td><strong>Resources</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computational Skills</td>
<td>0.92</td>
<td>0.27</td>
<td>11.66**</td>
<td>2.50</td>
</tr>
<tr>
<td>Heuristics (Set-up)</td>
<td>0.27</td>
<td>0.25</td>
<td>1.18</td>
<td>1.31</td>
</tr>
<tr>
<td><strong>Controls</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homework Grade</td>
<td>0.04</td>
<td>0.04</td>
<td>1.35</td>
<td>1.04</td>
</tr>
<tr>
<td>Time to Complete</td>
<td>0.00</td>
<td>0.01</td>
<td>.28</td>
<td>1.00</td>
</tr>
<tr>
<td>Math Self-regulation</td>
<td>0.03</td>
<td>0.27</td>
<td>.01</td>
<td>1.03</td>
</tr>
<tr>
<td><strong>Belief/Affects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>-0.38</td>
<td>0.29</td>
<td>1.68</td>
<td>0.60</td>
</tr>
<tr>
<td>Anxiety</td>
<td>0.70</td>
<td>0.27</td>
<td>6.71**</td>
<td>2.02</td>
</tr>
<tr>
<td>Performance Goals</td>
<td>0.43</td>
<td>0.22</td>
<td>3.85</td>
<td>1.54</td>
</tr>
<tr>
<td>Importance/Mastery</td>
<td>0.15</td>
<td>0.28</td>
<td>0.28</td>
<td>1.16</td>
</tr>
<tr>
<td>Ability Beliefs</td>
<td>0.40</td>
<td>0.25</td>
<td>2.45</td>
<td>1.49</td>
</tr>
<tr>
<td>Constant</td>
<td>-7.15</td>
<td>3.52</td>
<td>4.11</td>
<td>0.00</td>
</tr>
</tbody>
</table>

*Note. OR = Odds Ratio
*p < .05.  **p < .01.

Results showed that the logistic regression model was statistically significant, ($\chi^2(20) = 111.073, p < .001$), Nagelkerke $R^2 = .52$. The equation correctly classified 83%
of cases. From the model that analyzed all of the factors collectively, there were three significant predictors. Computational skills was a significant predictor of problem correctness ($B = 0.92$, OR = 2.50, $p < .01$); however when taking the other factors into account, problem-set up accuracy (heuristics) was not significant. The association between computational skills and correctly solving the unconstrained, non-routine problem indicated that students with stronger computational skills were 150% more likely to get the problem correct. In other words, those with higher computational skills are 2.5 times more likely to solve the problem correctly than those students with lower computational skills. Moreover, students’ course level, which indicated if the student was enrolled in the regular or honors math course also showed to be a significant predictor ($B = 2.16$, OR = 8.64, $p < .01$). This result indicates that students who were in the honors math class were over 8 times as likely than students in the regular math class to solve the problem correctly.

Mathematical anxiety was also a positive significant predictor of problem correctness ($B = 0.70$, OR = 2.02, $p < .01$) when collectively examining all of the variables; which means that students who were generally more anxious about mathematics were more likely to correctly solve the unconstrained, non-routine problem. The relationship between mathematical anxiety and correctly solving the unconstrained, non-routine problem implies that students with higher anxiety skills were 102% more likely to get the problem correct. In other words, those with higher anxiety were 2.02 times more likely to solve the problem correctly than those students who reported lower levels of anxiety.
The point-biseral correlation results imply that when individually examining what factors relate to problem correctness, computational skills, heuristics, homework grade, and ability beliefs showed positive correlations to varying degrees. There was also a positive correlation between socioeconomic status and importance/mastery with problem correctness. However, when examining the factors collectively, the factors significantly contributing variance in predicting problem correctness were limited. The logistic regression model showed that course level, computational skills, and mathematical anxiety were positive predictors of problem correctness.

_Correct Problem Set-up_

In order to identify what factors related to correct problem set-up on unconstrained non-routine tasks, it is important to differentiate between those students who set-up the problem correctly verses those who did not. As discussed in Chapter 3, students received a score of 1 for a correct problem set-up and a score of 0 for an incorrect set-up.

As mentioned when discussing problem correctness, there were nine different problem choices, three from each grade level; therefore, problem chosen was used as a control variable for the analysis. Table 4.5 shows the amount of students who chose that problem and the percentage of students who correctly set-up that particular problem. Due to these results, problem chosen was controlled for in the latter analyses in order to provide more accurate results with respect to the demographic, resource, control and belief/affect variables.

Table 4.5
_Correct Set-up Percentage Based on Problem Type  (N=242)_
Table 4.5, continued

<table>
<thead>
<tr>
<th>Problem Type</th>
<th># of Students</th>
<th>% Correct Set-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast Draw</td>
<td>37</td>
<td>86%</td>
</tr>
<tr>
<td>Four Four’s</td>
<td>28</td>
<td>89%</td>
</tr>
<tr>
<td>Goldbach’s</td>
<td>9</td>
<td>77%</td>
</tr>
<tr>
<td>Diagonals</td>
<td>38</td>
<td>89%</td>
</tr>
<tr>
<td>Martian Money</td>
<td>21</td>
<td>90%</td>
</tr>
<tr>
<td>Red Paint</td>
<td>30</td>
<td>93%</td>
</tr>
<tr>
<td>Cake</td>
<td>41</td>
<td>66%</td>
</tr>
<tr>
<td>Tiles</td>
<td>21</td>
<td>95%</td>
</tr>
<tr>
<td>Hiking</td>
<td>27</td>
<td>88%</td>
</tr>
</tbody>
</table>

After controlling for problem chosen, a point-biserial correlation was conducted to determine how each factor individually correlated with correct problem set-up. These results are displayed in Table 4.6. The results showed several significant correlations. Both resource factors (computational skills and heuristics) were significantly correlated with correct problem set-up. Similarly to problem correctness, this indicates that students who correctly set-up the problem instead of incorrectly tend to have significantly stronger computational skills \((r_{pb} = .15, p < .05)\) and were generally stronger at setting-up mathematics problems \((r_{pb} = .15, p < .05)\). The only control factor that significantly correlated with correct problem set-up was homework completion \((r_{pb} = .23, p < .01)\). This shows that students who set-up the problem correctly tended to have higher homework percentages which implies that those students who set-up the problem correctly tended to be more self-regulated. Results also showed a positive correlation
between ability beliefs and correct problem set-up ($r_{pb} = .15, p < .05$); which indicates that students who set-up the problem correctly tended to have higher beliefs in their mathematical ability. There also was a positive correlation between importance/mastery and correct problem set-up ($r_{pb} = .15, p < .05$). This indicates that students who solved the problem correctly may value mathematics more than students who solved the problem incorrectly. There was also a correlation between financial aid (measure for SES) and correct problem set-up ($r_{pb} = .17, p < .05$), which suggests that students who set-up the non-routine problems correctly were of higher SES. Lastly, in contrast to problem correctness, there was a significant correlation between mathematical interest and correct problem set-up ($r_{pb} = .15, p < .05$). This result indicates that students who set-up the problem correctly may have a higher interest in mathematics compared to the students who did not.

Table 4.6

*Point-Biserial Correlations with Independent Variables and Correct Problem Set-up (N = 242)*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Problem Set-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demographics</td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>-.04</td>
</tr>
<tr>
<td>Financial Aid</td>
<td>.17*</td>
</tr>
<tr>
<td>Resources</td>
<td></td>
</tr>
<tr>
<td>Computation</td>
<td>.15*</td>
</tr>
<tr>
<td>Heuristics</td>
<td>.15*</td>
</tr>
<tr>
<td>Controls</td>
<td></td>
</tr>
<tr>
<td>Math Self-regulation</td>
<td>-.03</td>
</tr>
<tr>
<td>Homework</td>
<td>.23**</td>
</tr>
<tr>
<td>Time</td>
<td>.08</td>
</tr>
<tr>
<td>Belief/Affects</td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>.15*</td>
</tr>
<tr>
<td>Anxiety</td>
<td>-.14*</td>
</tr>
</tbody>
</table>
Table 4.6, continued

<table>
<thead>
<tr>
<th>Performance Goals</th>
<th>-.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Importance/Mastery</td>
<td>.15*</td>
</tr>
<tr>
<td>Ability Beliefs</td>
<td>.15*</td>
</tr>
</tbody>
</table>

*Note. *p < .05 and **p < .01

Similarly to problem correctness, it is important to understand how the factors collectively work together to predict correct problem set-up. Another logistic regression was conducted with all of the factors included in the model, while controlling for problem chosen with correct problem set-up was the outcome variable. Table 4.7 shows the results of this analysis.

Table 4.7

*Logistic Regression Analysis for Variables Predicting Correct Problem Set-up (N=242)*

<table>
<thead>
<tr>
<th>Variable</th>
<th>B</th>
<th>SE</th>
<th>Wald</th>
<th>OR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>.06</td>
<td>0.59</td>
<td>.01</td>
<td>1.06</td>
</tr>
<tr>
<td>Fin. Aid (SES)</td>
<td>1.73</td>
<td>0.61</td>
<td>8.04**</td>
<td>5.65</td>
</tr>
<tr>
<td>Course Level</td>
<td>1.59</td>
<td>1.43</td>
<td>1.24</td>
<td>4.89</td>
</tr>
<tr>
<td><strong>Resources</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computational Skills</td>
<td>-.07</td>
<td>0.37</td>
<td>.03</td>
<td>0.94</td>
</tr>
<tr>
<td>Heuristics (Set-up)</td>
<td>0.28</td>
<td>0.387</td>
<td>.61</td>
<td>1.33</td>
</tr>
<tr>
<td><strong>Controls</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homework Grade</td>
<td>0.09</td>
<td>0.05</td>
<td>3.99*</td>
<td>1.07</td>
</tr>
<tr>
<td>Time to Complete</td>
<td>0.02</td>
<td>0.01</td>
<td>3.17</td>
<td>1.02</td>
</tr>
<tr>
<td>Math Self-regulation</td>
<td>0.62</td>
<td>0.36</td>
<td>2.91</td>
<td>1.87</td>
</tr>
<tr>
<td><strong>Belief/Affects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>0.61</td>
<td>0.39</td>
<td>2.37</td>
<td>1.83</td>
</tr>
<tr>
<td>Anxiety</td>
<td>-0.07</td>
<td>0.41</td>
<td>0.03</td>
<td>0.93</td>
</tr>
</tbody>
</table>
Results showed that the logistic regression model was statistically significant, ($\chi^2(20) = 67.950, p < .001$), Nagelkerke $R^2 = .50$. The equation correctly classified 90.5% of cases. From the model that analyzed all of the factors collectively, there were three significant predictors. The three predictive factors in the correct problem set-up model differed from those of the problem correct model. In contrast to the logistic regression on solving the problem correctly, both resource factors did not show any significance when predicting correct problem set-up; however, one control factors showed significance. Homework grade was a positive significant positive predictor of correct problem set-up ($B = 0.09$, OR = 1.07, $p < .05$). This indicates that students who were more self-regulated, as measured by homework completion, were 7% more likely to correctly set-up the non-routine problem. The belief/affect factor that showed significance in the correct problem set-up model was ability beliefs ($B = 0.41$, OR = 1.51, $p < .05$) and financial aid status ($B = 1.73$, OR = 5.65, $p < .01$). This implies that students who had higher ability beliefs towards mathematics were approximately 1.51 times more likely to set-up the problem correctly than those with lower levels of ability beliefs. In the problem correctness model, financial aid status did not show significance; however, when predicting correct problem set-up financial aid status positively predicted setting-up the problem correctly.
This result indicates that students who were of higher socio-economic status were more than 5 times as likely to appropriately set-up the unconstrained, non-routine task.

With regards to correct problem set-up, these results suggest that when individually examining what factors relate to correct problem set-up, financial aid status (SES), computational skills, heuristics, homework grade, interest, importance/mastery, and ability beliefs showed positive correlations to varying degrees. However, when examining the factors collectively, the significant factors on correct problem set-up tend to differ. Neither resource factors contribute significant variance when other factors are considered, but homework grade and ability beliefs showed to be significant predictors, as well as financial aid status (SES).

Problem Completion

To establish what factors relate to problem completion on unconstrained non-routine tasks, it is important to differentiate between those students who completed the problem verses those who did not. As discussed in Chapter 3, students received a score of 1 if they completed the problem either correctly or incorrectly and a 0 if they did not complete the problem. Problem completeness showed student persistence to solve an unconstrained non-routine task.

As mentioned when reporting the results on both problem correctness and correct problem set-up, there were nine different problem choices, three from each grade level. As it was done with problem correctness and correct problem set-up, problem chosen was used as a control variable. Table 4.8 shows the amount of students who chose that problem and the percentage of students who completed that problem. Due to these
results, problem chosen was controlled for in the latter analyses in order to provide more accurate results with respect to the independent variables.

Table 4.8

*Completion Percentage Based on Problem Type (N=242)*

<table>
<thead>
<tr>
<th>Problem Type</th>
<th># of Students</th>
<th>% Completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast Draw</td>
<td>37</td>
<td>76%</td>
</tr>
<tr>
<td>Four Four’s</td>
<td>28</td>
<td>64%</td>
</tr>
<tr>
<td>Goldbach’s</td>
<td>9</td>
<td>89%</td>
</tr>
<tr>
<td>Diagonals</td>
<td>38</td>
<td>84%</td>
</tr>
<tr>
<td>Martian Money</td>
<td>21</td>
<td>90%</td>
</tr>
<tr>
<td>Red Paint</td>
<td>30</td>
<td>87%</td>
</tr>
<tr>
<td>Cake</td>
<td>41</td>
<td>71%</td>
</tr>
<tr>
<td>Tiles</td>
<td>21</td>
<td>100%</td>
</tr>
<tr>
<td>Hiking</td>
<td>27</td>
<td>94%</td>
</tr>
</tbody>
</table>

After controlling for problem chosen, a point-biserial correlation was conducted to determine how each factor individually correlated with problem completion. These results are displayed in Table 4.9. The point-biserial correlations showed several significant results. Both resource factors (computational skills and heuristics) were significantly correlated with problem completion. Similarly to problem correctness and correct problem set-up, this indicates that students who completed the problem tended to have significantly stronger computational skills ($r_{pb} = .22, p < .01$) and were generally stronger at setting-up mathematics problems ($r_{pb} = .19, p < .01$). In addition to the resource factors, homework completion had a positive correlation with problem
completeness ($r_{pb} = .32, p < .01$). This shows that students who completed the problem tended to have higher homework percentages which implies that those students who completed the problem were more self-regulated. Results also showed a positive correlation between interest ($r_{pb} = .19, p < .01$), importance/mastery ($r_{pb} = .23, p < .01$), and ability beliefs ($r_{pb} = .26, p < .01$) with problem completeness. These results indicated that students who completed the non-routine problem showed higher levels of motivation. Lastly, similarly to both problem correctness and correct problem set-up, financial aid status (SES) was positively correlated with problem completeness ($r_{pb} = .22, p < .01$) such that students who completed the problem were of higher socioeconomic status.

Table 4.9

*Point-Biserial Correlations with Independent Variables and Problem Completion (N = 242)*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Problem Completion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>-.01</td>
</tr>
<tr>
<td>Financial Aid</td>
<td>.22**</td>
</tr>
<tr>
<td><strong>Resources</strong></td>
<td></td>
</tr>
<tr>
<td>Computation</td>
<td>.22**</td>
</tr>
<tr>
<td>Heuristics</td>
<td>.19**</td>
</tr>
<tr>
<td><strong>Controls</strong></td>
<td></td>
</tr>
<tr>
<td>Math Self-Regulation</td>
<td>-.18**</td>
</tr>
<tr>
<td>Homework</td>
<td>.32**</td>
</tr>
<tr>
<td>Time</td>
<td>.12</td>
</tr>
<tr>
<td><strong>Belief/Affects</strong></td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>.19**</td>
</tr>
<tr>
<td>Anxiety</td>
<td>-.14*</td>
</tr>
<tr>
<td>Performance Goals</td>
<td>-.09</td>
</tr>
<tr>
<td>Importance/Mastery</td>
<td>.23**</td>
</tr>
<tr>
<td>Ability Beliefs</td>
<td>.23**</td>
</tr>
</tbody>
</table>

*Note.* *p* < .05 and ** *p* < .01
Similarly to problem correctness and correct problem set-up, a logistic regression was conducted with all of the factors included in the model, while controlling for problem chosen with problem completion as the outcome variable. Table 4.10 shows the results of this analysis.

Table 4.10

*Logistic Regression Analysis for Variables Predicting Problem Completion (N=242)*

<table>
<thead>
<tr>
<th>Variable</th>
<th>B</th>
<th>SE</th>
<th>Wald</th>
<th>OR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>.10</td>
<td>.45</td>
<td>.05</td>
<td>1.11</td>
</tr>
<tr>
<td>Fin. Aid (SES)</td>
<td>1.27</td>
<td>.43</td>
<td>8.83**</td>
<td>3.56</td>
</tr>
<tr>
<td>Course Level</td>
<td>1.35</td>
<td>.84</td>
<td>2.56</td>
<td>3.84</td>
</tr>
<tr>
<td><strong>Resources</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computational Skills</td>
<td>-.14</td>
<td>.27</td>
<td>.26</td>
<td>0.87</td>
</tr>
<tr>
<td>Heuristics (Set-up)</td>
<td>0.44</td>
<td>.27</td>
<td>2.69</td>
<td>1.56</td>
</tr>
<tr>
<td><strong>Controls</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homework Grade</td>
<td>0.11</td>
<td>.04</td>
<td>9.36**</td>
<td>1.12</td>
</tr>
<tr>
<td>Time to Complete</td>
<td>0.01</td>
<td>.01</td>
<td>2.00</td>
<td>1.01</td>
</tr>
<tr>
<td>Math Self-Regulation</td>
<td>-0.09</td>
<td>.25</td>
<td>0.12</td>
<td>0.92</td>
</tr>
<tr>
<td><strong>Belief/Affects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>0.08</td>
<td>.28</td>
<td>0.09</td>
<td>1.09</td>
</tr>
<tr>
<td>Anxiety</td>
<td>-0.02</td>
<td>.28</td>
<td>0.01</td>
<td>0.98</td>
</tr>
<tr>
<td>Performance Goals</td>
<td>0.23</td>
<td>.27</td>
<td>0.76</td>
<td>1.26</td>
</tr>
<tr>
<td>Importance/Mastery</td>
<td>0.46</td>
<td>.29</td>
<td>2.52</td>
<td>1.58</td>
</tr>
<tr>
<td>Ability Beliefs</td>
<td>0.19</td>
<td>.26</td>
<td>0.57</td>
<td>1.21</td>
</tr>
<tr>
<td>Constant</td>
<td>-8.99</td>
<td>3.54</td>
<td>6.45</td>
<td>0.00</td>
</tr>
</tbody>
</table>

*Note. OR = Odds Ratio*

*p < .05. **p < .01.*
In the logistic regression model, homework completion \((B = 0.11, \ OR = 1.12, \ p < .01)\) and financial aid status \((B = 1.27, \ OR = 3.56, \ p < .01)\) showed to be significant predictors. The analysis shows that students who have a higher homework completion are 12\% more likely to complete the problem and students of higher socioeconomic status are approximately 3.56 times more likely to complete the problem.

The results from the bivariate correlations, point-biserial correlations, and logistic regressions indicated that the demographic, resource, control, and belief/affect factors have both individual and collective relationships to unconstrained non-routine problem solving. Chapter 5 discusses these results and how the results vary based on problem correctness, correct problem set-up, and problem completion. Chapter 5 also discusses the implications of these results and suggestions for further research.
CHAPTER 5
DISCUSSION

As history in the United States shows, problem solving has been a main focus in mathematics education for the last several decades (NCTM, 1980, 1989, 2000, 2010; Schoenfeld, 1992, 2007). Mathematics educators and policy makers strive to make children and adults alike better problem solvers; however, the problem solving process is complex. There is not simply one factor that predicts how students problem solve, but several characteristics that come together to shape an individual’s mathematical behavior. Much of the research on mathematical problem solving, in particular, examines one particular factor in depth, but there are seldom studies that explore several factors collectively. Drawing from Information Processing Theory and the work of George Pólya (1945), Alan Schoenfeld (1985, 1992) devised a framework for examining mathematical behavior to determine what people know and what people do as they work on mathematical problems. Schoenfeld (1985, 1992) designed his framework to best predict what takes place as a person attempts to find the solution to a mathematical problem. He argued that there are three categories of behavior and knowledge to best explain the problem solving process, which include: (a) Resources (the knowledge base), (b) Control, and (c) Beliefs and Affects. While research on problem solving takes place in different contexts with varying domains, the main goal of the current study was to identify the factors that related to unconstrained non-routine mathematical problem solving of middle school students. Chapter 2 discussed the literature based on Schoenfeld’s framework by analyzing the studies that focused on resource, control, and belief/affect factors and Chapter 4 presented the results on how those factors related to problem correctness, correct problem set-up, and problem completion on unconstrained
non-routine tasks. To mirror the framework of this paper, Chapter 5 is presented in a similar fashion. This chapter discusses how each category of Schoenfeld’s framework related to the outcomes of unconstrained non-routine problem solving followed by the limitations of the study, implications, and suggestions for further research.

The research questions were as follows: What resource (computational skills and heuristics), control (self-regulation), and belief/affect factors (motivation, mathematical anxiety, demographics) individually and collectively relate to unconstrained non-routine mathematical problem solving success of middle school students? This chapter discusses the results of the point-biserial correlations to explain how the factors individually related to the outcome variables and the logistics regressions to examine how the factors collectively related to problem correctness, correct problem set-up, and problem completion. Note that the discussions are all focused on the results after the analyses controlled for problem type.

Resources

In the literature, computational skills and heuristics have shown to affect both mathematics achievement and mathematical problems solving (Canobi, 2004, 2005; Canobi & Pattison, 2003; Hecht 1998; Huntly & Davis, 2008; Lannin, Barker, & Townsend, 2006; Montague, 1992; Pólya, 1945; Scheonfeld 1985, 2012). The main focus of research on computational skills and heuristics in mathematical learning has centered on various mathematical topics using routine problem solving tasks; however, this study examined how resource factors related to non-routine problem solving during unconstrained tasks. The results from this study suggest that the resource factors related to problem solving in various ways during the problem solving process. Correlations
showed that students who had stronger computational skills also had stronger problem set-up accuracy; however, both factors did not affect unconstrained non-routine problem solving in the same way. This section discusses how the resource factors related to and predicted problem correctness, correct problem set-up, and problem completion.

The student resource factors showed to have the most significance when it came to correctly solving the problem. The correlations showed computational skills and heuristics both significantly correlated with problem correctness. This indicates that there is a relationship between students who have stronger resource factors and students who solved the non-routine problem correctly. However, the logistic regression indicated that only computational skills were predictive of problem correctness. Students with stronger computational skills were more than twice as likely to solve the problem correctly. This study indicates that students with weaker computational skills on timed tasks, as measured by the placement exam, are still underperforming when they are given an unconstrained amount of time to solve the problem. As previous research shows, resource factors may influence problem solving success in various ways when individuals are put under different time constraints (Cifarelli & Cai, 2005; Montague & Bos, 1990). Even when given 14 days to solve a problem, students with weaker computational skills were significantly less likely to solve the problem correctly. This suggests that middle school students need to gain a stronger foundation of computational skills in order to not only successfully solve routine problems, but non-routine problems alike. Falkner, Levi, and Carpenter’s (1999) study showed that computational skills in young children may have an impact on algebraic achievement. This study adds to this literature by showing
that computational skills are, in fact, affecting Pre-algebra and Algebra students’ ability to correctly solve unconstrained non-routine problems.

However, with regards to correct problem set-up and problem completion, computational skills and heuristic measures did not show significance. In both the correct problem set-up and problem completion logistic regression models, both computational skills and heuristics were not significant predictors. These results imply that resource factors do not show significant importance when setting up and completing the non-routine problem, but instead, show significance when the computational skills must be executed during the problem solving process to obtain a correct answer.

Controls

To appropriately implement the resource factors (computational skills and problem solving strategies), Schoenfeld (1985, 1992) argues the importance of control or often called metacognition in the psychological literature. Being able to control one’s thoughts and actions during the problem solving process is impacted by an individual’s ability to self-regulate. For this study, there were three measures for self-regulation that included questions on a self-reported survey, students’ homework completion percentage, and the time it took to complete the unconstrained non-routine task. Correlations showed that the control measures in this study were correlated with each other, which means students who reported to be more self-regulated when it came to their mathematical studies tended to have significantly higher homework percentages and spent significantly more time on the non-routine task; however, the control measures affected non-routine problem solving outcomes in various ways.
Results showed that the only control factor individually correlated with problem correctness was the students’ homework completion grade. The correlation indicated that higher homework percentages related to problem correctness, but it was not a predictor. In contrast to problem correctness, homework completion significantly correlated with and predicted both correct problem set-up and problem completion. This indicates that students who tended to be more self-regulated when it came to their other assignments (i.e., homework) outside of the classroom, were more likely to correctly set-up the problem. Results also indicated that students who completed their homework more regularly were more likely to complete the non-routine task. The literature with regards to self-regulation and mathematics achievement suggests that students who are more self-regulated tend to perform better on mathematical tasks (Matthews, Ponitz, & Morrison, 2009; Montague, 1992; Shin, Jonassen, & McGee, 2002; Thronson, 2010); however homework completion does not always predict achievement (Cooper, Robinson & Patall, 2006). This study adds to that literature by showing that homework does not predict problem correctness, but instead problem persistence.

Generally in U.S. classrooms, homework has been a way for teachers to enhance student learning and achievement (Ramdass & Zimmerman, 2011); however, this study along with other literature (Pintrich, 2000; Trautwein & Köller, 2003) suggests that homework can affect more than mathematical understanding. While some researchers argue that homework has no effect on promoting study skills, independence, and self-discipline (Kohn, 2007; Kralovec & Buell, 2005), others suggest that homework improves self-regulation skills, time management, and self-efficacy (Pintrich, 2000; Trautwein & Köller, 2003). This study adds to that literature by showing that homework
completion does not relate to overall achievement on problem solving tasks, but can predict student persistence to complete long-term problem solving projects.

Furthermore, results indicated that there was no relationship between time spent on the non-routine tasks with problem correctness, correct problem set-up, or problem completion. This suggests that students may have taken various amounts of time to solve the problem based on their mathematical ability. Prior research suggests that time spent on task does not always predict achievement (Cooper, Robinson, & Patall, 2006). Since the PSP was completed outside of the classroom, students may have used varying amounts of time to solve the non-routine problem due to environmental factors such as distractions in study hall or distractions at home. Some students may have also taken the extra time to check their work, while others may not. The math self-regulation items on the questionnaire also showed no relationship with unconstrained non-routine problem solving. This may have occurred because students who reported higher levels of self-regulation towards their math class and their mathematical learning may not have taken the time to check their work on the problem solving project. Students also may have reported higher levels of self-regulation to simply please their teacher.

In summary, homework completion was a significant factor in predicting both correct problem set-up and problem completion. Students who had higher homework percentages were more likely to show persistence with regards to unconstrained non-routine problem solving. However, results indicate that when it comes to correctly solving the problem, it is not homework completion that is a predictor, but instead one’s computational skills.
Belief/Affects

As described in Chapter 2, beliefs and affects are an individual’s mathematical perspective or worldview (Schoenfeld, 1992). An individual’s beliefs and affects towards themselves, their mathematical experiences, and their environment can shape the way their resources and control factors work together, which in turn affects one’s mathematical behavior. Research shows that these factors which include mathematics anxiety and motivational factors such as ability beliefs, interest, performance goals, and importance/mastery have shown to make an impact on mathematics achievement (Andrews and Brown, 2015; McLeod and Adams’, 1989; Panaoura, 2013; Schoenfeld, 1992; Schommer-Aikins, Duell, and Hutter, 2005). Gender and socioeconomic status are also important factors to consider because it can affect an individual’s beliefs and affects towards mathematics (Barnes, 2000; Boaler, 2002). The current study examined how these factors both individually and collectively related to outcomes of unconstrained non-routine problem solving.

As mentioned in Chapter 3, this study measured the students’ belief and affect factors using a self-reported questionnaire. Demographic factors including socioeconomic status, which was measured by financial aid information, gender, and course level were also examined. The belief and affect factors were significantly correlated with each other along with the control and resource factors. There tended to be a relationship with one’s gender and homework, mathematical anxiety, and ability beliefs. Females tended to complete their homework significantly more than males and showed to be significantly more anxious; however, males tended to show stronger ability beliefs. Financial aid was also correlated with both resource factors, but the effect level
was low. Results also indicated that the students’ course level significantly correlated with several of the other variables and predicted problem correctness. Students that were enrolled in the honors math class showed to have significantly stronger resource factors, completed their homework more often, and reported to be more interested in mathematics. Students in the honors classes were also more than 8 times as likely to solve the non-routine problem correctly. These results align with the previous literature such that demographic factors have an effect on resources (Coe, Peterson, Blair, Schutten, & Peddie, 2013; College Board, 2007; Fennema, Carpenter, & Jacobs, 1998; Hyde et al., 1990, Maccoby & Jacklin, 1974), controls (Cadigan, Wei, & Clifton, 2013), motivation (Barnes, 2000; Fennema & Sherman, 1978; Shaw & Barbuti, 2010), and anxiety (Barnes, 2000; Cadigan, Wei, & Clifton, 2013).

Motivational components also tended to be correlated with each other and to resource and control factors. Interest, importance/mastery, and ability beliefs showed positive correlations with both computational skills and heuristics which indicated that students who reported to be more motivated towards mathematics have significantly stronger resource skills. The motivational components also positively correlated with self-regulation and homework grade. Mathematical anxiety negatively correlated with motivational components and both resource factors. These results suggest that students who were more anxious tended to have weaker computational skills and problem set-up accuracy on the placement exams and tended to be less motivated. From the results, it is clear that the belief/affect factors correlated with each other as well as resource and control factors; however, they affected the unconstrained non-routine problem solving outcomes in different ways.
In relation to problem correctness, the belief/affect factors that showed to individually correlate were socioeconomic status, importance/mastery and ability beliefs. These three factors positively correlated with problem correctness, but the only correlation that fell below the .01 significance level was ability beliefs. When all of the factors were entered into the logistic regression model for problem correctness, mathematical anxiety was the only significant predictors. In contrast to the literature (Andrews & Brown, 2015; Ashcraft & Krause, 2007; Guven & Cabakcor, 2013; Hoffman, 2010; Kaufmann & Vosburg.1997; Kyttälä, M., & Björn, 2014; Vukovic et al., 2012), results showed that mathematical anxiety was a significant positive predictor for problem correctness, but not for correct problem set-up or problem completion. This is quite different from results from previous research, where anxiety had a negative effect on individuals’ achievement during timed tasks (Miller & Mitchell, 1994). In fact, timed tasks have shown to negatively affect students’ attitudes towards mathematics starting at early ages (Popham, 2008; Scarpello, 2007; Thilmany, 2004; Tsui & Mazzocco, 2007). In general, mathematics anxiety does not come from the subject of mathematics itself, but instead the way the subject is presented to children (Stuart, 2000). Results from previous research and this current study suggest that mathematics anxiety may not have a negative impact on children’s problem solving ability when children are permitted to solve problems during unconstrained periods of time. During this study, it is possible that students with higher levels of anxiety went back to check their work on the non-routine problem more often because they were permitted the time to do so. Since they were given an unconstrained amount of time to solve the problem, students with higher levels of anxiety could have been more likely to catch computational errors. Research shows
that anxious children generally have to exert more effort to perform well on timed mathematical tasks because they are trying to manage their anxiety during the problem solving process (Owens et al., 2008); however, this study shows that anxiety can have a positive effect if students are working on tasks without time limits.

In addition to mathematical anxiety, socioeconomic status, as measured by financial aid status, was a significant predictor of correct problem set-up and problem completion; however it was not a significant predictor of problem correctness. These results indicate that students of higher socioeconomic status were significantly more likely to set-up the problem correctly and to complete the problem. These results are in line with previous research (Coe, Peterson, Blair, Schutten, & Peddie, 2013; Haveman & Wolfe, 1995; Klebanov, Brooks-Gunn, & Duncan, 1994; Sirin, 2005; Smith, Brooks-Gunn, & Klebanov, 1997) because it can be implied that students who are not correctly setting-up the problem or not completing the problem are underachieving during problem solving tasks.

Predicting Success at Different Stages in Unconstrained Non-routine Problem Solving

From these results, there were significant patterns that relate to problem solving during unconstrained non-routine tasks that differed during each stage of the problem solving process. This section will explain the patterns found at each state of the problem solving process and why particular factors are important during each respective stage.

**Problem Correctness**

In order to ultimately solve the problem correctly, computational skills, mathematical anxiety, and course were significant predictors. In addition to computational skills predicting routine problem solving ability (Canobi, 2004, 2005;
Canobi & Pattison, 2003; Hecht 1998), this study implies that computational skills are also predictors of correctly solving non-routine problem solving tasks. Computational skills are important at this stage of the problem solving process because even if a student correctly sets up and/or completes the problem, s/he may still make computational errors that restrain the student from solving the problem correctly. As Falkner, Levi, and Carpenter’s (1999) study suggested, computational skills in young children can have an impact on algebraic achievement. The current study indicates that computational skills are significant predictors of Pre-algebra and Algebra students’ ability to correctly solve unconstrained non-routine problems.

In contrast to the literature (Andrews & Brown, 2015; Ashcraft & Krause, 2007; Guven & Cabakcor, 2013; Hoffman, 2010; Kaufmann & Vosburg.1997; Kyttälä, M., & Björn, 2014; Vukovic et al., 2012; Miller & Mitchell, 1994), which shows that mathematical anxiety has a negative effect on mathematical achievement and problem solving, anxiety may have a positive effect during this stage of the non-routine problem solving process because students who were more anxious may have used the unconstrained time to their advantage to check their work. Since computational skill was a predictor during this stage, students with higher levels of anxiety may have made less computational errors, thus, were more likely to solve the problem correctly.

Lastly, with regards to problem correctness, course level was a significant predictor. Students who were enrolled in the honors class were more than 8 times as likely to solve the non-routine problem correctly. This may be due to the fact honors students had significantly stronger resource factors than the regular students. Honors students also tended to solve more critical thinking problems in class. Due to their daily
exposure to more difficult in-class and homework problems, their prior experience may have better prepared them to solve the unconstrained non-routine problem in this study.

**Correct Problem Set-up**

In contrast to problem correctness, there were different factors that predicted the problem set-up stage of unconstrained non-routine problem solving. To appropriately set-up the non-routine problem, homework grade, ability beliefs, and socioeconomic status were significant predictors. For this study, homework completion was an appropriate self-regulation measure because homework was often completed at home or during study hall periods. Given that students worked on homework during non-instructional time, it can be implied that homework required independence, planning, and time management (Ramdass & Zimmerman, 2011). Since the unconstrained non-routine problem was also solved outside of the classroom, it can be implied that it also required independence, planning, and time management. This suggests that students who were more self-regulated took the time chose an appropriate problem solving strategy before executing the mathematical procedures relative to his or her chosen problem.

In contrast to the problem correctness stage, ability beliefs was a predictor for correct problem set-up. Since ability beliefs have shown to positively predict mathematics achievement and problem solving ability on both routine (Cifarelli, Goodson-Epsy, and Chae, 2010; Panaoura, 2013; Peklaj, Podlesek, & Pečjak, 2014) and non-routine (Panaoura, 2013) tasks, this study adds to the literature by showing that mathematical ability beliefs are a predictor of correct problem set-up. In addition to examining problem performance, Panaoura (2013) also studied how students’ self-efficacy beliefs affected students’ ability to use representations on solving non-routine
geometrical tasks. This study showed that there was a relationship between self-efficacy and students’ geometric representations. The current study adds to the literature by showing that self-efficacy also has a positive effect on Pre-algebra and Algebra students’ ability to represent non-routine tasks. This result suggests that students who reported higher levels of self-efficacy may be more confident when approaching a non-routine problem, which improved their performance on setting up the problem.

Prior research indicates that socioeconomic status affects students’ mathematical achievement (Coe, Peterson, Blair, Schutten, & Peddie, 2013; Haveman & Wolfe, 1995; Klebanov, Brooks-Gunn, & Duncan, 1994; Sirin, 2005; Smith, Brooks-Gunn, & Klebanov, 1997); however, the research is limited on how SES affects unconstrained non-routine problem solving. This study implies that when all of the factors were examined collectively, SES affected non-routine problem solving during the set-up stage of the problem solving process; however SES was not a predictor of problem correctness. This implies that SES is a significant predictor during the non-routine problem solving process because it affects students’ ability to appropriately set-up a non-routine problem; however, when predicting problem correctness, computational skills and anxiety show to be predictors while SES loses its significance.

In contrast to the literature on routine problem solving (Hecht, 1998; Huntly & Davis, 2008; Lannin, Barker, & Townsend, 2006; Montague, 1992), neither resource factor had an influence on correct problem set-up when solving unconstrained non-routine tasks. This implies that when students were given an unconstrained amount of time, students of varying mathematical ability were able to set-up the problem appropriately. This result may be different than when students solve problems on time.
constrained tasks. Since the students completed the problem outside of the classroom, they may have asked a teacher or peer for help on getting the problem started; therefore, their resources did not predict setting up the problem appropriately.

Problem Completion

When collectively examining all of the factors, the two predictors of problem completion were homework completion and socioeconomic status. Similarly to correct problem set-up, it can be implied that the problem completion stage also required independence, planning, and time management. This suggests that students who were more self-regulated took the time to find a solution to the problem, whether it be right or wrong. In addition to correct problem set-up, socioeconomic status also showed to be a significant predictor of problem completion. It is possible that students of higher SES had parents or guardians that checked their child’s project to make sure that they started the problem correctly and completed all four problem solving logs; however, additional measures would be needed to determine if this was in fact the cause. These results indicate that self-regulation and SES are predictors of persistence during unconstrained non-routine problem solving; however, these factors do not show significance in predicting problem correctness.

These significance patterns during each stage of the non-routine problem solving process indicate that different factors play varying roles during the problem solving process on unconstrained non-routine tasks. When it comes to correctly setting-up and completing the problem, students of higher socioeconomic status, those who are more self-regulated, and show stronger characteristics of motivation tend to perform better. This study suggests that higher self-regulation and higher levels of value towards
mathematics had a positive effect on students persisting to complete the problem; however, it is clear that students’ students with higher levels of anxiety and stronger computational skills are significantly more likely to solve the problem correctly.

Implications

This study has several implications for both the persistence and success of children during mathematical problem solving. Since homework was a major predictor of unconstrained non-routine task set-up and completion, this implies that homework affects persistence on long-term problem solving projects. Homework can enhance the development of self-regulation, time management, and self-efficacy (Pintrich, 2000; Trautwein & Köller, 2003); therefore, educators need to help students who complete their homework less often become more self-regulated and accountable. If students who complete their homework less often begin completing their homework on a daily basis, it is possible that their self-regulation skills, time management, and planning will improve their persistence on long-term tasks. Studies have shown that self-regulation training with classroom tasks and homework activities in the elementary grades can help children learn time management skills, improve self-reflection, and enhance self-efficacy (Ormrod, 2006; Stoeger & Ziegler, 2008). If training can help improve students’ self-regulation skills on homework activities in the elementary grades, it can be suggested that these children will bring the skills with them as they enter middle school. Improving their self-regulation towards homework will hopefully help students persist on long-term problem solving tasks.

Aligned with prior research on middle school students (Zimmerman & Kitsantas, 2005), this study also implies that homework completion is related to motivational
components. Students who completed their homework more often tended to have higher ability beliefs, reported higher levels of interest towards mathematics, and believe that mathematics is important and valuable. Students can often see homework as a chore (McPherson & Zimmerman, 2002) and do not understand the importance behind the task. If educators can make homework assignments more engaging and meaningful to improve students’ motivation and self-regulation, it is possible that they can also improve students’ problem solving ability on unconstrained non-routine problem solving tasks.

While homework completion and socioeconomic status predict persistence on completing unconstrained non-routine tasks, it is one’s computational skills and mathematical anxiety that show the most importance for correctly solving non-routine problems. Mathematical anxiety generally has a negative effect on individuals during time constrained tasks, but had the opposite effect during unconstrained non-routine problem solving. While it is obvious that teachers and other educators should never employ tactics to make individuals more anxious about mathematics, this study does suggest that there may be some type of relationship between mathematical anxiety, computational skills, and the amount of time those with high levels of stress need to succeed when solving mathematics problems. In fact, there have been studies showing that low levels of anxiety may have a positive impact on students depending on the situation at hand (Ashcraft & Krause, 2007; Owens et al., 2014; Seipp, 1991). For example, Owens et al. (2014) showed that anxiety helped middle school students with higher levels of working memory capacity on test performance. The current study adds to the literature by suggesting that anxiety may also have an impact on students during
untimed tasks; however, more research is needed to determine how working memory capacity affects the variance and outcomes on unconstrained problem solving.

Teachers and educators also need to focus more on helping students improve their computational skills. This could be done by having educators in the elementary and middle grades employ different pedagogical strategies. For example, with the U.S. culture advancing technologically, younger students become more engaged when solving math exercises on laptops, tablets, and other computer-based devices, instead of using traditional worksheets (Chang, Yuan, Lee, Chen, & Huang, 2013; Pilli & Aksu, 2013; Zhang, Trussell, Gallegos, Asam, 2015). Zhang et al. (2015) found that the use of mathematics tablet applications helped improve decimal and multiplication skills of struggling fourth grade students. Since research shows that technology can help improve children’s arithmetic skills, schools should focus on utilizing tablet applications in the classroom on a more regular basis. In addition to implementing more technology into the classroom, research also shows that having students use worked examples for guided practice can help improve their procedural skills (Booth et al., 2013). Booth and colleagues (2013) found that having students explain both correct and incorrect worked examples during practice on linear equations can improve Algebra students’ procedural skills as well as develop their conceptual understanding. Furthermore, since the honors students in this study were more likely to solve the problem correctly and had more exposure to higher-level critical thinking problems that tended to be more open-ended and non-routine throughout the course of the academic year, it is possible that additional exposure to non-routine problems can help improve overall problem solving ability. If teachers of non-honors students can incorporate more thought-provoking, open-ended
problems into their daily teaching and student practice, lower level ability students may be more successful at solving non-routine unconstrained tasks.

Moreover, in order to stay globally competitive with other countries such as China and Japan, U.S. educators need to employ these various tactics to help improve computational ability. It is also important for educators to help students chose the best strategy to solve a problem in order to avoid computational errors. As previous research suggests, Chinese students use more abstract, complex strategies when solving problems (Cai, 2000a; 2000b). As problems become more abstract, students in Asian countries tend to out-perform U.S students. This is because studies (e.g., Cai, 2000a; 2000b) show that those who chose concrete strategies were more likely to make computational errors than those who chose abstract strategies. If educators can guide students to choose more appropriate problem solving strategies when solving mathematical tasks, they may be less likely to make computational errors during the problem solving process.

Limitations

Since this study was a secondary data analysis that was using the Academy’s pre-existing data, there were three main limitations. First, the motivation, self-regulation, and anxiety questionnaire were given to the students two months after they completed the problem solving project. In a perfect situation, they would have taken the survey during the time they were completing the project in order to understand their feelings and beliefs at that time, but the Academy’s mathematics department wanted to gain knowledge about their students’ feelings and beliefs closer to the end of the academic year. In addition, the students completed the placement exam approximately two months after the project. Once again, in an ideal situation, the students would take the placement exam during the
same time as the problem solving project, but due to the way the Academy’s department runs their curriculum, this was not possible.

The second main limitation was based on the problem solving project. Due to the design of the Academy’s problem solving project, the students had the option to choose from one of three non-routine tasks based on grade level. Since there were three problems per grade level, there were nine different non-routine tasks in the current study. The non-routine tasks required the students to draw on various types of resource factors. As mentioned, students who chose the Cake problem were more unlikely to solve the problem correctly and students who chose Martian Money were more likely to get the problem correct. Due to the variety of non-routine tasks, it was necessary to control for problem type during each analysis. It is unknown how the results would have differed if the students had only one problem to choose or if the students completed all three of the problems.

Lastly, on the motivation, self-regulation, and anxiety survey, there were only four questions to measure self-regulation. In order to gain a more comprehensive measure for the students’ self-reported self-regulation, it would have been ideal to have more questions that focused on that particular control factor. However, homework completion showed to be a sufficient measure for self-regulation because it measured their self-regulation on completing assignments outside of the classroom over the course of the entire academic year.

Suggestions for Future Research

This exploratory study was the beginning of a line of research on problem solving with unconstrained non-routine tasks. Results from this study give insight into what
factors relate to this specific type of problem solving; however, additional research is needed to confirm these results. In order to create appropriate interventions to help improve non-routine problem solving in the United States, a more clear understanding of the unconstrained non-routine problem solving process is necessary.

As mentioned in the limitations of this study, the design of the problem solving project allowed the students to choose from one of three problems to solve. The variation in problems required the students to draw on different resources and because of this, the control for problem type was necessary before each analysis. It would be beneficial to conduct further research such that all the participants solve the same problem to avoid having to control for problem type. Another suggestion is to have the students solve multiple unconstrained problems over the course of the academic year to get a better picture of the unconstrained non-routine problem solving process.

More research is also needed on the relationship between computational skills and mathematical anxiety on unconstrained tasks. Results showed that computational skills and higher levels of mathematical anxiety predicted problem solving correctness. It is unclear if there is a relationship between computational skills and mathematical anxiety when students solve problems outside of the classroom. Additional research is also needed to identify the specific computational errors Pre-Algebra and Algebra students continue to make when problem solving. Identifying the most common computational errors can aid researchers in the development of classroom interventions to improve middle school students’ mathematical learning and understanding.

Lastly, more research is needed with different populations. While this study examined the factors that relate to unconstrained non-routine problem solving of middle
school students, additional research is needed to see if the factors differ based on grade level. Differing factors may show significance with elementary, high school, or college students. It is also possible that factors may change based on content. More research is needed with varying mathematical topics other than Pre-Algebra and Algebra students.

Conclusion

The current study examined the varying factors that related to mathematical problem solving of middle school students on unconstrained non-routine tasks. Through correlations and logistic regressions, this study identified how computational skills, heuristics, control factors, and student beliefs/affects towards mathematics both individually and collectively related to the problem solving process. The results suggest that computational skills, self-regulation, varying motivational components, mathematical anxiety and socioeconomic status are significant predictors of successful problem solving during varying stages of problem solving process. These results have implications for improving students mathematical problem solving ability; however, more research needs to be conducted to gain better insight into what factors affect student problem solving success and to what extent in order to develop appropriate interventions.
REFERENCES


Handbook of infant, toddler, and preschool mental health assessment (pp. 491–521). New York: Oxford University Press.


GA Middle School Math Problem Solving Project

Purpose

Mathematicians often work hours, days, or even years on a single problem. The purpose of this project is to give you an opportunity to investigate a problem at length. The goal is not to solve the problem quickly; it is not even necessary to successfully solve the problem. You are to immerse yourself in the problem over the course of several days.

Project Directions

1. Select a problem from the list of options.
2. Work on the problem for a minimum of 15 minutes each day for a minimum of 4 days. These do not need to be consecutive days. Of course, you are encouraged to work a little more; these are minimums. The hope is that the problem will get inside your head.
3. Each day that you work, use the math log to track the date and time of your work, and keep notes on the work that you do.
   - If you solve the problem before the 4 days are up, you must extend the problem. Then you can look for a pattern, or make a generalization, or perhaps create a formula. If you have trouble thinking of a way to extend the problem, talk to your teacher.
   - If you do not solve the problem when the 4 days are up, that’s okay.
4. Complete a written summary and reflection about your work.
5. Your log and summary and reflection document are due on -
   ________________

Other Instructions

If you get stuck, try to come up with a different strategy for working on the problem. If you need help with this, ask your teacher first. Do not search for the answer using other resources such as books or websites. You
may discuss ideas with your classmates, but you may not copy another
student’s work and present it as your own. Remember that in this case,
"work" refers to your problem solving methods as well as the answer. Don’t
let someone else tell you how to solve the problem—that takes away the fun
of figuring things out yourself!
Directions for Log, Summary, and Reflection

To complete the project, you will turn in a log showing your work on the problem as well as a written summary and reflection about your work.

Log

Turn in your log (with dated entries) and work on the problem. Use the log pages provided and attach additional paper showing your work if necessary.

1) Each period of work should show the date and starting and ending times.
2) Use the section titled “Problem Solving” to show your work, including calculations, equations, etc.
3) Use the section titled “Reasoning” to explain your work, indicating why and how you did the work. You do not need to write complete sentences, but your teacher should be able to understand your thinking.

Summary

In full sentences, write about the mathematics that you did.

4) What methods did you use?
5) When did you get stuck? What did you do then? If you did not solve the problem, talk about the mathematics that you tried.
6) What was the solution? Did you solve the problem in more than one way?
7) Did you extend the problem? If so, how?
8) What new mathematical insights did you have as a result of working on this problem?

Reflection

In full sentences, write about the experience of working on a problem for several days.

1) What intrigued you about the problem? Why did you choose it?
2) What did you enjoy about the process?
3) What were your frustrations? How did you feel about getting stuck?
4) What did you learn about problem solving that will help you in the future?
### Rubric

<table>
<thead>
<tr>
<th></th>
<th><strong>Exceeds Expectations (5 pts)</strong></th>
<th><strong>Meets Expectations (3-4 pts)</strong></th>
<th><strong>Falls Below Expectations (0-2 pts)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Log: Problem Solving</strong></td>
<td>Work is clearly shown and organized. Problem solving strategies show particular creativity and/or insight into the problem. Several strategies are used if necessary.</td>
<td>Work is clearly shown and organized. Problem solving strategies are appropriate for the problem. Several strategies are used if necessary.</td>
<td>Work is unclear and/or disorganized. Problem solving strategies are not applied in a consistent or effective way.</td>
</tr>
<tr>
<td><strong>Log: Reasoning</strong></td>
<td>Explanations of reasoning are clear and thorough. If an answer is found, it is checked in multiple ways, and extensions to the problem are investigated in detail.</td>
<td>Explanations of reasoning are clear. If an answer is found, it is checked in some other way, and possible extensions to the problem are explored.</td>
<td>Explanations of reasoning are missing or unclear. If an answer is found, it is not checked, and/or possible extensions to the problem are not explored.</td>
</tr>
<tr>
<td><strong>Writing: Summary</strong></td>
<td>Summarizes the mathematics done in a thorough way, with details about each step. If a solution was not found, the strategies that were attempted are explained and ideas for further work are stated. If a solution was found, it is stated and extensions to the problem are discussed.</td>
<td>Summarizes the mathematics done. If a solution was not found, the strategies that were attempted are explained. If a solution was found, it is stated and possible extensions to the problem are discussed.</td>
<td>Summarizes some of the mathematics done, but is unclear and/or incomplete. If a solution was not found, the strategies that were attempted are not fully explained. If a solution was found, it is stated but possible extensions to the problem are not discussed.</td>
</tr>
<tr>
<td><strong>Writing: Reflection</strong></td>
<td>Describes the experience of working on the problem in a thorough way, with specific information including enjoyable and frustrating aspects, as well as learning about problem solving that occurred.</td>
<td>Describes the experience of working on the problem with specific information, including enjoyable and frustrating aspects.</td>
<td>Describes the experience of working on the problem in a limited way, without specific information.</td>
</tr>
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6th Grade Problems

Goldbach's Conjecture

Goldbach's conjecture is an idea about prime numbers that is believed to be true but has not been proved: Every even number greater than 4 can be written as the sum of two odd prime numbers. For example: \(8 = 3 + 5\). Both 3 and 5 are prime numbers. \(20 = 13 + 7 = 17 + 3\). \(42 = 23 + 19 = 29 + 13 = 31 + 11 = 37 + 5\). Notice that there can be more than one Goldbach pair. The conjecture says only that there is at least one, and has nothing to say about whether there may be more. Can you find Goldbach pairs for all even integers between 4 and 100? Which have more than one Goldbach pair? Can you find any patterns?

Fast Draw

Form exactly two squares by drawing five lines. By drawing six lines. By drawing seven lines. Extensions: 1. Can you draw two equilateral triangles using four lines? 2. Can you draw two equilateral triangles using five lines?

Four Fours

Using exactly four 4s and the operations addition, subtraction, multiplication and division, make equations that equal 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Remember to use the order of operations correctly! You may use parentheses. Extension: Can you make other numbers? Can you use 3s or another number for a similar activity?

7th Grade Problems

Red Paint

There are 27 small cubes arranged in a 3 by 3 by 3 cube. The top and sides of the large cube are painted red. How many of the 27 small cubes have 0 faces painted? 1 face? 2 faces? 3 faces? 4 faces? 5 faces? 6 faces?

Martian Money

On Mars, money is measured in reddies. The only coins available are a coin worth 5 reddies and a coin worth 11 reddies. What is the largest amount of money that a Martian CANNOT make using only these coins? (The Martian may use an unlimited number of coins, in any combination.)
Diagonals

How many diagonals does a polygon with 17 sides have? (Hint: Start with smaller polygons, look for a pattern.) Extension: Can you develop a formula for the number of diagonals in a polygon having a certain number of sides?

8th Grade Problems

A Piece of Cake

What is the greatest number of pieces you can cut a round cake into by making four straight cuts with a knife? Some possible extensions: 1. What is the maximum number of pieces produced by five cuts? Is there a pattern to these answers? 2. Is this the same problem as dividing a circle into the maximum number of areas by drawing four straight lines? (*Students were told by teachers to not stack the cake)

Tile Floor

I want to tile a rectangular floor with congruent square tiles. Blue tiles will form the border and white tiles will cover the interior. The number of blue tiles will equal the number of white tiles. What is the maximum area in square units that can be tiled? Some possible extensions: How would this problem be different if there were half as many blue tiles as white tiles? What if there were three times as many blue tiles as white tiles?

Hiking in the Desert

Three hikers head from home camp to an oasis. They can carry only 10 days supply of food and water in each of their packs. Since the oasis is more than 10 days away, they agree to try to get only hiker #1 to the oasis. After walking together for a time, hiker #3 refills the packs of hiker #1 and hiker #2 and returns home. Hiker #1 and hiker #2 continue, and later hiker #2 refills the pack of hiker #1 and returns home. Find the maximum distance from home to oasis such that hiker #1 can get there with hiker #2 and hiker #3 both safely home. Extension: If the oasis is 18 days from home and two hikers are expected to reach the oasis while the others return one at a time after refilling the packs of the others, find the least number of hikers needed.
APPENDIX B

Sixth Grade Placement Exam

Germantown Academy Mathematics

Placement Test: Entrance to Algebra A

You will have 40 minutes to work on this test. Some problems are very easy and some are quite difficult. Find a problem that you know how to solve and solve it. Move to another. Work quickly but carefully. Concentrate on what you are doing. At the end of the test period you should be puzzling over one of the more difficult problems. Do not be concerned if there are a few problems that you did not have time to solve.

For each of the following problems, show your work, express your answer in simplest form, and circle your answer.

A. Perform the indicated operations.

1. \[58 - 193 + 27\]
2. \[(-6)(-14)\]

3. \[17 - 6 \times 2\]
4. \[9 - 4(8 - 5)^2\]

5. \[\frac{5}{12} + \frac{3}{20}\]
6. \[-\frac{3}{4} \div \frac{1}{8}\]

7. \[17.4 - 0.06\]
8. Find four tenths of 5 thousandths.

B. Simplify each of the following expressions.

9. \[3x - 5y + x - 8y\]
10. \[9x^2 + 4x - x - 2x^2 + 6x - 5\]
11. \( n^5 \cdot n^{13} \)

12. \( \frac{(xy^2)^4}{x^3y^2} \)

C. Solve each of the following equations.

13. \( a + 12 = 35 \)

14. \( -4x = 76 \)

15. \( \frac{3}{5} = \frac{24}{x} \)

16. \( 3n - 28 = 17 \)

17. \( 6 - 3(2w + 5) = 9 \)

18. \( 7.8x - 4.5x + 3.9 = x - 18.24 \)

19. What number is 70% of 240?

20. Mr. Bingsley earns $12.50 per hour. How many hours does he have to work to earn $937.50?

21. Mr. Connor bought a dress for his wife on sale for $328.00. If the dress had been marked down 18%, how much did it cost before the sale?

22. Every whole number divisible by 1000 must also be divisible by which of the following numbers?

   a. 80   b. 125   c. 400   d. 625   e. 1001   f. 3
23. In a bag there are 5 red, 6 blue, 9 green, and 8 yellow marbles. If only one marble at a time can be drawn from the bag without looking, what is the least number of draws one must make to be sure of having at least 4 marbles of the same color?

24. A student finished ½ of his homework in the early afternoon and then ¼ of the remainder in the evening. What part of the homework still remained to be done?

25. What number is a million times as large as $10^{-4}$?
APPENDIX C

Seventh Grade Placement Exam
Germantown Academy Mathematics

Placement Test: Entrance to Algebra B

You will have 40 minutes to work on this test. Some of the problems are very easy and some are quite difficult. Find a problem that you know how to solve and solve it. Move to another. Work quickly but carefully. Concentrate on what you are doing. At the end of the test period you should be puzzling over one of the more difficult problems. Do not be concerned if there are a few problems that you did not have time to solve. Be sure to show your work and to express your answer in simplest terms.

1. Use the associative law of addition to write another expression equivalent to \( x + (y + 3) \).

2. Evaluate \( \frac{5b - 2a}{3} \) for \( a = 8 \) and \( b = 5 \).

3. Find the prime factorization of 144.

Perform the indicated operations and express in simplest form.

4. \( -\frac{5}{18} + \frac{4}{27} \)

5. \( -36 + 27 - 19 \)

6. \( 16 + 12 \div (-4) + 8 \)

7. \( -3(9) - |2(10) - 5^2| \)

8. The price of a boat was reduced to a sale price of $10,725. This was a 35% reduction. What was the original price?

Perform the indicated operation and simplify.
9. \((12x^2 - 3x + 5) - (3x^3 + 12x^2 - x)\)  
10. \((4n - 7)(n + 6)\)

11. If \(a\) is 40% of \(b\), \(b\) is what percent of \(a\)?

Solve each of the following for \(x\). Show your work.

12. \(8x - 13 = 39\)  
13. \(-7 + 3(x - 5) = 4x + 12\)

14. \(A = \frac{1}{2}xy\)  
15. \(\frac{a}{x} + 2b = 6\)

Simplify each of the following.

16. \(\frac{n^{12}}{n^3}\)  
17. \(6(x^4)(-15x^5)^3\)

18. Find the point-slope equation of the line containing the point with coordinates \((5, -1)\) and slope \(-\frac{2}{3}\).
19. Find \(m\) in \(y = mx + 4\) given that \((-2, 3)\) is a solution to the equation.

Sketch a graph of each of the following.

20. \(-3x + 4y = 12\)  
21. \(x + 2y \geq 6\)
22. The sum of the first 100 odd positive whole numbers is $100^2$. Find the sum of the first 100 even positive whole numbers.

23. The average of eight different positive whole numbers is 8. What is the largest possible value of any of these numbers?

24. If it takes 12 hours for 12 people to paint a barn, how long would it take 16 people to do the job?

25. The sum of the measures of complementary angles is 90°. If one angle measures 35° less than twice its complement, find the measure of each angle.
You will have 40 minutes to work on this test. Some of the problems are very easy and some are quite difficult. Find a problem that you know how to solve and solve it. Move to another. Work quickly but carefully. Concentrate on what you are doing. At the end of the test period you should be puzzling over one of the more difficult problems. Do not be concerned if there are a few problems that you did not have time to solve. Be sure to show your work and to express your answers in simplest terms.

1. Factor completely: 

   \[3x^2 + 20x + 12\]

2. Solve by factoring:

   \[x^2 - 5x - 14 = 0\]

3. Use the following graph to find the solutions to the equation \(-x^2 - x + 6 = 0\).
4. One leg of a right triangle is 3 more than the other. Find the length of the shorter leg if the hypotenuse is 15.

5. Divide, and if possible, simplify.

\[ \frac{x^2+4x+4}{2x^2} = \frac{x^2-4}{x} \]

6. Add, and if possible, simplify.

\[ \frac{1}{a} + \frac{3}{b} + 5 \]

7. Simplify.

\[ \frac{1-\frac{1}{n}}{1-\frac{1}{n^2}} \]

8. Solve for n.

\[ \frac{6}{n} - \frac{1}{4} = \frac{1}{8} \]

9. Solve by the elimination method.

\[ 2x + 3y = 9 \]
\[ 2(y + 4) = -6x \]
10. Sketch the graph of the solution set of

\[ y \leq 3x - 5 \]
\[ y \geq 2 \]

11. A chemist has a solution that is 30% acid and another that is 60% acid. How much of each is needed to make 120L of a solution that is 40% acid?

12. Multiply and simplify.

\[ \sqrt{6n} \sqrt{10nm^3} \]

13. Rationalize the denominator and simplify.

\[ \frac{1 - \sqrt{2}}{3 + \sqrt{2}} \]

14. Solve for \( n \).

\[ 2 + n = \sqrt{4 + 7n} \]

15. Solve using the quadratic formula.

\[ x^2 - 4x + 6 = 8 \]
16. How many 1-centimeter squares are required to make a border around the edge of the shaded square with side of 8-centimeters as shown in the figure below?

17. In the figure below, the area of the small square is $x$. If each short line segment in the figure has length $2y$ and every pair of intersecting segments is perpendicular, find the area of the shaded region in terms of $x$.

18. The figure below shows how a rectangular piece of paper is rolled to form a cylindrical tube. If it is assumed that the 4-centimeter sides of the rectangle meet with no overlap, what is the area, in square centimeters, of the circular base region enclosed.

(Remember, the area of a circle is $\pi r^2$, and the circumference of a circle is $\pi d$.)
19. In the figure below, a 5-meter pole and a 3-meter pole are tied together so that the length of the overlapping portion is 2 meters. What is the length x of the two poles combined in this way?

20. The figure below shows a rod with black and white beads. How many beads must be slid from the right to the left so that one-fourth of the beads on each side are black?
APPENDIX E

Motivation, Anxiety, and Self-Regulation Survey

Name: ____________________________________________

The following questions ask you to rate on a scale some of your feelings and beliefs towards mathematics. Answer each question by circling the number that best describes your particular feeling RIGHT NOW. There are no “right” or “wrong” answers, only answers that are most true for you. Please be completely honest.

1. In general, I find math assignments:

   Very Boring 1 2 3 4 5 6 7 Very Interesting

2. How good in math are you?

   Not at all 1 2 3 4 5 6 7 Very Good

3. When someone asks you questions to find out how much you know about mathematics, how much do you worry that you will do poorly?

   Not at all 1 2 3 4 5 6 7 Very Much

4. When I run into difficulty doing a math problem, I go back and work out where I went wrong.

   Not at all True 1 2 3 4 5 Very True

5. My aim is to completely master the material presented in this class.

   Strongly Disagree 1 2 3 4 5 Strongly Agree
6. My aim is to perform well relative to other students.

   Strongly Disagree 1 2 3 4 5  Strongly Agree

7. How well do you expect to do in math this semester?

   Not at all 1 2 3 4 5 6 7   Very Good
   Good

8. When I am taking a math test, I usually feel:

   Not at all 1 2 3 4 5 6 7   Very nervous
   nervous or and uneasy
   uneasy

9. When other students are distracting me in math class, I often find a way to keep concentrating on my work.

   Not at all True 1 2 3 4 5  Very True
   True

10. My aim is to avoid learning less than I possibly could.

    Strongly Disagree 1 2 3 4 5  Strongly Agree

11. Compared to other students, how good at math are you?

    Much worse 1 2 3 4 5 6 7   Much better

12. Compared to other students, how important is it to you to be good at mathematics?

    Not at all 1 2 3 4 5 6 7   Very
    important
13. When I notice that I haven’t been listening to my math teacher, I try to concentrate harder.

Not at all True 1 2 3 4 5 Very True

14. How much do you like mathematics?

Not at all 1 2 3 4 5 6 7 Very much

15. How good would you be at learning something new in mathematics?

Not at all Good 1 2 3 4 5 6 7 Very Good

16. My goal is to learn as much as possible.

Strongly Disagree 1 2 3 4 5 Strongly Agree

17. If you were to list all of the students in your class from worst to best in math, where would you put yourself?

One of the worst 1 2 3 4 5 6 7 One of the best

18. Taking math tests scare me.

I never feel 1 2 3 4 5 6 7 I very often feel this way

19. I dread having to do math.

I never feel 1 2 3 4 5 6 7 I very often feel this way

20. In general, how useful is what you learn in math?
21. Compared to most of your other activities, how much do you like math?
   Not as much 1 2 3 4 5 6 7 A lot more

22. I am striving to avoid an incomplete understanding of the course material.
   Strongly Disagree 1 2 3 4 5 Strongly Agree

23. My aim is to avoid doing worse than other students.
   Strongly Disagree 1 2 3 4 5 Strongly Agree

24. For me, being good at math is:
   Not at all important 1 2 3 4 5 6 7 Very

25. Compared to most of your other activities and classes, how useful is what you have learned about mathematics?
   Not at all useful 1 2 3 4 5 6 7 Very useful

26. It is very important for me to get good grades in math.
   I never feel this way 1 2 3 4 5 6 7 I very often this way
27. The topics in this class are important to my future career.

Not at all important 1 2 3 4 5 6 7 Very important

28. I am striving to understand the content of this course as thoroughly as possible.

Strongly Disagree 1 2 3 4 5 Strongly Agree

29. I am striving to do well compared to other students.

Strongly Disagree 1 2 3 4 5 Strongly Agree

30. Before I begin my math work, I think about the things I will need to do.

Not at all True 1 2 3 4 5 Very True

31. When I’m working on a math problem, I think about whether I understand what I’m doing.

Not at all True 1 2 3 4 5 Very True

32. My goal is to perform better than other students.

Strongly Disagree 1 2 3 4 5 Strongly Agree

33. When I finish my math work, I check it to make sure it’s done correctly.

Not at all True 1 2 3 4 5 Very True
34. In general, I find working on math assignments…

<table>
<thead>
<tr>
<th>Very boring</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Very Interesting</th>
</tr>
</thead>
</table>
APPENDIX F

Temple IRB Permission Letter

Research Integrity & Compliance
Student Faculty Center
3340 N. Broad Street, Suite 304
Philadelphia PA 19140

Institutional Review Board
Phone: (215) 707-3190
Fax: (215) 707-9100
e-mail: irb@temple.edu

PI: BOOTH, JULIE L
Committee: A2
Protocol Number: 23827
Project Title: An Exploratory Study of the Factors Related to Successful Mathematical Problem Solving on Non-routine Unconstrained Tasks

Date: 29-Jul-2016

Not Human Subject Research Determination

The proposed activity is not research involving human subjects as defined by DHHS or FDA regulations. Consequently, Temple IRB review and approval is not applicable. You are welcome to pursue the activity, obtaining any applicable administrative or departmental (non-IRB) approvals. This determination applies only to the activities described in this IRB submission and does not apply should any changes be made. Changes could affect this determination, therefore please contact the IRB for guidance.

DHHS Definitions:
Research - a systematic investigation, including research development, testing and evaluation, designed to develop or contribute to generalizable knowledge.
Human subject - a living individual about whom an investigator (whether professional or student) conducting research obtains:
(1) Data through intervention or interaction with the individual; or
(2) Identifiable private information.

FDA Definitions. Research - any experiment that involves a test article and one or more human subjects, and that either must meet the requirements for prior submission to the Food and Drug Administration.
Human subject - an individual who is or becomes a participant in research, either as a recipient of the test article or as a control. A subject may be either a healthy individual or a patient.

Please contact the IRB at (215) 707-3100 if you have any questions.