# OPTIMAL UNEMPLOYMENT INSURANCE IN A MODEL WITH SKILL LOSS AND MATCH QUALITY UNCERTAINTY

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## ABSTRACT

This dissertation makes a contribution to the question of how best to set the rate of unemployment compensation. Previous research on this topic has emphasized the behavioral response of non-workers to various incentives created by unemployment insurance. Recent work has emphasized two new features. One is the importance of including savings in the model, and the other the recognition that skills tend to rise during employment and fall during unemployment spells. This thesis seeks to combine all three features, search incentives, savings, and skill change effects.

The strategy is to develop an unemployment model with these features and to obtain parameters values from a variety of sources, including SIPP data and research by other authors on related questions. The model is then simulated for various ranges of policy choices. The primary policy choice is the benefit replacement ratio, a number that determines the actual level of unemployment compensation. Taxes are set under different assumptions. In some cases, taxes are set to achieve budget balance. In other cases, taxes are set independently of benefit levels. This feature assumes the possibility of a subsidy from other sources, but it allows for a study of the independent incentive effects of benefits and tax rates.

Results from the simulations using the most likely parameter specification indicate that a replacement ratio of 58% is best. A replacement ratio slightly higher than the optimal ratio can lead to a large decrease in average utility, and is problematic. When human capital changes are relatively less responsive to unemployment and employment duration, longer unemployment spells are more desirable as they lead to

better matches. When the effect of taxes and benefits are looked at separately, the benefit ratio aspects matters more than the tax rate.

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# CHAPTER 1

# INTRODUCTION

This dissertation focuses on two competing ideas in the realm of unemployment policy. One is that if people are out of work for a longer period of time, they can use that additional time to find a better matching job. Longer unemployment spells are associated with better overall job matches, a desirable effect. This idea is developed by Marimon and Zilibotti (1999), Chetty (2008), Burdett (1979) and Acemoglu and Shimer (2000). Henceforth it will be referred to as the match quality hypothesis. The competing idea is that when people are out of work, their work related skills diminish. Longer unemployment spells are thus associated with reduced employment outcomes, an undesirable effect. This idea is associated with Ljungqvist and Sargent (1998), as well as Pissarides (1992). Henceforth it will be referred to as the human capital hypothesis. These forces have opposite effects on the employment outcomes of a job loser. The relative magnitude of the two effects is important in interpreting optimal unemployment insurance policy. More generous unemployment benefits lead to longer unemployment spells. If the match quality hypothesis is stronger, then the generosity of benefits should be high. If the human capital hypothesis is stronger, then the generosity of benefits should be low.

Studying unemployment insurance is important because it is a large social program in developed countries that provides consumption smoothing for unemployed at the expense of introducing perverse incentives. The perverse incentives caused by paying people not to work include not accepting jobs that an unemployed worker would have otherwise accepted and reducing search effort in looking for a job. The positive aspects of unemployment insurance include consumption-smoothing for the unemployed and macroeconomic stabilization effects.

In the United States, the unemployment insurance system is a main source of income stabilization for unemployed workers. According to the Congressional Budget Office (2011), the median contribution to the income of families that receive unemployment compensation was $6000 in 2010. This payment accounts for 11% of income for all unemployed families and 22% of income for families with members who have been unemployed for more than 26 weeks. The total unemployment compensation paid out in the United States in 2010 was $160 billion, up from $120 billion in 2009, and $35 billion in 2007. The Congressional Budget Office estimates that without the unemployment compensation system, the poverty rate in the United States would be 15.4% instead of 14.3%.

This dissertation attempts to answer the question of how match quality and human capital affect the unemployed worker. To do this, a model is created and simulations are run. The model created in this dissertation is a dynamic optimization labor supply model where workers move between employed and unemployed states. The model relies on a limited number of parameters from which optimal unemployment benefits can be determined. To simulate the model, three inputs to the model must be calibrated: a) the rate at which human capital depreciates while unemployed; b) the rate at which human capital appreciates when employed; and c) and the variance of match quality at the time of job offer. These inputs are important to the results of the model, but may be difficult to find. For this reason, calibration is handled in three ways: (a) by estimating the terms using available micro-data; (b) choosing values based on what previous authors have used; and (c) by varying input values used in the model solution.

The calibrated model is solved using two policy environments. First, an actuarially fair budget balance is required. Second, an environment where the budget is not balance is studied. Any budget shortfalls in the second environment are not modeled. The first approach is standard. The second approach is necessary for three reasons: a) it reflects how unemployment compensation is funded in the real world; b) it shows the independent impacts of taxes and benefits, each of which has a negative impact on the desire to work that cannot be seen when they rise and fall in tandem; and c) due to a potential Laffer curve effect, it is possible that there is no equilibrium in the budget balanced world when benefit generosity is high. In the budget balance environment, it is also possible that there is no tax rate that can balance the budget when benefit levels are high. The environment where the budget is not required to balance allows for a wider range of benefit levels to be analyzed.

Results from the budget balance model show that the optimal benefit, paid as a replacement rate of previous wages is 0.58. The analysis performed by varying the calibrated variables shows that this rate could range from 0.39 to 0.70. These replacement rates align with real-world policy in developed nations. The budget balance model shows the trends between economic variables and the benefits generosity that is expected, mainly that increasing benefit generosity makes average match quality in the economy increase and average human capital decrease. The match quality hypothesis and the human capital hypothesis are verified in these results. An increase in benefit generosity is accompanied by an increase in taxes in the budget balance analysis. There is a large decrease in total utility in the economy when benefits are set slightly above the optimal rate. An increase from the optimal replacement rate of 0.58 to 0.60 leads to a .57% decrease in total utility. If the optimal benefit is set too low, the decrease in total utility is much smaller: a replacement rate of 0.56 leads to only a .023% decrease in utility versus the optimal replacement rate.

When the budget is not required to balance, the expected results hold: the lower the tax rate, the higher the total utility. The analysis shows that there is the presence of a Laffer curve in this model. On the right side of the Laffer curve, the behavior of economic variables when benefits are increased is the reverse from what is expected. An increase in taxes on the right side of the Laffer curve leads to a decrease in unemployment, an increase in human capital, and a decrease in match quality.

# CHAPTER 2

# BACKGROUND AND LITERATURE REVIEW

There are a few common threads that exist in the literature on unemployment insurance. First, unemployment insurance is considered to be pure insurance, not a transfer of wealth. It is modeled using an actuarially fair tax system where the present value of tax receipts for the government is equal to the present value of benefits that are paid out to the unemployed. Employed workers fund benefits for unemployed workers. There is no budget balance over an individual worker's life-cycle making it a possibility that some workers will pay more into the system than they receive in benefits. The budget is not required to balance in each period. Second, macroeconomic models of unemployment insurance have to consider the underlying trade-off between the reduced risk that benefits provide for workers and the higher unemployment rate that is a result of providing insurance. The higher rate of unemployment occurs because unemployment benefits subsidize unemployment, making unemployment more desirable to the worker and the hazard rate out of unemployment decrease. The decrease in the hazard rate is caused my the moral hazard that paying unemployment insurance benefits creates.

## 2.1 Real-World Unemployment Policy

The current unemployment insurance system in the United States was founded as part of the Social Security Act of 1935 as a program that provided partial, time-limited replacement of wages to workers who lost their job through no fault of their own. The policy debate around unemployment insurance focuses on the amount of the payments, the duration of payments, and the circumstances on which the benefits are paid.

A motivation for making unemployment payments is that a sudden income reduction due to a job loss could be painful for the worker, his or her dependents and for the economy as a whole. A second motive is that unemployment can arise because of circumstances that are not attributable to the worker. Some protection for the worker is therefore justified. The argument against unemployment compensation or for less generous benefits in the form of lower payments or shorter duration is based on three incentive effects. First, when unemployed workers receive benefits, those benefits are typically thought to come from a pool of money derived from workers who are employed. High unemployment benefit levels or long benefit durations will impose a burden on existing workers and hence a disincentive to work. Second, workers who are unemployed are less inclined to generate job offers through search or accept jobs when offered if they are eligible for unemployment benefits. The third reason against high unemployment benefits is it removes the incentive to save for the possibility of an unemployment spell. The policy debates are centered around the competing effects of pain and dislocation that comes with low benefits against the various adverse incentive effects that come with higher benefits.

In a survey of the unemployment insurance literature, Nicholson and Needels (2006) outline the main goals of unemployment insurance programs as they are implemented in developed countries. First, the programs aim to sustain consumption for job losers and their families. Second, unemployment programs help job losers make efficient choices about re-employment during a period of financial distress. Third, programs aim to minimize the adverse incentives created by paying workers not to work. Fourth, the programs aim to stabilize the overall economy.

In many of the models of unemployment insurance, there is a reservation wage that arises. The reservation wage is a theoretical object that is defined as a threshold wage above which wage offers are accepted and below which wage offers are rejected. Often, the focus of unemployment policy is discussed in terms of how it impacts the reservation wage. Reservation wages were used by McCall (1970) and explicitly applied to unemployment insurance models by Hopenhayn and Nicolini (1997). The role of reservation wages is especially important in the model of Shimer and Werning (2007) who claim that the reservation wage is the only signal needed to calculate the optimal level of unemployment benefits. The reservation wage minus taxes needed to fund the unemployment compensation program translates directly to workers' utility. The higher the after tax reservation wage the higher utility is for the worker.

#### *2.1.1 Aspects of Unemployment Insurance Models*

There are nine aspects of the real world that models of unemployment insurance may contain: 1) concavity of the utility function, 2) precautionary savings, 3) search effort, 4) budget balance, 5) human capital changes, 6) match quality, 7) benefits contingent of search and labor force participation, 8) life-spans, and 9) business cycle considerations. Ideally, a model would contain all nine aspects but in practice adding additional complexity make models more difficult to solve or to analyze the model's results.

Most unemployment insurance models have concave utility. Concave utility is necessary so that agents are risk-averse. As consumption approaches zero for the agent, the utility becomes very low when the utility function is concave. Without concave utility, the need for unemployment insurance as a means of consumption smoothing is lost in the model. Acemoglu and Shimer (1999) show that an economy with risk-neutral agents achieves maximum output without unemployment insurance, but an economy with risk-averse agents requires a positive level of unemployment compensation to reach maximum output.

Precautionary savings against a possible unemployment spell can be thought of as a substitute for unemployment insurance. Precautionary savings, savings by an individual to protect against the future loss of an income stream, is a self-insurance mechanism. According to estimates by Engen and Gruber (2001), the presence of unemployment insurance lowers the precautionary savings of the worker. Hansen and Imrohoroglu (1992) state that even in an economy with precautionary savings unemployment insurance is still welfare increasing because the unemployed lack access to capital markets.

Models of unemployment insurance can have different types of search effort and search cost. In most models of unemployment insurance search is costly and an agent experiences a decrease in utility if search is undertaken. Search effort can be continuous or discrete. For example, Hopenhayn and Nicolini (1997) specify a model with continuous search effort, but when the model is parameterized and solved, search effort is limited is made discrete, search or no search. Changing a continuous search variable to discrete often aids with solving an unemployment insurance model.

Most unemployment insurance models have some sort of budget balancing mechanism. Most models balance the amount of payments paid into the unemployment insurance system through taxes with the amount paid out as benefits in present value. Each period is not expected to be in budget balance, but the system is expected to be in balance through time. Occasionally, a different budget balance mechanism is used. For example, Lentz (2009) forces the amount paid into the system to equal the amount taken from it for each individual, before relaxing that assumption.

Changes in a worker's human capital during employment or unemployment, or a structural change in the economy that changes the return to human capital of a worker are important to consider in an optimal unemployment insurance model. If skills decline during an unemployment spell, it is optimal to get workers back to work quickly before skills drop too far. If human capital increases while a work is employed, more time in a job is better for the productivity of the economy. Ljungqvist and Sargent (1998) model an economy with skills that appreciate during employment and depreciate during unemployment. They find that workers set reservation wages based on the ending wage of the job lost before an unemployment spell. When skills decline during unemployment, the worker does not re-evaluate the reservation wage in the face of diminishing wage offers. The presence of skill loss during unemployment means that benefits should be less generous than they would be in the absence of human capital change, because workers are predisposed to reject jobs that should be accepted.

Match quality in this dissertation does not refer to a matching function. Following Pissarides (1992), many models use a matching function to determine the probability that worker and firm match. This framework has been extended by Boone et al. (2007) where instead of the overall unemployment and vacancies determining the match probability, the number of applicants for a job opening determines the probability of a match. In this dissertation, match quality refers to the productivity between worker-firm pairs. Two equally skilled workers might have different wages at the same firm because one worker is a better match for that firm. One paper that explores the role of match quality in determining the optimal level of unemployment benefits is Marimon and Zilibotti (1999). In their paper, firm-worker pairs have different productivities depending on their location around a unit circle. Higher levels of unemployment benefits leads to better matches, because workers are unemployed for a longer period of time and can sample more jobs to find a more productive match.

In the United States, unemployment benefits are contingent on the unemployed worker having satisfied some income and work requirements during a set period before applying for unemployment benefits. Most states require at least 6 months of work in the previous year to the unemployment spell or have a minimum income requirement, in that time period. Benefits are also contingent on the unemployed worker searching for a job. Failure to search for a job or accept a job that is acceptable is grounds for termination of unemployment benefits. One place where models try to add to the policy discussion is the shape of the benefit curve, should benefits increase, decrease or stay the same during an unemployment spell. Hopenhayn and Nicolini (1997) find that it is optimal for benefits to decrease over the unemployment spell. Decreasing unemployment benefits counteract some of the moral hazard of paying workers to remain unemployed, and give an incentive for them to go back to work. Shimer and Werning (2006) find that when the social planner faces a group of homogeneous workers the path of benefits does not depend on the amount of time unemployed. When workers are heterogeneous or skills decline during unemployment, the optimal path of benefits is downward sloping.

Some unemployment insurance models deal with life spans. The relevant policy question about life spans are whether to pay young workers a different level of benefits compared to old workers and whether or not to pay workers with no experience a subsidy to search for a first job. This can be done in a two period model as in Baily (1978) or a model with more periods as in Michelacci and Ruffo (2011) and Hairault et al. (2007). These papers find that because of the shorter time-horizon of older workers, benefits should be lower for older workers and more generous for younger workers.

The final aspect of unemployment insurance models is business cycle considerations. Current U.S. policy extends the duration, though not the replacement rate of unemployment benefits during economic downturns. The reason for extended benefits is to provide consumption smoothing to workers who may take longer to find a job during a recession. Alternatively, Mitman and Rabinovich (2012) find that unemployment benefits should be pro-cyclical, positively correlated with productivity and employment. In response to a negative productivity shock, the optimal level of benefits should increase at the time of the shock, but then quickly fall below the pre-recession level. The argument for the pro-cyclical benefits is that increasing the workers' value of being unemployed during a recession is the wrong policy to increase employment.

## 2.2 Review of Theoretical Models

This literature review categorizes the writings about unemployment policy into three distinct eras starting with early search models like McCall (1970). First, early papers try to estimate level of the optimal unemployment insurance replacement rate. These early papers explicitly modeled the trade-off between the moral hazard created by unemployment insurance and the lowered risk that the benefits provide. Second, researchers integrated other aspects of the labor market such as search effort. Integrating a more realistic view of the labor search process necessitated the introduction of a reservation wage into the model. Third, researchers became interested in the macroeconomic effects of unemployment insurance, studying such things as international comparisons of unemployment insurance systems and overall productivity gains that an economy can get with an efficient insurance system. It is in this third area that this dissertation aims to contribute to the literature.

### *2.2.1 Early Models*

An early labor search model was described in McCall (1970). In his model, an unemployed agent gets a job offer from a uniform wage distribution every period. The offer can be rejected and the worker remains unemployed the next period, or the offer can be accepted and the worker remains on that job forever. There is no job separation in this model. Workers are paid a fixed benefit when they are unemployed, but are not taxed while employed. McCall's model is not an actuarially fair unemployment system such as those that are standard in later models. What McCall's model does predict is that a reservation wage, the wage where the worker is indifferent between remaining unemployed for one more period and accepting the job that lasts forever, naturally arises out of the model. The job search is modeled to be sequential, the worker who rejects a job is offered another job in the next period. McCall's model does not have most of the features of more modern models such as search effort, search cost, savings, human capital, and time dependent benefits. McCall's model was also subject to the Diamond Paradox. In models where firms are perfectly informed, the wage offer by the firm will be lower than the search costs of the workers. Low offers would make workers not search. In practice workers do search and firms offer a range of wages, hence the paradox.

The first attempts at computing an optimal replacement rate for unemployment benefits were not undertaken until the late 1970s. Models by Baily (1978), Fleming (1978) and Shavell and Weiss (1979) contemporaneously tried to estimate a replacement rate. These three papers were the first to explicitly model the trade-off between higher unemployment rates and the reduced risk that is generated by providing unemployment insurance.

Baily (1978) presents a principal-agent problem of unemployment insurance. The two period model has the worker start out employed in the first period and then face some probability of job separation in the second. The worker can save in the first period to protect themselves against the possible loss in earnings if laid off in the second period. From this set-up, the Baily test is found. According to the Baily test, the optimal level of unemployment insurance benefits is set when the drop in risk adjusted consumption resulting from a loss in a job is equal to the elasticity of duration of unemployment, assuming an actuarially fair tax system. This paper is important because it introduces many aspects of the study of unemployment insurance. Savings, search effort, and an optimal rule for unemployment insurance were found. Baily's rule depends on some things that are not easily observable in data, most notably the elasticity of the duration of unemployment with respect to benefits. The difficulty in measuring this elasticity has led to try to mitigate this need, most recently Chetty (2006) and Shimer and Werning (2007).

Like Baily, Fleming (1978) analyzes the problem in an optimal taxation framework. He creates two models, one with perfect access to capital markets for the unemployed worker and one with no lending or borrowing. The argument here is that unemployed workers would not have access to capital markets as no lender would risk loaning an unemployed worker the funds. Fleming also extends the Baily model from two periods to infinite time. Shavell and Weiss (1979) depart from fixed levels of benefits throughout the unemployment spell and allow benefits to fluctuate. Their main finding is that benefits should be downward sloping during the unemployment spell if workers do not have savings which provides an incentive to look for a job.

These early papers highlight the trade-off between higher unemployment rates and the advantages of offering some workers protection against loss of income. They do not contain the features that are found in later models. The definition of optimal benefits requires concepts that are not easily measured in the data.

#### *2.2.2 More Complex Models*

After the simple models of the 1970s, research turned to more realistic models of the unemployment insurance system. Theorists studied other aspects of the job search process and drew conclusions about how unemployment benefits affect the behavior of unemployed workers. One important paper by Hopenhayn and Nicolini (1997) used the repeated principal-agent framework to compute the optimal insurance contract. Similar to Shavell and Weiss (1979), the optimal contract involves a replacement ratio that decreases throughout the unemployment spell. This result was true even when a more realistic model was used. Hopenhayn and Nicolini introduced unobservable search effort, and varying levels of search. The addition of unobserved search effort requires that there be an incentive compatibility constraint added to model so the agent chooses the optimal search effort. Fredriksson and Holmund (2001) show that a two-tier unemployment system is close to optimal, and confirm the result of Hopenhayn and Nicolini that benefits should decrease with longer unemployment durations.

Hansen and Imrohoroglu (1992) use a general equilibrium framework to model the effect of unemployment insurance for workers that are allowed to reject suitable jobs. The general equilibrium framework allows them to add in the trade-off between work and leisure that was not a part of early models. Davidson and Woodbury (1997) study the optimal amount of time that unemployment benefits should be paid, along with the shape of benefits. Wang and Willimanson (1996) study how the presence of unemployment insurance changes the in job behavior of employed workers. Shimer and Werning (2008) show that if job losers have access to a risk-free asset and have constant relative risk aversion preferences then the optimal policy is a constant benefit during unemployment and a constant tax during employment.

### *2.2.3 Macroeconomic Models*

In this section, papers that study macroeconomic outcomes of different unemployment insurance policies are discussed. Specifically, this section will summarize the papers that deal with human capital accumulation and match quality changes that affect an unemployed worker. First, two papers that come to different conclusions about the efficacy of generous unemployment benefits will be discussed. These two papers form the theoretical question that this dissertation is trying to answer. Second, more recent papers dealing with the match quality and human capital issues will be summarized. Finally, the model presented in this dissertation will be put into the context of previous models.

The two papers that form the theoretical basis for this dissertation are Ljungqvist and Sargent (1998) and Marimon and Zilibotti (1999). These two papers come to opposite conclusions about the economic consequences of generous unemployment benefits. Ljungqvist and Sargent (1998) claim that more generous benefits are bad for the economy because workers lose skill while they are unemployed, and therefore set their reservation wage too high and do not take job offers that they should. Marimon and Zilibotti (1999) claim that more generous unemployment benefits are good for the economy because they allow unemployed workers to sample more job matches, eventually finding a more productive match. Sorting out these contradicting forces is the aim of this dissertation.

The model that Ljungqvist and Sargent (1998) use is a partial equilibrium search model. The recursive nature of the problem puts the workers in one of three states, employed, unemployed and not eligible for benefits, or unemployed with benefit eligibility. Employed workers face a probability that they die and don't continue to the next period. They also face an exogenous probability that they lose their job. Employed workers pay a tax on their wages. Wages are based on the worker's skill level. If the worker remains employed, skill level increases. If the worker is separated from her job she becomes unemployed with benefits that are paid as a replacement rate of her previous wage. Benefit eligible workers pay a tax, and choose a search effort. The higher the search effort, the more likely benefit eligible workers are to get a job offer that they can accept or reject. There is dis-utility to searching. If they reject a job offer that the social planner deemed sufficiently high, the worker stays unemployed, but loses benefits. If the worker stays unemployed, skill level decreases going into the next period. Unemployed workers without benefits must choose a search effort, which increases the probability of getting a job. Benefit ineligible workers accept all jobs that are offered.

Marimon and Zilibotti (1999) present a search and matching model of unemployment insurance. In their model, workers and firms are distributed uniformly around a unit circle. Productivity is inversely related to the distance between the worker and the firm. There is a cost for firms to open a vacancy. Unemployed workers sample vacant jobs and can accept or reject them. The more jobs that unemployed workers sample, the more likely they are to match with a job that is productive. Higher benefits lead to better matches in this model which is welfare improving because of the productivity gains from sampling many jobs.

The two competing forces of match quality and human capital depreciation have been studied in individually and in combination. Pavoni (2009) shows that when there is human capital depreciation, the social planner must guarantee a minimum level of utility. If this minimum is not guaranteed, generous benefits will cause utility to become infinitely low. This phenomenon is seen in the simulations presented in this dissertation. The optimal contract may be a wage subsidy for long-term unemployed workers in Pavoni's model. Pollak (2008) presents a model with heterogeneous agents and studies the transitional dynamics of moving to an optimal unemployment system. Shimer and Werning (2006) present a model with skill deterioration and draw some conclusions about the optimal duration of unemployment benefits.

### 2.3 Numerical Model Problems

The complexity of models about unemployment insurance is often limited by how easily the model can be solved for real policy conclusions. The goal of most models is to find an optimal level of unemployment insurance given the assumptions made about the economy in the model. The complexity of the economy is limited by the numerical methods used to solve the problem. Most models are solved using numerical methods, such as value function iteration or log-linearizion. Models often do not have a closed form solution. The limitations of the numerical methods limit the complexity of the model, especially when value function iteration is the chosen method of solving the model. Value function iteration is done by solving the optimal behavior for each point in the state-space by creating a grid of those points. As the size of the grid grows, the number of calculations that must be performed increases.

The number of points in the state-space dictates the computational complexity of the model for value function iteration. The state-space is the set of values that describe an agent in the model. If all workers are the same except for their unemployment status then the state-space is two. Early models had small state-spaces, which made it so that the models could be solved by hand, without the help of computational power that was not available or expensive to implement. As more variables that describe the worker are added to the model, the state-space grows. For instance, when savings is added to a model with two states, the values of savings must be kept track of for both the employed and the unemployed group. Adding three levels of savings increases the state-space from two to six. Further addition of state variables leads to the curse of dimensionality. The curse of dimensionality is when the addition of a state variable increases the grid of values that must be solved for exponentially.

Lentz (2009) uses policy function iteration, a close alternative to value function iteration to solve his structural estimation. While his goal is to perform a structural estimation that is not attempted in this dissertation, the model limitations are similar except that Lentz's structural estimation requires a much finer solution set than is required here. In order to overcome the limits of computational power, he uses cubic spline projections to interpret the space in-between the discretized values that are solved for. Lentz does this because his preliminary experimentation revealed that the number of points in the state-space required to do his estimation where such that the computation time to solve the model was too long.

Other researchers limit the number of values that each state can take, thus limiting the grid size of the problem. For instance, Ljungqvist and Sargent (1998) have a state-space that is made up of values for employment status, search effort, previous wage and level of human capital. In order to get around the curse of dimensionality, they limit the size of the grid. For instance, they choose 21 different skill levels that a worker can have, and only two levels of search effort. Hopenhayn and Nicolini (1997) also limit the size of the grid, choosing only two levels of possible search effort in their simulation.

As computers become more powerful and cheaper, the limits on the complexity of models caused by the curse of dimensionality will decrease. There are two options available right now to speed up large optimization problems. First, the programs can be written so that calculations are done in parallel to take advantage of multiple processor cores at once. To implement parallel programming, code written to do serial calculations must be vectorized. Second, graphical processing units (GPUs) are much faster at floating-point mathematical operations than the central processing unit (CPU). Moving mathematical calculations from the main processor to the graphics card can increase the speed of solving a model with a large state-space.[1]

### 2.4 Some Results

In this section, a summary of results of unemployment insurance models will be summarized. There are four areas where results could be considered relevant to this dissertation. First, the optimal level of unemployment insurance is important for this dissertation because a main goal of this study is to see how competing effects influence the optimal rate. Second, the optimal shape of the benefit curve is important. Third, the optimal length that payments should be made to workers is summarized. Fourth, the macroeconomic effects of paying unemployment benefits is important to this research as it is something that will be tracked using the model results. This dissertation focuses on the optimal level of benefits and the macroeconomic outcome of applying those benefits. Shape and duration of benefit payments are not determined by the model solution, they are assumptions that are fed into the model.

The optimal replacement of wages is estimated to be between 20 and 85 percent of gross wages. This falls in line with actual replacement rates discussed in the next section. The low estimate of 20 percent was postulated by Fleming (1978) where unemployed workers have access to borrowing. Baily (1978) suggested that replacement rates should be around 65 percent. More recent estimates put the optimal

[1]For more information on setting up Matlab to run certain calculations on the GPU, see Aldrich et al. (2010)

unemployment replacement rate around 55 percent. Chetty (2006) shows that the optimal rate should be greater than 50 percent. Hopenhayn and Nicolini (1997) suggest that replacement rates should start around 85 percent and fall with additional weeks of unemployment.

Hopenhayn and Nicolini (1997) show that the replacement rate should fall during an unemployment spell. The system they designed would have the replacement rate fall every additional week that a worker is unemployed. Fredriksson and Holmund (2001) show that one drop in replacement rates during an unemployment spell is close to optimal, and is an easier policy to implement. The finding that replacement rates should fall during an unemployment spell is still in dispute. Shimer and Werning (2006) and (2008) show that when a planner is faces with a homogeneous labor force and constant absolute risk aversion utility function, the optimal benefit policy is a fixed replacement rate while unemployed and a fixed tax rate while employed. Away from that benchmark, the gains from implementing an optimal policy of decreasing benefits are small.

The optimal length of time that unemployment benefits should be paid out is an important policy consideration, but has received relatively less study than some other aspects of unemployment insurance. Davidson and Woodbury (1997) show that the optimal length should be infinite when an actuarially fair tax is placed on employed workers. No moral hazard is created by paying workers during unemployment if they are forced to pay a higher tax rate when employed. Wang and Willimanson (2002) compute that benefits should be lower for unlimited durations than for limited durations, 24 percent vs 60 percent.

The fact that higher unemployment insurance leads to higher unemployment rates is well established. Unemployment benefits provide an incentive for the unemployed to reject jobs, and not drop out of the labor force when they otherwise would. There are other macroeconomic effects that do not have a clear pattern. Ljungqvist and

Sargent (1998) show that with human capital depreciation, a generous unemployment insurance system has detrimental effects to the economy. Their simulations indicate that GNP per capita, worker productivity, average skill level of workers, and lifetime consumption all decrease, while the unemployment rate and duration of unemployment increase in a system with more generous unemployment benefits. Marimon and Zilibotti (1999) show that in a model with match quality, the average productivity per worker increases under a more generous unemployment system.

## 2.5 Real World Patterns

Real world considerations for the design of unemployment insurance systems include the conditions that must be met to qualify for unemployment insurance, the duration that benefits are paid, whether or not those benefits fall during an unemployment spell, and the base wage that benefits are paid on. Most OECD countries have an unemployment compensation system that is compulsory for workers, but a few have systems that are voluntary and are augmented by an unemployment assistance program. Countries with voluntary programs include Denmark, Finland, and Sweden. Table 2.1 summarizes the real world considerations of unemployment benefits.

Table 2.1: A comparison of benefit policies in selected OECD countries with compulsory insurance systems.

| Country | Employment Conditions | Contribution Conditions | Maximum Duration | Initial Replacement Rate | Ending Replacement Rate | Replacement Wage Base |
|---|---|---|---|---|---|---|
| Austria | Yes | Yes | 9 months | 55 | 55 | Net |
| Belgium | Yes | Yes | Unlimited | 60 | 50 | Gross |
| Canada | Yes | Yes | 8 months | 55 | 55 | Gross |
| Czech Republic | Yes | Yes | 6 months | 50 | 45 | Net |
| France | No | Yes | 23 months | 57-75 | 57-75 | Gross |
| Germany | Yes | No | 12 months | 60 | 60 | Net |
| Greece | Yes | Yes | 12 months | 19 | 19 | Gross |
| Italy | No | Yes | 7 months | 50 | 40 | Gross |
| Japan | Yes | Yes | 9 months | 50-80 | 50-80 | Gross |
| Netherlands | Yes | Yes | 38 months | 75 | 70 | Gross |
| Spain | Yes | No | 24 months | 70 | 60 | Gross |
| United Kingdom | No | Yes | 6 months | Fixed | Fixed | n/a |
| United States | Yes | No | 6 months | 53 | 53 | Gross |

Source: OECD 2007

The countries listed in table 2.1 all have compulsory insurance for unemployed workers. The compulsory insurance is the standard type of insurance where workers pay a tax while employed that finances benefits for the unemployed. All of the countries have some sort of conditions that must be met in order to qualify for benefit. These conditions can be employment conditions or contribution conditions. Employment conditions are the conditions that an unemployed worker must have worked a certain amount of time before becoming unemployed in order be eligible for benefits. Contribution conditions require the unemployed worker to have made a certain amount of payments into the system to be eligible for benefits. In many places self-employed workers and contract workers do not contribute to the unemployment insurance system, and therefore are not eligible for unemployment payments.

The length of time that benefits are paid out also varies greatly between countries. Belgium pays out benefits to unemployed workers in perpetuity. The United States, United Kingdom, and the Czech Republic pay out benefits for the shortest amount of time, 6 months. In most countries, there are extensions of benefits that kick in beyond the maximum allowed duration when the economy is bad. These extension are usually triggered by the unemployment rate going over a certain threshold.

The way that unemployment benefits are paid is different across countries. Most pay a replacement rate based on the previous gross wage that a worker was earning before the job separation. A few countries pay out a replacement of net wages, which translates into

less generous benefit in those countries. The United Kingdom is unique in that it pays out a fixed wage based on the average wage in the country. Some countries, like Czech Republic, Italy, and the Netherlands reduce benefit payouts during the unemployment spell. This reduction of payments follows the prescription of Hopenhayn and Nicolini (1997) among others. Most countries do not reduce benefits during the unemployment spell, a policy that Shimer and Werning (2008) conclude is close to optimal.

One other aspect of unemployment insurance that might prove fruitful for future research is the payout of benefits to workers who are new to the labor force. Those workers may include recent graduates or workers who were not in the labor force. Belgium is one such country where such a program exists. Belgium has an unemployment assistance program that pays a lump sum to workers making the transition from school to the labor force. The efficacy of such a system hinges on the job matching effect. If those eligible for the program use the money to find a better job, efficiency gains may offset the cost to the government. If these workers merely delay entry into the labor force without finding a more productive job, then the system is not effective.

# CHAPTER 3

# THEORETICAL MODEL

The model consists of a continuum of agents of unit mass. Agents can be in one of two labor force states: employed or unemployed. Agents move between the employed and unemployed states through time.

Employed agents are described by the amount of wealth, human capital, and the match quality for the job currently held. The job pays the employed agent a wage based on human capital of the agent and match quality of the job. The employed agent pays a proportional tax that funds the unemployment compensation program. The employed agent chooses how much to consume and how much savings to take into the next period. Employed agents face an exogenous risk of losing the job and becoming unemployed. Agents who remain employed may experience an increase in human capital.

Unemployed agents are described by the amount of human capital and wealth, as well as the ending wage of the job held before the unemployment spell. Unemployed agents are paid a benefit that is a replacement rate of the wage of the previous job. Unemployed agents choose how much to consume, how much to save, and how much effort to search for a job. Search is costly to the unemployed agent. A higher search effort leads to a greater probability that a job will be offered. If a job is offered, the unemployed agent must choose whether to accept or reject the job. If the offer is accepted, the agent moves to the employed state. If the offer is rejected or no job is offered, there is a chance that human capital will decrease and the agent remains in the unemployed state.

### 3.1 Model Specification

The model is comprised of a unit mass of agents, $i$, who are ex-ante identical. Each period, with probability $\alpha$, the agent dies and is replaced by another agent. The mass of agents in the model remains constant. Each agent in the model is in one of two labor force states, employed or unemployed. The decisions of the employed and unemployed agents are slightly different.

Agent $i$ who is employed has state variables for the amount of wealth, $k_{i,t}$, the amount of human capital accumulated, $h_{i,t}$, and the match quality to the job currently held, $\theta_{i,t}$. Agent $i$ who is employed receives wage $w_{i,t}$ and pays a tax on income, $\tau$, that funds the unemployment benefit system. The employed agent $i$ chooses the amount of wealth to take into the next period, $k_{i,t+1}$, and the amount to consume, $c_{i,t}$. The employed agent $i$ faces an exogenous probability, $\delta$, of being separated from the job. The employed wage, $w_{i,t}$ is based on the amount of human capital, $h_{i,t}$, and match quality with the current job, $\theta_{i,t}$, specifically:

$$w_{it} = \theta_{it} h_{it}. \tag{3.1}$$

The job is held until exogenous separation occurs. Exogenous separation occurs with probability $\delta$. Quits are not allowed. If an employed agent $i$ is not separated from the job, human capital increases with probability $\pi_e$ by a fixed amount $\Delta h_e$, unless $h_{i,t}$ is already at the maximum, in which case the increase is zero. This process is denoted $g_e(h_{i,t})$.

Agent $i$ who is unemployed is described by state variables for wealth, $k_{i,t}$, human capital, $h_{i,t}$, and the ending wage of the previous job, $\hat{w}_{i,t}$. The agent $i$ that is unemployed receives an unemployment benefit. The benefit paid is $b\hat{w}_{i,t}$, where b is a replacement rate for all people. It is a choice variable for the government. The agent must choose how much to consume, $c_{i,t}$, how much wealth to take into the next period, $k_{i,t+1}$, and how much effort to expend searching for a job, $s_{i,t}$. Searching for a job is costly for the agent. The search cost is determined by the function $e(s_{i,t})$, which is increasing in $s_t$. The probability that a job is offered, $\pi(s_{it})$, is increasing in $s_{it}$. The job offer is a match quality, $\theta_{i,t}$, drawn from the distribution $\Gamma_\theta$. A job offer can be accepted or rejected by the agent. If that agent rejects the job offer, or does not receive a job offer, then three things happen. First, the agent enters the unemployed state. Second, with probability $\pi_u$, human capital either declines by $\Delta h_u$ or, if $h_{it}$ is already at the minimum possible value, remains constant. This process is denoted $g_u(h_{i,t})$. With probability $(1 - \pi_u)$, it also remains constant. Third, the unemployment benefit is paid according to the replacement ration $b$ and the most recently held wage, $\hat{w}_{i,t}$.

Death, which occurs with probability $\alpha$, occurs for agents immediately after the consumption and savings decisions and after the search effort decision. An agent that dies is replaced in the continuum by an unemployed agent with the lowest level of previous wage, wealth, and match quality. Assets from dead agents do not go to the government. Wealth, $k_{i,t}$, earns interest rate, $r$. Agents seek to maximize the expected value of a discounted infinite sum of utility:

$$E \sum_{t=0}^{\infty} \frac{(1-\alpha)^t}{(1+\rho)^t} [u(c_{i,t}) - e(s_{i,t}) - n_{i,t} \cdot d]. \tag{3.2}$$

where $n_{i,t}$ is an employment indicator that is equal to 1 when the worker is employed and 0 when the worker is not employed. The disutility of work is denoted $d$ and $\rho$ is the subjective time rate of preference. The maximization is subject to the following constraints:

$$k_{i,t+1} = (1+r)k_{i,t} + (1-\tau)(w_{i,t} \cdot n_{i,t}) + b \cdot w_{i,t} \cdot (1-n_{i,t}) - c_{i,t} \tag{3.3}$$

$$g_e(h_{i,t}) = \begin{cases} \Delta h_e & \text{if } h_{i,t} < h_{\max} \\ 0 & \text{if } h_{i,t} = h_{\max} \end{cases} \tag{3.4}$$

$$g_u(h_{i,t}) = \begin{cases} \Delta h_u & \text{if } h_{i,t} < h_{\min} \\ 0 & \text{if } h_{i,t} = h_{\min} \end{cases} \tag{3.5}$$

$$h_{i,t+1} = \begin{cases} h_{i,t} + g_e(h_{i,t}) \text{ with probability } \pi_e \text{ if employed} \\ h_{i,t} \text{ with probability } 1-\pi_e \text{ if employed} \end{cases} \tag{3.6}$$

$$h_{i,t+1} = \begin{cases} h_{i,t} + g_u(h_{i,t}) \text{ with probability } \pi_u \text{ if unemployed} \\ h_{i,t} \text{ with probability } 1-\pi_u \text{ if unemployed} \end{cases} \tag{3.7}$$

$$k_{i,t} \geq 0 \tag{3.8}$$

$$c_{i,t} \geq 0 \tag{3.9}$$

$$s_{i,t} \in [0,1] \tag{3.10}$$

$$n_{i,t} \in \{0,1\} \tag{3.11}$$

$$\theta_{i,t+1} = \theta_{i,t-1} \text{ if } n_{i,t} = 1 \text{ and } n_{i,t-1} = 1 \tag{3.12}$$

$$\theta_{i,t+1} \text{ drawn from: } \Gamma(\theta_t) \text{ if } n_{i,t} = 0 \text{ and } n_{i,t+1} = 1 \tag{3.13}$$

$$\Pr(n_{t+1} = 1 | n_t = 1) = 1 - \delta, \Pr(n_{t+1} = 0 | n_t = 1) = \delta \tag{3.14}$$

$$\Pr(n_{t+1} = 1 | n_t = 0) = \pi(s_t) \cdot \Pr(\theta_{i,t+1} \geq \theta^R(k_{i,t+1}, h_t, \hat{w}_{i,t})) \tag{3.15}$$

$$\Pr(n_{t+1} = 0 | n_t = 0) = 1 - \Pr(n_{i,t+1} = 1 | n_{i,t} = 0) \tag{3.16}$$

$$w_{i,t} = \theta_{i,t} h_{i,t} \tag{3.17}$$

$$\hat{w}_{i,t} = w_{i,t-x} \text{ for the smallest x such that } n_{i,t-x} = 1 \text{ and } n_{i,t-x+1} = 0. \tag{3.18}$$

## 3.2 Recursive Specification

The problem can be written recursively with values functions for the employed agents, $V_E(k_t, h_t, \theta_t)$, and for the unemployed agents, $V_U(k_t, h_t, \hat{w}_t)$. The recursive formulation is what is programmed in to the computer to solve the optimization problem. The recursive specification also makes the order of events in the model more clear.

### *3.2.1 Employed Value Function*

Figure 3.1 shows the decisions of the workers as events positioned below the line, and events that occur because of nature positioned above the lines. The figure starts at time $t$ and shows the order of events up to time $t+1$. The first event that comes from nature is that the employed worker is paid a wage. The income received is $w_t$ which is equal to the level of human capital times the match quality to the job, $h_t\theta_t$. Next, the worker chooses how much to consume, $c_t$ and how much to save, $k_{t+1}$. These decisions are made simultaneously by the worker and are the only decisions that are made in the employed state. Next, the worker faces a probability of death, $\alpha$. In the case of death, the worker receives utility $V_0 = 0$. If the worker remains alive, the worker faces an exogenous probability $\delta$ that the job currently held will be lost. If the job is lost, the worker retains the current level of human capital, $h_t$, and starts the next period unemployed. The value function for the worker who loses the job is $V_U(k_{t+1}, h_t, \theta_t h_t)$, where $\theta_t h_t$ is the wage, $w_t$, that is used to determine the amount of benefits paid out in the next period. If the worker does not lose the job, which occurs with probability $1-\delta$, human capital in the next period might increase. The probability that human capital increases is $\pi_e$. Human capital increases according to the function $g_e(h_t)$. The continuation utility for the worker who does not lose employment is $\mathbb{E}_t[V_E(k_{t+1}, h_{t+1}, \theta_t)]$.

The value function that corresponds to the figure is displayed in equation 3.19.

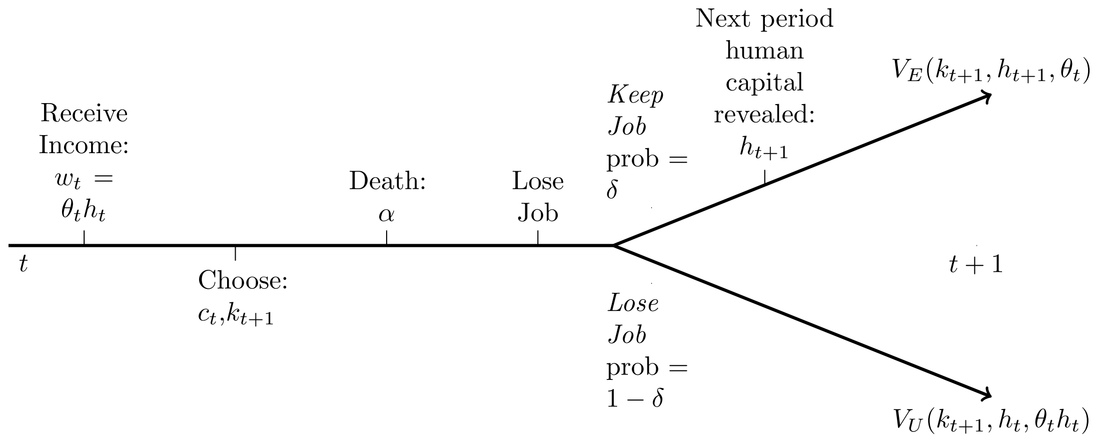


Figure 3.1: The order of the decision (below the line) and how nature affects the worker (above the line) of the employed worker between time $t$ and the time $t + 1$.

The employed worker has state variables for the amount of wealth, $k_t$, the amount of human capital, $h_t$, and the match quality to the current job, $\theta_t$. The worker maximizes over choice of consumption $c_t$ and savings $k_{t+1}$. In the current period, the worker receives utility based on consumption, $u(c_t)$. If the worker stays alive, which occurs with probability $(1-\alpha)$, the worker discounts the next period by $\frac{1}{1+\rho}$. With probability $\delta$, the worker loses the job and gets the utility of being unemployed, $V_U(k_{t+1}, h_t, \theta_t h_t)$. With probability $(1 - \delta)$, the worker keeps the job. The Bellman equation for the worker who is employed at the beginning of the period is: $\mathbb{E}_t [V_E(k_{t+1}, h_{t+1}, \theta_t)]$.

$$\begin{aligned} V_E(k_t, h_t, \theta_t) = \max_{\{c_t, k_{t+1}\}} \{ & u(c_t) + (1-\alpha)\frac{1}{1+\rho} \\ & [\delta V_U(k_{t+1}, h_t, \theta_t h_t) + (1-\delta)\mathbb{E}_{h_t}[V_E(k_{t+1}, h_{t+1}, \theta_t)]]\} \\ & \text{subject to (3.22) - (3.29)} \end{aligned} \tag{3.19}$$

For the worker who remains unemployed, human capital might increase. The expectation on human capital is as follows:

$$\mathbb{E}_{h_t}[V_E(k_{t+1}, h_{t+1}, \theta_t)] = (1-\pi_e)V_E(k_{t+1}, h_t, \theta_t) + \pi_e V_E(k_{t+1}, h_t + g_e(h_t), \theta_t) \tag{3.20}$$

With probability $(1 - \pi_e)$, human capital does not change and the worker carries the current level of human capital into the next period. With probability $\pi_e$, human capital changes. Human capital changes according to the function $g_e(h_t)$, which maps the current period human capital to the change in human capital for the next period.

### *3.2.2 Unemployed Value Function*

The unemployed workers problem is more complex. Figure 3.2 shows the order of decisions and events for the unemployed worker from time $t$ to time $t+1$. First, the unemployed worker receives a benefit that is a replacement rate of the wage held at the previous job, $b\hat{w}_t$. The unemployed worker then simultaneously chooses how much to consume this period, $c_t$, and how much savings to take into the next period, $k_{t+1}$. Next, the unemployed worker chooses how much effort to expend looking for a job, $s_t$. With probability $\alpha$, the worker dies and receives $V_0 = 0$ utility. Next, a job may be offered. The probability of a job offer, $\pi(\lambda s_t)$ depends on the search effort chosen. If a job is not offered, the worker remains unemployed. Human capital for the worker that remains unemployed might decrease according the function $g_u(h_t)$. The value of remaining unemployed is $V_U(k_{t+1}, h_{t+1}, \hat{w}_t)$.

For workers who receive a job offer, a match quality, $\theta_{t+1}$ is drawn from the distribution $\Gamma(\theta_{t+1})$. The worker then chooses to accept or reject that match quality offer based on a reservation match quality. The worker accepts jobs that are greater than or equal to a reservation match quality, $\theta^R(k_{t+1}, h_t, \hat{w}_t)$. The worker is assumed to use the reservation strategy as optimal, this will be proven later. If the job is accepted, $\theta_{t+1} \geq \theta^R(k_{t+1}, h_t, \hat{w}_t)$, then the worker receives the value of accepting the job, $V_A(k_{t+1}, h_t, \theta_{t+1}) = V_E(k_{t+1}, h_t, \theta_{t+1})$. If the job offer is rejected, $\theta_{t+1} < \theta^R(k_{t+1}, h_t, \hat{w}_t)$, then the worker remains unemployed. Human capital might decrease according to the function $g_u(h_t)$. The worker gets the value of rejecting the job, $V_R(k_{t+1}, h_t, \hat{w}_t) = \mathbb{E}_t[V_U(k_{t+1}, h_{t+1}, \hat{w}_t)]$.

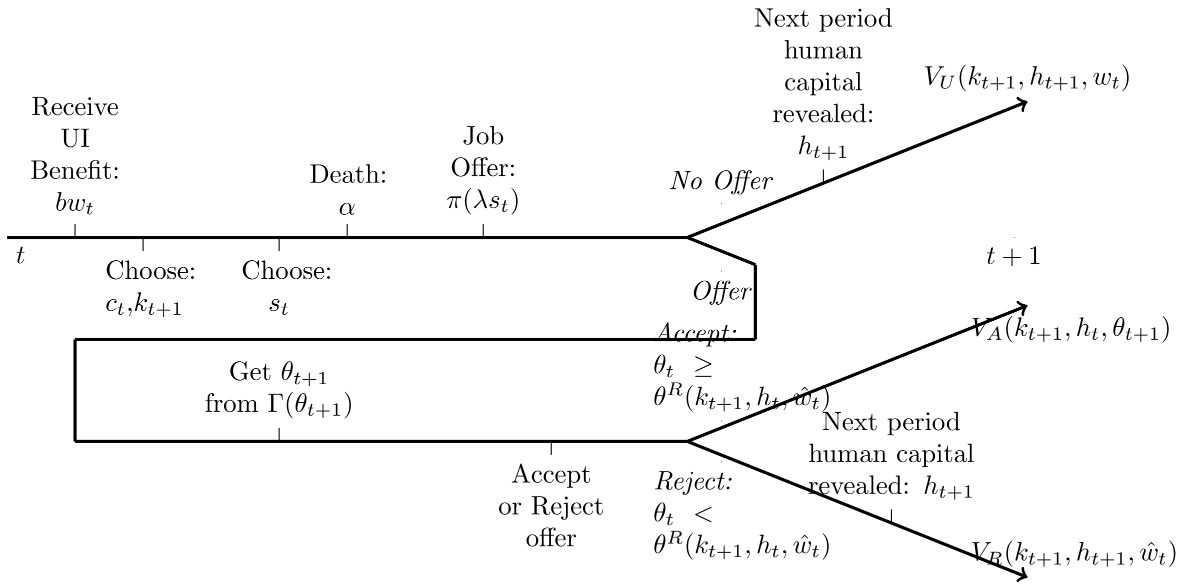


Figure 3.2: The order of the decision (below the line) and how nature affects the worker (above the line) of the unemployed worker between time $t$ and the time $t+1$.

Equation 3.21 is the Bellman equation for unemployed worker. The unemployed worker has state variables for wealth, $k_t$, human capital, $h_t$, and the wage of the previous job, $\hat{w}_t$. The unemployed worker maximizes over choice of consumption, $c_t$, savings, $k_{t+1}$, and search effort, $s_t$. In the current period the unemployed worker gains utility from consumption, $u(c_t)$, and has disutility in search, $e(s_t)$. With probability $\alpha$, the worker dies and receives $V_0 = 0$. With probability $(1-\alpha)$, the worker does not die and moves into the next period which is discounted by subjective time rate of preference, $\rho$. Search effort, $s_t$, and the base arrival rate, $\lambda$, are inputs to the function $\pi(\lambda s)$, which determines the probability of receiving a job offer. With probability $\pi(\lambda s)$, the worker receives a job offer which is drawn from the cumulative distribution of $\Gamma(\theta_t)$. Offers greater than or equation the reservation match quality $\theta^R(k_{i,t+1}, h_t, \hat{w})$ are accepted. Offers less that $\theta^R(k_{i,t+1}, h_t, \hat{w})$ are rejected. $V_A(k_{t+1}, h_t, \theta_{t+1})$ is the value of accepting a job, $V_R(k_{t+1}, h_t, \hat{w}_t)$ is the value of rejecting the job. With probability $(1-\pi(\lambda s_t))$, the worker does not receive a job offer and goes into the next period unemployed with human capital that may have decreased.

$$\begin{aligned}
V_U(k_t, h_t, \hat{w}_t) = \max_{\{c_t, k_{t+1}, s_t\}} \{ & u(c_t) - e(s_t) + (1-\alpha)\frac{1}{1+\rho}\Big[ \\
& \pi(\lambda s_t) \int_{\theta_{t+1}\in\Theta} \max\{V_A(k_{t+1}, h_t, \theta_{t+1}), V_R(k_{t+1}, h_t, \hat{w}_t)\} d\Gamma(\theta_{t+1}) \\
& + (1-\pi(\lambda s_t))\mathbb{E}_{h_t}[V_U(k_{t+1}, h_{t+1}, \hat{w}_t)]\Big]\} \\
& \text{subject to (3.22) - (3.29)}
\end{aligned} \tag{3.21}$$

The value of rejecting the job offer is expectation over human capital $h_t$ of remaining unemployed. The value of accepting the job is the value of being employed in the next period with $k_{t+1}$, $h_t$ human capital and $\theta_{t+1}$ match quality.

$$V_R(k_{t+1}, h_t, \hat{w}_t) = \mathbb{E}_{h_t}[V_U(k_{t+1}, h_{t+1}, \hat{w}_t)] \tag{3.22}$$

$$V_A(k_{t+1}, h_t, \theta_{t+1}) = V_E(k_{t+1}, h_t, \theta_{t+1}) \tag{3.23}$$

Expectation of human capital in the next period, $h_{t+1}$, is related to $h_t$ in two ways. First, there is a probability that human capital will decrease, $\pi_u$. If human capital does decrease, the change in human capital is determined by the function $g_u(h_t)$. The function $g_u(h_t)$ states that human capital decreases by some amount until a lower bound is reached. At that lower bound, there is no more decrease in human capital from the worker staying unemployed.

$$\mathbb{E}_{h_t}[V_U(k_{t+1}, h_{t+1}, \hat{w}_t)] = (1-\pi_u)V_U(k_{t+1}, h_t, \hat{w}_t) + \pi_u V_U(k_{t+1}, h_t + g_u(h_t), \hat{w}_t) \tag{3.24}$$

The reservation match quality strategy is optimal because $V_A(k_{t+1}, h_t, \theta_{t+1})$ is increasing in $\theta^R(k_{i,t+1}, h_t, \hat{w}_t)$, and $V_R(k_{t+1}, h_t, \hat{w}_t)$ is constant.

### *3.2.3 Constraints*

The constraints for both value functions are displayed in equations (3.25)-(3.32).

$$k_{t+1} = (1+r)k_t + (1-\tau)(w_t)n_t + (1-n)b\hat{w}_t - c_t \tag{3.25}$$

$$g_e(h_t) = \begin{cases} \Delta h_e & \text{if } h_t < h_{\max} \\ 0 & \text{if } h_t = h_{\max} \end{cases} \tag{3.26}$$

$$g_u(h_t) = \begin{cases} \Delta h_u & \text{if } h_t < h_{\min} \\ 0 & \text{if } h_t = h_{\min} \end{cases} \tag{3.27}$$

$$w_t = \theta_t h_t \tag{3.28}$$

$$c_{i,t} \geq 0 \tag{3.29}$$

$$k_i, t \geq 0 \tag{3.30}$$

$$s_{i,t} \in [0,1] \tag{3.31}$$

$$n_t \in \{0,1\} \tag{3.32}$$

### 3.3 Budget Balance and Optimal Benefits

The budget balance is such that the present value of benefits must equal the present value of taxes. The economy is started at an initial position and the behavior of the agents is used to move the economy through time until a steady state is reached. When the budget is in balance, the value of all benefits paid out over time is equal to the value of taxes received in terms of present value. Calculating the budget balance over time takes into account the full transitional dynamics that are influenced by the benefit scheme. Taking into account the transitional dynamics avoids a downward bias in the optimal benefit rate that occurs when just the steady-state utilities are compared.[1] This method of finding the optimal level of unemployment benefits assumes that the social planner is able to commit to the benefit scheme even though

[1] See Joseph and Weitzenblum (2003) for a discussion of downward bias that arises from comparing steady states instead of transitional dynamics.

the states of agents in the economy are changing. The choice of starting point, while arbitrary, does influence the steady state found in the end. Choosing a starting point with a higher or lower unemployment rate will change the long term budget balance.

Let $q_t^e(k, h, \theta)$ represent the mass of employed agents at each possible combination of human capital and match quality at time $t$, and $q_t^u(k, h, \hat{w})$ be the mass of unemployed agents at each possible human capital previous wage at time $t$. For one period, tax revenue from employed workers is:

$$TR = \tau \sum_k \sum_h \sum_\theta q^e(k, h, \theta) \cdot h\theta. \tag{3.33}$$

The benefits paid by the government are:

$$TO = b \sum_k \sum_h \sum_w q^u(k, h, \hat{w}) \cdot \hat{w}. \tag{3.34}$$

The actuarially fair tax system requires that the present value of tax revenue and benefit outlays to be equal. The structure of this optimization means that over a lifetime, an agent may receive more in benefits than was paid in taxes. The following condition must be satisfied:

$$\sum_{t=0}^{\infty} \left[\frac{1}{1+r}\right]^t [TR_t - TO_t] = 0 \tag{3.35}$$

The optimal benefit level is the benefit level that maximizes the expected utility of all the agents at the ergodic steady state. The optimal benefit-tax pair, $(b, \tau)$, is the one that maximizes:

$$S(b, \tau) = \sum_k \sum_h \sum_\theta \sum_w \{V_e(k_t, h_t, \theta_t | b, \tau) q_0^e + Vu(k_t, h_t, \hat{w}_t | b, \tau) q_0^u\} \tag{3.36}$$

subject to 3.35 and the condition that all agents optimize each period.

# CHAPTER 4

# SOLUTION TECHNIQUE

The solution to the dynamic optimization involves solving for the agents' behavior using value function iteration and then simulating the economy through time to find out the tax rate that balances a budget given a benefit rate. A range of benefit tax pairs are computed and then compared against each other to find the optimal rate of benefits. A second analysis that relaxes the budget balance requirement eases the computation burden by not requiring that a budget balancing tax rate be found.

## 4.1 Solution Discussion

The first step to program the value functions into the computer, is vectorizing all of the state and choice variables in the model. Vectorizing the variables gives the computer a finite set of possibilities that it will loop over to solve the value functions. The variables for wealth, human capital, match quality, previous wage, and search effort have to be vectorized. Once the vectorization is done, the model is solved by value function iteration. Value function iteration is a solution technique where the Bellman equations are estimated by guessing a solution and then checking to see if substituting the guess into the value functions is the correct answer. If the answer is not correct, the guess is updated and the value functions are checked until they converge. It is necessary to re-iterate the value function when the policy variables of benefit replacement rate and tax rate are adjusted.

The solution to the optimal unemployment insurance rate is done in two parts. First, the computer program chooses a benefit level and then loops through a vector of successively higher tax rates to find the budget balancing tax rate. The budget balancing tax rate is found by interpolating between the values of the tax rate when the first positive budget balance is found. After the budget balancing tax rate is interpolated, the model is run again using that interpolated tax rate to find the characteristics of the economy with the chosen benefit level. These two steps are repeated for each benefit level chosen to solve for. When all benefit levels are solved, the steady states of the economy for each benefit level are compared and graphed.

Figure 4.1 displays a flowchart of how the model is solved. Starting at the top of the figure, the first row of the flowchart shows what must be defined by the user in order for the computer model to have enough information for a solution. The parameters must be defined from values obtained by calibration. The vectors for human capital, match quality, wealth, and search effort must next be defined. The model depends on a finite number of elements in these sets and solves for all possible solutions using a grid method. Next, moving right on the flowchart, the functional forms for utility, cost of search and probability of finding an job must be defined. Next, the different levels of benefits that the model will loop through are defined. The last piece of information that has to be defined is the possible levels of taxes that the model will test to see if the budget balances. The vector of taxes must be defined such that the lowest tax level will result in a budget deficit and the highest must result in a budget surplus for all chosen benefit levels. This guarantees that the budget balancing tax rate can be interpolated from the possible tax levels that are defined.

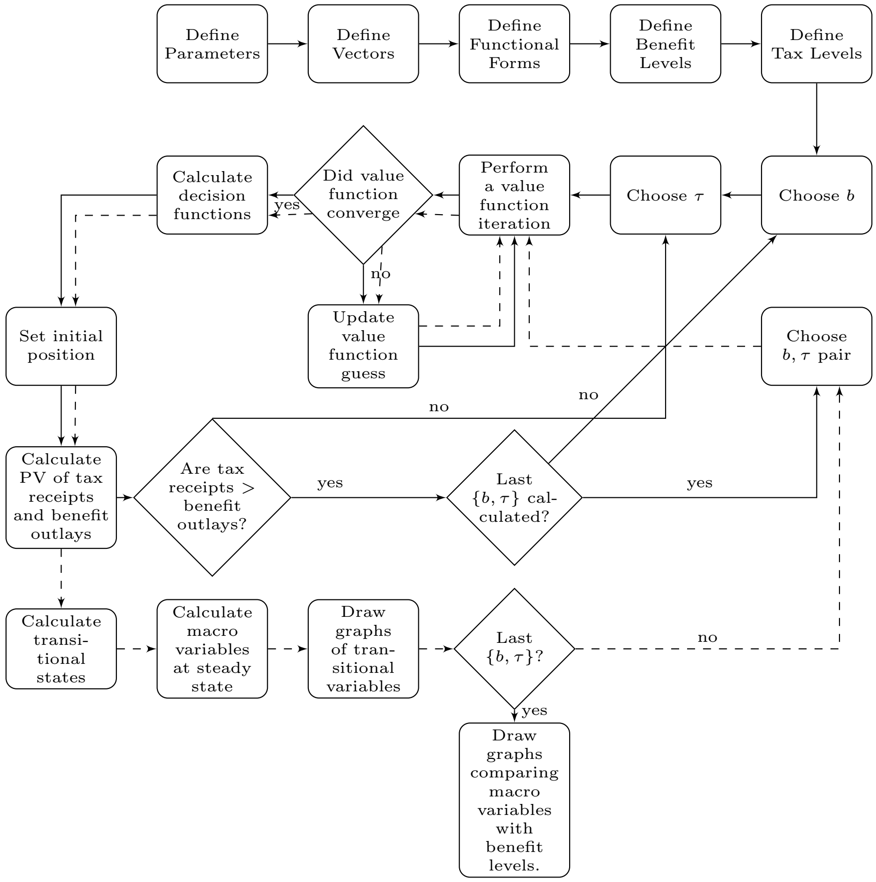


Figure 4.1: A flowchart of the solution algorithm implemented in Matlab. The solid lines indicate the first run through the model where the computer finds the budget balancing tax rate. The dotted lines refer to the second run through the model using the interpolated tax rate found in the first run. The second run is use to find the behavior of the agents.

On the second row of the flowchart in figure 4.1, the computer starts the optimization problem. User input is not needed after this step. The computer chooses the first benefit level $b$ and the first tax rate $\tau$ and performs the value function iteration until the value functions converge. When the value function converges, the decision functions for the employed and unemployed worker are calculated. The decision matrices store the behavior of the agent at each position in the value matrices.

Following the solid line around the flowchart, the program next sets an initial position for the economy. The initial position chosen has the probability that the agent is employed equal to one and has the probability that the agent is at any node in the employed value matrix equal to 1 divided by the number of nodes in the matrix. Once the initial position is set, the present value of tax receipts and benefit outlays for the current benefit and tax pair is calculated. If receipts are not greater than outlays, a deficit still exists and the next tax rate is chosen and the process is started over again. Because changing the tax rate might change the behavior of the agent at different nodes in the value functions, the value function iteration has to be re-run to calculate the new decision of the agents. Once a budget balancing tax rate is found, the program sets out to find the budget balancing tax rate for the next benefit value in the set of possible benefits.

Once the budget balancing tax rate for all the chosen benefit levels is found, the model is run another time for each $\{b, \tau\}$ pair. The results of the model using these pairs are compared to find the optimal unemployment replacement rate and the characteristics of the economy. In the flowchart in figure 4.1, the dotted lines indicate the second round of the solution process for the budget balancing benefit and tax pairs. Once the decision functions are calculated, the program finds the steady state for variables in the economy and draws transitional and comparison graphs. This process is repeated until the last $\{b, \tau\}$ pair is calculated. When the final benefit-tax pair is found, the computer draws graphs comparing the steady states of all the benefit-tax pairs.

## 4.2 Matlab Code Discussion

The matlab code that solves the optimization problem is listed in Appendix A. Table 4.1 lists the programs that are listed in the appendix[1].

[1]The programs that graph the results were omitted from the appendix because they are not necessary to solve the model.

Table 4.1: The list of programs that solve the model. The `main.m` file calls all other functions.

| Program Name | Function |
|---|---|
| `main.m` | Main program that calls all other programs |
| `value_iter.m` | Performsp value function iteration and creates decision functions |
| `initial_position.m` | Creates initial position matrices |
| `law_of_motion.m` | Changes position matrices based on model structure. |
| `present_value.m` | Calculates present value of taxes and benefits. |
| `graph_decision.m` | Graphs 3-d decision functions. |
| `graph_transition.m` | Graphs transition over time. |
| `graph_macro.m` | Graphs benefits and macro variables. |

The Matlab code is written in a modular format so that different parts of code can be reused. Each file listed in table 4.1 is a function that takes certain values and returns others. All the functions are called by running the `main.m` file. The `main.m` file is where the parameters for the model are defined. The parameters are defined as global variables to minimize the number of arguments that have to be passed to each function. No files other than `main.m` need to be changed in order to run different simulation.

After the parameters are defined at the top of the `main.m` file, the solution is started. The files that the `main.m` file call run different parts of the solution. The `main.m` file runs through the solution twice, once to get the budget balancing tax rate through grid search and interpolation and once to solve the model with each benefit and tax pair. The program `value_iter.m` solves the value of being at each node in the value matrices for the employed and unemployed agent. This program also returns the decisions that the agent would make at each point in the value matrices. At this point in the process, the decisions and value of each point is known, but the overall behavior of the economy is not known because the mass at each node is not known. To know the overall behavior in the economy, a simulation must be performed. The program `initial_position.m` declares a distribution of workers across each state. The program assumes that the mass of agents in unemployed states is zero, and that agents are evenly distributed among the employed states.

After the initial position is set, the present value of being in that position is tabulated. This is done by starting the simulating of the economy and computing a present value. The file `law_of_motion.m` takes the current distribution of the agent over the nodes of the value matrices and uses the decision functions to return the distribution of the agent in the next period. This function is repeated many times when calculating present value. The `present_value.m` file returns the present value to the `main.m` file which then checks the budget balance. If the budget balance has been satisfied and all benefit tax pairs have been solved for, the graph programs are called. There are three graph programs, `graph_decision.m`, `graph_transition.m` and `graph_macro.m`. The `graph_decision.m` produces three dimensional graphs of the decision functions that are useful to show the decisions of agents at different states. These graphs show individuals' behavior in the economy. The `graph_transition.m` file graphs how each variable in the model evolves over time, starting with the initial position. These graphs are an important diagnostic tool to make sure that the steady state is being found by the program. The file `graph_macro.m` compares the steady state macroeconomic variables for each benefit level. These graphs are useful for finding the optimal unemployment benefit replacement rate and also for comparing the effect of different benefits levels on the economy.

## 4.3 Non-Budget Balance Analysis

For the analysis where the budget is not required to balance, the programs are modified slightly. Instead of looping through different tax rates for a given benefit level until the budget balancing tax rate is found, the program solves the model for each benefit and tax pair and records the results. The second step of re-solving the model with the interpolated tax rate is not necessary. Different graphing routines are call in the non-budget balance analysis that compare steady-state economic variables for different tax levels.

# CHAPTER 5

# CALIBRATION

In this dissertation, three variables are of central importance: $\mu_u$, the rate of human capital depreciation for non-working individuals, $\mu_e$, the rate of human capital appreciation for working individuals, and $\sigma_\theta$, the standard deviation of match quality offers. In general, with higher levels of $\mu_u$, being out of work is more costly to the worker and to society and suggests that incentives should be designed to ensure that unemployment spells are short. A high value for $\sigma_\theta$ indicates a high variation in job offers and hence a higher value to waiting. Incentives in this case should be designed to allow for longer unemployment spells. The impact of $\mu_e$ is ambiguous. On one hand, a higher level of $\mu_e$ makes it easier to recover lost human capital and hence the cost of losing human capital while unemployed is lower. This suggests that incentives should be designed to allow for longer unemployment. On the other hand, a higher level of $\mu_e$ means that those worker are gaining more human capital, so incentives should be designed to expedite entry into employment.

Because of the importance of these three parameters, three tacts are taken in obtaining values for them. The first is to create a calibration model and estimate it with a large data-set. The second is to draw upon values used by other authors in previously published research. The third approach is to vary the values used in set of simulations and thereby allow for deficiencies in the previous two approaches.

## 5.1 Calibration Model

The first approach to obtaining values for the parameters involves an estimation. The model used in this estimation, described below, is considerably more simple than the model used in the simulations. Ideally, the model used for the estimation and the one used for the full-scale simulations would be the same. Computation costs makes this impossible. Using a simpler model facilitates estimation of the central parameters.

One of the main three parameters to estimate is $\mu_u$, the rate of decline of human capital for people out of work. In seeking an estimate of this number, attention is focused on people who are not working. Therefore, within the SIPP data, a subgroup is selected that has entered unemployment. That subgroup is examined to see what happens to them over time. These unemployed workers have a wage, $w_{iT}$ prior to job loss. It is assumed that the wage is a function of three variable:, human capital, which is not observable; a vector $X_i$ of attributes which is observable; and a match quality, $\theta_{it}$, which is not observable. In the simplest form, the wage is:

$$w_{it} = \beta X_{it} + h_{it} + \theta_{it} \tag{5.1}$$

The second of the three main parameters to estimate is the standard deviation of match quality offers, $\sigma_\theta$. This parameter is estimated as the error term in the wage equation. Match quality offers are not observed at the lower end of the distribution. Since the equations are estimated using the normal distribution, a truncation problem occurs. The estimates for standard deviation are adjusted because of this truncation problem.

The third of the three main parameters to estimate is $\mu_e$, the rate at which human capital increases while a worker is employed. The data used in the estimations does not contain information on what happens to the workers when they are employed, so $\mu_e$ is estimated from sources outside the model.

The model has some limiting assumptions. The out of work individual is assumed to search for a job, and to receive offers based on current characteristics and a match that is drawn. It is also assumed that if human capital falls below a threshold level, it will be impossible to draw a match that is sufficiently high to induce job offer acceptance from the unemployed worker. At this point, the worker becomes discouraged[1].

The SIPP data has information on both finding a job and becoming discouraged. The people in the subgroup selected from the SIPP data: a) have a work history and a prior wage; b) lost their job within the time horizon of the SIPP data; and c) engaged in some job search activity after losing their job. Hence, a subgroup is selected that were working and lost their job within the first six months of each SIPP data-set. For a breakdown of how the data-set was created, see appendix C.

Figure 5.1 illustrates the path of wages for a worker who is unemployed under the assumption of a uniform match quality distribution. Wages are measured on the y-axis and the amount of time unemployed is measured along the x-axis. The range of wages that an individual may be offered is the difference between the two lines that cross the y-axis at $Q_H$ and $Q_L$. These lines decrease through time because human capital decreases every period for the unemployed worker. The slope of lines that cross the y-axis at $Q_H$ and $Q_L$ is $\mu_u$. The line that crosses the y-axis at $Q_R$ represents the reservation wage of the worker. Wage offers between $Q_R$ and $Q_H$ are accepted. Wage offers lower than $Q_R$ are rejected. There exists a wage where the cost of searching for a job offer is greater than the potential gain from accepting a job offer. At that point, the worker becomes discouraged and stops looking for a job. The wage at which discouragement occurs is $Z$. Workers who do not become discouraged never have all of their offers drop below $Z$. At time $T_0$ some wage offers that the worker might receive are lower than the discouragement wage. At time $T_1$, all possible wage offers are lower than the discouragement wage and the worker stops looking for a job. Between times $T_0$ and $T_1$, the fraction of jobs rejected increases compared to the rejection rate between time 0 and time $T_1$.

[1]It is necessary to use a uniform distribution in the discouragement model because under a normal distribution, all wage offers would not be below a certain wage, only the probability of wage offers being above a threshold would become low.

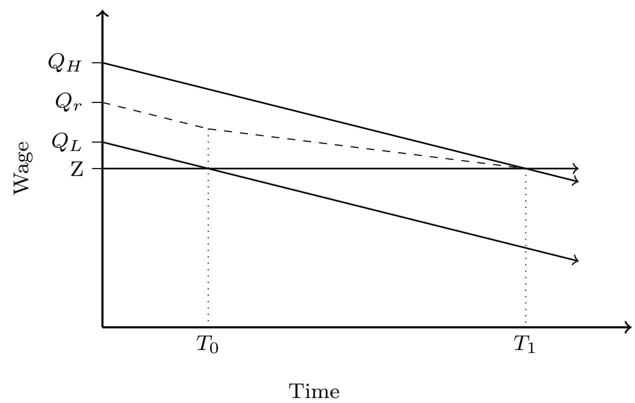


Figure 5.1: Shape of wages through unemployment for discouraged workers and the impact of the reservation wage in the estimation model. The vertical distance between $Q_H$ and $Q_L$ is the match quality range of job offers. The lines that cross the y-axis at $Q_H$ and $Q_L$ slope downward because skill deteriorates during unemployment. The $Q_R$ lines represents the reservation wage. $Z$ represents the wage offers that will induce a worker to leave the labor force.

The estimation is broken down into three parts, based on the sub-group of the SIPP data that is used. First, the group of workers that find a job are used to estimate $\mu_u$, and $\sigma_\theta$ in one equation, and the probability of getting a job offer, $\pi_o$, and the probability of rejecting a job, $\pi_r$ in another. Second, the group of workers who become discouraged is used to estimate the rate of decrease in human capital, $\mu_u$ and the variance of match quality, $\sigma_\theta$. The third part is to estimate a model that takes into account both discouraged workers and workers that find a job and combines all the estimates into one model.

### *5.1.1 Workers who find a job Model*

The first two equations estimated are performed over the data where a worker finds a job. The first equation estimates the variance of match quality, $\sigma_\theta^2$, and the skill loss rate, $\mu_u$, on this subset of data. Let $t_f$ be the month in which the worker finds a job. At every time $t$, the wage that the worker will be offered is:

$$w_{it} = \beta X_{it} + h_{it} + \theta_{it}. \tag{5.2}$$

The wage offer is a combination of the known return to attributes that were estimated in the wage equation, the worker's human capital at time t, $h_{it}$, and the match quality offered at time $t$, $\theta_{it}$. Similarly, the wage that the worker had at the end of the job lost before the unemployment spell is:

$$\hat{w}_{iT} = \beta X_{iT} + h_{iT} + \theta_{iT}. \tag{5.3}$$

if the job was lost in period $T$.

Exploiting the fact that the human capital at the end of the old job is the human capital at time $t = 0$ of the unemployment spell, subtracting equation 5.3 from 5.2 is:

$$w_{it} - \hat{w}_{iT} = (h_{it} - h_{i0}) + (\theta_{it} - \theta_{iT}). \tag{5.4}$$

The term $(h_{it} - h_{i0})$ is the amount of human capital lost and is equal to $-\mu_u t_f$, the amount of time that a worker takes to find a job times the rate of human capital depreciation. Rearranging and substituting $\epsilon_\theta$ for $(\theta_{it} - \theta_{iT})$:

$$\epsilon_{\theta i,t} = w_{it} - \hat{w}_{iT} + \mu_u t_f. \tag{5.5}$$

Equation 5.5 is the first form to be estimated using maximum likelihood estimation. The error term $\epsilon_\theta$, is distributed normally with mean 0 and standard deviation $\sigma_\theta^2$, $e \sim N(0, \sigma_\theta^2)$. The likelihood function to be estimated is:

$$L(w_{it}, \hat{w}_{iT}, t_f; \sigma_\theta, \mu_u) = L_{1i,t} = \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\{\frac{-(w_{it} - \hat{w}_{iT} + \mu_u t_f)^2}{2\sigma_\theta^2}\}. \tag{5.6}$$

The second equation to be estimated using the set of workers who find a job estimates the job rejection probability $\pi_r$, and the probability that a job will be offered, $\pi_o$. From figure 5.1, the distance between $Q_r$ and $Q_L$ is probability of rejecting a job, if the uniform distribution is assumed. For the worker who accepts a job at time $t_t$, that worker spent $t_f - 1$ periods looking for a job where an offer may or may not have been given, and all offers given were rejected. At time $t_f$ it is know that an offer was given and was accepted. The sub-set of data used here is only workers that found a job, so the combination of the probability of these events is equal to 1

$$1 = [(1 - \pi_{oi,t})(\pi_{oi,t})\pi_{ri,t})]^{t_{fi}-1} \cdot \pi_{oi,t}(1 - \pi_{ri,t}) \tag{5.7}$$

Estimating this equation yields the second estimation equation taken from the subset of data where workers find a job:

$$L(\pi_o, \pi_r) = L_{2i,t} = (t_{fi} - 1)[\log((1 - \pi_{oi,t}]) + \pi_{oi,t}(\pi_{ri,t})] + \log(\pi_{oi,t} + (1 - \pi_{ri,t})). \tag{5.8}$$

This equation estimates the probability of being offered a job and the probability of rejecting the job for the group of workers who eventually accept a job. It is not possible to estimate this equation from workers who become discouraged.

### *5.1.2 Discouraged Worker Model*

The object of this section is to describe the way to create estimates of $\mu_u$ and $\sigma_\theta$ from the SIPP Data when the worker becomes discouraged. From figure 5.1, it can be seen that the estimation model assumes that workers who become discouraged do so because human capital for those workers drops below a threshold, the $Z$ line in the figure. This fact can be exploited to estimate the variance of match quality, $\sigma_\theta^2$, and the rate of human capital loss, $\mu_u$. In the calibration model, workers are out of work and have a wage history that is the ending wage of the previous job. The worker exerts effort in a job search. The job search may yield one job offer each period from a particular firm. The period length is one month. The job offer is in the form of paying the worker for the amount of human capital at time $t$, and a match quality between the firm and the worker, $\theta_{it}$, drawn from a distribution. The firm observes the match quality and the worker's human capital and computes the marginal product exactly and offers a job with a wage exactly equal to the marginal product of the worker with that firm. The wage offered at time $t$ is equal to:

$$w_{it} = \beta X_{it} + h_{it} + \theta_{it} \tag{5.9}$$

where $X_{it}$ is a vector of characteristics of the worker.

The worker compares the offer to the reservation wage. If the wage offer exceeds the reservation wage of the worker, $w_{it} \geq w_{iRt}$, the worker accepts the wage and works at that job for the indefinite future. If the wage offer is below the reservation wage, $w_{it} < w_{iRt}$, the worker rejects the offer and continues searching, drawing a new firm with a new match in the next period. The human capital of the worker decreases by:

$$\Delta h_{it} = t_{di} \mu_u \tag{5.10}$$

where $\mu_u$ is the rate at which human capital depreciates per period.

Workers who are discouraged do not get a new wage, but they still have the information gained from the wage of the previous job, $\hat{w}_{iT}$, which is the same as the human capital at the start of the unemployment spell, $h_{i0}$. The wage for the job lost before the unemployment spell is:

$$\hat{w}_{iT} = \beta X_{iT} + h_{i0} + \theta_{iT}. \tag{5.11}$$

The discouragement wage, $Z_t$ is:

$$Z_t = \beta X_{it} + h_{i0} - t_{di} \mu_u \tag{5.12}$$

where $t_d$ is the time that workers are discouraged. Equation 5.12 says that at the discouragement date, $t_{di}$, the worker has lost $t_{di}\mu_u$ human capital. $Z_t$ is therefore the wage at the time of discouragement after that amount of human capital is lost.

Subtracting 5.12 from 5.11 and rearranging yields:

$$\theta_{it} = \hat{w}_{iT} - Z_t + t_{di}\mu_u. \tag{5.13}$$

If $\theta_{it} \sim N(0, \sigma_\Theta^2)$, equation 5.13 can be estimated with maximum likelihood estimation as follows:

$$L(\hat{w}_{i,t}, t_d; \sigma_\theta, \mu_u) = L_{3i,t} = \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\{\frac{-(\hat{w}_{iT} - Z_t + t_d\mu_u)^2}{2\sigma_\theta^2}\}. \tag{5.14}$$

For this estimation it is necessary to make assumptions about the value of $Z_t$. It is assumed that $Z_t$ is equal to 80% of the previous wage, $Z_t = .80\hat{w}_{iT}$. The choice of $Z_t$ effects the results of the estimation. 80% was chosen through experimentation and looking at the data.

*Final Estimation*

The final estimation takes into account the three previous estimations, $L_1$, $L_2$, and $L_3$. The three estimations are simultaneously estimated and the parameter estimates are used for the final calculations. The system of equations to estimate is:

$$\begin{aligned}
L_{1i,t} &= \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\{\frac{-(w_{it} - \hat{w}_{iT} + \mu_u t_{fi})^2}{2\sigma_\theta^2}\}. \\
L_{2i,t} &= (t_{fi} - 1)[\log((1 - \pi_{oi,t}) + \pi_{oi,t}(\pi_{ri,t})] + \log(\pi_{oi,t} + (1 - \pi_{ri,t})) \\
L_{3i,t} &= \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\{\frac{-(\hat{w}_{iT} - Z_t + t_d\mu_u)^2}{2\sigma_\theta^2}\}.
\end{aligned} \tag{5.15}$$

with $L_1$ and $L_2$ estimated over the group of workers in the SIPP data that find a job, and $L_3$ estimated over the set of workers who do not find a job. The computer fits the maximum likelihood estimation for the system. $L_1$ and $L_3$ assume a normal distribution, $L_2$ assumes a uniform distribution. The computer algorithm is able to solve the system with two different distributions.

#### *5.1.3 Estimation Results*

Four models are estimated, with the results of the last model used for model calibration. The reason four separate models are estimated is to see if consistent estimates are found when using the job finding group and the discouraged group separately. The fourth equation estimates the wage and probability equations simultaneously for both groups of unemployed workers.

Table 5.1 shows estimation results for the maximum likelihood estimation. Model (1) estimates wage equation 5.6 for just the unemployed workers who find jobs. Model (2) estimates the probability equation 5.8 on workers who find a job. It is not possible to estimate probabilities on the set of discouraged workers because the probabilities of getting an offer and the fraction of those offers rejected are only observed when a job is taken. Model (3) estimates the wage equation 5.14 which takes into account the discouragement wage $Z$. This model is only estimated on the discouraged workers. Model (4) combines models (1), (2), and (3) and estimates them simultaneously.

Table 5.1: Maximum likelihood results for all four forms described in this section.

| Parameter | $(L_1)$ | $(L_2)$ | $(L_3)$ | $(L_4)$ |
|---|---|---|---|---|
| $\sigma_\theta^2$ | 0.305 | | 1.418 | 0.745 |
| | (0.0123) | | (0.608) | (0.301) |
| $\mu_u$ | 0.0203 | | 0.035045 | 0.0342 |
| | (0.00418) | | (0.0107) | (0.0101) |
| $\pi_o$ | | 0.309 | | 0.486 |
| | | (0.00781) | | (0.00814) |
| $\pi_r$ | | .0708 | | 0.410 |
| | | (.00260) | | (0.00671) |

The results for variance are higher for the model estimated with the predicted wage than the model estimated with the actual new job wage. The rate of skill loss was also higher for the model estimated with predicted wage. The final calibration model shows that skill loss is around 3.4% of wage a month. The variance of wages, interpreted to be match quality is small at 0.745. The probability that a job offer is received is .49, for all levels of search effort. The probability that a job offer is rejected is .41.

The estimation for variance suffer from a bias in this estimation. New wages are only observed when match quality is at the high end of the distribution. Low match quality offers are rejected and not observed in the SIPP data. The distribution is truncated. It might be possible to use assume a distribution where truncation does not matter, for instance the Pareto distribution. Wage offers are not observed when the they are below the $Q_r$ line. To correct for the truncation problem, $\sigma_\theta^2$ will be divided by 0.7 in the remainder of the analysis.

## 5.2 Base Calibration

Using the estimation from SIPP data and other authors' calibrated parameters, a base calibration for the model that matches the U.S. economy is created. The main parameters to be calibrated are for human capital and match quality. The model was calibrated to a monthly time period.

### *5.2.1 Human Capital*

The human capital calibration requires three items be calibrated: a) functions that describes how human capital changes while employed, $g_e(h_t)$, and while unemployed, $g_u(h_t)$; b) the probability that human capital increases when employed, $\pi_e$; and c) the probability that human capital decreases while unemployed, $\pi_u$. These three items make up the human capital component of the model.

Following Ljungqvist and Sargent (1998) the functions $g_e$ and $g_u$ that describes how human capital changes for the employed and unemployed agents simply moves the agents' human capital up or down one value along the vector of possible human capital values, $H$. The vector for human capital follows from Ljungqvist and Sargent and is bounded by 1 and 2. There are 5 evenly spaced elements in the vector. The number of elements in the vector is chosen to be 5, in part because of the computational complexity that adding more values of human capital creates[2]. Sargent and Ljungqvist used 21 possible values of human capital in their parameterization and bucketed previous wages into 5 buckets.

In the model, for every period that a worker remains in the employed state there is a probability that the worker experiences a human capital increase. For every period that a worker remains in the unemployed state, there is a probability that human capital decreases. These probabilities are denoted $\pi_e$ and $\pi_u$, respectively. The probabilities give the amount of time that it takes a worker to go from the lowest level of human capital to the highest. In their parameterization, Ljungqvist and Sargent chose a probability such that it took 7.69 years to go from the lowest level of human capital to the highest level of human capital while employed. They chose a probability that was twice as high for the unemployed worker It only takes 3.35 years of unemployment to go from the highest skill level to the lowest skill level in their model.

Kambourov and Manovskii (2009) estimate occupation specific human capital returns to tenure to be about 16% after 5 years of employment. This translates into a $\mu_e$ estimate of .00269 which is much lower than the estimate for $\mu_u$ in the model. The $\mu_u$ estimate is about 12 times greater than Kambourov and Manovskii (2009). If human capital in an economy is constant, and more workers in the economy are employed than unemployed, then the estimate for $\mu_u$ should be higher than $\mu_e$. The estimates from Kambourov and Manovskii and from the SIPP estimation imply that

[2] In the parameterization of this model, each human capital/match quality pair is a wage. Those wages make up the basis for the unemployment benefit which is made up of the replacement rate. Adding one level of human capital increases the number of wages not by one level but by the number of elements in the match quality vector.

the unemployment rate would be around 7.5%. Kambourov and Manovskii break down human capital in their paper occupational, firm-specific and individual specific components. Occupational specific human capital is the most directly related to this dissertation. Firm-specific human capital is captured in the match quality component of this model. Individual specific human capital is captured in the predicted wage equation.

In the estimated model using the SIPP data, the coefficient on $\mu_u$ is .0342, indicating that workers in SIPP data use on average about 3.4% of their skills every month. After 20 months of unemployment, the a worker in this data-set will have 49.86% of the human capital that they brought into the unemployment spell. This is consistent with Sargent and Ljungqvist.

The vector chosen for the computer simulations for human capital is bounded by 1 and 2 and has 5 elements. To account for the different rates of skill gain while employed and skill loss while unemployed, the probability of moving from one point in the vector to the next is different for employed and unemployed workers. The probability of human capital increase while employed is set to .1, the probability of human capital decrease is set to .2. With the time period calibrated to monthly, it takes on average 3.33 years (40 months) to go from the lowest level of human capital to the highest. It takes 20 months to go from the highest level to the lowest level while unemployed. The lowest level of human capital is half the highest level of human capital. The estimates from the SIPP data and the calibration taken from Ljungqvist and Sargent (1998) are very close in this regard.

### *5.2.2 Match Quality*

The second important parameter to calibrate is the standard deviation of match quality offers, $\sigma_\theta$. The wage in the model is multiplicative, human capital is multiplied by match quality. The vector of possible match qualities $\Theta$ is centered around 1 so that it scales up or down the human capital of the agent. The vector length for $\Theta$ is chosen to be five. To find the high and low values of the vector, the maximum likelihood estimation results for the variance were used. The coefficient for $\sigma^2$ in the estimation results was 0.745. The estimation is biased because wages are only observed at the high end of the distribution. The wages that are accepted are observed in the SIPP data. Following convention, the variance is divided by .7 to correct for this bias:

$$\sigma^2 = \frac{0.745}{0.7} = 1.064 \tag{5.16}$$

To find the range, the adjusted variance number was used. Define $D$ to be the range of possible match quality variables centered around 1:

$$D \equiv \frac{\Theta_H - \Theta_L}{2} \tag{5.17}$$

The expectation of $X^2$ over the range of match qualities $R$ is:

$$\begin{aligned} E[X^2] &= \int_{-D}^{D} x^2 \text{dx} \qquad (5.18) \\ &= \frac{D^2}{12} \end{aligned}$$

The variances of $X$ is equal $E[X^2]$ and is estimated to be 1.064.

$$\text{Var}(X) = E[X^2] = \frac{D^2}{12} = \sigma^2 = 1.064 \tag{5.19}$$

Solving for $D$ yields $D = 3.5737$. $D$ is the log hourly range in wages of the match quality vector. This corresponds to about a 40% of wages. A vector of possible match qualities is centered around zero and with a range of 40%. The final match quality vector is:

$$\Theta = [.8, .9, 1.0, 1.1, 1.2]$$

### *5.2.3 Other Parameters*

The other parameters in this model were chosen based on convention, outside sources, or experimentation with the model. The interest rate for the model, $r$, was selected to be a 4% annualized. The subjective discount rate, $\rho$, was set to be higher than the interest rate at a 6% annualized rate. Job separation occurs every two years in the model, which corresponds to a job separation rate, $\delta$, of $\frac{1}{24}$. The average worker works 40 years in the labor market. That corresponds to a death rate, $\alpha$, of .0208. The dis-utility of work, $d$, was set to be 40% of the median wage at .4. The range of the wealth vector, $K$, is from 0 to 4. The vector has 5 elements.

The final set of values calibrated from the maximum likelihood estimation are the coefficient on search cost and the search elasticity. To calculate these values, the data from the calibration and results from the optimization model were used. First, the values of possible search efforts $S$ where normalized to be between 0 and 1. The vector $S = \{0, 1\}$ was chosen. Then, at the median search effort $s = .5$, the value of search elasticity $\lambda$ was set so that the probability of finding a job was equal to the coefficient on the job finding probability $\pi$ from the maximum likelihood estimation:

$$\begin{aligned} \pi(s) &= 1 - e^{-\lambda s} \\ \pi(.5) &= 1 - e^{-\lambda} = .486 \end{aligned} \tag{5.20}$$

Solving for $\lambda$ yields a search effort elasticity of $\lambda = 1.3250$. With the values of $S$ and $\lambda$ set, the computer model used to calculate $A$, the coefficient on the search cost function. To do this, the model was run with the median value of search effort $s = .5$ and the calculated value of $\lambda$ for many values of $A$. The value of $A$ that made the agent indifferent from working and not working was the one chosen for the calibration. Through simulation and experimentation, it was found that this value of $A$ was .1. The median search effort was used for this calibration because through experimentation, it was shown that not all workers engaged in search, so using 1 for search effort was not realistic.

### *5.2.4 Functional Forms*

The chosen calibration values depend on the functional forms of the model. There are three functions that were chosen to be the same as in Lentz (2009). The utility function chosen is the content relative risk aversion (CRRA) utility function with risk coefficent $\phi$:

$$u(c_t) = \frac{c_t^{1-\phi}}{1-\phi} \tag{5.21}$$

The probability of getting an offer function the search effort of the unemployed agent $s$ into the probability of getting a job offer given the elasticity of search effort $\lambda$:

$$\pi(\lambda s) = 1 - e^{-\lambda s_t} \tag{5.22}$$

This function has the property that any input will return a value between 0 and 1.

The third function is the cost of search function that maps the search effort to the dis-utility of searching for a job. The functional form is quadratic:

$$e(s_t) = As_t^2 \tag{5.23}$$

These functional forms are used throughout the rest of the calibration process.

The form of the wage equation has to be set. In this model, wages are multiplicative such that:

$$w(h_t, \theta_t) = h_t\theta_t. \tag{5.24}$$

## 5.3 Parameter Summary

Table 5.3 shows the final calibrated values and the source for the values. In cases where the author used a different calibration time period than 1 month, the values were adjusted to reflect a one month time period.

Table 5.2: Final calibrated parameters.

| Parameter | Description | Value(s) | Source |
|---|---|---|---|
| $\phi$ | Coefficient of Risk Aversion | 1.5 | Lentz (2009) uses $\phi \in \{1.5, 2.5\}$, 1.5 chosen by convention. |
| $r$ | Interest Rate | $1.04^{\frac{1}{12}} - 1$ | Lentz (2009) uses 1.05, Ljungqvist and Sargent (1998) use 1.04. |
| $\rho$ | Subjective Discount Rate | $1.06^{\frac{1}{12}} - 1$ | Chosen so that it is slightly greater than the interest rate. |
| $\delta$ | Job Separation Probability | $\frac{1}{24}$ | Chosen from SIPP data. Most jobs last around 2 years in the SIPP data. Ljungqvist and Sargent (1998) use 0.09, Lentz (2009) uses $\frac{1}{20}$ |
| $\lambda$ | Search Effort Elasticity | 1.325 | Calibrated using Matlab. Lentz (2009) uses an average $\lambda =$ .09826 |
| $A$ | Coefficient on Search Cost | .1 | Calibrated using Matlab. |
| $d$ | Disutility of work | .40 | Corresponds to about 26 % of average wage in the data. |
| $w$ | Base Wage | 1 | Normalized to one, workers are paid based on human capital and match quality. |
| $\alpha$ | Probability of Death | .0020833 | Author's calculation, average working life of an agent is 40 years. Ljungqvist and Sargent (1998) use 41.7 years. |
| $\mu_e$ | Probability of Human Capital Increase | .1 | Author's calculation so that it takes 5 years to go from the lowest human capital level to the highest. Ljungqvist and Sargent (1998) use 7 years 8 months. Kambourov and Manovskii (2009) say it takes 5 years to reach similar returns in human capital. |
| $\mu_u$ | Probability of Human Capital Decrease | .2 | Author's calculation. Computed so that the rate of decline makes it so the overall skill level in the economy stays constant. |
| $H$ | Vector of possible human capital values | {1 1.2 1.4 1.6 1.8 2.0} | Author's calculations |
| $\Theta$ | Vector of possible human capital values | {.8, .9, 1, 1.1, 1.2} | Author's calculations |
| $K$ | Vector of assets | {0,1,2,3,4} | Authors calculations |
| $S$ | Vector of search efforts | {0,1} | Authors calculations |

Table 5.3 shows a set of alternative parameters that were create by adjusting the original parameterization. This was done to account for possible measurement error in the original set of values. For the high human capital growth run the probabil-

ities that human capital increases during employment $\pi_e$ and decreases during an unemployment $\pi_u$ spell were doubled. Changing the probability that human capital will increase or decrease is equivalent to changing the amount that human capital changes during employment, $\mu_e$, and during unemployment, $\mu_u$. The new value of $\pi_u$ corresponds to a period of 20 months to go from the lowest level of human capital to the highest during employment, and 10 months to go from the highest to the lowest during unemployment. The low human capital growth run halved $\pi_u$ and $\pi_e$ relative to the base level. This doubles the amount of time that it takes for human capital to appreciate and depreciate.

Table 5.3: Table of final calibrated values and some alternatives. Values in the table show where parameters were changed from the original values. All other parameters remain the same as the base case for each run.

| Parameter | Description | Calibration | Higher $\mu_e$ and $\mu_u$ | Lower $\mu_e$ and $\mu_u$ | Higher $\sigma_\theta$ | Lower $\sigma_\theta$ |
|---|---|---|---|---|---|---|
| $\Theta$ | Match Quality Vector | [.8,.9,1,1.1,1.2] | | | [.5,.75,1.0,1.25,1.5] | [.9,.95,1,1.05,1.1] |
| $\pi_e$ | Probability of HC Increase | .1 | .2 | .05 | | |
| $\pi_u$ | Probability of HC Increase | .2 | .4 | .1 | | |

For the final two alternative analysis runs, the value of the standard deviation of match quality offers, $\sigma_\theta$. Different extreme values in the match quality vector $\Theta$ were created. Changing $\sigma_\theta$ has the effect of changing the range of possible match quality offers.

# CHAPTER 6

# SIMULATION AND RESULTS

Simulations are run under two distinct assumptions, first with the budget required to balance, second with no budget balance requirement. The analysis where the budget is not required to balance is non-standard in the unemployment insurance literature. There are three reasons that justify this approach: a) It reflects reality; b) it shows the independent impacts of taxes and benefits, each of which has a negative impact on the desire to work that cannot be seen when they rise and fall in tandem; and c) due to a potential Laffer curve effect, it is possible that there is no equilibrium in the budget balanced world when benefit levels are high. Not balancing the budget reflects reality because there is nothing that prevents the government from taxing a different source in order to pay for unemployment insurance benefits. When the budget is required to balance a change in benefit level necessarily means that there is a change in tax rate. It is impossible to determine if the change in behavior of the workers is due to the change in benefits or the change in taxes. The budget balance requirement is that the government is required to balance the budget of the unemployment insurance system, not that individuals must balance their budget over time. The potential Laffer curve effects means that an increase in taxes may result in lower government revenues to fund the unemployment insurance system.

The first assumption requires the government budget to balance at each policy choice. Under the first assumption, the optimal benefit rate is benefit replacement rate that maximizes total utility in the economy. The second assumption is that the tax rate is constant. Thus, with $b$ as the replacement ratio, the first assumptions assumes implicitly the existence of a function $\tau(b)$ with the property that the policy pair $(b, \tau(b))$ will balance the budget. The second assumption considers pairs of $(b, \tau)$, where $\tau$ does not depend on $b$. The budget balance is defined as the present value of government payments of unemployment insurance must be equal to the present value of taxes collected. Figure 6.1 shows the relationship between benefits and taxes. At benefit replacement rates above a certain level, employment drops to zero and a tax rate that balances the budget cannot be found. At this tax rate, the overall utility of workers in the economy becomes very low.

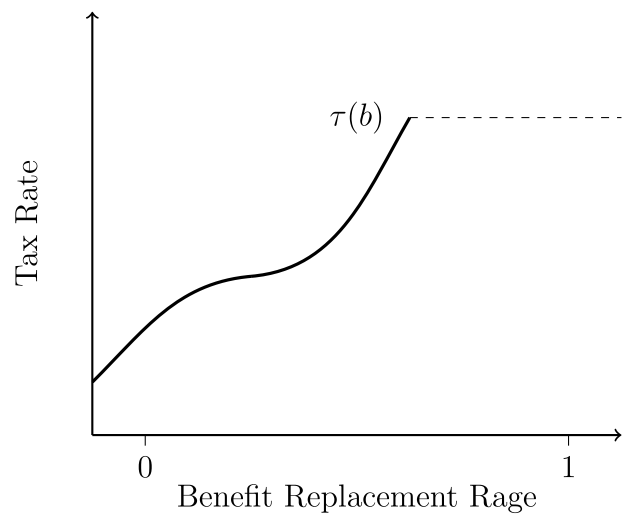


Figure 6.1: The tax rate increases with the benefits replacement rate. At a certain point, a tax rate that balances the budget cannot be found. This area is represented by the dotted line in the graph.

For the parameters chosen in the model, benefit replacement rates above 0.65 are not possible because there is no tax rate that balances the budget. When human capital appreciates and depreciates more slowly, the benefit rate at which the budget can no longer be balanced is lower, at 0.42. When the standard deviation of match quality is lower than the calibrated value, the benefit rate at which the budget can no longer be balanced is higher, 0.71.

The first assumption, where the budget is required to balance, is a special case of the second assumption, where the budget is not required to balance. Assumption two can have a balance budget by setting the benefit replacement rate, $b$, and varying $\tau$ until the budget balance is found. Any implication of $(b, \tau(b))$ will be exhibited by one point of the $(b, \tau)$ graphs. Assumption one is more restrictive, but also more common.

## 6.1 Budget Balance Results

The budget balance analysis using an actuarially fair tax system is the standard assumption in most unemployment insurance models. In such a system, the present value of benefits paid out over time must equal the present value of taxes collected. The budget must balance across all agents in the model. Each agent is not require to pay in taxes the amount received in benefits. While this analysis is standard it has the downside of not being able to study the behavior of agents in the model when either the replacement rate or tax rate changes, since the budget balance requires that benefits and taxes move in tandem.

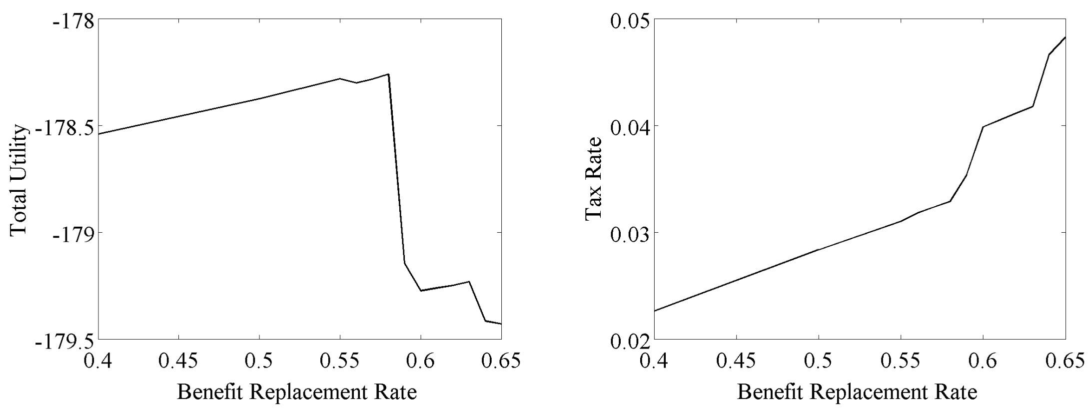


Figure 6.2: Average utility graph. The optimal unemployment insurance replacement rate is the maximum of this graph, which is at a benefit level of 0.58.

The left panel of Figure 6.2 shows the total utility level for all agents in the model graphed against the benefit level. The maximum point of the line is the optimal unemployment insurance replacement ratio. The optimal unemployment insurance replacement rate in the base calibration is 0.58. Utility increases until it reaches the optimal replacement rate. After that level utility falls. At benefit replacement rates of above 0.65, a $\tau(b)$ that balances the budget cannot be found. The right panel of Figure 6.2 shows the budget balancing tax rate, $\tau(b)$ for a range of benefit replacement rates. At replacement rates above 0.58, the tax rate that is required to balance the budget becomes steeper, as more workers are opting to not return to the labor force after an unemployment spell.

Figure 6.3 shows the unemployment rate when each benefit level is offered. At the optimal replacement rate of 0.58, the unemployment rate is 5.82%. Higher benefit levels cause higher unemployment because workers spend more time in unemployment, rejecting jobs that would be accepted under a less generous benefit policy. The difference in the unemployment rate cause by offering a less generous replacement rate than optimal is small. Once the generosity of benefits is slightly higher than optimal, the unemployment rate goes up, because of the lower search effort that the higher benefit induces. The right panel of 6.3 shows that search effort is decreasing from 1 at low benefit levels to 0.96 at the optimal benefit level to 0.68 at a benefit level slightly higher than optimal and then eventually to zero at high benefit levels. Zero search effort at high benefit levels is what makes it impossible to compute $\tau(b)$ for very generous benefits. The model has exogenous layoffs, without search effort agents do not return to the labor force.

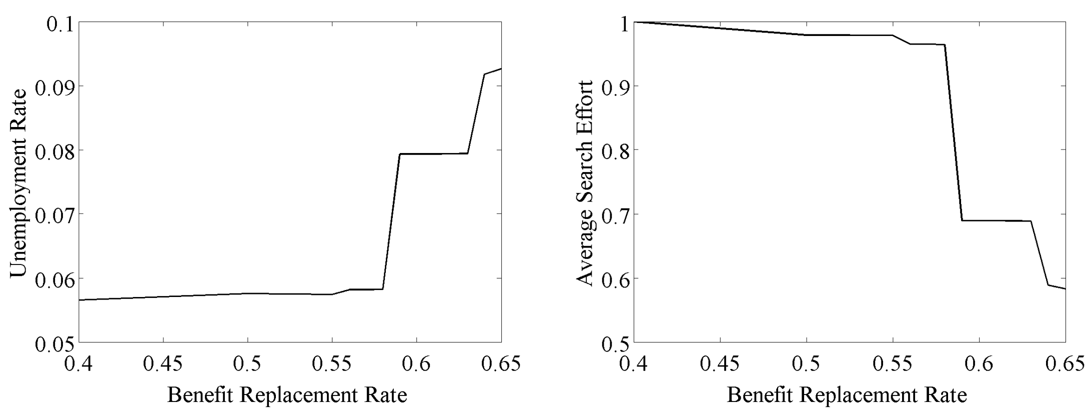


Figure 6.3: The unemployment rate and average search effort for different benefit levels. The unemployment rate increases with benefit generosity because of lower search effort, rejecting more jobs, and spending more time in the unemployed state.

Figure 6.4 shows the average wealth in the economy and the average after tax wage in the economy for the different budget balancing benefit levels. Agents on average save money and keep wealth high until a spell of unemployment when savings is spent down. The model has concave utility, which induces agents to spend down savings very quickly when faced with the prospect of low income. The after tax wage is falling as benefit replacement ratio increases for multiple reasons. First, the tax rate is increasing to fund the higher benefit levels. Second, human capital for workers is lower as unemployment spells are longer and workers have seen more skill depreciation. This means wages are lower.

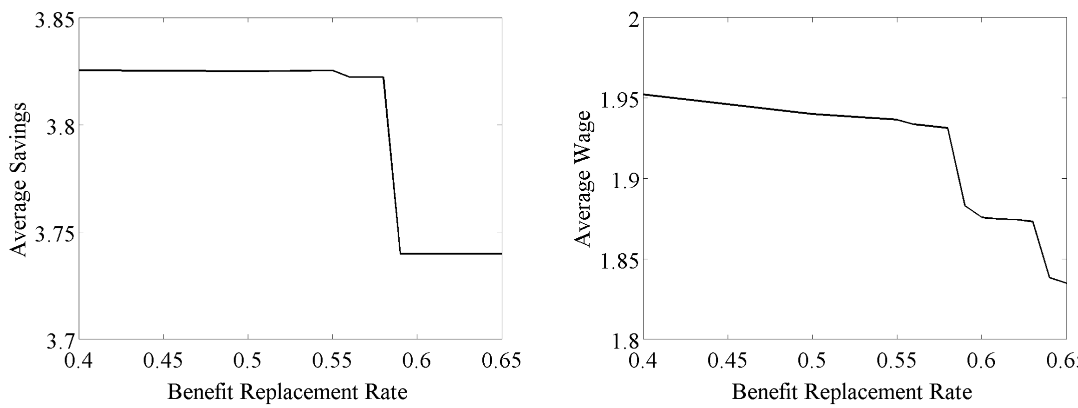


Figure 6.4: The average wealth and average wage for different benefit levels. More generous benefits induce workers to hold less wealth.

The effect of benefits levels on match quality and human capital are shown in figure 6.5. Match quality increases as the benefit replacement rate rises. Match quality rises slowly with benefits until the replacement rate is higher than the optimal level of 0.58, at which point the match quality increases quickly. Human capital decreases slowly until the optimal replacement rate of 0.58. At benefit replacement rates higher than the optimal, workers sit out of the labor force for longer periods of time waiting for better matches at the expense of a loss in human capital.

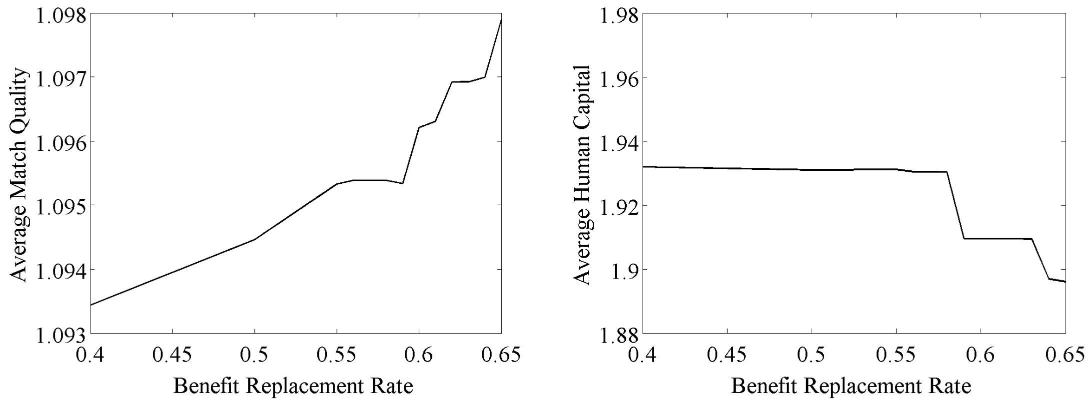


Figure 6.5: Average match quality and human capital for different benefit levels. Match quality is increasing with benefit generosity as workers wait for the best job or reject good jobs. Human capital declines with benefit generosity because more time out of the employed state means more skill deterioration.

Figure 6.5 shows the trade-off that occurs between human capital match quality in this model. Waiting for the best match is linked with a decrease in human capital because time spent out of the labor force means that workers lose skill. The graphs show that human capital is very sensitive to this waiting effect. The negative effect from waiting on human capital dominates the positive effect on match quality that waiting causes. The optimal benefit replacement rate is the point that an increase in match quality caused by a more generous benefit is less than the loss to the economy because of human capital loss.

A set of alternative simulations with varied values of standard deviation of match quality, $\sigma_\theta$, and human capital change, $\mu_e$ and $\mu_u$ where run to compare the results of different economies to the calibrated one. Table 6.1 shows the comparison of economy variables at the optimal benefit replacement rate.

Table 6.1: Comparison of results at the optimal benefit level for different values of the parameters. The table compares the optimal benefit level, the budget balancing tax rate, the unemployment rate, the average human capital, the average match quality, the average search effort, and the total utility in the economy.

| | Benefit Level | Tax Rate | Unemp Rate | Average Human Cap | Average Match Qual | Average Search Eff | Total Utility |
|---|---|---|---|---|---|---|---|
| Calibrated Economy | 0.58 | 0.0329 | 0.0582 | 1.9303 | 1.0954 | 0.9639 | -178.2554 |
| High $\mu_e$ and $\mu_u$ | 0.62 | 0.0365 | 0.0629 | 1.9511 | 1.0997 | 0.8886 | -176.909 |
| Low $\mu_e$ and $\mu_u$ | 0.39 | 0.0224 | 0.0579 | 1.8809 | 1.0548 | 0.9748 | -182.6076 |
| Low $\sigma_\theta$ | 0.70 | 0.0430 | 0.0690 | 1.9195 | 1.0301 | 0.8059 | -179.6522 |
| High $\sigma_\theta$ | 0.48 | 0.0402 | 0.1081 | 1.8801 | 1.2981 | 0.4642 | -176.1408 |

Table 6.1 shows that the range of optimal unemployment replacement rates may be between 0.39 and 0.70. With higher values of $\mu_e$ and $\mu_u$ the optimal replacement rate is slightly higher than the calibrated case. Even with the higher unemployment rates in this case, the average human capital is still higher than in the calibrated economy. The quicker recovery of human capital makes the penalty for sitting out of employment smaller for workers in this economy. Search effort is lower in this economy as well. When $\mu_e$ and $\mu_u$ are lower than the calibrated economy, the optimal replacement rate is lower. When the standard deviation of match quality offers, $\sigma_\theta$, is smaller, the optimal replacement rate is higher. When $\sigma_\theta$ is higher the optimal replacement rate is lower than the calibrated economy. The higher the range of match quality, the higher the average match quality in the economy will be. There is more reward to waiting for the best match when the range is higher, so workers have a higher reservation wage.

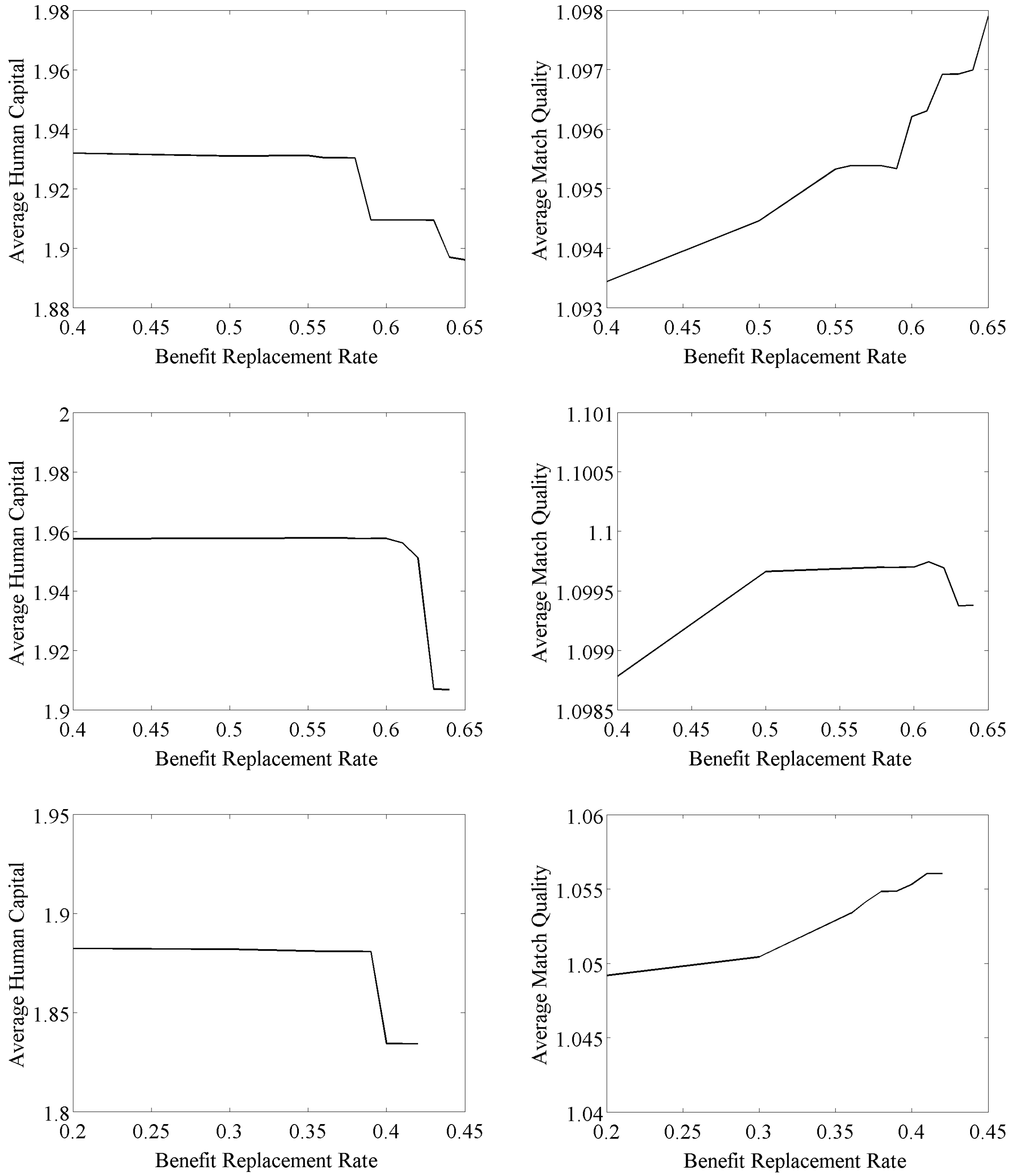


Figure 6.6: A comparison of average human capital and match quality for original run (top two graphs), high human capital growth (middle two graphs), and low human capital (bottom two graphs).

Figure 6.6 compares the average human capital and average match quality for the base case and the two human capital analyses. The graphs shows that when the average human capital in the economy starts to decline, that benefit level coincides with the optimal benefit. For each of the three analyses, the point at which human capital starts to decline is the same point as when match quality increases. This is confirmation of the finding that the trade-off between match quality gains and human capital loss is such that human capital losses are more detrimental to the economy than match quality gains.

When the match quality range was reduced or expanded there are some some interesting results. When the match quality range is wider, at the optimal level of benefits, more jobs are rejected and search effort is lower than the base case. The returns to match quality begin to occur at lower benefit levels and waiting becomes more desirable at a lower benefit level. The returns to match quality are still not high enough to overcome the detrimental effect to the economy of lower human capital, therefore the optimal rate is lower. The opposite is true when the range of possible match qualities is tighter than the base case. The return to waiting does not occur until higher benefit levels than the base level, so human capital remains high for greater benefit levels.

### 6.2 Non-budget balancing requirement

The second assumption is that the budget is not required to balance. In the budget balancing analysis it is not possible to compare the optimal benefit rate across different parameterizations because the tax rates are different for those optimal rates. The non-budget balancing analysis allows for the comparison of behavior at the different benefit replacement rate because the tax levels can be fixed.

The possibility of a Laffer effect is one of the reasons that the no balance budget analysis is run. Figure 6.7 shows the presence of a Laffer curve in this model. Higher tax rates lead to higher government tax receipts up to a certain point. After that point, the government receipt go down. When taxes are high, working is undesirable so the unemployment rate is higher than at lower tax rates. When fewer people are working, fewer people pay taxes and tax receipts for the government go down.

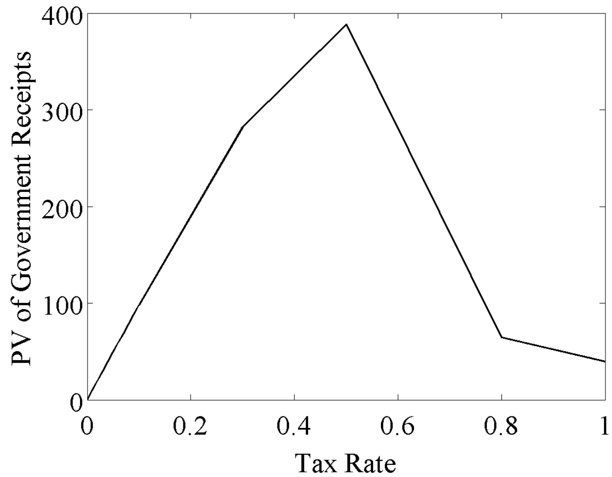


Figure 6.7: The present value of government tax receipts plotted against the tax rate.

Figure 6.8 displays graphs of the total utility and budget balance plotted against benefit level. The left panel shows that when the tax rate is lower, total utility is higher. The right panel of figure 6.8 shows that as the tax rate increases, the budget balancing point occurs at a higher benefit level. The budget balancing point is where each of the lines goes from positive to negative on the y-axis.

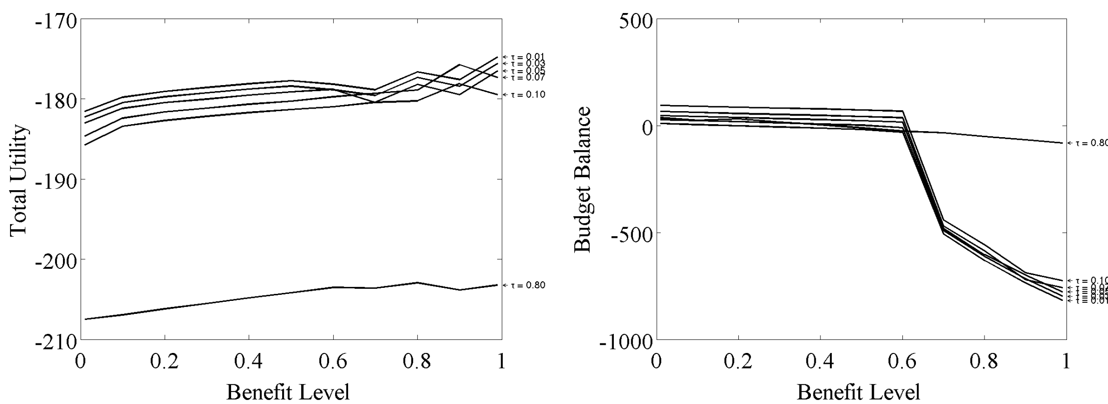


Figure 6.8: Total utility and budget balance for various tax and benefit pairs. Lower tax rates lead to higher utility. Higher tax rates lead to a higher budget balancing benefit level.

The right panel of figure 6.9 shows the average match quality in the steady state for various tax-benefit schemes. For reasonable tax rates, those under .10, the effect on match quality is not very strong. In the realistic range of benefit levels, between 0.40 and 0.70, higher tax rates lead to slightly lower match quality. For the very high tax rate, 0.80 increasing the benefit level decreases match quality. The right panel shows the average human capital for various tax-benefit pairs. The tax rate does not have a big effect in the amount of human capital in the economy at reasonable tax levels. At very high tax levels, an increase in benefits increases human capital.

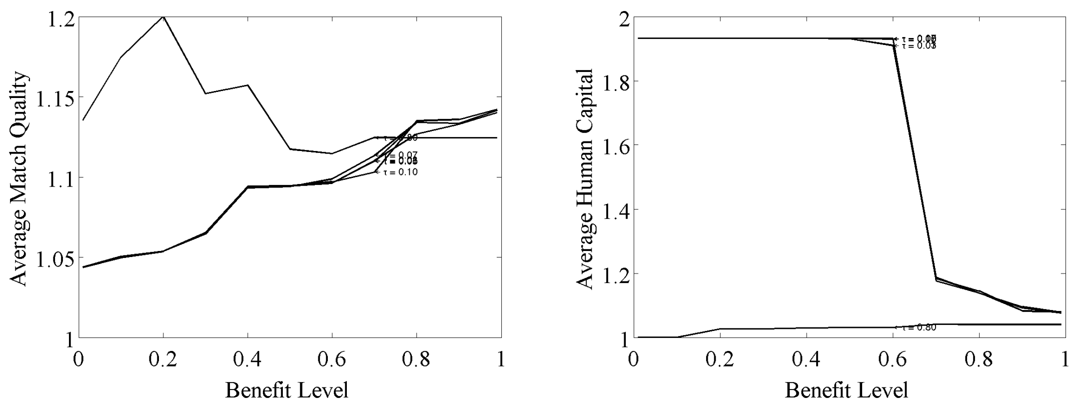


Figure 6.9: Average match quality and human capital for various tax and benefit pairs.

Figure 6.10 shows the search effort and unemployment rate for various tax-benefit pairs. Agents in the model decrease search effort at lower benefit levels when they are faced with lower taxes leading higher budget deficits in the economy. This behavior makes the unemployment rate higher for a given benefit level when tax rates are lower. When the tax rate is set very high, agents choose not to search, causing the unemployment rate to become very high. The incentive to rejoin the labor force is low, and search is costly so no search takes place under very high tax rates even with very high benefit replacement rates.

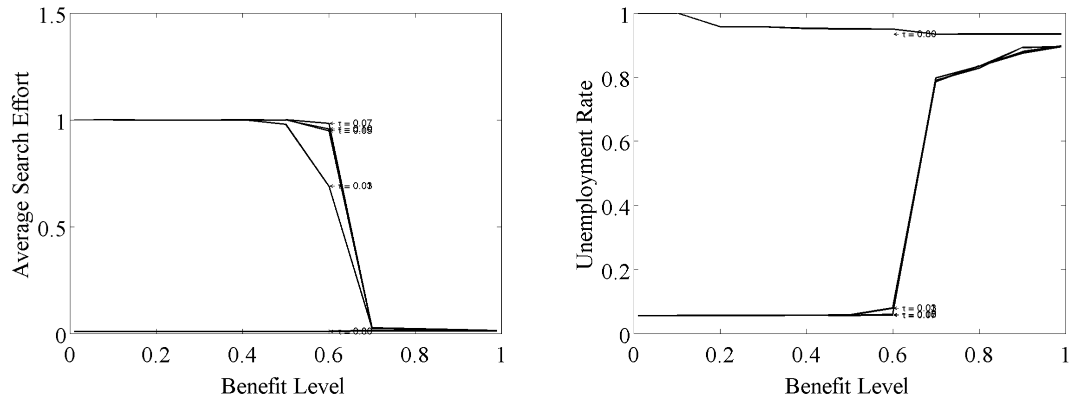


Figure 6.10: Average search effort and unemployment rate for various tax and benefit pairs.

Figure 6.11 shows the average wealth in the economy under different benefit tax pairs. The savings decisions of agents in the economy depends largely on the budget balance. With higher tax rates, and therefore a larger budget deficit, the agents start to save less at lower levels of benefits than they do under lower tax rates.

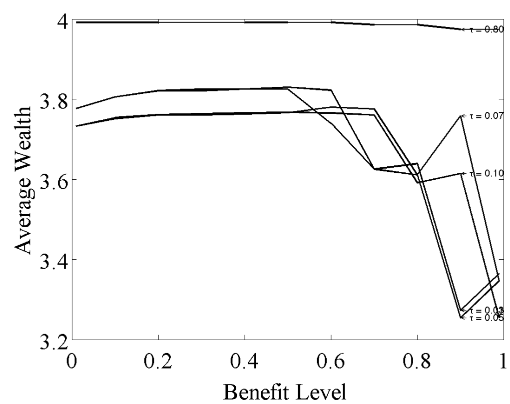


Figure 6.11: Average wealth for various tax and benefit pairs.

# CHAPTER 7
# CONCLUSION

This dissertation studied the effects of match quality and human capital effects on unemployed workers. While workers are unemployed, there are two forces that can act upon them. One, they lose skill, which they regain while employed. Two, workers and jobs have different levels of match quality. Unemployment allows workers to sample more jobs and perhaps find an employer with a good match. The challenge of the social planner is to design an unemployment compensation system that is best for the economy while taking into consideration that time spent unemployed can be bad or good for the worker and the economy. The goal of the paper is to point out that both of these effects are important, and to try to untangle which effect dominates and what the optimal amount of unemployment compensation should be.

To study the problem, a model was presented that could be simulated using a limited number of parameters. The model follows agents between employed and unemployed states. Workers lose jobs through no fault of their own. They receive benefits based on a previous wage and search for a new job. When working, human capital for the worker rises. While unemployed, human capital falls. Workers receive job offers that have different match qualities, and therefore different wages and productivity. This model is used to study the effects of human capital change and match quality for unemployed workers on the whole economy.

Finding the correct parameters is important because all conclusions that the model will show will be affected by the starting parameters. For this reason, three strategies were used for calibration of the parameters. First, an estimation model was created and estimated using data from the Survey of Program Participation (SIPP). Second, a review of the literature was done to find values that other authors have used for similar models. Third, the values found in the first two approaches were varied to account for any mismeasurement.

Results from simulating the model were analyzed in two ways: a) with the budget for the government required to balance; and b) with the budget for the government not require to balance. Under the budget balance assumption, there are some benefit levels that a budget balancing tax rate cannot be found. This is one reason that the assumption of budget balance was relaxed.

Results from the simulations show that the policy maker needs to be careful not to set the unemployment insurance rate too high in a world where there is both the human capital and match quality effect. Setting the policy even slightly greater than the optimal rate leads to a large decrease in total utility in the economy. This effect is seen under the budget balance assumption but is not seen when the budget is not required to balance. The cost of providing benefits greater than the optimal is not high if the budget is not required to balance and leads to an increase in match quality in the economy.

There are some items left for future research. Limiting the time that workers can collect unemployment benefits could be implemented into this model. Whether benefits should be provided to workers who have never been in the labor force. This group may include college graduates or individuals who have been out of the labor force for some time. The effects of the business cycle on the optimal amount of unemployment insurance is also a possible avenue for future research.

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# APPENDIX A

# MATLAB CODE

Listing A.1: main.m

```
tic
%%%%%%%%%%%%%%%% notes %%%%%%%%%%%%%%%%
%%% This program calls the following programs (functions):
  % value_iter.m -
  % initial_position.m -
  % law_of_motion.m -
  % present_value.m -
  % graph_decison.m -
  % graph_transition.m -
  % graph_macro.m -
%%%%%%%%%%%%%%%% end notes %%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%% global variables %%%%%%%%%%%%%%%%
global rho Δ lambda r w a d phi K H Theta S u e mu What;
global valTol pvTol;
global nK nH nTheta nS nWhat nB nTau;
global rundir pHe pHu pD;
%%%%%%%%%%%%%%%% end global variables %%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%% declarations %%%%%%%%%%%%%%%%
%%% define constants
  rho = 1.06^(1/12) - 1;        % subjective discount rate
  Δ = 1/24;                 % job loss probability
  lambda = 1.325;               % elasticity of search effort
  r = 1.04^(1/12) - 1;          % interest rate
  w=1;                          % base wage rate
  a = .1;                       % coefficient search cost function
  d= .40 ;                       % disutility of working
  phi = 1.5;                    % coefficient of risk aversion
  pHu = ones(1,5)*.2;          % probability of human capital decrease
  pHe = ones(1,5)*.1;          % probability of human capital increase
  pD = .0020833;                % probability of death

%%% define vectors;
  K = linspace(0,4,5);                % values of savings
  H = linspace(1,2,5);             % values of human capital
  Theta = [ .8  .9 1 1.1 1.2 ];            % values of match quality
  S = [ 0.01 1];                   % values of search effort

%%% define functional forms;
  u =@(c)c.^(1-phi)/(1-phi);      % utility function
  e =@(s)a.*s.^2;                 % cost of effort function
  mu =@(s)1-exp(-lambda.*s);      % probability of getting an offer
```

```
    function

%%% give a name to the output
  runname = 'base008';

%%% choose benfit levels, guess tax rate for lowest benefit level
  B = [.4 .5 .55 .56 .57 .58 .59 .60 .61 .62 .63 .64 .65 .7 .8]
  B = .58;
  Tau =[.015 .020 .030 .032 .034 .036 .038 .040 .042 .046 .050 .1 .4 .6
      .8 1];
  Tau =[.032 .034 .036 .038 .040 .042 .046 .050 .1 .4 .6 .8 1];

%%% set convergence criteria
  valTol = .001;     % Tolerence of value function

%%%%%%%%%%%%%%%% end declarations %%%%%%%%%%%%%%%;


%%%%%%%%%%%%%%%% preallocate variables %%%%%%%%%%%%%%%;
%%% create directories for output;
  rundir = ['./graph/',runname];
  mkdir(rundir);

%%% find size of vectors;
  nK = length(K);
  nH = length(H);
  nTheta = length(Theta);
  nS = length(S);
  nB = length(B);
  nTau = length(Tau);

%%% create vector and size of previous wage
  [gH gTheta] = ndgrid(H,Theta,1);
  What = w.*gH.*gTheta;
  What = reshape(What,1,nH,nTheta);
  What = unique(What);
  %What = [0 What];
  nWhat = length(What);

%%% variables needed to calcuate present value;
  n = 0;
  pvChk = pvTol+1;
  PVi1 = ones(nB,nTau)*999;
  PVo1 = ones(nB,nTau)*999;
  PVo = zeros(nB,nTau);
  PVi = zeros(nB,nTau);
  budBal = zeros(nB,nTau);
%%%%%%%%%%%%%%%% end preallocate variables %%%%%%%%%%%%%%%;

%%%%%%%%%%%%%%%% run optimization problem %%%%%%%%%%%%%%%;
```

```
for iB = 1:nB
  fTau = 0;
  iTau = 1;
  %loop through tax rates until budget balance is found, reduce tax
     choices
  while (fTau < 1 & iTau ≤ nTau);
    [Ve,Vu,DeKp,DuKp,DuS,DuThetar,iDeKp,iDuKp,iDuThetar,iDuS] =
       value_iter(B(iB),Tau(iTau));
    [Pe,Pu] = initial_position();
    t = 1;
    pvChk = pvTol+1;
    pvI = 0;
    pvO = 0;
    %while (pvChk>pvTol);
    for i = 1:500;
  [pvO,pvI] = present_value(t,Pe,Pu,B(iB),Tau(iTau),pvO,pvI,DuS);
      [Pe,Pu] = law_of_motion(Pe,Pu,B(iB),Tau(iTau),iDeKp,iDuKp,
         iDuThetar,iDuS,DuS);
      t = t+1;
    end
    budBal(iB,iTau) = pvI-pvO
    if (iTau > 1 & budBal(iB,iTau-1)<0 & budBal(iB,iTau)>0)
      fTau = 1;
      balTax(iB) = interp1(budBal(iB,1:iTau),Tau(1:iTau),0,'linear');
      Tau = Tau(max(1,iTau-3):nTau);
      nTau = length(Tau);
    end
  iTau = iTau + 1;
  toc
  end
end

for iB = 1:nB;
  [Ve,Vu,DeKp,DuKp,DuS,DuThetar,iDeKp,iDuKp,iDuThetar,iDuS] =
     value_iter(B(iB),balTax(iB));
  [Pe,Pu] = initial_position();
  t = 1;
  pvChk = pvTol+1;
  pvI = 0;
  pvO = 0;
  %while (pvChk>pvTol);
  for i = 1:500;
    [pvO,pvI,unemp,avgWage,avgK,avgH,avgTheta,avgS] = present_value(t,
       Pe,Pu,B(iB),balTax(iB),pvO,pvI,DuS);
    [Pe,Pu] = law_of_motion(Pe,Pu,B(iB),balTax(iB),iDeKp,iDuKp,
       iDuThetar,iDuS,DuS);
    t_unemp(t+1) = unemp;
    t_avgWage(t+1) = avgWage;
    t_avgK(t+1) = avgK;
    t_avgH(t+1) = avgH;
    t_avgTheta(t+1) = avgTheta;
    t_avgS(t+1) = avgS;
    t_v(t+1) = sum(sum(sum(Pe.*Ve)))+sum(sum(sum(Pu.*Vu)));
    t = t+1;
```

```
    end
    b_pvI(iB) = pvI;
    b_pvO(iB) = pvO;
    budBalFinal(iB) = pvI-pvO;
    b_unemp(iB) = unemp;
    b_avgWage(iB) = avgWage;
    b_avgK(iB) = avgK;
    b_avgH(iB) = avgH;
    b_avgTheta(iB) = avgTheta;
    b_avgS(iB) = avgS;
    b_v(iB) = sum(sum(sum(Pe.*Ve)))+sum(sum(sum(Pu.*Vu)));
    graph_transition(B(iB),t_unemp,t_avgWage,t_avgK,t_avgH,t_avgTheta,
        t_avgS,t_v);
    graph_decision(B(iB),DeKp,DuKp,DuS,DuThetar);
  end
   graph_macro(B,balTax,b_unemp,b_avgWage,b_avgK,b_avgH,b_avgTheta,
       b_avgS,b_v,budBalFinal,b_pvI,b_pvO);

finaltime = toc
save([rundir,'/',runname]);
```

Listing A.2: value_iter.m

```
function[Ve,Vu,DeKp,DuKp,DuS,DuThetar,iDeKp,iDuKp,iDuThetar,iDuS] =
    value_iter(B,Tau)
    global rho Δ lambda r w a d phi K H Theta S What;
    global valTol pvTol;
    global nK nH nTheta nS nWhat;
    global mu u e pHe pHu;

   %% preallocate value matrices
    Ve = zeros(nK,nH,nTheta);
    Vu = zeros(nK,nH,nWhat);
    Ve1 = Ve;
    Vu1 = Vu;
    valChk = valTol+1;

  %% preallocate function solutions
    pMu = mu(S);
    pE = e(S);

  while (valChk>valTol);
    %% Value function iteration;
    for iK = 1:nK,
      for iH = 1:nH,
        % employed case
        for iTheta = 1:nTheta,
          Vp = 1:nK;
          for iKp = 1:nK,
            chat = (1+r)*K(iK) + (1-Tau)*(w*H(iH)*Theta(iTheta)) + (K(iK
                )-K(iKp)) ;
            chat = max(.00001,chat-d);
            uhat = u(chat);
```

```
        iHp = min(iH+1,nH);
        Vp(iKp) = uhat + (1+rho)^-1*((1-Δ)*(pHe(iH)*Ve(iKp,iHp,
            iTheta)+(1-pHe(iH))*Ve(iKp,iH,iTheta))+Δ*Vu(iKp,iH,find(
            abs(What-(H(iH)*Theta(iTheta)*w))<.01,1)));
      end
      % employed decision functions
      [Vep,Iep] = max(Vp);
      Ve(iK,iH,iTheta) = Vep;
      DeKp(iK,iH,iTheta) = K(Iep);
      iDeKp(iK,iH,iTheta) = Iep;
    end
    %unemployed
    for iWhat = 1:nWhat,
      Vup = zeros(nK,nS,nTheta);
      for iKp = 1:nK
        for iS = 1:nS
          chat = (1+r)*K(iK)+B*(1-Tau)*What(iWhat)+(K(iK)-K(iKp));
          chat = max(.00001,chat);
          uhat = u(chat);
          for iTheta = 1:nTheta,
            sQ = 0;
            for iQTheta = iTheta:nTheta,
              sQ = sQ+Ve(iKp,iH,iQTheta);
            end
            iHp = max(1,iH-1);
            Q = nTheta;
            Vup(iKp,iS,iTheta) = uhat - pE(iS) + (1+rho)^-1*(pMu(iS)
                *(1/nTheta)*sQ + pMu(iS)*((iTheta-1)/nTheta)*(pHu(iH
                )*Vu(iKp,iHp,iWhat)+(1-pHu(iH))*Vu(iKp,iHp,iWhat)) +
                 (1-pMu(iS))*(pHu(iH)*Vu(iKp,iHp,iWhat)+(1-pHu(iH))*
                Vu(iKp,iH,iWhat)));

          end
        end
      end
      % Unemployed decision functions
      Vu(iK,iH,iWhat) = median(max(max(Vup)));
      Vup1 = repmat(Vup(:,:,2),[1 1 nTheta]);  % Take out effect of
          match quality for K and S decision;
      VupV = reshape(Vup1,1,nK*nS*nTheta);
      [m,I] = max(VupV);
      sz = [nK nS nTheta];
      [Ikp Is Itheta] = ind2sub(sz,I);
      DuKp(iK,iH,iWhat) = K(Ikp);
      DuS(iK,iH,iWhat) = S(Is);
      iDuS(iK,iH,iWhat) = Is;
      iDuKp(iK,iH,iWhat) = Ikp;
    end
  end
end
valChk = max(abs(max(max(max(Ve-Ve1)))), abs(max(max(max(max(Vu-Vu1)
    )))));
Ve1 = Ve;
Vu1 = Vu;
```

```
  end

for iK = 1:nK;
  for iH = 1:nH;
    for iWhat = 1:nWhat;
      for iTheta = 1:nTheta;
        Emp(iK,iH,iWhat,iTheta) = Ve(iDuKp(iK,iH,iWhat),iH,iTheta) > Vu(
            iDuKp(iK),iH,iWhat);
end;
end;
end;
end;

% Solve for reservation match quality.
for iK = 1:nK;
  for iH = 1:nH;
    for iWhat = 1:nWhat;
      if max(Emp(iK,iH,iWhat,:)) == 0
    iDuThetar(iK,iH,iWhat) = nTheta+1;
    DuThetar(iK,iH,iWhat) = 0;
      else
    iDuThetar(iK,iH,iWhat) = squeeze(min(find(Emp(iK,iH,iWhat,:)>0)));
        DuThetar(iK,iH,iWhat) = Theta(iDuThetar(iK,iH,iWhat));
      end;
    end;
end;
end;
```

Listing A.3: initial_position.m

```
function[Pe,Pu] = initial_position();
global nK nTheta nH nWhat nB nTau;
% Set initial positions of economy;
  Pe = ones(nK, nH, nTheta);
  Pu = zeros(nK, nH, nWhat);
  Pe = Pe / (nK*nH*nTheta);
```

Listing A.4: law_of_motion.m

```
function[Pe,Pu] = law_of_motion(Pe,Pu,B,Tau,iDeKp,iDuKp,iDuThetar,iDuS,
    DuS)

global rho Δ lambda r w a d phi K H Theta S u e mu What;
global valTol pvTol;
global nK nH nTheta nS nWhat nB nTau;
global pHe pHu pD pU;

% Preallocate transition matrices;
Transeu = zeros(nK,nH,nTheta,nK,nH,nWhat);
Transue = zeros(nK,nH,nWhat,nK,nH,nTheta);
Transuu = zeros(nK,nH,nWhat,nK,nH,nWhat);
Transuu1 = zeros(nK,nH,nWhat,nK,nH,nWhat);
```

```
Transuu2 = zeros(nK,nH,nWhat,nK,nH,nWhat);
Transee = zeros(nK,nH,nTheta,nK,nH,nTheta);
Transee1 = zeros(nK,nH,nTheta,nK,nH,nTheta);
Transee2 = zeros(nK,nH,nTheta,nK,nH,nTheta);

% Speedup;
pMu = mu(S);

%%%%%%%%%%%%%%%% calculate transition matrices %%%%%%%%%%%%%%%;
% employed transitions
for iK = 1:nK
  for iH = 1:nH
    for iTheta = 1:nTheta
      Transee1(iK,iH,iTheta,iDeKp(iK,iH,iTheta),min(iH+1,nH),iTheta) = (
          pHe(iH)*(1-Δ));
      Transee2(iK,iH,iTheta,iDeKp(iK,iH,iTheta),iH,iTheta) = ((1-pHe(iH)
          )*(1-Δ));
      Transeu(iK,iH,iTheta,iDeKp(iK,iH,iTheta),iH,find(What==H(iH)*Theta
          (iTheta)*w)) = Δ;
    end
  end
end
Transee = Transee1+Transee2;

% Unemployed Transitions;
for iK = 1:nK;
  for iH = 1:nH;
    for iWhat = 1:nWhat;
      for iTheta = 1:nTheta;
        numThetas = nTheta - iDuThetar(iK,iH,iWhat)+1;
        Transue(iK,iH,iWhat,iDuKp(iK,iH,iWhat),iH,iTheta) = (pMu(iDuS(iK
            ,iH,iWhat))*((iTheta-iDuThetar(iK,iH,iWhat))≥0))/max(
            numThetas,1);
      end;
    end;
  end;
end;

for iK = 1:nK;
  for iH = 1:nH;
    for iWhat = 1:nWhat;
      Transuu1(iK,iH,iWhat,iDuKp(iK,iH,iWhat),iH,iWhat) = (1-pHu(iH))*(
          (1-pMu(iDuS(iK,iH,iWhat))) + (pMu(iDuS(iK,iH,iWhat))*((iTheta-
          iDuThetar(iK,iH,iWhat))<0)));
      Transuu2(iK,iH,iWhat,iDuKp(iK,iH,iWhat),max(1,iH-1),iWhat) = pHu(
          iH)*( (1-pMu(iDuS(iK,iH,iWhat))) + (pMu(iDuS(iK,iH,iWhat))*((
          iTheta-iDuThetar(iK,iH,iWhat))<0)));
    end;
  end;
end;
Transuu = Transuu1+Transuu2;
%%%%%%%%%%%%%%%% end calculate transition matrices %%%%%%%%%%%%%%%;

%%%%%%%%%%%%%%%% apply transition matrices to position matrices
```

```
    %%%%%%%%%%%%%%%%;
Pe1 = Pe;
Pu1 = Pu;

for iK = 1:nK;
  for iH = 1:nH;
    for iTheta = 1:nTheta;
      Pe = Pe+Pe1(iK,iH,iTheta)*squeeze(Transee(iK,iH,iTheta,:,:,:));
      Pu = Pu+Pe1(iK,iH,iTheta)*squeeze(Transeu(iK,iH,iTheta,:,:,:));
      Pe(iK,iH,iTheta) = Pe(iK,iH,iTheta)-Pe1(iK,iH,iTheta)*(sum(sum(sum
          (squeeze(Transee(iK,iH,iTheta,:,:,:)))))+sum(sum(sum(squeeze(
          Transeu(iK,iH,iTheta,:,:,:))))));
    end;
  end
end;

for iK = 1:nK;
  for iH = 1:nH;
    for iWhat = 1:nWhat;
      Pu = Pu+(Pu1(iK,iH,iWhat)*squeeze(Transuu(iK,iH,iWhat,:,:,:)));
      Pu(iK,iH,iWhat) = Pu(iK,iH,iWhat)-Pu1(iK,iH,iWhat)*(sum(sum(sum(
          squeeze(Transuu(iK,iH,iWhat,:,:,:))))));
    end;
  end;
end;

for iK = 1:nK;
  for iH = 1:nH;
    for iTheta = 1:nTheta;
      for iWhat = 1:nWhat;
      Pe(iK,iH,iTheta) = Pe(iK,iH,iTheta)+(Pu1(iK,iH,iWhat)*Transue(iK,
          iH,iWhat,iDuKp(iK,iH),iH,iTheta));
      Pu(iK,iH,iWhat) = Pu(iK,iH,iWhat)-(Pu1(iK,iH,iWhat)*Transue(iK,iH,
          iWhat,iDuKp(iK,iH),iH,iTheta));
    end;
  end;
end;
end;
%%%%%%%%%%%%%%%% apply transition matrices to position matrices
    %%%%%%%%%%%%%%%%;

%%%%%%%%%%%%%%%% death %%%%%%%%%%%%%%%%;
PeD = pD*Pe;
PuD = pD*Pu;

Pe = Pe - PeD;
Pu = Pu - PuD;

Pu(1,1,1) = Pu(1,1,1) + sum(sum(sum(PeD)))+sum(sum(sum(PuD)));
%%%%%%%%%%%%%%%% end death %%%%%%%%%%%%%%%%;
```

Listing A.5: present_value.m

```
function[pvO,pvI,unemp,avgWage,avgK,avgH,avgTheta,avgS] = present_value(
    t,Pe,Pu,B,Tau,pvO,pvI,DuS)

global rho Δ lambda r w a d phi K H Theta S u e mu What;
global valTol pvTol;
global nK nH nTheta nS nWhat nB nTau;

%%% create the income and outlay matrices;
  [gK,gH,gWhat] = ndgrid(K,H,What);
  Go = (gWhat.*B.*(1-Tau)).*Pu;
  [gK,gH,gTheta] = ndgrid(K,H,Theta);
  Gi = (Tau.*(w.*gH.*gTheta)).*Pe;

% Calcuate Present value
  tpvO = sum(sum(sum(Go)));
  tpvI = sum(sum(sum(Gi)));

  pvO = pvO + (1/(1+r)^t)*tpvO;
  pvI = pvI + (1/(1+r)^t)*tpvI;

% Calculate Stats at each time period
  unemp = sum(sum(sum(Pu)));
  avgWage =sum(sum(sum((1-Tau).*(w.*gH.*gTheta).*Pe)));
  avgK = sum((sum(sum(Pe,2),3)+sum(sum(Pu,2),3)).*K');
  avgH = sum((sum(sum(Pe,1),3)+sum(sum(Pu,1),3)).*H);
  avgTheta = sum(squeeze(sum(sum(Pe,1),2)).*Theta')/sum(sum(sum(Pe)));
  avgS = sum(sum(sum(Pu.*DuS)/sum(sum(sum(Pu)))));
```

# APPENDIX B

## WAGE EQUATION

This appendix describes the how the predicted ending wage was estimated for the SIPP data. The predicted wage is used to determine the match quality of the respondent. A regression with the wage of the job lost by the SIPP respondent as the dependent variable and the characteristics of the workers as the independent variables was run on the entire SIPP sample. The regression coefficients found in this analysis were applied to the 968 observations used in the calibration and are summarized in the $\hat{w}_1$ column of table C.1.

Table B.1 shows descriptive statistics of the variables used in the wage equation. Overall, there were 152,260 respondents that were employed during the first wave of their respective surveys. The data was analyzed as a cross-section, but a regression on panel data could also have been performed. The disadvantage of performing a fix-effects regression on the panel data is that all static variables, such as age and gender, would be absorbed into the fixed effect.

The `Years of Education` variable was recoded so that 12 referenced a high school diploma and 16 a college diploma. `Labor market experience` was created as potential labor market experience, the number of years that the respondent could potentially have been in the labor market. This is not an indication of actual number of years the respondent had a job, which is not available in the SIPP data. Labor market tightness was calculated taking the number of unemployed in the state where the respondent lived, but perhaps not where they worked, divided by the national help wanted advertisers index available from the Conference Board. `Gender`, `race` and `Marital status` were taken from the survey respondents self-stated answers from the first wave of the SIPP survey. The number of dependents used were dependents under the age of 18, whether they were related to the respondent or not. Adult de-

Table B.1: Descriptive statistics for variables used in the wage equation.

| Variable | Description | N | Mean | St Dev |
|---|---|---|---|---|
| Years of Education | Number of years of schooling, high school diploma = 12 | 152,260 | 13.13 | 2.68 |
| Labor Market Experience | Potential Labor Market Experience: Age - Years of Education + 6 | 152,260 | 17.73 | 12.74 |
| Labor Market Tightness | For the state of the respondent, the number of unemployed people divided by the national help wanted advertisers index | 152,600 | 1.38 | 0.69 |
| Gender | 0 = male, 1 = female | 152,260 | 0.47 | 0.30 |
| Race | 0 = white, 1 = non-white | 152,260 | 0.19 | 0.39 |
| Marital Status | 0 = not married, 1 = married | 152,260 | 0.52 | 0.50 |
| Number of Dependents | Number of dependents under the age of 18 | 152,260 | 0.75 | 1.10 |

pendents were considered. Time dummies for each month of the survey were added to account for inflation.

The functional form of the regression equation is as follows:

$$\begin{aligned}\ln(w_1) = {} & \beta_0 + \text{Time Dummies} + \beta_1 \times \text{Years of Education} + \beta_2 \times \text{labor market experience}+ \\ & \beta_3 \times \text{labor market experience}^2 + \beta_4 \times \text{labor market tightness}+ \\ & \beta_5 \times \text{gender} + \beta_6 \times \text{race} + \beta_7 \times \text{marital status}+ \\ & \beta_8 \times \text{number of dependents} + \beta_9 \times \text{gender} \times \text{number of dependents}.\end{aligned} \tag{B.1}$$

Estimation results are displayed in Table B.2. Regression results show the expected results for `Years of Education` and `Labor Market Experience`. The coefficient on the squared `labor market experience` shows that the return to experience is decreasing as experience goes up. The tighter the labor market, the higher the wage is. Women and non-whites have lower wages. Respondents that are married and respondents that have dependents have higher wages, but women with children have lower wages. The coefficients from this regression were applied to the 968 respondents that were used in the calibration exercise to determine the predicted wage.

Table B.2: OLS estimation of predicted wage based on the characteristics of the worker.

| Variable | Estimate |
|---|---|
| Intercept | -57.5243 |
| | (91.4892) |
| Time Dummies | Various |
| Years of Education | 0.09521 |
| | (0.000518)*** |
| Labor Market Experience | 0.03195 |
| | (0.0004323)*** |
| (Labor Market Experience)$^2$ | -0.0004210 |
| | (0.00000773)*** |
| Labor Market Tightness | .00168 |
| | (0.00264) |
| Gender | -0.1817 |
| | (0.00322)*** |
| Race | -0.08216 |
| | (0.00340)*** |
| Marital Status | 0.1018 |
| | (0.00305)*** |
| Number of Dependents | 0.02571 |
| | (0.00171)*** |
| Gender × Number of Dependents | -0.03653 |
| | (0.00243)*** |
| Time Dummies | 6.16112- 44.1511 |
| Adjusted R-squared: 0.3527 | |
| Dependent variable: $\ln(w_1)$ | |
| $***p < .01$, $**p < .05$ | |

# APPENDIX C

## SIPP DATA

SIPP is a national longitudinal survey of the labor market behavior of households that is conducted by the U.S. Census Bureau. SIPP specifically focuses on households' participation in government income and work-support programs, making it a good data source for the study of unemployment insurance. Topics in the survey include income maintenance programs such as welfare and unemployment compensation, employment and earnings, health insurance coverage, and household assets. SIPP tracks all individuals in a household month-to-month for a period of up to four years.

Each SIPP panel lasts between two to four years. The survey began in 1984 when the first panel was surveyed. A redesign in 1996 introduced data collection by computer which claims greater accuracy in job matching. Computer entry also coincided with an expansion of the basic SIPP data allowing for more robust analysis and the elimination of overlapping panels. Data is collected in waves. Every calendar month, one-forth of respondents are surveyed and answer questions about the preceding four months. Each respondent is interviewed three times per year.

The SIPP has some advantages over other longitudinal data-sets. The amount of households covered is much larger than the National Longitudinal Study of Youth (NLSY). The SIPP 1996 panel covered 40,188 households and 115,999 individuals, the SIPP 2004 panel 40,600 households and 104,054 individuals of all ages. In contrast, the NLSY 1979 panel contained only 12,686 young men and women. The Current Population Study (CPS) covers a nationally representative sample of households of comparable size to the SIPP, but only tracks households for 8 months. The SIPP follows households after they move, or when a new household is created out of an old household.

There are several disadvantages to using the SIPP. First, oversampling of low income workers can bias estimation results. Second, like the NLSY, the SIPP suffers from attrition of survey respondents. The complicated nature of the survey means that data quality for some variables is poor as some respondents do not want to answer some questions or have trouble recalling details that have happened over the last four months. The recall leads to a seam bias problem in the SIPP data. Seam bias is the phenomenon where estimates of change measured across two interviews exceed estimates of change recorded in one interview. Seam bias is a form of measurement error. This paper does not correct for seam bias, but does throw out wage changes that seem to be unrealistic.

For the calibration estimation, the 1996, 2001 and 2004 panels are used. The 1996 panel is the longest of the recent SIPP panels, lasting 4 years. The 2001 and 2004 panels last 3.5 and 2.5 years, respectively. A 2000 panel was started, but was abandoned due to lack of funding after 6 months and was not used in this study. The data is presented in person-month format, with the Census Bureau combining the 4 month recall of each respondent.

*Data Selection*

Labor force history is constructed for each respondent in the SIPP survey. The labor force history that is needed for this analysis are wages before and after an unemployment spell. Respondents who do not lose a job in the first six months of their survey were removed from the analysis. Respondents who do not find a new job or drop out of the labor force are also removed. Wages at the old job and the new job are recorded. The characteristics of the respondent are recorded at the time of job loss.

A strict selection criteria is necessary in order to overcome some limitations of the SIPP. The short panel of the SIPP means that workers who lose their job in a later wave of the survey will not have the required information about the unemployment spell and subsequent labor force outcomes that are needed for this analysis. For that reason, the SIPP data is subset so that only workers who start the first wave with a job and then lose that job within the first six months of the survey are considered job losers. This lowers sample size, but guarantees that the outcome of the unemployment spell is observed for the remaining workers. There were no workers who lose a job in the first six months and remain unemployed throughout the length of the SIPP survey. Matching the job lost to the job's characteristics is done using a combination of variables. When the respondent has two jobs and loses them both in one wave, it is sometimes difficult to determine the main job or the job that is lost second. Using wages, recorded end dates of the job, and the weekly labor force history the jobs are matched to the characteristics of the job when possible.

The weekly labor force attachment variable from the SIPP is used to track the labor force history of job losers. A respondent that is not working can be coded in one of two ways in the SIPP survey, unemployed or out of the labor force. Sometimes workers that are coded as out of the labor market should be coded as unemployed, as they move directly from the out of the labor market state into the employed state. Wherever possible, wrongly coded data is corrected. For example, in the weekly labor force attachment variable in the SIPP data, if there is one week of out of the labor force surrounded by weeks of unemployment, the erroneous week is recoded as being unemployed. That worker was not considered discouraged. The respondent leaves the unemployed state in one of two ways, finding a job or becoming discouraged.

Some data points are excluded because they lacked the data needed for parts of the analysis. This may be because job characteristics can not be matched or because the respondent lacked information from the first interview when key variables such as age are recorded. Some of the wage data indicates the respondent might be an outlier. When wages are particularly high or particularly low, or rose or fell rapidly within a job, the respondent is removed. The extreme wages may be a result of a wage rate being coded incorrectly. The final data-set had 968 observations of unemployment spells. Of the 968 spells, 250 ended in discouragement and 718 ended in finding a job. Table C.1 shows a monthly summary of the SIPP calibration data.

Table C.1: The data used in the calibration summarized by the month the respondent left the unemployed state. The respondent may have become employed or discouraged during the month. The column $w_1$ is the average wage at the job that was lost, $\hat{w}_1$ is the predicted wage for that job, $w_2$ is the average starting wage of the job received after the unemployment spell.

| Month | Number Discouraged | Number Find | Percent Discouraged | Percent Find | $w_1$ | $\hat{w_1}$ | $w_2$ | Job Stability | Wage Growth |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 141 | 0.1% | 14.6% | 10.37 | 10.26 | 9.50 | 43.2 | 1.04 |
| 2 | 3 | 188 | 0.3% | 19.4% | 10.06 | 9.02 | 9.13 | 37.1 | 1.06 |
| 3 | 14 | 129 | 1.4% | 13.3% | 10.49 | 10.27 | 9.84 | 37.5 | 1.04 |
| 4 | 12 | 92 | 1.2% | 9.5% | 9.63 | 9.70 | 9.05 | 37.7 | 1.01 |
| 5 | 47 | 42 | 4.9% | 4.3% | 9.43 | 10.09 | 9.69 | 37.9 | 1.02 |
| 6 | 7 | 40 | 0.7% | 4.1% | 10.68 | 10.37 | 9.86 | 39.0 | 1.18 |
| 7 | 9 | 29 | 0.9% | 3.0% | 10.61 | 10.68 | 8.04 | 36.2 | 1.04 |
| 8 | 6 | 17 | 0.6% | 1.8% | 10.33 | 9.78 | 9.70 | 37.4 | 1.00 |
| 9 | 29 | 16 | 3.0% | 1.7% | 9.09 | 9.05 | 9.68 | 35.2 | 1.02 |
| 10 | 13 | 4 | 1.3% | 0.4% | 8.08 | 9.46 | 9.32 | 30.6 | 0.94 |
| 11 | 10 | 9 | 1.0% | 0.9% | 8.59 | 9.51 | 9.01 | 27.1 | 1.05 |
| 12 | 3 | 4 | 0.3% | 0.4% | 11.35 | 10.95 | 15.41 | 27.9 | 0.68 |

The *month* column in table C.1 refers to the month since the unemployment spell started. The *Number Discouraged* refers to the number of respondents who became discouraged that month out of the 968 that are in the data-set. *Number Find* indicates the number of respondents that found a job that month. The column $w_1$ is the average hourly wage of the job that was lost prior to the unemployment spell for respondents that either found a job or lost a job that month. The column $\hat{w}_1$ is the predicted wage for the lost job given the respondents' characteristics. This column was computed using a wage equation that is described in Appendix B. The column $w_2$ is the average hourly wage for respondents at the job subsequent to the unemployment spell for respondents that found a new job that month. *Job Stability* is the average number of weeks the subsequent job lasted for the respondent that found a job in that month. The column *Wage Growth* ratio of the starting wage at the job subsequent to the unemployment spell to the wage three months later for respondents that found a job in that month.

The monthly summary in Table C.1 shows some interesting patterns in the data. Only one worker became discouraged in the first month of looking for a job, but many became discouraged after the fifth month of looking. This corresponds to most states having unemployment benefits that lapse after six months of unemployment. The job finding rate is highest in the second month of looking for a job and declines sharply after the fourth of looking. This suggests that there is an optimal amount of time that a worker should look for a job and public policy could create incentives for workers to look for a period of about four months. The wages at the new job are highest for workers that finds a job in the third or sixth month. Wages are fairly stable for other months suggesting that for this cohort, there is not much of a stigma effect from being unemployed. Job stability at the job subsequent to an unemployment spell is highest for workers that find a job in the first month, but appears to be about equal for all other unemployment durations except for very long durations. Wage growth at the job subsequent to an unemployment spell is highest for workers who find a job after the sixth month of unemployment. This high wage growth may be a result of small sample size as respondents who find a job in the first 3 months also have above average wage growth.