MODELING OF THERMO-MECHANICAL BEHAVIOR OF NITINOL ACTUATOR FOR SMART NEEDLE APPLICATION

A Thesis
Submitted to
the Temple University Graduate Board

in Partial Fulfillment
of the Requirements for the Degree
MASTER OF SCIENCE IN MECHANICAL ENGINEERING

by
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May, 2012

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ABSTRACT

A large and increasing number of cancer interventions, including both diagnosis and therapy, involve precise placement of needles, which is extremely difficult. This challenge is due to lack of proper actuation of the needle (i.e., actuated from the proximal end, which is far away from the needle tip). To overcome this challenge, we propose to bend the needle using a smart actuator that applies bending forces on the needle body; thereby, improving the navigation of the needle. The smart actuator is designed with shape memory alloy (SMA) wires, namely Nitinol, due to their unique properties such as super-elasticity, shape memory effect, and biocompatibility.

For accurate steering of the smart needle, there is a need to understand Nitinol thermo-mechanical behaviors. Various existing SMA constitutive models were investigated and compared. Since SMA is used as an actuator in this project, only one dimensional constitutive models are considered. Two distinct models with different phase transformation kinetic approaches were chosen. The first model was proposed by Terriault and Brailovski (J. Intell. Mat. Systems Structures, 2011) using a modified one dimensional Likhachev formulation. The second model was developed by Brinson (J. Intell. Mat. Systems Structures, 1993).

Since all SMA constitutive models are empirically based, several important materials’ constants such as Phase Transformation Temperatures are needed. The four Transformation Temperatures are: Martensite start (M_s), Martensite finish (M_f), Austenite start (A_s), Austenite finish (A_f). Differential Scanning Calorimetry (DSC) was used to obtain these constants. These temperatures are also influenced by stress, defined by the
Clausius-Clayperon coefficients. The coefficients were obtained by measuring Nitinol temperature and displacement response under various constant stress conditions.

In order to study its actuation behavior, Nitinol wires under constant strain configuration and resistance heating were tested for their force response. The thermo-mechanical responses were then compared with numerical simulations. While Terriault and Brailovski resistance heating formulation agrees strongly with temperature responses, the model cannot be used to simulate the actuator mechanical responses. Brinson model simulations of the force responses were found to agree well with experimental results. In conclusion, Terriault and Brailovski resistance heating formulation should be coupled with Brinson model to accurately simulate Nitinol actuation behavior for the smart needle.
ACKNOWLEDGEMENT

First and foremost, I would like to thank my advisor, Dr. Parsaoran Hutapea, for his guidance and support. I would also like to thank my Thesis Committee Member, Dr. Kurosh Darvish and Dr. Shriram Pillapakkam for their valuable inputs.

I also like to thank Dr. Naresh Datla for all of the valuable discussions and for his kind assistance in designing and running the experiments. I would like to thank the Composites Laboratory team members: Joe Koo, Daniel Choi, Georgiy Diloyan, Bardia Kohn, and Mohammad Honarvar for their collaborative efforts.

Last but not least, I would like to thank my family; especially my parents, my uncle Tuan and my aunt Hue for their unconditional support and encouragements.
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CHAPTER 1

INTRODUCTION

1.1 Motivation

Prostate cancer is the most common non-cutaneous cancer in American men. In 2009, the American Cancer Society (ASC) has estimated 1,922,280 new prostate cancer cases and prostate cancer is one of the leading causes of cancer death [1]. The existing prostate cancer treatments include radical prostatectomy, brachytherapy, external beam radiation therapy (EBRT), hormonal therapy, and cryotherapy. Among these, brachytherapy with permanent radioactive seed implant is a popular treatment choice for patients with prostate cancer [2]. It is an outpatient procedure and in many cases has been contributed to lower long term risks of urinary incontinence and impotence when compared to radical prostatectomy or cryotherapy. Moreover, the procedure is better tolerated by patients with serious comorbidities [3]. It is a non-surgical procedure that, unlike EBRT, does not require multiple treatments. The implantation of seeds is also less technically challenging for the surgeon as compared to radical prostatectomy [4].

In brachytherapy, radioactive seeds are permanently implanted in the prostate for delivering a tumorocidal dose to the prostate tissues. The challenge in most of the existing needle based procedures is the safe and accurate navigation of the needle through tissue to the desired target. The seeds provide a significant radiation dose to a relatively small volume, requiring the seeds to be placed accurately to ensure complete treatment. Sensitivity of the urethra and rectal mucosa to radiation are other factors that heighten the need for careful placement of seeds.
Therefore, Podder et al. [7] proposed to develop a new type of needle, a self-actuating (“smart needle”, i.e., needle having distributed actuators and sensors along the needle’s body/ shaft) needle, with greater flexibility and maneuverability for accurate and easy access to the target by avoiding obstacles and with fully conformity of the anatomical structure. The smart needle will be developed using the concept of smart structures and actuators. It will have sensors to track the needle tip and to monitor the needle configuration, as well as distributed actuators along the needle shaft/body to respond to an undesirable structural response.

As shown in Figure 1.1, the novel curvilinear approach [7], as opposed to the rectilinear approach, reduces the required number of needles and seeds (or activity) by 30.1% and 10.6%, respectively. Overall, clinical implementation of the proposed smart needle can potentially improve radiation dose distribution and reducing dose of critical organs and thereby would potentially improve quality of life and survival of the prostate cancer patients.

Figure 1.1: Comparison between (a) the conventional needle approach and (b) the proposed smart needle [7].
The current proposed design is Longitudinal Body Segment design in which the Nitinol wires are attached along the needle. Depends on the need for needle deflection, different segment of the needle can be activated separately or simultaneously.

![Current proposed design of the smart needle](image)

**Figure 1.2 Current proposed design of the smart needle [7].**

To efficiently design the needle, there is a need to first understand the basic characteristics of smart actuator and then use Finite Element (FE) analysis. There are three factors that affect Shape Memory Alloys behaviors: temperature, stress and strain. Shape Memory Alloys have two unique capabilities: isothermal, super-elasticity and shape memory effect. Super-elasticity refers to the actuator ability to withstand large elastic strains, considerably larger than conventional materials. Furthermore, shape memory effect enables the actuator to recover its original shape by thermal activation, which generates actuation force. Different FE software was explored in term of their ability to simulate the behavior of SMAs. The software has not to only simulate the isothermal superelasticity, but more importantly the two way shape memory effect. ABAQUS has no documented Shape Memory Alloy constitutive model. ANSYS and LSDYNA have the same Shape Memory Alloys material model. The model was developed by Auricchio et al. [17]- [18] and has the unique capability to simulate both
tensile and compressive superelasticity behavior. This is especially useful for modeling devices such as superelastic needle or flexible glass frame [37]. The model requires a set of five different constants for each specific temperature, up to six different temperatures: martensite start transformation stress $\sigma_{AS}^S$, martensite finish transformation stress $\sigma_{AS}^f$, austenite start transformation stress $\sigma_{SA}^S$, austenite finish transformation stress $\sigma_{SA}^f$, and theoretical maximum residual strain ($\varepsilon_L$) (Fig. 1.3).

Figure 1.3: Illustration of different constant inputs required by ANSYS and LSDYNA Shape Memory Alloy model [14]-[16]

The model, however, does not have the capability to simulate shape memory effect. That means the strain can only be induced by mechanical loading and not by thermal loading. The goal of this thesis, therefore, is to develop a unified 1-D constitutive equation of SMAs.
Since the smart actuator is activated using resistance heating, the relationship between SMA mechanical responses (force and strain) to applied electrical current/temperature must be investigated. This thesis has two specifics aims

The first aim is to study and compare different constitutive models. The studied models have to satisfy the following criteria

- In this project, Nitinol is utilized for its actuation capability, therefore, only one dimensional constitutive models are studied
- The models have to be robust to implement in order to implement into commercial FE software (ANSYS) via its USETMAT architecture
- The constitutive models have to simulate both Nitinol unique characteristics of superelasticity and shape memory shape memory effect.
- At least of the models should have a component of resistance heating formulation for this project application purpose.

In order to compare and verify the chosen constitutive models, different experiments have to be conducted. There are two sets of experiments that have to be performed

- Material properties characterizations: Nitinol thermo-mechanical behaviors can be influenced by various factors such as: chemical composition, heat treatments, shape memory training procedures… Therefore, Nitinol wires from different manufacturers behave differently. There is a need thus to characterize the specific Nitinol materials used in this project. There are three different prominent characterization methods for Nitinol properties: constant stress experiment to measure temperature response; heat flow of Nitinol phase
transformation via Differential Scanning Calorimetry (DSC); and isothermal mechanical testing. This thesis would only focus on the first two and compare its result to determine the best method to characterize Nitinol behavior.

- Numerical verification: In Nitinol actuation configuration, the wires are constraint along its longitudinal body. Therefore, constant strain experiments are conducted to study Nitinol force and temperature response. These experimental results are also used to verify the models’ simulations.

1.2 Literature Survey

1.2.1 Shape Memory Alloys

Shape Memory Alloys has its name from its most special characteristic: shape memory effect. The phenomenon of the shape memory effect governed by the thermoelastic behavior of the martensite phase was widely reported by Kurdjumov and Khandros [21] and also by Chang and Read [22]. Since then, shape memory alloys with various materials composition have been widely used. Wuttig et al. [23] explored CO\textsubscript{2}Ni\textsubscript{1-x}Ga\textsubscript{1+x}, as a possible ferromagnetic shape memory alloys. Oikawa et al. [24] studied Ni-Co-Al alloy as a promising ferromagnetic shape memory alloy system, while Murray et al. [25] investigated large field induced strain in Ni-Mn-Ga alloys for magnetic application. Among them, Nitinol is the most popular and extensively studied. Nitinol is an alloy of roughly 50% Nickel and 50% Titanium. Discovered in 1960 as part of a Naval research project by Buehler and his co-workers [26], Nitinol bears its name from its chemical components and its founders: Ni (Nickel)+Ti(Titanium)+NOL(Naval Ordinance Laboratory). Nitinol has good biocompatibility [27]-[28] and good magnetic resonance imaging opacity [29]-[30]. Coupled with unique superelasticity and shape memory effect,
Nitinol have been widely studied for medical applications such as orthodontic wire, biomedical stents, and steerable catheter [29]-[31].

1.2.1.1 Microstructural Phases of SMAs

Nitinol has two basic microstructural phases: high temperature austenite, simple cubic B2 structure (Fig. 1.4a); and low temperature martensite monoclinic B19 (Fig. 1.4b and Fig. 1.4c) [32].

![Figure 1.4](image)

Figure 1.4: Different phases of Nitinol: (a) High temperature austenite, (b) Low temperature martensite, and (c) stress induced martensite [32].

Martensite can also exist in two different forms: multivariant twinned martensite, or temperature induced (Fig 1.4b); and thermodynamically stable single variable detwinned martensite, or stress induced (Fig. 1.4c). In twinned martensite, its variants are crystallographically equivalent, differing only by habit plane indices, and each variant consists of two twin-related martensites [34], hence the term twinned martensite. Twinned martensite can be detwinned by mechanical stress loading. During this loading, stress reorients the martensitic variants lattices into the one single direction, thus making it thermodynamically more stable.
Nitinol phase transformation is the main factor for Nitinol unique behaviors. Nitinol constitutive behavior is driven by mechanical properties of each of its phases. At a particular temperature and stress state, Nitinol has different volume fraction of martensite and austenite. Therefore, it is crucial to understand the transformation condition of each phase.

The evolution of Martensite is dictated by four temperatures: Austenite finish temperature \((A_f)\), Austenite start temperature \((A_s)\), Martensite start temperature \((M_s)\), Martensite finish temperature \((M_f)\). It is experimentally observed that the transformation temperature is influenced by the stress state of the material [33]. The higher the stress state is, the higher its transformation temperature. The relationship between the stress state and the transformation is graphically depicted in Fig. 1.5.

![Figure 1.5: Relationship between the transformation temperatures (T) and stress state (σ), characterized by two Clausius-Clayperon coefficients, \(C_M\) for martensite and \(C_A\) for austenite [33].](image)
Consider a Nitinol wire is initially in a stress free state with temperature above Austenite finish temperature \((T > A_f)\). At this state, the wire is in complete austenite phase. The wire is then subsequently cooled down, move along the x axis. The wire only starts to transform back to martensite once the temperature fall below the martensite start temperature \((T < M_s)\). The martensitic transformation would be completely once the temperature reaches below Martensite finish temperature \((T < M_f)\). During this process, there is no incurrence of transformation strain due to the formation of multivariant twinned martensite. This phenomenon is called “self accommodation” [34]. On the reverse process, the same principle applies, in which Austenitic transformation begin once the temperature reach \(A_s\), and complete once the temperature reaches \(A_f\) again.

1.2.1.2 Superelasticity

Superelasticity, as defined above, is the ability of Nitinol to elastically deform more than conventional material. Theoretically, Nitinol can elastically deform to nearly 11% strain [30]-[31] at temperature above \(A_s\), compared to most metal at 1% strain. This property is illustrated clearly in Fig. 1.6.
Figure 1.6 can be explained as follows. At temperature above $A_f$, the wire is initially at zero stress, or point (a). Mechanical stress is subsequently applied by stretching the wires. The wire then elastically deform until point (b). This means that between point (a) and point (b), the wire is completely in austenite phase, thus Fig. 1.6 reflects Young’s modulus of austenite phase at the line from (a) to (b). At point (b), the applied stress is large enough to start reorient the lattices, thus effectively start to transform the wire from martensite to austenite. Due to the martensitic transformation, continuous stretching of the wire (increasing strain) only induce very minimal stress, as observed in Fig. 1.6. The transformation completed at point (c). The wire is now completely in detwinned martensite state (or stress induced martensite). Further stretching beyond point (c) would induced more stress to point (d) (before plastic deformation). The slope from point (c) to point (d) now reflects the Young’s modulus of martensite. At point (d), the wire is slowly released until it reaches point (e) in Fig. 1.6.
The slope from point (d) to point (e) is the same as point (c) to point (d) since the wire is still completely in detwinned martensite phase. At point (e), the wire enter start to transform back into austenite. Similarly to the loading curve, from point (e) to point (a), we can see very little stress response with large strain recovery. Once the transformation is completed, complete release of the wire constrain would return it to point (a).

As observed in Figure 1.6, the loading and unloading path of Nitinol is different from each other. This phenomenon is called pseudoelasticity (or hysteresis). Pseudoelasticity can only be apparent in high temperature, specifically in temperature above $A_f$ and below critical stress induced martensite temperature ($M_d$) [35].

1.2.1.3 Shape Memory Effect

Shape Memory Effect refers to the material to remember its shape at different temperature. There are two different types of shape memory effect: one way and two ways. One way shape memory effect is graphically depicted in Fig 1.7.

![Illustration of one way shape memory effect response of Nitinol](image)

Figure 1.7: Illustration of one way shape memory effect response of Nitinol [35]
In Figure 1.7, consider a Nitinol wire under zero stress with temperature above \( A_f \), point (a). When the wire is cooled down below \( M_f \) under zeros stress, or along the \( z \) axis in Fig. 1.6, from point (a) to point (b), the wire transform completely into twinned martensite. Under an isothermal condition, applied mechanical transforms the wire from twinned martensite to detwinned martensite, as observed at point (c). The transformation is completed at point (d), also required a large strain with very little stress induced. Since the single martensitic variant is thermodynamically stable, upon unloading of the stress to point (e), there would be no reconversion back to multivariants martensite, thus a large residual strain occurs. The residual strain can be recovered by simply heating up the material, allow it to transform back into austenite, point (e) to point (a). This strain however, cannot be recovered until the wires are heated above \( A_s \), or point (f).

Two way shape memory effect refers to SMAs ability to transform directly from point (a) to point (e) at zero stress (plane XZ in Fig. 1.7) without the intermediate mechanical loading. This is done via thermal cycle training [36]. In most actuation applications, two-way shape memory effect is required, thus essential for this project.

### 1.2.2 Constitutive Models

Since its invention, numerous models have been proposed to simulate the stress-strain-temperature relationships of Nitinol. As mentioned in section 1.2.1, Nitinol unique behaviors derived from its unique phase transformation kinetic. Therefore, the main difference between all the models is its derivation of the phase transformation kinetic. Graesser and Cozzarelli [38] in 1994 used a three dimensional evolutionary plasticity to model the stress-strain-temperature relationship. However, only isothermal
superelasticity could be simulated. Another notable model was proposed by Malorvh and Gandhi [39], but it cannot be used to explain shape memory effect.

The most prominent approach is by using energy derivation to describe the micro and macro behavior of Nitinol. These approaches use temperature and stress to describe Nitinol phase transformation kinetic. Kamita and Matsuzaki [40] proposed a lumped parameter pseudoelastic model of an SMA uniform wire. The model describes dissipation energy during the phase transformation, called the phase interaction energy function (PIEF). Boyd and Lagoudas [41] use thermodynamics approach utilizing Gibbs free energy and dissipation potential to derive a three dimensional constitutive relation. Another noticeable model was proposed by Ivshin and Pence [42] utilizing thermodynamics principle to derive constitutive relations from Maxwell relation. The models mentioned above follow rigorous mathematical approach, with very promising results. However, they are very difficult to implement due to the models requirements of many material constants, some of which are difficult to obtain.

The second approach to model Shape Memory Alloys constitutive behavior by utilizing the Clausius-Duhem inequality and Helmholtz free energy approach. The models introduced below are all one dimensional. The first model was proposed by Tanaka [43] then later refined by Liang and Rogers [33]. These two models have the same constitutive relation as a function of SMAs martensite volume fraction. The biggest distinction between them is their phase transformation kinetic rule. Tanaka [43] proposed phase transformation kinetic as a function in exponential form of both temperatures and stress. Liang and Rogers [33] proposed phase transformation kinetic as a function in cosine form. In most loading cases, the two models yield almost the same results. Both
models, however, cannot be used to simulate the one way shape memory effect (the XY plane of figure 5b) [34] at low temperature due to its formulation of the constitutive equation. It is noteworthy that as complicated as the Boyd and Lagoudas model and Ivshin and Pence are, they can be simplified and yield almost identical results to the Tanaka model [43]. Based on the same principle of Tanaka [43] and Liang and Rogers[33], Brinson [34] introduced the separation of different types of martensite: stressed induced and temperature induced into the constitutive equation. Brinson model therefore overcomes the first two models’ limitation and successfully simulates the shape memory effect. The model also adopts the different phase transformation kinetic rule to accommodate the introduction of different martensite types.

The aforementioned models present major efforts of different researchers in modeling Shape Memory Alloys constitutive behavior. Most of the models presented above follow similar approach of describing the phase transformation kinetics using stress and temperature as input parameters. Therefore, another model with different approach is also studied. A one dimensional modified Likhachev model [50] is thus chosen and examined for its rather simple and physically grounded formulation. The model is then compared to more popular Brinson [34] model for its ability to simulate the complex thermo-mechanical behavior of SMAs.
CHAPTER 2

METHODS

2.1 Constitutive Models

This chapter introduces Likhachev [50] and Brinson [34] model. Since this thesis only concentrates on modeling one dimensional behavior of Nitinol for its actuation force; it is important for the readers to know that modeling effort is only concentrate on round Nitinol wires only. The constitutive models only consider the stress and strain behavior along the wire longitudinal axis.

2.1.1 Brinson Model

Brinson model is perhaps the most prominent one dimensional constitutive model. It is a result of previous outstanding efforts of Tanaka [43] and Liang and Rogers [33]. Tanaka [43] first derived the constitutive relation for shape memory alloys using thermodynamic principles Clausius-Duham inequality and Helmholtz free energy

$$\sigma - \sigma_o = E(\xi)(\varepsilon - \varepsilon_o) + \Omega(\xi)(\xi - \xi_o) + \beta(T - T_o)$$

[2.1]

Where $\sigma$ and $\varepsilon$ denote the current stress and strain state, respectively. $E(\xi)$, $\Omega(\xi)$, $\beta$ is the Young’s Modulus, phase transformation tensor, and thermoelastic coefficient, respectively. Their mathematical definitions mean that they are functions of martensite volume fraction. The subscript $o$ in $\xi_o$, $T_o$, $\sigma_o$, $\varepsilon_o$ indicates the initial values those variables.

Tanaka [43] also proposed the phase transformation kinetic rule using exponential functions. Liang and Rogers [33], based on Tanaka’s work, used the same constitutive equation. They, however, described the phase transformation kinetic as cosine functions,
motivated from cosine function simulate much better agreement with experimental finding. The two models are widely recognized for its ability to simulate superelasticity accurately via experimental validation, as shown by Prahlad and Chopra [51]. Prahlad and Chopra [51] also confirmed that using cosine functions, Liang and Rogers [33] model demonstrate better agreement than Tanaka model [43]. The two models, however, cannot simulate one way shape memory effect at low temperatures, as analytically demonstrated by Brinson [34] and numerically confirmed by Zak et al. [52].

Based on the works done by Tanaka [43] as well as Liang and Rogers [33], Brinson proposed a new model by introducing the separation of martensite volume fraction ($\xi$) into stress induced ($\xi_s$) and temperature induced ($\xi_t$) [34]. This model thus effectively overcame Liang and Rogers[33] model limitation of describing shape memory effect at low temperature [34].

Since its introduction, Brinson model has been implemented to study the general thermo-mechanical behaviors of shape memory alloys due to its robust and easy to implemented formulation. Utilized Brinson model, Ucardi [53] studied the deformation of a composite cantilever beam using embedded Shape Memory Alloys actuators. Sun [54] studied the curvature response of elastic beam with embedded shape memory alloy wires via numerical simulation of the Brinson model. Pae et al. [55] also used the Brinson model to study the use of SMA wires in order to control higher modes of deformation in beams.
2.1.1.1 Formulation

At anytime, the martensite volume fraction can be described as:

$$\xi = \xi_s + \xi_T$$  \[2.2\]

Where $\xi$ is the global martensite volume fraction, $\xi_s$ denotes the stress induced martensite and $\xi_T$ denotes the temperature induced martensite.

Due to the introduction of separation of different types of martensite, equation [2.1] by Tanaka has to be re-defined. Follow the same thermodynamics approach of Tanaka, Brinson derived the new constitutive relation, as mathematically defined as follow

$$\sigma - \sigma_o = E(\xi)\varepsilon - E(\xi_o)\varepsilon_o + \Omega(\xi)\xi_s - \Omega(\xi_o)\xi_{so} + \beta(T - T_o)$$  \[2.3\]

Where the Young Modulus and the Transformation Tensors are defined as

$$E(\xi) = E_A + \xi(E_M - E_A)$$  \[2.4\]

$$\Omega(\xi) = -\varepsilon_{max}E(\xi)$$  \[2.5\]

$E_A$ denotes the Young’s Modulus of austenite and $E_M$ is the Young’s Modulus of martensite while $\varepsilon_{max}$ is the maximum transformation strain. In this thesis, $\beta$ refers to the thermoelastic term, which is assumed to be constant for simplicity in implementation.

Based on the kinetic transformation cosine function of Liang and Rogers[33], as well as the critical stress definition, Brinson redefined the kinetic phase transformation to accommodate newly defined $\xi_T$ and $\xi_s$ [34]. The transformation zones are graphically represented in Fig. 2.1.
In Figure 2.1, $\sigma_s^{cr}$ and $\sigma_f^{cr}$ denotes the start and finish critical stress for conversion to detwinned martensite. Regions [1], [2], and [4] mark the transformation zones from either twinned martensite or austenite to detwinned martensite. Region [3] marks the transformation zone from austenite to twinned martensite. Region [5] marks the transformation zone from either twinned or detwinned martensite to austenite. The rest of the regions have no phase transformation.

The phase transformation kinetic is mathematically defined as follow. At all time, equation [2.2] must be satisfied.

For $T<M_s$ and $\sigma_s^{cr}<\sigma<\sigma_f^{cr}$, regions [1]-[2]

$$\xi_s = \frac{1-\xi_{so}}{2} \cos \left( \frac{\pi}{\sigma_s^{cr}-\sigma_f^{cr}} (\sigma - \sigma_f^{cr}) \right) + \frac{1+\xi_{so}}{2}$$

[2.6]
\[ \xi_T = \xi_{T0} - \frac{\xi_{T0}}{1-\xi_{so}} (\xi_s - \xi_{so}) + \Delta T_\xi \]  

[2.7]

Where, if \( M_f \) < \( T \) < \( M_s \), and \( T \) < \( T_o \), or region [2]

\[ \Delta T_\xi = \frac{1-\xi_{T0}}{2} \left( \cos \left( a_M (T - M_f) \right) + 1 \right) \]  

[2.8]

else for regions [1]

\[ \Delta T_\xi = 0 \]  

[2.9]

For \( T > M_s \) and \( \sigma_{cr}^M (T-M_s) < \sigma < \sigma_{cr}^f + C_M(T-M_s) \), or region [4]

\[ \xi_s = \frac{1-\xi_{so}}{2} \cos \left( \frac{\pi}{\sigma_{cr}^f - \sigma_{cr}^M} \left( \sigma - \sigma_{cr}^M (T-M_s) \right) \right) + \frac{1+\xi_{so}}{2} \]  

[2.10]

\[ \xi_T = \xi_{T0} - \frac{\xi_{T0}}{1-\xi_{so}} (\xi_s - \xi_{so}) \]  

[2.11]

For \( T > A_s \) and \( C_A(T-A_f) < \sigma < C_A(T-A_s) \), or region [5]

\[ \xi = \frac{\xi_{so}}{2} \left( \cos \left( a_A \left( T - A_s - \frac{\sigma}{C_A} \right) \right) + 1 \right) \]  

[2.12]

\[ \xi_s = \xi_{so} - \frac{\xi_{so}}{\xi_o} (\xi_o - \xi) \]  

[2.13]

\[ \xi_T = \xi_{T0} - \frac{\xi_{T0}}{\xi_o} (\xi_o - \xi) \]  

[2.14]

Where \( a_M \) and \( a_A \) are mathematically defined as:

\[ a_M = \frac{\pi}{M_s - M_f} \]  

[2.15a]

\[ a_M = \frac{\pi}{A_f - A_s} \]  

[2.15b]
2.1.2 Modified Likhachev Model

The tensorial three dimensional constitutive model was first proposed by V.A Likhachev in 1995 [45]. The work was published almost exclusively in Russian. Therefore, it is much less popular in the US than other three dimensional models such as of Boyd and Lagoudas [41]. The Likhachev model, however, consists of physically grounded equations as well as simple material constants inputs. Likhachev model has been studied by Pushtshaenko et al. [46] to simulate the damping behavior of Shape Memory Alloy Rod. Evard and Volkov used the model to study the martensite accommodation effect on mechanical behavior of shape memory alloys [47]. Likhachev model specially been studied extensively by the Shape Memory and Intelligent Systems Research Laboratory of Ecole De Technologie Superieure, Canada. Wong et al. [48] first introduce the hybrid micro-macro-mechanical constitutive model for shape memory alloys utilizing Likhachev model. Paradis et al. [49] investigated the residual strain accumulation of Nitinol via this model. Terriault and Brailovski [50] then modified it into a 1D constitutive model and thus presented here in this thesis. This section also presents a resistance heating formulation, which is proposed by Terriault and Brailovski.

2.1.2.1 Formulation

The model first calculates the strain correspond to the material temperature and stress state. The strain of Shape Memory Alloys can be broken down into four parts:

\[ \varepsilon = \varepsilon_{el} + \varepsilon_{tr} + \varepsilon_{th} + \varepsilon_{pl} \] \hspace{1cm} [2.16]

Where \( \varepsilon \) denotes the material total strain state, \( \varepsilon_{el} \) is the elastic strain induced by the stress state, \( \varepsilon_{tr} \) is the strain induced by the phase transformation, \( \varepsilon_{th} \) is the strain caused by the material thermal expansion and \( \varepsilon_{pl} \) is the strain caused by irreversible plastic
transformation (trapped martensite). The actuator, however, is trained to work in many cycles; thus plastic strain is not considered in this model. The thermal strain is also not considered in this case since it is negligible compared to the large transformation strain. Therefore, equation [2.16] can be rewritten as:

$$\varepsilon = \varepsilon_{el} + \varepsilon_{tr}$$  \[2.17\]

The elastic strain is mathematically defined as:

$$\varepsilon_{el} = \left(\frac{1-\Xi}{E_A} + \frac{\Xi}{E_M}\right)\sigma$$  \[2.18\]

Where $E_A$ and $E_M$ are the Young’s modulus of austenite and martensite, respectively; $\Xi$ is Nitinol global volume fraction of martensite; $\sigma$ is the stress of the material. The following conditions must be applied for the volume fractions:

$$0 \leq \Xi \leq 1$$  \[2.19\]

The wire is now divided into $N$ number of difference equal volumes (Fig 2.2). The regions can be interpreted as an assembly of all the grains that has similar crystallographic orientation. This assumption main purpose is to simulate multivariant twinned martensite. Since each pair of the twinned martensite has different habit plane of indices, the orientation of the maximum transformation strain is also different.

Figure 2.2: Schematic representation of Nitinol wire divided in to N series of equal regions [50]
In Figure 2.2, $\theta_i$ denotes the regions maximum strain orientation with respect to the wire longitudinal axis. $\xi_i$ denotes the local martensite volume fraction. $\varepsilon_{\max}$ denotes theoretical maximum transformation strain of Nitinol.

Based on Figure 2.2, the wire transformation strain along the wire longitudinal axis can be calculated as:

$$\varepsilon_{tr} = \frac{1}{N} \sum_{i=1}^{N} \xi_i \varepsilon_{\max} \cos(\theta_i) \tag{2.20}$$

The biggest difference between Likhachev model compared to other models is its unique approach in modeling the phase transformation. Since each of the wire small regions has different maximum strain orientation, thus it also has different rate of phase transformation. This is especially true under mechanical loading. Likhachev [50] introduced a new concept called equivalent temperature for each small region. Equivalent temperature is a function of both the wire temperature and its stress state, as mathematically defined:

$$T_i^* = T_w - \frac{\sigma \cos(\theta_i)}{C_i} \tag{2.21}$$

Where $T_i^*$ denotes equivalent temperature at the $i^{th}$ region, $T_w$ is the wire global temperature and $C_i$ are the Clausius-Clapeyron coefficients. The Clausius Claypeyron coefficients represent the slope of transformation temperature increase due to stress influence ($C_M$ and $C_A$). By obtaining the equivalent temperatures of all the regions, the respective local martensite volume fraction can be calculated using equation [2.22] and graphically depicted in Figure 2.3.
In equation [2.22], $d\Phi$ indicates the change of local volume martensite fraction from previous condition to current condition. Similarly, $dT^*$ indicates the change in equivalent temperature. The first pair of terms $\frac{dT^*}{M_f-M_s}$ and $\frac{dT^*}{A_s-A_f}$ are the slope of local martensite volume fraction in respect with cooling and heating of the equivalent temperature. ($H(x)$ is the Heaviside function that yields $H(x)=0$ for $x<0$ and $H(x)=1$ for $x>1$ or $x=0$. Therefore, the second pair of terms $H(-dT^*)$ and $H(dT^*)$ indicates the cooling and heating part of the equation. $H(\Phi_{max}-\Phi)$ and $H(\Phi-\Phi_{min})$ limits the local martensite volume fraction to be between the maximum and the minimum theoretical local
martensite volume fraction. The final pair of Heaviside term specifies the temperature at which direct phase transformations occur.

The global martensite volume fraction can be then be mathematically defined as the average of all the local martensite volume fractions

$$\Phi_M = \sum_{i=1}^{N} \frac{\phi_i}{N}$$  \hspace{1cm} [2.23]

### 2.1.2.2 Resistance Heating Formulation

Since our actuators are activated using resistance heating, it is important to study the relation of the actuators heat transfer mechanisms. Terriault and Braislovski [50] also proposed a complete iteration scheme to find the change of temperature in the actuators caused by dominant heat transfer mechanism such as: environmental convection, resistance heating and even latent heat difference due to phase transformation. The formulation is described as followed.

The energy generated in the wire are contributed by resistance Joule heating ($E_{JE}$) and latent energy of transformation ($E_{LH}$), and mathematically defined as

$$E_G = E_{JE} + E_{LH}$$  \hspace{1cm} [2.24]

With the resistance Joule heating defined as

$$E_{JE} = R i^2 \Delta t$$  \hspace{1cm} [2.25]

where $i$ is the average current applied during the time increment and $R$ is the resistance of the wire. $R$ is a given by the mixing law of the global phase volume fraction based on the electrical resistivity of austenite and martensite.

The latent heat of transformation can be calculated using the increment of the global volume fraction of martensite ($\Delta \Xi_M$), the latent heat of transformation ($Q_{PT}$), and
the actuator volume. The actuator volume is a product of the wire cross sectional area ($S$) and its length ($L$).

$$E_{LH} = Q_{PT}SL\Delta \varepsilon_M$$  \hspace{1cm} [2.26]

The energy generated can be lost due to convection ($E_C$) or the energy stored by the wire ($E_s$).

$$E_G = E_C + E_S$$  \hspace{1cm} [2.27]

The convection energy can be described as a function of the actuator convection coefficient ($h$), length of the wire ($L$), the perimeter of the wire ($P$)

$$E_C = hLP(T_w + T_{AMB})\Delta t$$  \hspace{1cm} [2.28]

where $T_{AMB}$ and $T_w$ is the average ambient air temperature and average wire temperature during the time increment ($\Delta t$), respectively. From equation 10-14, the wire temperature change during the time increment can be derived as

$$\Delta T_w = \frac{Ri^2\Delta t + Q_{PT}SL\Delta \varepsilon_M - hLP(T_w - T_{AMB})\Delta t}{\frac{hLP\Delta t}{2} + C_{pd}LS}$$  \hspace{1cm} [2.29]

2.2 Experiments

Brinson and modified Likhachev model, as most Shape Memory Alloys constitutive models, are empirical based, therefore, many materials constants are needed to obtain. Nitinol materials properties are extremely sensitive compared to other materials. Depends on its chemical composition, manufacturing process, heat treatment conditions, different Shape Memory Alloys can have different material properties. In this thesis, there are six important material properties are obtained via two different experiments: four Transformation Temperatures and the two Clausius-Clayperon
coefficients. Other less sensitive material properties are either obtained by widely accepted literatures or parametric studies of the models.

There are three different sets of experiments conducted in this thesis. The first set of experiments measure Nitinol temperature and strain response under various constant stress levels. This set of experiments was designed to obtain all six aforementioned material properties. The second set of experiments was design to measure Nitinol heat flow due to phase transformations via Differential Scanning Calorimetry (DSC). Base on different peaks of the heat flow, the Transformation Temperatures are obtained. This set of experiment is used to compare and confirm the Transformation Temperatures obtained from the constant stress experiments.

The Nitinol wires are obtained for Dynalloys Inc. (Tustin, CA). The wires are commercially known as Flexinol. Only Low Temperature (LT) Flexinol wires were ordered with various diameters, ranging from 0.48 mm to 0.025 mm, with identical length of 100 mm. They all have a composition of roughly 50% Nickel and 50% Titanium. Each wire has a crimp attached to each of its ends, thus allow easy attachment mechanism in the experiments.
2.2.1 Constant Stress Experiment

The Clausius-Clayperon coefficients are used as an indication of effect of stress on Transformation Temperatures, with the common unit of MPa/°C. The common practice for measuring the Clausius-Clayperon coefficients are applied thermal loading (heating-cooling) of the Nitinol wires under various constant-stress states and measure its strain response.

The set up can be described as followed. A Nitinol wire is hung vertically and under uniaxial tension loading. The load is applied using a weight hanger (Fig. 2.5). The weight hanger also connected to a free hanging magnetic transducer rod. The magnetic transducer rod is inside of a Linear Variable Differential Transducer (LVDT) core. The LVDT core composed of 3 magnetic cores place vertically. The primary core is between two secondary cores. When the Nitinol wires are thermally activated, it contracts, moving...
the weight hanger-transducer rod upward. The transducer rod’s movement induces a voltage signal that is output by two secondary cores. The schematic for the experiment is presented in Fig. 2.5.

![Schematic for the LVDT experiment.](image)

Figure 2.5: Schematic for the LVDT experiment.

The LVDT used is HSD 750-500 manufactured by Macro Sensor (Pennsauken, NJ) with nominal range of ±12mm with scale factor of 0.8V/mm. The primary coil is excited by ±15VDC. The Nitinol wires are thermally activated by resistance heating. The current is supplied as a ramp function using a programmable DC power supply (BK Precision 1696, Yorba Linda, CA). A bare 0.003 in J-thermocouple (Omega Engineering, Stamford, CT) were attached to the top of Nitinol wires. The output signal of both the thermocouple and the LVDT was collected using SCXI-1321 terminal block (National Instrument, Austin, TX).
Two Nitinol wires with 0.48 mm diameter and 0.31 mm diameter were tested. In each wire, to ensure complete austenitic transformation, the wires were heated up to 90-100°C. For each constant stress level of each wire, three repetitions were performed to ensure material stable behavior as well as statistically significant data are obtained. Five constant stress levels were tested for each wire diameter to ensure sufficient data.

2.2.2 Measurement of Transformation Temperatures

Differential Scanning Calorimetry (DSC) is the most common method to obtain the true zero stress transformation temperatures of Shape Memory Alloys. Samples are put into a small aluminum pan, then heated and cooled down at a constant rate. During these periods, DSC measures the heat flow of the samples due to material phase transformations. The experiments were performed in using a DSC 2920CE machine (TA Instrument, New Castle, DE), as shown in Fig. 2.6. In all of the experiments, liquid Nitrogen was used as both the cover and purge gas.

Figure 2.6: TA DSC 2920 machine used to obtain the transformation temperatures.
There were two experiments performed. The first experiment was performed on a 0.48 mm wire diameter in its as-received conditions. The second experiment was performed on a 0.48 mm wire diameter that was previously annealed.

ASTM-F2004-05 [62] standard suggests the Nitinol samples annealed in an inert environments to avoid oxidation of the samples. Therefore, the wire was first cut up into smaller samples with less than 5 mm long. These pieces then were put into a small pre-made quartz tube (5mm x 7mm IDxOD). The pre-made quartz tube is sealed at one end. A vacuum pump, Maxima C Plus manufactured by Fisher Scientific (Waltham, MA) was used to pump air and moisture out of the tube. The tubes with samples inside have to be vacuumed for a few minutes to ensure complete air removal. The pump suction hose to pass through a buret submerged in liquid nitrogen to ensure complete moisture removal. The tube was then sealed using a high temperature glass burner. The experimental set up for annealing process is illustrated in Fig. 2.7.

Figure 2.7: Experimental setup of annealing sample preparation process.
The samples inside the tubes (Fig. 2.8) were annealed in Thermolyne 1500 furnace at 850°C for 45 mins, follow by water quenching.

![Flexinol samples](image)

Figure 2.8: Nitinol samples in the premade quartz tube.

### 2.2.3 Constant Strain Experiment

The project application is to model the Nitinol wires’ for its actuation capability. The actuation capability can be described analogously as a fixed end cantilever beam. When thermally activated, the actuator contracts to its higher temperature (shorter) shape. Since the actuator would be fixed at both ends by the plastic collet (Fig. 1.2), there would be reaction force at the collet, or actuation force. Therefore, a fixed-strain experiment was conducted to simulate the “actuation force” response of Nitinol. The experimental setup is illustrated in Fig. 2.9.
The Nitinol wires were also activated using resistance heating by applying current as a ramp function. Force response of the Nitinol wire is measured using a 50lbs load cell (Futek Advance Sensor Tech, Irvine, California). The load cell signal was collected using SCXI-1314 DAQ system. Similarly to the constant stress experiment, a J thermocouple also was attached on the top of the wires. A linear stage (Edmund Optics, Barrington, NJ) were used to control pre-strained amount of the Nitinol wires.

Similarly to constant stress experiment, two wire diameters of 0.48 mm and 0.31 mm were selected to verify the constitutive models simulation.
CHAPTER 3

RESULTS AND DISCUSSION

3.1 Constant Stress Experiment

A typical result of the constant stress experiment is illustrated in Fig. 3.1. In order to get the Transformation Temperatures, tangent line on the plateau and the transformation curve should be drawn. The intersections between these lines denote the four Transformation Temperatures are then obtained.

Figure 3.1: Typical results of constant stress experiment. Results of strain are plotted against temperatures.

For each constant stress level of each wire diameter, 3 repetitions were performed to ensure stability of the material as well as for statistical analysis. A typical result for three different repetitions are plotted in Fig. 3.2
Figure 3.2 shows a very good, stable behavior of the Nitinol wire for three different repetitions.

Figure 3.3 compares strain-temperature response for 0.48 mm wire diameter under various constant stress levels for the third repetitions. The y-axis was normalized by taking the initial position of the wire when the weight hanger-transducer rod first hung. The initial room temperature strain of each stress level is different due to the mechanical loading of the wires. The higher the stress level, the more it stretches the wires. This can be shown by higher stress level has higher positive strain. When the wires are heated, it reverses back to its high temperature shorter shape, hence the decrease in strain for the heating curve. Figure 3.3 shows very good agreement with literature findings [52], [59].
Figure 3.3: Constant stress results for 0.48 mm Nitinol wire under various stress level for the second repetition

For each repetition of each wire under a constant stress level, the four Transformation Temperatures are obtained with methods illustrated in Fig. 3.1. Compiled data of 0.48 mm are tabulated in Table 3.1 with all the Transformation Temperatures for all five constant stress levels and all the repetitions. The stress levels are plotted against transformation temperatures for all the repetitions (Fig. 3.1). Linear curve fit then was performed for all the Transformation Temperature. Based on the curve fit slope, the Clausius-Clayperon coefficients are obtained as 5.75 MPa/°C, 6.09 MPa/°C, 8.43 MPa/°C, and 9.15 MPa/°C for $M_f$, $M_s$, $A_s$, and $A_f$ respectively. Since both constitutive models require only one constant for martensitic transformation ($C_M$) and austenitic transformation ($C_A$), averages those coefficients are taken. Therefore, $C_M$ is 5.92 MPa/°C and $C_A$ is 8.79 MPa/°C. These values are recorded in Table 3.1. Moreover, based on the linear curve fit equations, the zero stress values of Transformation Temperatures can also
be interpolated. They are $28.02^\circ$C, $37.23^\circ$C, $49.73^\circ$C, $60.17^\circ$C for $M_f$, $M_s$, $A_s$, and $A_f$ respectively. These values are also tabulated in Table 3.1

Table 3.1: Compiled data for three repetitions of constant stress experiment for Nitinol wire of 0.48 mm diameter

<table>
<thead>
<tr>
<th>Stress (MPa)</th>
<th>$M_f$ (°C)</th>
<th>$M_s$ (°C)</th>
<th>$A_s$ (°C)</th>
<th>$A_f$ (°C)</th>
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<td>Clausius-Clayperon coefficients</td>
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<td>8.79</td>
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</tr>
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Figure 3.4: Linear curve fit of compiled Transformation Temperatures for Nitinol wire diameter of 0.48 mm.

Similarly to wire diameter of 0.48 mm, the results for 0.31 mm are tabulated and graphically illustrated in Table 3.2 and Figure 3.5.
Table 3.2: Compiled data for three repetitions of constant stress experiment for Nitinol wire of 0.31 mm diameter

<table>
<thead>
<tr>
<th>Repetition</th>
<th>Stress (MPa)</th>
<th>$M_f$ (°C)</th>
<th>$M_s$ (°C)</th>
<th>$A_s$ (°C)</th>
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<td>5.92</td>
<td>8.79</td>
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</table>
A key observation in here is while the $R^2$ values for the linear curve fits. While the curve fits of 0.48 mm has extremely high $R^2$ values that indicate very good agreement and reliable data; $R^2$ value for the 0.31 mm data cannot follow the same trend. Except for $M_s$, the rest of the data does not have very high agreeable values. This case is particularly true for $A_s$ data, as its $R^2$ value is around 0.43. Another mean of verification is also to compare the Transformation Temperatures between the 0.48 mm wire diameter with the 0.31 mm diameter (Table 3.1 and Table 3.2). Values of $M_f$, $M_s$, and $A_f$ for 0.31 mm show very good agreement with results of 0.48 mm, with very small differences of 5°C. The $A_s$ values however, are more than 10°C different.
3.2 Measurement of Transformation Temperatures

ASTM standard suggests intersecting the baseline of the cooling and heating curve with the tangent lines of the heating and cooling spikes, as graphically depicted in Fig. 3.6.

Figure 3.6: Typical results for DSC experiment. There are three peaks in this result correspond to three different phases.

On the heating curve, the heat flow peak corresponds to austenite transformation. In some results, as shown in Fig. 3.7 for as received wires, there are two different heat peaks in the cooling portion of the curve, correspond to a higher temperature Rhombohedral phase, or R-phase, and a lower temperature martensite phase. R-phase is a distortion of the cubic austenite phase. R-phase most commonly appears during the cooling of Nitinol from austenite to martensite (Fig. 3.7). R phase only responsible for about 0.5% of recovery strain observed in shape memory effect [55]. The present of R-phase in Fig. 3.7 can be interpreted as a product of cold work [20]-[57]. This is also an evidence for R-phase existence in obtained in constant stress experiment. However,
through literature survey, there is no clear indication on R-phase’s effect on the Nitinol mechanical behavior. R-phase biggest effect is on the resistivity of Nitinol [58]. Modeling efforts only include simulating resistance evolution in Shape Memory Alloys [59]. Matsuzaki and Naito, however, did propose a modification to their mechanical constitutive model to accommodate R-phase. However, the lack of numerical simulations as well as experimental evidence hinders a valid conclusion on R-phase’s influence of Nitinol mechanical behavior.

Figure 3.7: Comparison of DSC results for as received and annealed wires of 0.48 diameter.

When compare between as-received and annealed wires, the annealed wire has only one clear peak with much higher peak value than as received wire. Once the wire is fully annealed, the material recrystallizes, thus relieving any residual stresses introduced by manufacturing processes.

Table 3.3 shows the comparison obtains by both DSC and constant stress method for 0.48 mm wire diameter (Table 3.1)
Table 3.3: Comparison of all Transformation Temperatures obtained by DSC and constant stress experiment

<table>
<thead>
<tr>
<th></th>
<th>( M_f (^\circ C) )</th>
<th>( M_s (^\circ C) )</th>
<th>( R_f (^\circ C) )</th>
<th>( R_s (^\circ C) )</th>
<th>( A_s (^\circ C) )</th>
<th>( A_f (^\circ C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>As received</strong></td>
<td>0.00</td>
<td>28.00</td>
<td>40.00</td>
<td>76.00</td>
<td>60.00</td>
<td>72.00</td>
</tr>
<tr>
<td><strong>Annealed</strong></td>
<td>28.00</td>
<td>45.00</td>
<td>N/A</td>
<td>N/A</td>
<td>60.00</td>
<td>80.00</td>
</tr>
<tr>
<td><strong>Constant Stress</strong></td>
<td>28.02</td>
<td>37.2</td>
<td>N/A</td>
<td>N/A</td>
<td>49.73</td>
<td>60.17</td>
</tr>
</tbody>
</table>

Table 3.3 clearly shows that for as received wire, DSC yields very unrealistic results for martensitic Transformation Temperatures. Annealed wires, for \( M_f \) can be compatible to the constant stress experiment; there is still a big difference between them. The austenitic Transformation Temperatures show a big difference between the constant stress and DSC experiments results. As shown in section 3.3, the simulation using the material properties obtained from constant stress show much more agreeable results.

There is no repetition done on DSC results since it has been established by ASTM for Nitinol [62]. Base on the study, the standard deviation for the Transformation Temperatures within 1\(^\circ\)C between repetitions conducted in the same laboratory.

### 3.3 Constant Strain and Numerical Simulations

In order to simulate the actuator mechanical responses, other less sensitive materials properties are also needed. These materials are either obtained from widely accepted literature or by parametric studies, and tabulated in Table 3.4 for Brinson model. There is a noted difficulty in modified Likhachev model, therefore, even though it has been extensively studied in this thesis, the model simulation results are not presented.
Table 3.4: Additional Material Properties Input for Brinson model simulations

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical start transformation stress ($\sigma_s$)</td>
<td>0.5 MPa</td>
</tr>
<tr>
<td>Critical finish transformation stress ($\sigma_f$)</td>
<td>50 MPa</td>
</tr>
<tr>
<td>Martensite Young’s Modulus ($E_M$)</td>
<td>31.10 GPa</td>
</tr>
<tr>
<td>Austenite Young’s Modulus ($E_A$)</td>
<td>69.60 GPa</td>
</tr>
<tr>
<td>Maximum residual strain ($\varepsilon_{max}$)</td>
<td>0.07</td>
</tr>
</tbody>
</table>

The Four Transformation Temperatures inputs as well as two Clausius-Clayperon coefficients for both wire diameter are obtained from the constant stress experiments (Table 3.1 and Table 3.2). The values use are all the average of all the repetitions. Since there is a relative low standard deviations, the assumption of using average values are valid.

For each wire diameter, the experiments were repeated 3 times to ensure good repeatability (Fig. 3.8-3.11).
Figure 3.8: Average temperatures response of all three repetitions for 0.48 mm wire diameter, illustrated along with standard deviation.

Figure 3.9: Average force responses for three repetitions for Nitinol wires of 0.48mm diameter, illustrated along with standard deviation.
When compare between Fig. 3.9 and Fig. 3.7, the temperature repeatability of 0.31 mm is better than of 0.48 mm. Similar trend for the force repeatability when comparing Fig. 3.8 and Fig. 3.10. The high force hysteresis can be explain due to the attachment of
the crimps on the wires. It was noticed that after several repetitions, the crimps started to loosen. Therefore, the inconsistency in force response can be due to the movement of the wires breaking away from the crimp for 0.48 mm wire diameter. On the other hand, crimp attachments on 0.31 mm wire diameter is much more stable, thus giving it a much repeatable results. In the case of second and third repetition, both the force and temperature response of 0.31 mm matches very well. It is also noted that the crimps are the same size regardless of wire diameter.

The temperature hysterisis between repetitions for 0.48 mm can be explain by the attachment of thermocouples as well as the crimp attachment. As aforemention, the crimps insufficient grips on 0.48 mm wire diameter can lead to movement of the wires during phase transformation process. This can also means that bad contacts between the thermocouple and the 0.48 mm wire is a great possibility, making much harder to measure wire temperatures, lead to bigger hysterisis compare to 0.31 mm wire diameter results.

Since Brinson model only used temperatures and not resistance heating convection cooling as input, comparison can only be used for force vs. temperature response. Furthermore, to better quantify between different wire diameters, the force response will be normalized by dividing it by the cross sectional area to get stress response. The stress vs. temperature response are done by using the average values as depicted in Fig. 3.8 to Fig. 3.11.
Figure 3.12: Comparison between the Brinson model and experimental actuation response for Nitinol wires of 0.48 mm diameter.

Figure 3.13: Heating curve comparison between the Brinson model simulation and experimental actuation response for Nitinol wire of 0.48 mm diameter.
Figure 3.14: Cooling curve comparison between Brinson model simulation and experimental actuation response for Nitinol wire of 0.48 mm diameter.

Figure 3.15: Comparison between the Brinson model and experimental actuation response for Nitinol wires of 0.31 mm diameter.
Figure 3.16: Heating curve comparison between the Brinson model simulation and experimental actuation response for Nitinol wire of 0.31 mm diameter
Figure 3.17: Cooling curve comparison between the Brinson model simulation and experimental actuation response for Nitinol wire of 0.31 mm diameter

When compare the response for the heating portion, the 0.48 mm wire diameter simulation can be correctly simulated (Fig. 3.13). This can be explained by the $A_s$ values to be corrected (Table 3.1). The contrast is shown for 0.31 mm wire diameter (Fig. 3.16) due to possible incorrect $A_s$ value, as explained in section 3.1. The cooling portion of both wire diameter display the same trend for both wire diameters. In general, the Brinson model does have the capability to simulate the actuation force response of Nitinol wire. Most of the mismatch between numerical simulations and experimental results come from difficulty with the current experimental setup, especially crimp attachment and thermocouple attachment.

Teriault and Brailoski resistance formulation, however, can accurately simulate the heat transfer mechanism of Nitinol actuator mechanism (Fig. 3.18). Most of the thermal values used in this resistance heating simulation are obtained from the manufacturer,
with the exception of the convection coefficients obtained from [58]. These values are tabulated in Table 3.5.

Table 3.5: Thermal properties of the Flexinol wires

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistivity of austenite ($\Omega_{A}$)</td>
<td>1.0e-6 Ω.m</td>
</tr>
<tr>
<td>Resistivity of martensite ($\Omega_{M}$)</td>
<td>8.0e-5 Ω.m</td>
</tr>
<tr>
<td>Density of the wire (D)</td>
<td>6450 kg/m$^3$</td>
</tr>
<tr>
<td>Latent heat of phase transformation ($Q_{PT}$)</td>
<td>156e6 J/m$^3$</td>
</tr>
<tr>
<td>Specific heat ($C_{p}$)</td>
<td>837.17 J/kg/oC</td>
</tr>
<tr>
<td>Convection coefficient (h)</td>
<td>85 W/m$^2$/oC</td>
</tr>
</tbody>
</table>

Figure 3.18 Temperature response using Terriaault and Brailovsky resistance heating formulation (1.5A for 15s, D=0.48 mm).
CHAPTER 4

CONCLUSIONS AND FUTURE WORK

Two one-dimensional models were studied and compared for their capability to simulate Nitinol actuation response. Brinson model, while lacks resistance heating formulation, can be used to simulate the thermo-mechanical behavior of Nitinol. There is a challenge in implementing the modified Likhachev model by Terriault and Brailovski, thus there is no simulation results presented in this thesis. Terriault and Brailovski however proposed a coupled resistance formulation that taken into account Nitinol major heat transfer mechanism: Joule heating, convection cooling/heating, and especially latent heat due to phase transformation. Therefore, in order to efficiently design the actuation system, a coupled algorithm utilized Terriault and Brailovski resistance heating formulation and Brinson model should be used.

There is a difficulty in measuring the true temperature of the Nitinol wires. This specially amplified in measuring the temperatures of 0.48 mm due to the insufficient gripping of the crimps.

Two methods were used to obtain materials constant inputs for the two models. It is found that only constant stress experiments should be used to obtain the four Transformation Temperatures as well as the Clausius-Clayperon coefficients. There should be more experiments conducted to find much better statistically agreeable results. Different wire diameters should also be tested to study the clear effect of different wire diameters on the Transformation Temperature of Nitinol wires.
For Brinson model inputs, this thesis did not obtain $\sigma_s^{cr}$ and $\sigma_f^{cr}$ experimentally. Therefore, future work should include conducting the isothermal mechanical testing to not only verify the superelasticity capability of Brinson, but also to obtain the two missing material inputs in Brinson model.

Furthermore, the Brinson model coupled with Terriault and Brailovski resistance heating formulation should be translated into Fortran and implement in ANSYS USERMAT subroutines for Finite Element Analysis of the smart needle actuation system.
REFERENCES


[31] Wilton W. Webster, Jr., inventor, Steerable Catheter Having Puller Wire with Shape Memory, USA Patent number 5383923


APPENDIX A

Brinson Model algorithm used to simulate the force response of Nitinol wire.

```matlab
function [T,sig]=brinson(N,ep,Tstart,Tend)

% Necessary inputs are
% N: number of space the stress, strain, temperature matrix should have
% ep: The amount of prestrain the wire should have. This is kept constant
% throughout the calculation
% Tstart: Initial temperature, usually for convection (room) temperature
% Tend: Maximum temperature that the wire reach
% Several material constants must first be defined
% ----------------------- Material constants ----------------------

% Young's Modulus
Ea=69.643e9; % Austenite Young's Modulus [Pa]
Em=33.14e9; % Martensite Young's Modulus [Pa]
theta= 0.7e6; % Coefficient of thermo expansion [Pa/C]
ep_l=0.07; % Maximum residual strain

% Stress related constants
Cm= 5.91e6; % Clausius-Clapeyron coefficient of Martensite [Pa/C]
Ca=8.78e6; % Clausius-Clapeyron coefficient of Austenite [Pa/C]
sig_s=5e5; % Critical start stress below Ms temperature [Pa]
sig_f=40e6; % Critical finish stress below Ms temperature [Pa]

% Temperature related constants
Mf=28.02; % Martensite Finish Temperature [C]
Ms=37.23; % Martensite Start Temperature [C]
As=49.73; % Austenite Start Temperature [C]
Af=60.17; % Austenite Finish Temperature [C]

% Heating curve

% Initialize all the vectors
T_lo=linspace(Tstart,Tend,N); % Temperature profile
sig_lo=zeros(size(T_lo)); % Stress profile
ep_lo=ep*ones(size(sig_lo)); % Strain profile
ep_temp=zeros(size(sig_lo)); % Trial strain
D_lo= zeros(size(sig_lo)); % Young's Modulus profile
Ohm_lo=zeros(size(sig_lo)); % Transformation profile
exci_tlo=zeros(size(sig_lo)); % Temperature induced martensite
exci_slo=zeros(size(sig_lo)); % Stress induced martensite
```

---
% Global martensite
% --------------------- Initial Conditions -------------------
-
T_lo(1)=Tstart; % Initial temperature
ep_lo(1)=ep; % Initial strain
sig_lo(1)=0; % Initial stress
exci_tlo(1)=0.85; % Initial temperature induced martensite
martensite
% Initial temperature induced martensite
exci_lo(1)=zeros(size(sig_lo)); % Global martensite

% -------------------- Calculation of the model -------------------
-
for jj=2:length(T_lo)
    sig_temp=sig_lo(jj-1);
    while abs(ep_temp(jj)-ep_lo(jj)) > 1e-4
        [exci_lo(jj),exci_tlo(jj),exci_slo(jj)]=kinetic1(sig_temp,
            ... T_lo(jj),T_lo(1),exci_slo(1),exci_tlo(1),exci_slo(1),
            ... sig_s,sig_f,Mf,Ms,As,Af);
        D_lo(jj)=Ea+exci_lo(jj)*(Em-Ea);
        Ohm_lo(jj)=-ep_l*D_lo(jj);
        ep_temp(jj)=1/D_lo(jj)*(sig_temp-sig_lo(1)-
            ... exci_slo(jj-1),exci_tlo(jj-1),exci_lo(jj-1),Cm,Ca,
            ... sig_s,sig_f,Mf,Ms,As,Af);
        D_lo(jj)=Ea+exci_lo(jj)*(Em-Ea);
        Ohm_lo(jj)=-ep_l*D_lo(jj);
        ep_temp(jj)=1/D_lo(jj)*
            ... (sig_temp-sig_lo(1)-exci_slo(jj)) ...
            ... -theta*(T_lo(jj)-T_lo(1))+Cons);
        sig_temp=sig_temp+0.01e5;
    end
    sig_lo(jj)=sig_temp;
end

% Since the constitutive equations are based on initial condition
% calculation of the cooling and heating curve has to be separated
% --------------------- Cooling curve ----------------------
-
% Initialize all the vectors
T_un=linspace(Tend,Tstart,N); % Temperature profile
sig_un=sig_lo(end)*ones(size(T_un)); % Stress profile
ep_un=ep*ones(size(sig_un)); % Strain profile
ep_t=zeros(size(sig_un)); % Trial strain
D_un=zeros(size(sig_un)); % Young's Modulus profile
Ohm_un=zeros(size(sig_un)); % Transformation profile
exci_tun=zeros(size(sig_un)); % Temperature induced martensite
martensite
exci_sun=zeros(size(sig_un)); % Stress induced martensite
exci_un=zeros(size(sig_un)); % Total martensite

% -------------------------- Initial Conditions --------------------------
--

T_un(1)=Tend; % Initial temperature
ep_un(1)=ep; % Initial strain
sig_un(1)=sig_lo(end); % Initial stress
exci_tun(1)=exci_tlo(end); % Initial temperature induced martensite
exci_sun(1)=exci_slo(end); % Initial stress induced martensite
exci_un(1)=exci_sun(1)+exci_tun(1); % Initial martensite volume fraction
D_un(1)=Ea+exci_un(1)*(Em-Ea); % Initial young modulus
Ohm_un(1)=-ep_l*D_un(1); % Initial transformation tensor
Cons_un=D_un(1)*ep_un(1)+Ohm_un(1)*exci_sun(1); % Initial term of cons. model
% -------------------------- Calculation of the model ---------------------------
--

for jj=2:length(T_un)
    sig_t=sig_un(jj-1);
    while abs(ep_t(jj)-ep_un(jj)) >1e-3
        [exci_un(jj),exci_tun(jj),exci.sun(jj)]=kinetic1(sig_t, ... 
            T_un(jj),T_un(1),exci_sun(1),exci_tun(1),exci_un(1), ... 
            exci.sun(jj-1),exci_tun(jj-1),exci.un(jj-1),Cm,Ca, ... 
            sig_s,sig_f,Mf,Ms,As,Af);
        D_un(jj)=Ea+exci_un(jj)*(Em-Ea); % Initial transformation tensor
        Ohm_un(jj)=-ep_l*D_un(jj); % Initial transformation tensor
        ep_t(jj)=1/D_un(jj)*(sig_t-sig_un(1)-Ohm_un(jj)*exci_sun(jj) ... 
            -theta*(T_un(jj)-T_un(1))+Cons_un);
        sig_t=sig_t-0.01e5;
    end
    sig_un(jj)=sig_t;
end

% Combine output temperature and stress response
T=[T_lo,T_un];
sig=[sig_lo, sig_un];

% Nested Phase Transformation Kinetic Formulation
function [exci,exci_t,exci_s]=kinetic1(sig,T,To,exci_so,exci_to,exci_o ... 
    ,exci_st,exci_tt,exci_m,Cm,Ca,sig_s,sig_f,Mf,Ms,As,Af)
% ------------------ Define material constants ------------------
\[ a_A = \pi / (A_f - A_s); \]
\[ a_M = \pi / (M_s - M_f); \]

% ------------------- Conversion to austenite -------------------

if \( T > T_o \)
    if \( T > A_s \) \&\& \( T < A_f \)
        if \( \sigma > C_a (T - A_s) \)
            \( \text{exci} = \text{exci}_m; \)
            \( \text{exci}_s = \text{exci}_st; \)
            \( \text{exci}_t = \text{exci}_tt; \)
        elseif \( \sigma < C_a (T - A_s) \)
            \( \text{exci} = \text{exci}_o / 2 \cdot (\cos(a_A(T - A_s - \sigma / C_a)) + 1); \)
            \( \text{exci}_s = \text{exci}_so - \text{exci}_so / \text{exci}_o (\text{exci}_o - \text{exci}); \)
            \( \text{exci}_t = \text{exci}_to - \text{exci}_to / \text{exci}_o (\text{exci}_o - \text{exci}); \)
        end
    elseif \( T > A_f \)
        if \( \sigma < C_a (T - A_s) \) \&\& \( \sigma > C_a (T - A_f) \)
            \( \text{exci} = \text{exci}_o / 2 \cdot (\cos(a_A(T - A_s - \sigma / C_a)) + 1); \)
            \( \text{exci}_s = \text{exci}_so - \text{exci}_so / \text{exci}_o (\text{exci}_o - \text{exci}); \)
            \( \text{exci}_t = \text{exci}_to - \text{exci}_to / \text{exci}_o (\text{exci}_o - \text{exci}); \)
        elseif \( \sigma < C_a (T - A_f) \)
            \( \text{exci} = \text{exci}_m; \)
            \( \text{exci}_s = \text{exci}_st; \)
            \( \text{exci}_t = \text{exci}_tt; \)
        end
    elseif \( T < A_s \)
        \( \text{exci} = \text{exci}_m; \)
        \( \text{exci}_s = \text{exci}_st; \)
        \( \text{exci}_t = \text{exci}_tt; \)
end

% --------------------------------- Conversion to martensite ---------------------------------

if \( T < T_o \)
    if \( T > M_s \)
        if \( \sigma < \sigma_s + C_m (T - M_s) \)
            \( \text{exci} = \text{exci}_m; \)
            \( \text{exci}_s = \text{exci}_st; \)
            \( \text{exci}_t = \text{exci}_tt; \)
        elseif \( \sigma_s + C_m (T - M_s) < \sigma \) \&\& \( \sigma < \sigma_f + C_m (T - M_s) \)
            if \( \sigma_s = 0 \) \&\& \( \sigma_f = C_m (M_s - M_f) \)
                \( \text{exci}_s = (1 - \text{exci}_so) / 2 \cdot \cos(a_M(T - M_f - \sigma / C_m)) \ldots \)
                \( + (1 + \text{exci}_so) / 2; \)
            elseif \( \sigma_s = (1 - \text{exci}_so) / 2 \cdot \cos(pi() / (\sigma_s - \sigma_f) * (\sigma \ldots \)
                \( - \sigma_f - C_m (T - M_s)) + (1 + \text{exci}_so) / 2; \)
            end
        end
    end
end
\[
\text{exci}_t = \text{exci}_t - \text{exci}_t/(1-\text{exci}_so) * (\text{exci}_s - \text{exci}_so);
\]
\[
\text{exci} = \text{exci}_t + \text{exci}_s;
\]
\[
\text{elseif} \quad \text{sig} > \text{sig}_f + Cm \times (T - \text{Ms});
\]
\[
\text{exci} = \text{exci}_m;
\]
\[
\text{exci}_s = \text{exci}_st;
\]
\[
\text{exci}_t = \text{exci}_tt;
\]
\[
\text{end}
\]
\[
\text{elseif} \quad T < \text{Ms}
\]
\[
\text{if} \quad \text{sig} > \text{sig}_f
\]
\[
\text{exci} = \text{exci}_m;
\]
\[
\text{exci}_s = \text{exci}_st;
\]
\[
\text{exci}_t = \text{exci}_tt;
\]
\[
\text{elseif} \quad \text{sig} < \text{sig}_s
\]
\[
\text{exci} = \text{exci}_m;
\]
\[
\text{exci}_s = \text{exci}_st;
\]
\[
\text{exci}_t = \text{exci}_tt;
\]
\[
\text{elseif} \quad \text{sig} > \text{sig}_s \&\& \text{sig} < \text{sig}_f
\]
\[
\text{exci}_s = (1 - \text{exci}_so)/2 \times \cos(\pi/((\text{sig}_s - \text{sig}_f)/(\text{sig}_f - \text{sig}_s)) \times ((\text{sig}_f - \text{sig}_s)/2) + (1 + \text{exci}_so)/2;
\]
\[
\text{if} \quad \text{Mf} < T \&\& \text{T} < \text{Ms}
\]
\[
\text{deltaT} = (1 - \text{exci}_to)/2 \times (\cos(aM*(T - \text{Mf}))/2);
\]
\[
\text{else}
\]
\[
\text{deltaT} = 0;
\]
\[
\text{end}
\]
\[
\text{exci}_t = \text{exci}_to - \text{exci}_to/(1 - \text{exci}_so) * (\text{exci}_s - \text{exci}_so) + \text{deltaT};
\]
\[
\text{exci} = \text{exci}_t + \text{exci}_s;
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]