ESSAYS ON RISK FINANCE AND INCENTIVE CONTRACTING

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ABSTRACT

This thesis consists with three topics. Chapter 1 Incentive Contracting with an Independent Underwriter: Does It Benefit Insurers? proposes an analytical model to investigate the decision factors of an insurance company when choosing between direct writing and independent underwriter as distribution channel. It also explores the impact of contingent commissions on the underwriting performance of insurance companies. To count for the impact of policy renewal, this paper measures the difference of underwriting performance between using independent underwriter and direst writing in the single-period model, as well as in the multi-period model. It is found that the key decision factors of distribution system include: underwriting risk, underwriting task complexity, underwriting cost, as well as policy renewal.

Chapter 2 Risk Finance for Catastrophe Losses with Pareto-Calibrated Levy-Stable Severities proposes a risk finance paradigm for catastrophe losses. The conventional risk finance paradigm of enterprise risk management identifies transfer, as opposed to pooling or avoidance, as the preferred solution. However, this analysis does not necessarily account for differences between light- and heavy-tailed characteristics of loss portfolios. Of particular concern are the decreasing benefits of diversification (through pooling) as the tails of severity distributions become heavier. In the present article, a loss portfolio characterized by nonstochastic frequency and a class of Lévy-stable severity distributions calibrated to match the parameters of the Pareto II distribution is investigated. Then a conservative risk finance paradigm is proposed. It can be used to

prepare the firm for worst-case scenarios with regard to both (1) the firm's intrinsic sensitivity to risk and (2) the heaviness of the severity's tail.

Chapter 3 A Risk-Based Risk Finance Paradigm proposes an alternative to the conventional risk finance paradigm of enterprise risk management that accounts for not only a loss portfolio's expected frequency and expected severity, but also its "risk" as captured by an appropriate measure of dispersion/spread. This new paradigm is based upon four distinct properties of a loss portfolio that enhance the benefits of diversification: (1) a high expected frequency; and (2) less than perfect positive correlations between individual severities; (3) light-tailed severities; and (4) a predictable (i.e., non-erratic) frequency.

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THIS DISSERTATION IS DEDICATED TO MY HUSBAND RONG GAO.

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CHAPTER 1

INCENTIVE CONTRACTING WITH AN INDEPENDENT UNDERWRITER: DOES IT BENEFIT INSURERS?

1.1. Introduction

In insurance industry, the distribution system varies along a spectrum from the use of a professional employee sales force, to contracting with independent sales representatives, to direct response methods such as mail, telephone and most recently, internet solicitation. The choice of distribution system, the structure of agent compensation, and the regulatory of insurance distribution activities remain the three major economic issues in the insurance industry distribution. Although the interaction between regulation and choice of distribution system cannot be overseen, the regulation factor is considered as exogenous and not being discuss here. This paper focuses on insurance company's choice of the distribution system and compensation scheme in the context of property-liability insurance market.

1.1.1. Main property-liability distribution channels

A wide variety of distribution methods are used in the property-liability insurance industry. This includes the use of: (1) exclusive professional employee sales force who dedicate to sell products of a single insurance company; (2) independent agencies who sell products of several competing insurers and owns the customer list upon the termination of the contractual relationship; (3) brokers who are also independent businesses who may sell products from various insurers; (4) direct response methods such as mail, telephone and internet solicitation. For most of the insurance companies, they use multiple distribution methods, which will benefit them in marketing relationships and alliances with non-insurance concerns.

Figure 1. shows the breakdown of different distribution systems in the property-casualty insurance industry in 2011[1].

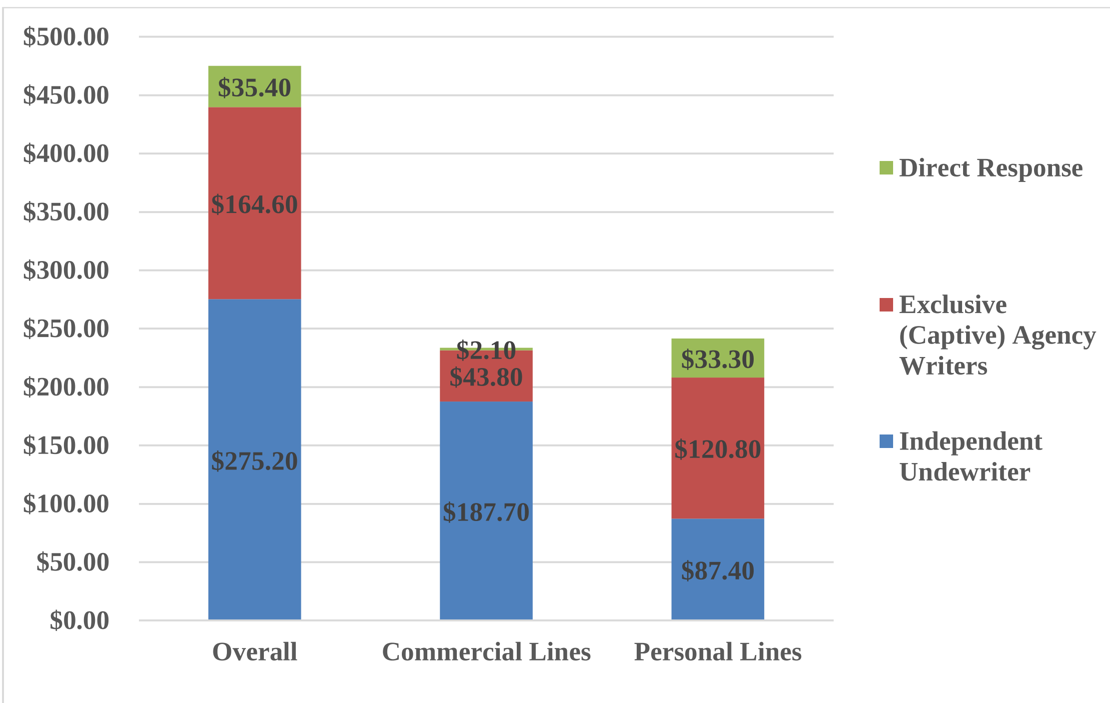

Figure 1. Market Share of Distribution System _ 2011 P-C Insurance

Source: IIABA Market share Report, 2013

In Billions of dollars of direct premium written

[1] Here independent underwriter refers to both independent agency and broker

Figure 1. documents that in 2011, independent underwriter takes the largest market share overall in the property-casualty insurance market, with a 275.2 billion of direct premium written; exclusive agency follows with a 164.6 billion of direct premium written. Firms that distribute primarily through direct response achieve a 35 billion of direct premium written. The market share of independent agency distribution system also varies by lines of business written. It is larger for commercial lines, but smaller in personal lines of business. Over the years, the independent agency distribution system has seen a slow drop in its control of the overall commercial lines market, with 2.1 percentage points dropped since 2006, as shown in figure 2 below.

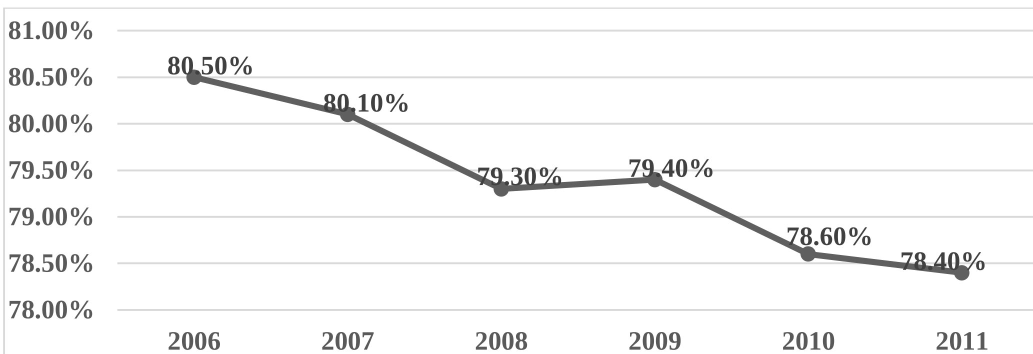

Figure 2. 5-Year View of Independent Underwriter Distribution System Market Share in Commercial Line

Source: IIABA Market share Report, 2013

Historically, the main distinction between independent agents and brokers is that brokers don't have formal contractual relationships with insurers, so that brokers serve and represent the insurance purchaser. Independent agents, however, have formal contractual relationships with the insurers and therefore are considered as representatives of the insurance companies. This set-up of contractual relationship means that a broker cannot commit an insurer to provide insurance without the insurer's specific approval of the policy, whereas many independent sales agents can bind the insurer to offer a policy.

However, the difference between independent agents and brokers is blurred since 2004 with the adoption of the NAIC'S Producer Licensing Model Act (PLMA) by a majority of states. Under this act, both independent agents and brokers are referred as insurance "producers". In practice, brokers differentiate from independent agents by severing clients' need of more complex insurance product, whereas independent agents usually sell products in personal and small to mid-size commercial line of business. In addition, brokers may sometimes offer other fee-basis services such as risk management or loss control consulting, which are generally not provided by independent agencies. In this paper, brokers and independent agents are both referred as independent underwriter.

1.1.2. Coexistence of Multiple Distribution Systems

There are extensive literatures focusing on the questions regarding the efficiency of distribution systems and the theoretical and empirical reasons of the coexistence of multiple distribution systems. These studies usually group the various distribution systems into two main categories, based on the degree of vertical control of the sales force, as "direct writer" and "independent underwriter". Direct writer includes direct marketing and exclusive agent. Independent underwriter includes broker and independent agent.

After careful comparison between the direct writer distribution system and independent underwriter distribution system, the previous researches have consistently found that independent underwriter system associates with relatively higher cost than direct writing, and are sometimes with higher profitability. With these observations, if the independent underwriter provides similar service with high cost, economic theories would then predict the non-existence of the independent underwriter, according to long-run competitive equilibrium. However, market observations shows that independent underwriter remains as one of the most important distribution systems in commercial property-casualty insurance market. Therefore, previous researches have attributed the existence of the independent underwriting system to either market-imperfections or higher product quality.

Initially, the market-imperfection hypothesis states that independent underwriting system is inefficient compared to direct writing system. This hypothesis was more prevail in the 70's and 80's when independent underwriting system lost its dominance in the insurance market. The market-imperfections hypothesis claims that independent underwriter insurers survive with cost inefficiency (Cummins, 1977) because of market imperfections, such as price regulation (Joskow, 1973; Cummins & VanDerhei, 1979; Weiss, 1990), slow diffusion of information in insurance markets (Dahlby & West, 1986), or search costs that permit inefficient firms to survive alongside efficient firms (Dahlby & West, 1986). In addition, with more precise identification of firms by marketing strategy, Barrese & Nelson (1992) found that in private passenger automobile insurance market, independent underwriters continue to lose market share, indicating that independent underwriters' service differential is not sufficient to offset their higher cost in personal line of business.

However, the product quality hypothesis argues that higher cost of independent agencies is associated with higher product quality, greater service intensity, or better mitigation of principle-agent problems between insurer and insurance buyers. It is claimed that independent underwriter can provide higher product quality by assisting customers with superior claims settlement service, offering more product choices to customers, reducing policyholders' search costs and therefore enjoys greater customer satisfaction (Barrese et.al. 1995). The product quality hypothesis also argues that independent underwriters may, to some extent, solve principle-agent problems such as company/buyer conflicts, by credibly threatening to shift business to an alternate insurer. For example, Kim, Mayers and Smith (1996) found that independent agents may deal more effectively with agency conflicts between policyholder and insurer.

In terms of coexistence, Posey & Yavas (1995) proved that there exists equilibrium where the independent agency and direct writer marketing systems coexist. Using pure price search model, characteristics of such equilibrium has been further analyzed by Posey & Tennyson (1998). They found that direct writing tend to be used when producers associate with low production cost, and consumers associate with low search cost. Using frontier efficiency methods, Berger et al., (1997) estimated both cost and profit efficiency for direct-writing and independent-agency insurers. They found that although independent agencies are cost inefficient, most of the average cost-efficiency difference between using independent agencies and direct writing distribution system does not carry through as a profit-efficiency difference. Therefore, higher costs of independent-agency firms appear to be due primarily to the provision of higher-quality services, which are compensated for by additional revenues.

The choice of whether to use independent underwriter or use direct writing usually depends on the difficulty of measuring outcome, the importance of non-selling activities, and the nature of insurance target. When there is perceived difficulty of measuring outcome (Anderson & Schmittlein, 1984), or when there is a higher importance of non-selling activities, firms will tend to use home contractor (Anderson, 1985). Independent underwriter, however, is more aggressively used when the products are nonstandard items such as umbrellas and earthquake insurance, which requires more effort in exposure identification (Cummins & Weisbart, 1977). Regan & Tennyson (1996) found that insurers tend to use direct writing when policyholders can be easily sorted without sales agent's participation in screening. Otherwise, when agent information is important for risk placement, independent underwriter may be preferred. Empirical support of this theory was obtained from analysis of compensation contracts and market shares in the context of different marketing forms. Direct writing were found to be prevalent in relatively standardized, homogeneous product lines and markets, and their agents receive less profit-based compensation than those of independent underwriter insurers. In addition, Regan (1997) found that the independent agency system will be preferred by insurers marketing complex products or operating in lines or markets where uncertainty is higher.

1.1.3. Compensation Systems

1.1.3.1 Agency Costs associated with independent Underwriter

There are two types of agency costs associates with insurance distributions: agency cost arising from the asymmetric information between the insurer and the agent; as well as the agency cost arising from the asymmetric information between insurer and the insured.

The first type of agency cost is known as moral hazard: if not properly monitored, the agent will choose to exert the lowest (unverifiable) amount of effort. Ideally, this problem could be mitigated by flat salary based on the volume of business placed, as long as monitoring and enforcement of agent behavior is available. However, the stochastic nature of output precludes the use of direct monitoring. In this case, incentive compensation system with volume based contingent commissions may be paid in addition to flat salary to provide financial incentives to motivate the agent to act in the interest of the insurer. Such volume based contingent commission scheme allows insurer to capture economies of scale, and to benefit from portfolio diversification by adding more individual risks to the portfolio.

The second type of agency cost is known as adverse selection: the real risk type of potential insured is unrevealed to the insurer. Theoretically, without a truth-revealing mechanism, the high risk type will tend to hide behind the low-risk type, or purchase more insurance to benefit from lower premium. In this case, insurer will either operate with loss, or charge every potential insured high premium to cover the potential loss due to adverse selection, which in turn, is unfair for the low-risk type. Therefore, it is very important to determine risk class in the underwriting process and apply them to appropriate price so that actual loss is consistent with expected loss. For risks with simple nature such as auto insurance and homeowners insurance, insurer is able use classification variables that are observable and verifiable at low cost to sort different risk types apart. However, for risks with complex nature, such as inland marine insurance and commercial multiperil insurance, the cost of risk misclassification is so high that the above method may no longer be feasible. In this case, independent underwriter can be used to gather and verify risk information that is otherwise costly for the insurer to obtain.

1.1.3.2 Compensation systems for various distribution systems

There are two types of compensation schemes in insurance distribution systems: straight commission (including volume based contingent commission) and profit-contingent commission. The straight commission is usually a flat-rate that only associates with the premium volume. The profit-contingent commission is based on premium volume and loss ratio of the business sold for the insurer.[2] The choice of compensation scheme is usually correlated with the type of distribution system used.

Exclusive agents are generally paid by straight commission. Although sometimes they can be compensated by contingent commission, evidence shows that exclusive agents are less likely to be compensated by contingent commission than independent agents (Regan & Tennyson, 1996). Besides straight commission, exclusive agents may also benefit from retirement plans, certain training and support from the insurer, and may be compensated partially by salaries, depending on the contract between exclusive agents and insurer.

Independent underwriter may be compensated by either straight commission or contingent commission, or both. Paying straight commission to independent underwriter will create severe moral hazard problem: independent underwriters with premium volume as only goal may not be motivated to put enough effort in the underwriting process to fully determine the risk characteristics of the insureds, thus posing a significant threat to insurer (Cheng & Powers, 2008). Contingent commission is usually based on the profitability of the independent underwriter's business placed with the insurer, the persistency rate, and/or on the volume of business, where the commission rate varies

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[2] For ease of exposition, profit-contingent commission is referred as contingent commission, and volume based contingent commission is referred as straight commission below.

across insurance products. According to previous research, the use of contingent commission can encourage independent underwriters to better match consumers with products (Focht et.al. 2013), and is found to be positively associated with underwriting performance (Regan & Kleffner, 2010). Although contingent commissions arrangements varies widely, the majority of the revenues of independent underwriters are are profit-based (Cummins & Doherty, 2006).

There has been debate on whether insurer should use contingent commission to compensate independent underwriters. The opponents of contingent commission argue that it: (1) increases insurer's underwriting expenses, at the cost of increased premium passed to the policyholders, as showed by Cummins & Doherty (2006) that commissions paid by insurers are mostly passed through to policyholders in premiums charged; (2) creates conflicts of interests between independent agent and clients, by placing business with the insurer who offers the highest compensation package, rather than the one that is most suitable for the client, if the client is unaware of such payment arrangement. The recent investigation into insurance broker Marsh's big-rigging activities to steer clients to insurers that paid relatively higher contingent commissions is one famous case supporting this view. As a consequence, many property-liability insurers have agreed to eliminate the use of contingent commissions in their compensation formulas.

However, it has been found that the use of contingent commission is the optimal agent compensation scheme, and is natural development of the competitive insurance marketplace that helps to make insurance widely available and affordable (Carson, et al., 2007) because it: (1) encourages agent to exert more effort on risk assessment and match clients with appropriate insurers (Regan & Kleffner, 2010); (2) assists the insurer in retaining existing business and promote the growth goals of the insurer(Hoyt et al., 2006); (3) mitigates adverse selection problem from the asymmetric information between clients and the insurer by providing information to insurers about the risk type of the applicants that the insurer would find costly or difficult to verify (Regan & Tennyson,1996; Regan, 1997); (4) provides loss mitigation services to reduce loss ratios of the portfolio (Hoyt et al., 2006). After the New York legislation on Marsh case, studies have found significant negative returns for insurers following the announcement of the investigation, with larger discounts for insurers that use relatively more contingent commissions (Ghosh & Hilliard, 2006; Cheng et al., 2007). Since compensating independent underwriter with contingent commission is more consistent with modern managerial compensation mechanisms since performance based payment should be prompted in response to higher quality projects (Bernardo et al., 2001), contingent commission serves as the basis in the current research.

The underlying research considers the condition for an insurer to use independent underwriter, and a compensation scheme in which the independent underwriter is compensated by a proportion of net profit realized at the end of a business period. First an analytical method is used to examine the condition when insurer should use an independent underwriter. It is found that cost advantage is not a necessary condition for the insurer to use an independent underwriter. The most important decision factor for the insurer when he chooses a distribution system is underwriting risk factor. With higher underwriting risk and more complex underwriting task, insurer is more willing to use independent underwriter, while with lower underwriting risk and less complex underwriting task, the insurer will tend to use direct writing instead. Then this research continues to model an optimal incentive contract between an independent underwriter and an insurer. It is found that the underwriting risk factor plays a key role in optimal compensation scheme: with larger underwriting risk factor, the independent underwriter will be compensated more heavily. A dynamic contracting setting considering policy renewal is also investigated, and it is found that independent underwriter tends to be used when renewal is less important.

The importance of this study is twofold. First, it enriches the literature on the coexistence of independent underwriting insurer and direct writing insurer. In contrast to previous studies, this research stands out with a clear analytical result stating the condition of when it is optimal for the insurer to use independent underwriter. Second, this research proposes an optimal contract that insurer and independent underwriter can both benefit in equilibrium.

The research is organized as follows. Sections 1.2 and 1.3 derive the formal equilibrium optimal contract; Section 1.4 discusses insurer's decision of whether or not to use independent underwriter analytically; Section 1.5 discusses independent underwriter's decision of whether or not to enter the contract, and the optimal effort to exert; Section 1.6 derives single-period results and empirical implications of the optimal contract; Section 1.7 investigates multi-period contract; Sections 1.8 and 1.9 conclude the research.

1.2. Model and equilibrium

1.2.1. Model Set Up

Consider a commercial insurance market with a single mono-line risk-neutral insurer (the "seller") operating on a direct-writing basis. This insurer faces N potential insureds (the "buyers"), each of which is either of type H (high-risk) or type L (low-risk). Assume each buyer's type is known only to himself, but the overall proportion of high-risk buyers ρ_H is common knowledge to the market. So $\rho_H N$ represents the set of high-risk buyers in the market and $(1-\rho_H)N$ represents the set of low-risk buyers in the market.

Assume that the insurer can either assess the risk of potential buyers by himself and use direct distribution channel, or retain a risk-neutral independent underwriter, to evaluate all potential buyers, and set up two subsidiary pools, one for high-risk buyers and the other for low-risk buyers. Under the independent-underwriter system, the responsibility of underwriting/sorting the two different risk groups falls to the independent underwriter, and the independent underwriter is able to obtain each buyer's risk profile with certain cost. Then the only unresolved issue in the underwriting process is whether or not and to what extend the independent underwriter is willing to exert effort to underwrite correctly.

Denote the proportion of correctly underwritten high-risk buyers as q_H , the proportion of correctly underwritten low-risk buyers as q_L . Both q_H and q_L are unobservable by the insurer. Independent underwriter is able to influence q_H and q_L by exerting a certain amount of underwriting effort at a cost. Let $N_{HH} = \rho_H q_H N$ be the set of buyers underwritten as high-risk and are truly high-risk, $N_{HL} = (1-\rho_H)(1-q_L)N$ be the set of buyers underwritten as high-risk and are truly low-risk, $N_{LL} = (1-\rho_H)q_L N$ be the set of buyers underwritten as low-risk and are truly low-risk, and $N_{LH} = \rho_H(1-q_H)N$ be the set of buyers underwritten as low-risk and are truly high-risk. For ease of exposition, N is normalized as $N = 1$ for the rest of the article.

Premium is set according to expected loss λ per insurance target of the underwritten risk category. Thus the expected loss of high-risk type is denoted as λ_H and the expected loss of low-risk type is denoted as λ_L. By definition, the expected loss of high-risk type should be higher than the expected loss of low-risk type. Therefore, λ_H is larger than λ_L. Premium loading β is set according to expense loading e and profit loading π. Let p_H and p_L denote the actuarially fair premium per business of high-risk category and low-risk category respectively:

$$p_H = \frac{\lambda_H}{1-e-\pi} = \beta\lambda_H, \qquad \text{whereas } \beta = \frac{1}{1-e-\pi}, \beta > 1 \tag{1}$$

$$p_L = \frac{\lambda_L}{1-e-\pi} = \beta\lambda_L, \qquad \text{whereas } \beta = \frac{1}{1-e-\pi}, \beta > 1 \tag{2}$$

Therefore, insurer's before-compensation profit can be written as

$$\begin{aligned} Profit &= Income - Loss = Y - L = Y_{HH} + Y_{LL} + Y_{LH} - L_{HH} - L_{LL} - L_{LH} \\ &= (\beta-1)\lambda_H(\rho_H q_H) + (\beta-1)\lambda_L(1-\rho_H)q_L + (\beta\lambda_L - \lambda_H)\rho_H(1-q_H) \end{aligned} \tag{3}$$

Low-risk buyer is made worse off when assigned to the high-risk category. Assume that the buyers are rational and are well informed of their own risk type, the low-risk type will not buy insurance from the insurer if they are recognized as high-risk type and unfairly charged with higher premium. Therefore, equation (3) does not have the term related with N_{HL} (those who are underwritten as high-risk but are actually low-risk) buyers.

Independent underwriter is compensated by contingent commission in a scheme given by: α (*Total Net Profit*). Commission loading α will be modeled as an explicit compensation strategy of the seller. Meanwhile, the outside option for the independent underwriter is given as $\gamma = 0$, which is treated as constraint of the compensation.

Cost is incurred during the underwriting process by the independent underwriter. Specifically, the cost should be different for sorting a low-risk type from high-risk type and sorting a high-risk type from low-risk type. Restricting $C(q_L, q_H)$ to be simple form that induces close form interior solutions [3], without losing generality, independent underwriter's underwriting cost can be written as:

$$C(q_L, q_H) = c_L\left[(1-\rho_H)q_L\right]^2 + c_H(\rho_H q_H)^2 \tag{4}$$

Insurer's underwriting cost, if the insurer decides to underwriter by himself, takes the same format but with different parameters:

$$C^I(q_L^I, q_H^I) = c_L^I\left[(1-\rho_H)q_H^I\right]^2 + c_H^I(\rho_H q_H^I)^2 \tag{5}$$

1.2.2. Structure and Timing of the Game

The insurer designs a contract that compels the independent underwriter to exert the desired level of effort to maximize insurer's net profit (incentive compatibility), and to voluntarily enter into the contract (individual rationality).

[3] Holmstrom and Milgrom (1991) provide a number of interesting insights by considering a variety of cost functions. A simple quadratic form is used here since it is adequate to provide a number of interesting insights into the impact of performance measures.

The timing of the compensation scheme is shown as follows. At the beginning of the contract period, the insurer designs a compensation mechanism contingent on the incentive compatibility constraint and the individual rationality constraint. The insurer also makes decision of whether to underwrite the underlying business by himself or to use independent underwriter at this point. Based on the properties of the mechanism, the independent underwriter then chooses the optimal underwriting accuracy q_L and q_H, and whether to enter into the contract. No compensation is paid at this moment. At the end of the period, insurer's net profit is realized. Independent underwriter is then compensated with a proportion of the net profit α(*Total Net Profit*).

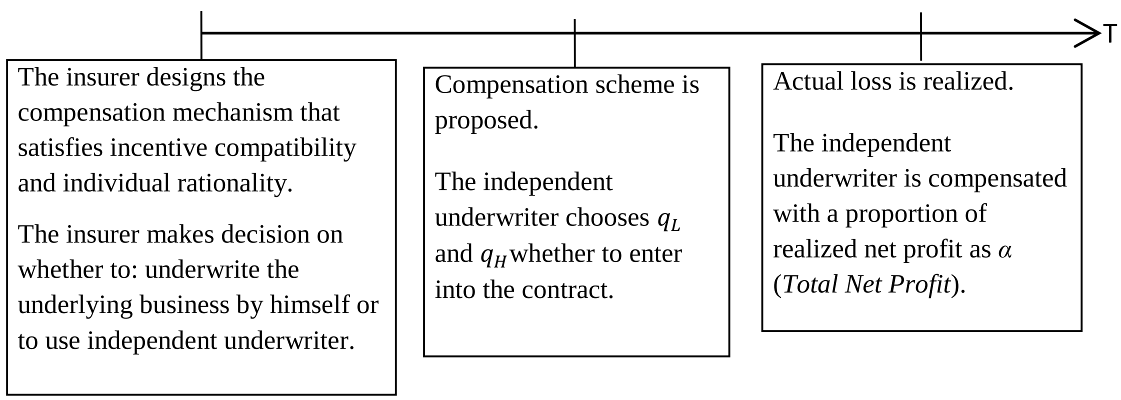


Figure 3. The Timing of the Game

Assume that the insurance company accepts all business generated and underwritten by the independent underwriter. There is no cash flow at the beginning of the contract period although sometimes the independent underwriter is compensated with a certain amount of overheads, which is unrelated with final net profit. At the end of the contract period, the independent underwriter is fully compensated with a pre-negotiated proportion of realized net profit of the underwritten business. Since profit-contingent commission usually exists between insurance company and large and well-experienced brokerage firm (independent underwriter), both insurer and independent underwriter are assumed to be risk neutral here.

1.2.3. The Mechanism-Design Problem

The compensation of the independent underwriter can be written as:

$$\begin{aligned} Compensation &= \alpha\left[Y_{HH} + Y_{LL} + Y_{LH} - \left(L_{HH} + L_{LL} + L_{LH}\right)\right] \\ &= \alpha\left[\beta\rho_H\left(\lambda_H - \lambda_L\right)q_H + \left(\beta - 1\right)\lambda_L\left(1 - \rho_H\right)q_L + \rho_H\left(\beta\lambda_L - \lambda_H\right)\right] \end{aligned} \tag{6}$$

The insurer's mechanism design problem is to maximize his expected payoff:

$$\begin{aligned} &\underset{q_L,\, q_H,\alpha}{Max}\left\{Profit - Compensation\right\} \\ &= \underset{q_L,\, q_H,\alpha}{Max}\left\{(1-\alpha)\left[\beta\rho_H\left(\lambda_H - \lambda_L\right)q_H + \left(\beta - 1\right)\lambda_L\left(1 - \rho_H\right)q_L + \rho_H\left(\beta\lambda_L - \lambda_H\right)\right]\right\} \end{aligned} \tag{7}$$

Subject to the incentive compatibility [*IC*] constraint of the independent underwriter:

$$\begin{aligned} q_L, q_H &= \underset{\hat{q}_L,\hat{q}_H}{\arg\max}\left(Compensation - Cost\right) \\ &= \underset{\hat{q}_L,\hat{q}_H}{\arg\max}\left\{\alpha\left[\beta\rho_H(\lambda_H - \lambda_L)\hat{q}_H + (\beta-1)\lambda_L(1-\rho_H)\hat{q}_L + \rho_H(\beta\lambda_L - \lambda_H)\right] - c_L\left[(1-\rho_H)\hat{q}_L\right]^2 - c_H(\rho_H\hat{q}_H)^2\right\} \end{aligned} \tag{8}$$

And the individual rationality [*IR*] constraint of the independent underwriter:

$$\begin{aligned} &Compensation - Cost \geq Outside\ Option \\ &\alpha\left[\beta\rho_H(\lambda_H - \lambda_L)q_H + (\beta-1)\lambda_L(1-\rho_H)q_L + \rho_H(\beta\lambda_L - \lambda_H)\right] - c_L\left[(1-\rho_H)q_L\right]^2 - c_H(\rho_H q_H)^2 \geq 0 \end{aligned} \tag{9}$$

The individual rationality [*IR*] constraint (9) ensures that the independent underwriter voluntarily enters into the contract. Assume that outside option is 0, so that the underwriter will not enter the contract with negative net income. The incentive compatibility [*IC*] constraint (8) requires that the independent underwriter finds it optimal to exert the optimal level of effort given by $\hat{q}_L$ and $\hat{q}_H$ according to the compensation scheme and the underwriting cost.

1.2.4. Equilibrium

Following Holmstrom (1979)'s approach in solving agent's moral hazard problem, the compensation scheme can be formally written as:

$$\begin{cases} \max\limits_{\{q_L, q_H, \alpha\}} (1-\alpha)\left[\beta\rho_H(\lambda_H-\lambda_L)q_H+(\beta-1)\lambda_L(1-\rho_H)q_L+\rho_H(\beta\lambda_L-\lambda_H)\right] \\ q_L, q_H = \arg\max\limits_{\hat{q}_L, \hat{q}_H}\left\{\alpha\left[\beta\rho_H(\lambda_H-\lambda_L)\hat{q}_H+(\beta-1)\lambda_L(1-\rho_H)\hat{q}_L+\rho_H(\beta\lambda_L-\lambda_H)\right]-c_L(1-\rho_H)^2\hat{q}_L{}^2-c_H\rho_H{}^2\hat{q}_H{}^2\right\} \\ \alpha\left[\beta\rho_H(\lambda_H-\lambda_L)\hat{q}_H+(\beta-1)\lambda_L(1-\rho_H)\hat{q}_L+\rho_H(\beta\lambda_L-\lambda_H)\right]-c_L(1-\rho_H)^2\hat{q}_L{}^2-c_H\rho_H{}^2\hat{q}_H{}^2 \geq 0 \end{cases} \quad (10)$$

Solving the above problem, [*IC*]’s can be obtained as:

$$q_L = [\alpha(\beta-1)\lambda_L]/[2c_L(1-\rho_H)] \quad (11)$$

$$q_H = [\alpha\beta(\lambda_H-\lambda_L)]/[2c_H\rho_H] \quad (12)$$

From [*IC*]’s (equation 11 and equation 12), it is clear that with fixed commission loading α, the underwriting accuracy of each risk type is: (1) positively related with profit loading β, and the expected loss of each risk type given by λ_L and λ_H; (2) negatively related with the proportion of each risk type (i.e. $(1-\rho_H)$ for low-risk type and ρ_H for high-risk type), and underwriting cost c of each risk type.

These observations are due to the fact that: (1) compensated by a proportion of insurer’s net profit, the underwriter is more profitable with higher profit loading β, thus will be more willing to exert higher effort. With larger expected loss, the accuracy of underwriting becomes more important. Therefore, independent underwriter needs to exert more effort to underwrite accurately; (2) when the underwriting cost of a specific risk type or/and the proportion of this risk type in the market pool increases, the independent underwriter will incur more cost in the underwriting process. In this case, holding commission loading α as fixed, the independent underwriter will tend to slack and lower the effort in the underwriting task.

The second part of observation (1) is very intuitive, because the purpose of contingent commission is to encourage the independent underwriter to correctly underwrite the business. This observation is consistent with Cheng & Powers (2008), and Dutta (2008) finding that performance and pay-performance sensitivity are positively correlated.

1.3. Optimal Compensation Schemes

The insurer (principal) chooses the commission loading α to maximize expected net profit, subject to [*IC*] constraint, [*IR*] constraint and independent underwriter's choice of the level of effort (q_L^* and q_H^*) to exert.

The insurer (principal)'s problem is to maximize virtual surplus:

$$\max_{\{\alpha\}}(1-\alpha)[\beta^2(\lambda_H-\lambda_L)^2\alpha/2c_H+(\beta-1)^2\lambda_L{}^2\alpha/2c_L-\rho_H(\lambda_H-\beta\lambda_L)] \tag{13}$$

Equation (13) consists of 4 parts: the first part $(1-\alpha)$ is the virtual surplus retained by insurer (the principle), as a proportion of net profit gained; the second part $\beta^2(\lambda_H-\lambda_L)^2\alpha/2c_H$ is the "high-risk underwriting gross profit", which shows the total gain from correctly underwritten high-risk business. This part can also be considered as the total savings from the worst-case scenario of underwriting activity (all high-risk are incorrectly underwritten as low risk type); the third part $(\beta-1)^2\lambda_L{}^2\alpha/2c_L$ is "low-risk net profit" that refers to the total net profit insurance company can obtain from low-risk business; the fourth part $\rho_H(\lambda_H - \beta\lambda_L)$ stands for the "complexity of underwriting task", in which $(\lambda_H - \beta\lambda_L)$ is considered as "underwriting risk factor". The "underwriting risk factor" can be either positive or negative. When the "underwriting risk factor" is positive, the expected loss of high-risk type given by λ_H exceeds the expected revenue of the low-risk type given by "actuarially fair premium" $\beta\lambda_L$. In this case, there will be a net loss if high-risk is incorrectly underwritten as low-risk, where higher $(\lambda_H - \beta\lambda_L)$ associates with larger net loss. Therefore, the "complexity of underwriting task" $\rho_H(\lambda_H - \beta\lambda_L)$ is actually the bottom line of the worst case scenario: the potential loss from the underlying business if all high-risks are incorrectly underwritten as low risk. Note that in virtual surplus, all terms are with positive sign except for the "complexity of underwriting task" $\rho_H(\lambda_H - \beta\lambda_L)$. So holding other factors as constant, insurer's total net profit is negatively related with complexity of underwriting task.

LEMMA 1: Assume equations (10-13) hold, the optimal compensation mechanism of the independent underwriter will be at equilibrium $\{\alpha^*, q_L^*, q_H^*\}$ in which:

$$\alpha^* = 0.5 + [\rho_H(\lambda_H - \beta\lambda_L)]/[\beta^2(\lambda_H - \lambda_L)^2 / c_H + (\beta-1)^2 {\lambda_L}^2 / c_L] \tag{14}$$

$$\begin{aligned} q_L^* &= [(\beta-1)\beta^2\lambda_L(\lambda_H - \lambda_L)^2 / c_H + (\beta-1)^3 {\lambda_L}^3 / c_L \\ &+ 2\rho_H(\beta-1)\lambda_L(\lambda_H - \beta\lambda_L)] / \{4c_L(1-\rho_H)[\beta^2(\lambda_H - \lambda_L)^2 / c_H + (\beta-1)^2 {\lambda_L}^2 / c_L]\} \end{aligned} \tag{15}$$

$$\begin{aligned} q_H^* &= [\beta^3(\lambda_H - \lambda_L)^3 / c_H + \beta(\beta-1)^2 {\lambda_L}^2 (\lambda_H - \lambda_L) / c_L \\ &+ 2\rho_H\beta(\lambda_H - \beta\lambda_L)(\lambda_H - \lambda_L)] / \{4\rho_H c_H[\beta^2(\lambda_H - \lambda_L)^2 / c_H + (\beta-1)^2 {\lambda_L}^2 / c_L]\} \end{aligned} \tag{16}$$

Proof: see Appendix A

LEMMA 1 gives the optimal commission loading $\alpha *$ at equilibrium. Given this optimal commission loading, independent underwriter will choose his optimal amount of effort to exert, given by the optimal underwriting accuracy of the low-risk type q_L^* and the optimal underwriting accuracy of the high-risk type q_H^*. LEMMA 1 indicates that the independent underwriter's underwriting accuracy q_L^* and q_H^* are both positively related with underwriting risk factor $(\lambda_H - \beta\lambda_L)$, as well as the complexity of the underwriting task $\rho_H(\lambda_H - \beta\lambda_L)$, as shown in equations (15) and (16). Detailed analysis can be found in proposition 8.

1.4. Insurers' Rationale

Given the optimal compensation scheme, insurer would make a decision between the two distribution channels: direct writing and independent underwriter.

Proposition 1 It is profitable (not necessarily optimal) for the insurer to use direct writing if and only if complexity of underwriting task $\rho_H(\lambda_H - \beta\lambda_L)$ is no larger than

$$[\beta^2(\lambda_H - \lambda_L)^2]/4c_H^I + [(\beta-1)^2 \lambda_L{}^2]/4c_L^I \tag{17}$$

Proof: see Appendix A

This proposition shows the region where it would be profitable for the insurer to use direct writing: when the underlying business is not too complex. Otherwise, the insurer could either use an independent underwriter, or not to assume the underlying risk. Note that this condition does not provide optimal region to use direct writing. The potential profit when using independent underwriter can still be higher than when using the direct writing channel. This observation is further discussed in proposition 6.

Lemma 2 Profit margin for insurer to use independent underwriter instead of underwriting by himself can be formally written as:

$$\frac{1}{4}\left\{\beta^2(\lambda_H-\lambda_L)^2(\frac{1}{2c_H}-\frac{1}{c_H^I})+(\beta-1)^2\lambda_L{}^2(\frac{1}{2c_L}-\frac{1}{c_L^I})+2\rho_H(\lambda_H-\beta\lambda_L)+\frac{\rho_H{}^2(\lambda_H-\beta\lambda_L)^2}{\beta^2(\lambda_H-\lambda_L)^2\frac{1}{2c_H}+(\beta-1)^2\lambda_L{}^2\frac{1}{2c_L}}\right\}$$

(18)

Proof: see Appendix A

This profit margin compares insurer's profit of using independent underwriter versus direct writing. It is obtained by subtracting insurer's profit when using direct writing from the profit when using independent underwriter. Important insight can be observed from this condition: independent underwriter is preferred when this profit margin is positive.

Proposition 2 Insurer's profit margin from using independent underwriter is positively related with insurer's costs in direct writing c_H^I and c_L^I, negatively related with independent underwriter's underwriting costs c_H and c_L.

Proof: see Appendix A

According to this proposition, underwriting cost is one of the advantage provided by independent underwriter: if the independent underwriter is more efficient in underwriting, indicated by smaller underwriting costs c_H and c_L, insurer can enjoy higher profit margin.

Proposition 3 With positive underwriting risk factor $(\lambda_H-\beta\lambda_L)$, insurer's profit margin is positively related with proportion of high risk business in the market.

Proof: see Appendix A

This proposition shows the impact of underwriting risk factor on insurer's choice of distribution channel: if underwriting risk factor is positive, the higher proportion of high risk business there is in the market, the more benefit can the independent underwriter provide the insurer.

Proposition 4 Holding other factors as fixed, independent underwriter distribution channel is more beneficial when underwriting risk factor $(\lambda_H - \beta\lambda_L)$ is larger.

Proof see Appendix A

Underwriting risk factor $(\lambda_H - \beta\lambda_L)$ compares actuarial fair premium generated by low-risk buyers $\beta\lambda_L$ and expected loss of high-risk buyers λ_H : when underwriting risk factor $(\lambda_H - \beta\lambda_L)$ is positive, premium collected $\beta\lambda_L$ will not be enough to compensate the expected loss λ_H incurred, when a high-risk is incorrectly underwritten as a low-risk. Without a truth-revealing mechanism, high-risk buyers will have higher incentive to hide their true risk-type, because they may enjoy larger benefit from hiding their information with larger underwriting risk factor. In this case, using independent underwriter can be considered as a risk sharing mechanism where the independent underwriter handles the underlying risk and shares potential profit/loss as well. This is consistent with Regan and Tennyson (1996) empirical finding that independent underwriter may be preferred when policyholders are not easily sorted without sales agent participation in screening, and when agent information is important for risk placement.

To further illustrate this finding, a restriction on the cost function is posed so that the insurer and the independent underwriter have the same cost. Results are demonstrated in Proposition 5 and Proposition 6.

Proposition 5 Given independent underwriter has the same cost as the insurer, with positive underwriting risk factor $(\lambda_H - \beta\lambda_L)$, when the underlying business is unprofitable to use direct distribution channel, it may still be profitable to use independent underwriter.

Proposition 5 shows that cost advantage is not a necessity for the existence of independent underwriter. In contrast to conventional rationale that independent must have cost advantage to, independent underwriter is found to benefit the insurer here (makes the insurer more profitable), with the same exact cost as the insurer. Other than cost advantage, independent underwriter can make a market niche by advantageous functions: risk sharing. Underwriting risks in specialty areas such as inland marine insurance and commercial multiperil insurance, independent underwriter has larger pool of similar risk, thus is able to better diversify risks (assuming the underlying risks are diversifiable) within at a larger scope.

Proposition 6 Given independent underwriter has the same cost as the insurer, it is optimal for insurer to use independent underwriter if and only if the product of underwriting risk factor $(\lambda_H - \beta\lambda_L)$ and $(\alpha^* - 0.5)$ is positive:

$$(\lambda_H - \beta\lambda_L)(\alpha^* - 0.5) > 0 \tag{19}$$

Proof: see Appendix A

This is an interesting and neat observation: as noted above, underwriting risk factor $(\lambda_H - \beta\lambda_L)$ compares the expected loss of high-risk type and the expected profit from low-risk type. If the underlying business is risky (i.e. with positive underwriting risk factor $(\lambda_H - \beta\lambda_L)$), the insurer will need to compensate the independent underwriter more than half of the total profit (i.e. $\alpha^* - 0.5 > 0$) to stay profitable. Otherwise, the independent underwriter should be compensated by less than half of the total profit.

1.5. Independent Underwriter's Rationale

To insure the independent underwriter enter into the contract, individual rationality constraint is required. Since it is assumed that the independent underwriter will enter the contract as long as positive profit is obtained, the following proposition can be found:

Proposition 7

The independent underwriter will enter the contract if and only if the following conditions are satisfied:

$$\begin{cases} (\lambda_H - \beta\lambda_L) \geq 0 \\ 1/2 < \alpha \leq 2/3 \end{cases} \text{ or } \begin{cases} (\lambda_H - \beta\lambda_L) < 0 \\ \alpha < 1/2 \quad or \quad \alpha > 2/3 \end{cases} \tag{20}$$

Proof: see the Appendix A

Proposition 7 states that when the underlying business is risky (with positive underwriting risk factor $\lambda_H - \beta\lambda_L$), the independent underwriter expects to receive more than 50% but no more than 2/3 of the profit share as their compensation. Explanation of this condition will be that for risky business, large profit share is necessary to (1) incentivize the independent underwriter and (2) make independent underwriter stay profitable. But since the profit loading α also represents risk, the independent underwriter will avoid excessive risk by requiring profit share to be less than 2/3.

However, when the underlying business is less risky (with negative underwriting risk factor $\lambda_H - \beta\lambda_L$), the independent underwriter can be either compensated by less than 50% of the total profit, or by more than 2/3 of the total profit. When compensated by 2/3 of the total profit, the underlying business can be essentially considered as primarily belong to the independent underwriter rather than the insurer.

Another implication of Proposition 7 is risk sharing: when the underlying business is risky, risk sharing become more important: neither party would be willing to assume the entire risk. And since via the independent underwriter has better and more direct control of the risk via underwriting performance, independent underwriter's risk share is expect to be heavier than the insurer (insurer takes at most 1/2 of the risk whereas independent underwriter takes at most 2/3 of the risk); when the underlying business is not risky, risk sharing become less important: either party would be willing to assume the entire risk.

Proposition 8

Independent underwriter's choice of optimal underwriting accuracy q_L^* and q_H^* are positively related with the complexity of the underwriting task $\rho_H(\lambda_H - \beta\lambda_L)$

Proof see Appendix A

This proposition implies that when underwriting task is more complex, the independent underwriter will choose to exert more effort in the underwriting process to boost underwriting accuracy. Regan (1997) studied insurance distribution system choice based on 1990 accounting data from a sample of 149 insurance groups and found that if independent agency insurers operate in more complex lines of business, then they should exert relatively more efforts in underwriting inspections and audits as measured by the proportion of the firm's expenses allocated to surveys and audits.

1.6. Characteristics of the Optimal Contract

If the insurer decides to use independent underwriter, the independent underwriter will be offered with contract of optimal compensation scheme stated in section 3. This optimal contract is in equilibrium with the following characteristics:

1.6.1. Market Characteristic and the Optimal Contract

Proposition 9 In equilibrium, commission loading α^* exceeds 0.5 if and only if underwriting risk factor $(\lambda_H - \beta\lambda_L)$ is positive

Proposition 10 In equilibrium, commission loading α^* increases with underwriting risk factor $(\lambda_H - \beta\lambda_L)$

Proof: see Appendix A

Propositions 9 and 10 state that in equilibrium, insurance company should compensate the independent underwriter with commission loading α^* that exceeds one half of the total profit, when the premium collected from incorrectly underwritten high-risk $\beta\lambda_L$ is not enough to cover the expected loss λ_H. When this gap gets larger, the independent underwriter is expected to get higher complementation (larger α^*) to be incentivized to exert more effort to underwrite correctly.

This finding complies with Dutta (2008) that risk and incentives are positively associated for general managerial expertise and Bernardo et al. (2001) that "…optimal sharing-rule increases in importance of managerial effort…" In addition, the larger is the underwriting risk factor, the higher value can the independent underwriter potentially provide, such as Regan & Tennyson (1999) and Posey & Tennyson (1998) claimed that independent underwriters' higher costs are associated with the potential added values they could provide.

1.6.2. Underwriting Cost and the Optimal Contract

Proposition 11 In equilibrium, commission loading α^* is positively related with independent underwriter's cost c_H and c_L, if and only if underwriting risk factor $(\lambda_H - \beta\lambda_L)$ is positive.

Proof: see the Appendix A

This proposition states that in equilibrium, commission loading α^* should increase with underwriting cost when there is positive underwriting risk factor $(\lambda_H - \beta\lambda_L)$. This finding is different with previous accounting literature stating compensation is negatively related with cost when agent is risk averse. (Dutta, 2008; Holmstrom, 1979; Holmstrom & Milgrom, 1991). In Holmstrom & Milgrom (1991) framework, the agent is risk averse so shifting the risk to the agent entails an efficiency cost. Therefore, the optimal contract in that framework reflects a tradeoff between the incentive benefit of conditioning the agent's compensation on the observed level of profit, and the cost of inefficiency allocating the risk to the risk-averse party. In the model of this paper however, independent underwriter is assumed to be risk neutral. The result is therefore different with previous accounting literature. As have stated previously, positive underwriting risk factor means potential loss when a high-risk is incorrectly underwritten as low-risk. In this case, it is necessary to incentivize the independent underwriter to underwrite correctly. And when underwriting cost goes up, the independent underwriter will tend to exert less effort to maintain profitability. Therefore, the independent underwriter should be incentivized more heavily with increased cost.

1.7. Multi-Period Contracting with Policy Renewal

Ownership of renewal grants independent underwriter contracting advantage ever since the second period on. In practice, independent underwriter has the right to put renewal policy with any insurer. Therefore, policy renewal is modeled as auction process in which insurer bids against other insurers to win the policy renewal as auction target, as shown by Figure 4 and Figure 5.

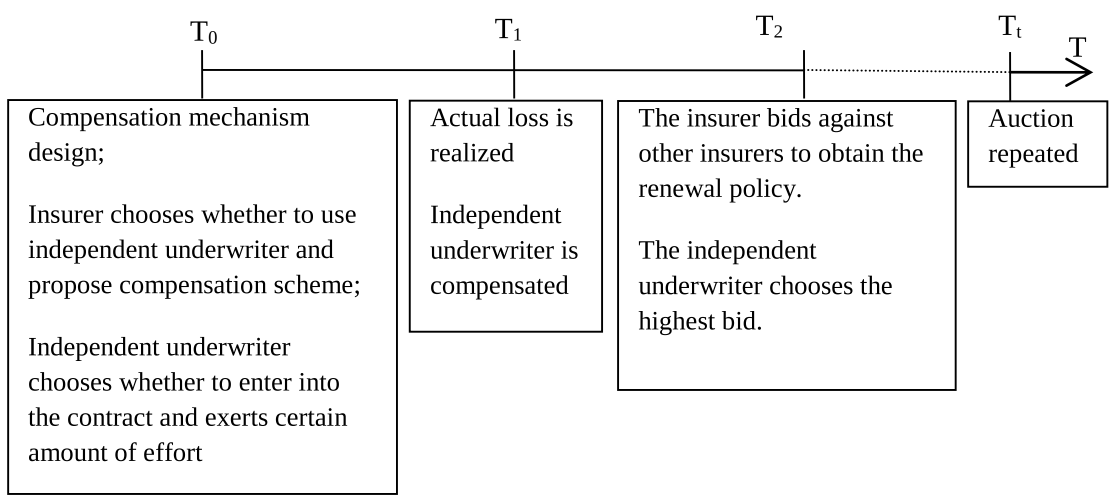
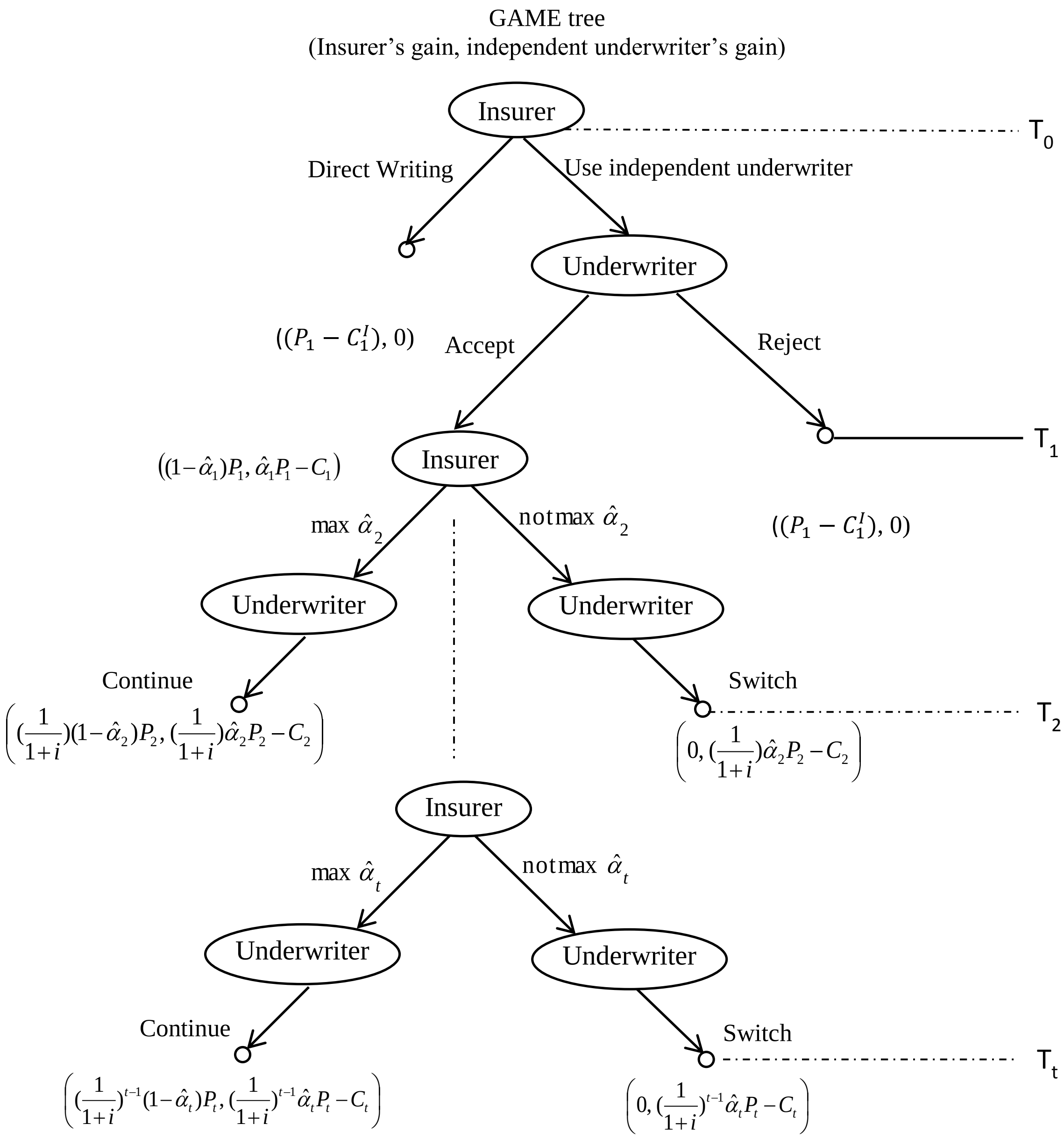


Figure 4. The Timing of the Game of t Periods

In each period, contracting scenario can be demonstrated in the following GAME tree:



Underwriter: independent underwriter; $(\frac{1}{1+i})$: discounting factor; P_t: total profit obtained in the t^{th} period; C_t^I: underwriting cost in the t^{th} period; $\hat{\alpha}_t$: profit loading in the t^{th} period

Figure 5. The Multi-period Game Tree

1.7.1. Auction

From the second period on, the insurer will need to bid against other insurers for the renewal. For ease of exposition, assume that there are only two identical insurers competing for the renewal, each of them makes a sealed bid of profit share α, highest bidder wins. Also assume that the probability of winning is uniformly distributed in which:

$$\Pr(win) = \Pr(other\ insurer\ 's\ bid < \alpha) = F(\alpha) = \alpha \tag{21}$$

$$Insurer's\ expected\ \ payoff = Pr\ (win) \times (1- \alpha\ ) \times Profit + Pr\ (loss) \times 0 = \alpha = \alpha(1-\alpha) \times Profit \tag{22}$$

Proposition 12 From period 2 on, insurer will offer α =1/2 to maintain renewal as a best response[4].

Proof: see Appendix A

For period $t \geq 2$, independent underwriter's optimal underwriting accuracy satisfies:

$$q_{Lt}, q_{Ht} = \underset{\hat{q}_{Lt},\hat{q}_{Ht}}{\arg\max} \left\{ \frac{1}{2} \left[\beta\rho_H (\lambda_H - \lambda_L)\hat{q}_{Ht} + (\beta - 1)\lambda_L (1-\rho_H)\hat{q}_{Lt} + \rho_H (\beta\lambda_L - \lambda_H) \right] - c_{Lt} (1-\rho_H)^2 \hat{q}_{Lt}{}^2 - c_{Ht} \rho_H{}^2 \hat{q}_{Ht}{}^2 \right\} \tag{23}$$

With optimal underwriting accuracy being:

$$q_{Lt} = \begin{cases} \dfrac{(\beta-1)\lambda_L}{4c_{Lt}(1-\rho_H)} & if \quad \lambda_L < \dfrac{4c_{Lt}(1-\rho_H)}{\beta - 1} \\ 1 & if \quad \lambda_L \geq \dfrac{4c_{Lt}(1-\rho_H)}{\beta-1} \end{cases} \quad \text{and} \quad q_{Ht} = \begin{cases} \dfrac{\beta(\lambda_H - \lambda_L)}{4\rho_H c_{Ht}} & if \quad (\lambda_H - \lambda_L) < \dfrac{4c_{Ht}\rho_H}{\beta} \\ 1 & if \quad (\lambda_H - \lambda_L) \geq \dfrac{4c_{Ht}\rho_H}{\beta} \end{cases} \tag{24}$$

[4] It is fairly straightforward to show that with generic α, the results found in Proposition 13 and Proposition 14 still hold.

1.7.2. Insurer's Rationale

In a multi-period policy contracting setting, insurer will use independent underwriter rather than underwriting by himself if and only if he finds doing so brings him larger profit, from a multi-period contracting prospective. There are two scenarios associated with this set-up:

- Scenario 1: insurer uses independent underwriter:

Insurer's total payoff when he uses independent underwriter is obtained as:

$$Insurer's\ Total\ Profit_{use\ independent\ underwriter}$$
$$=\frac{1}{2}\sum_{t=1}^{T}\{(\frac{1}{1+i})^{t-1}[\beta^2(\lambda_H-\lambda_L)^2\frac{1}{4c_{Ht}}+(\beta-1)^2\lambda_L{}^2\frac{1}{4c_{Lt}}-\rho_H(\lambda_H-\beta\lambda_L)]\}+\frac{1}{2}\frac{\rho_H{}^2(\lambda_H-\beta\lambda_L)^2}{\beta^2(\lambda_H-\lambda_L)^2\frac{1}{c_{H1}}+(\beta-1)^2\lambda_L{}^2\frac{1}{c_{L1}}} \quad (25)$$

- Scenario 2: insurer uses direct writing

Insurer's total payoff when he uses direct writing distribution channel can be written as:

$$Insurer's\ Total\ Profit_{use\ direct\ writing}=\sum_{t=1}^{T}(\frac{1}{1+i})^{t-1}[\beta^2(\lambda_H-\lambda_L)^2\frac{1}{4c_{Ht}^I}+(\beta-1)^2\lambda_L{}^2\frac{1}{4c_{Lt}^I}-\rho_H(\lambda_H-\beta\lambda_L)] \quad (26)$$

Lemma 3 Insurer will decide to use independent underwriter if the total discounted profit margin is positive:

$$Total\ Profit\ Margin=Total\ Profit_{use\ independent\ underwriter}-Total\ Profit_{use\ direct\ writing}$$
$$=\frac{1}{4}\sum_{t=1}^{T}\{(\frac{1}{1+i})^{t-1}[\beta^2(\lambda_H-\lambda_L)^2(\frac{1}{2c_{Ht}}-\frac{1}{c_{Ht}^I})+(\beta-1)^2\lambda_L{}^2(\frac{1}{2c_{Lt}}-\frac{1}{c_{Lt}^I})+2\rho_H(\lambda_H-\beta\lambda_L)]\} \quad (27)$$
$$+\frac{\rho_H{}^2(\lambda_H-\beta\lambda_L)^2}{\beta^2(\lambda_H-\lambda_L)^2\frac{2}{c_{H1}}+(\beta-1)^2\lambda_L{}^2\frac{2}{c_{L1}}}$$

Proposition 13 Insurer will be more willing to use independent underwriter with a shorter term of renewal.

Proof: see Appendix A

Proposition 14 Insurer will be more willing to use independent underwriter with a larger discount rate.

Proof: see Appendix A

Propositions 13 and 14 show the relationship between the importance of renewal and insurer's decision. When policy period is shorter and/or the discount rate is larger, renewal becomes less important. In either case, insurer will be more willing to use independent underwriter.

1.8. Empirical Implications

This research provides a guideline for the insurer when choosing between the distribution system of independent underwriter and direct writing, as well as the optimal contract between an insurer and independent underwriter, in both single period and multi-period setting, with the following conclusions:

When choosing distribution systems, the decision rules are as follows:

- It is profitable for insurer to underwrite by himself only when the complexity of underwriting task is below a certain threshold, as shown in Proposition 1.
- Costs are indeed important decision factors for insurer, when choosing distribution system, as demonstrated in proposition 2: holding other conditions as fixed, cost advantage makes independent underwriter more preferable for the insurer.
- Holding other conditions as fixed, larger underwriting risk factor makes independent underwriter more preferable, as demonstrated in proposition 3 and proposition 4. This finding explains paradox mentioned in previous literature: even

without cost advantage, independent underwriter can still be an attractive option for the insurer through the risk sharing mechanism provided by the independent underwriter.

To gain a better insight of this finding, a restriction has been posted on the cost functions assuming that the independent underwriter has the same cost as the insurer, with the following conclusions:

- When insurer finds it unprofitable to use direct writing, using independent underwriter can still be profitable, even though the independent underwriter may not have cost advantage, as demonstrated in proposition 5. This observation can help to explain the paradox of coexistence between independent underwriting and direct writing distribution system, when the independent underwriter does not have cost advantage.
- When the underlying business is risky (i.e. with positive underwriting risk factor), the independent underwriter is expected to be compensated by more than one half of the total profit, as demonstrated in proposition 6.

From the independent underwriter's prospective, it will be optimal to accept insurer's offer if individual rationality restriction is satisfied. And the independent underwriter will choose to exert the optimal amount of effort accordingly:

- When the underlying business is risky, the independent underwriter should be compensated by more than 1/2 but no more than 2/3 of the total profit, to ensure that the underlying business is profitable but not too risky for the independent underwriter. When the underlying business is not risky, the independent underwriter can be compensated by either less than 50% or more than 2/3 of the total profit, as demonstrated in proposition 7.

- Independent underwriter’s choice of optimal underwriting accuracy is found to be positively related with complexity of the underwriting task, as demonstrated in proposition 8.

If the insurer decide to use independent underwriter, and the independent underwriter is willing to join the business, the optimal compensation contract should have the following properties:

- Holding other factors as fixed, the optimal commission loading exceeds 0.5 as long as underwriting risk factor is positive, and is positively related with underwriting risk factor and independent underwriter's underwriting cost, as demonstrated in propositions 9 through 11.

When considering policy renewal, insurer's decision of whether to use independent underwriter has the following properties:

- Independent underwriter will be utilized more when renewal is less important and competition is less severe, as demonstrated in proposition 13 and proposition 14.

1.9. Conclusions

In this article, a principal-agency model has been used to study the interactive behavior of a risk-neutral insurer and a risk-neutral independent underwriter in property-liability lines of business. It is found that: (1) the insurer chooses between direct writing distribution channel and independent underwriter, based on underwriting risk factor and the importance of renewal (in a multi-period policy setting); (2) the insurer can implement an optimal contract to maximize his profit, while the independent underwriter also maximizes her profit under this contract; (3) the optimal contract has certain characteristics that complies with business scenario.

One very interesting finding is that underwriting risk factor $\lambda_H - \beta\lambda_L$ is the contracting key. It compares the expected loss of high-risk type λ_H and the premium generated from low-risk type $\beta\lambda_L$, when a true high-risk is incorrectly underwritten as a low-risk. It is found that this underwriting risk factor is the key to (1) insurer's decision of channel; (2) independent underwriter's decision of optimal effort to exert; (3) optimal contract in equilibrium.

This paper presents a formal mathematical modeling to: (1) solve the paradox of coexistence of independent underwriting insurer and direct writing insurer; (2) present an analytical boundary within which it is optimal for the insurer to use independent underwriter and vice versa; (3) present an optimal contract in which both insurer and independent underwriter benefit in an equilibrium.

CHAPTER 2

RISK FINANCE FOR CATASTROPHE LOSSES
WITH PARETO-CALIBRATED LÉVY-STABLE SEVERITIES

2.1. Introduction

2.1.1. The Risk Finance Paradigm

In the field of enterprise risk management, the random losses to which a firm is exposed in a given time period are commonly analyzed within a two-dimensional space spanned by expected frequency and expected severity (see, e.g., Zuckerman, 2010). The former quantity, usually plotted along the horizontal axis, denotes the expected number of occurrences of a particular type of event during the given time window, whereas the latter quantity, plotted along the vertical axis, denotes the expected value of the individual severities arising from events of a certain type.

This two-dimensional space is analogous to, but technically different from the probability-versus-severity space often used to analyze risk in fields such as public health and environmental science (see, e.g., Cox, 2008). Essentially, the expected frequency-versus- expected severity formulation is appropriate when the individual points in the two- dimensional plane are viewed as loss portfolios, consisting of sums of (possibly random) numbers of severities, whereas the probability-versus-severity framework is appropriate when the individual points are viewed as single random events that either do or do not occur (i.e., Bernoulli random events).

After identifying and assessing risks in terms of their frequency and severity components, enterprise risk managers embark on the more complex and costly steps of risk control and risk finance. Risk control typically prescribes frequency mitigation for exposures with high expected frequencies and low expected severities, severity mitigation for exposures with high expected severities and low expected frequencies, and avoidance for exposures with both high expected frequencies and high expected severities (see, e.g., Kwon and Skipper, 2007, p. 22 and p. 306). The textbook approach to risk finance is given by the paradigm of Figure 6 (see, e.g., Baranoff et al., 2009, p. 96).

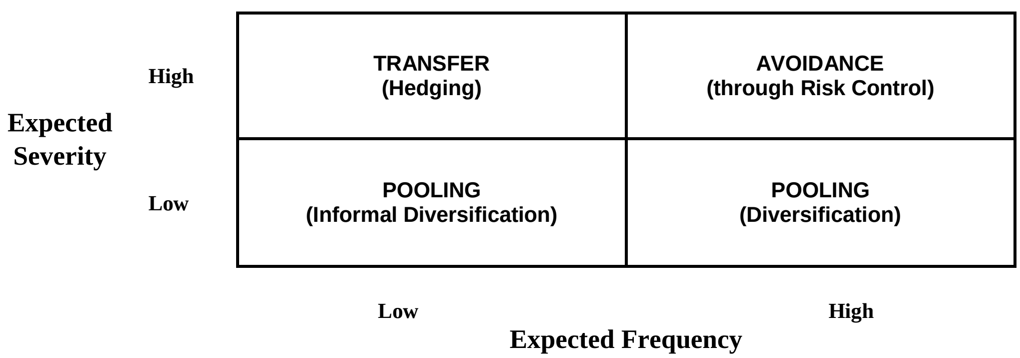


Figure 6. Conventional Risk-Finance Paradigm

Having avoided, as a matter of risk control, those unacceptably costly exposures with both high expected frequencies and high expected severities, the firm can focus on the exposures exhibiting either high expected frequencies or high expected severities alone. In the former case, the likely presence of a great number of similar and statistically independent claims presumably permits the company to take advantage of diversification by retention (i.e., pooling). In the latter case—which includes regional catastrophes such as hurricanes, earthquakes, and even terrorist attacks—there presumably is an insufficient number of claims to diversify successfully, and so the company is more likely to employ hedging by purchasing insurance (i.e., transfer). For exposures with both relatively low expected frequencies and relatively low expected severities, the company will continue to pool these risks, but will do so informally (i.e., with less explicit contemplation of the benefits of diversification).

2.1.2. Problems of Catastrophe Losses

Although the scheme indicated by Fig. 6 seems intuitively plausible, it presents two problems for firms considering the financing of catastrophe losses:

(1) By indicating transfer for loss portfolios with low expected frequencies but arbitrarily high expected severities, the conventional paradigm suggests that there will always be some insurance company willing to assume (and presumably pool) those catastrophe exposures. However, this is inconsistent with actual market experience, in which certain types of catastrophe losses—for example, damages from terrorist attacks—are not readily covered by private insurance. Given that such exposures are not insurable, it may make sense to amend Fig. 6 by adding an avoidance rectangle above the transfer rectangle.

(2) There is an inconsistency between the effect of increasing expected frequencies on losses with low expected severities, and the effect of increasing expected frequencies on losses with high expected severities. In the former case, diversification applies because pooling is effective for higher expected frequencies, whereas in the latter

case, diversification does not appear to work because the firm must resort to avoidance. One possible explanation for this discrepancy is that losses with higher expected severities also possess heavier tails, thus inhibiting diversification. However, there is no obvious theoretical or empirical reason why this should be true.

Rather than simply tweaking the conventional paradigm in an *ad hoc* manner, this paper proposes to address the above two problems through the formal mathematical modeling of loss portfolios and the careful study of the boundaries separating the regions of pooling, transfer, and avoidance. Given the potentially significant role of severity tails, this research will pay particular attention to the decreasing benefits of pooling as the tails of severity distributions become heavier.

2.2. The Basic Model

2.2.1. A Lévy-Stable Loss Portfolio

Let's begin by positing a simple loss portfolio, $\mathrm{L} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{\mathrm{i}}$, in which the frequency component (i.e., the number of loss-generating events) is a nonstochastic positive integer, n, and the severity component (i.e., the individual loss amounts) is given by a sequence of independent and identically distributed (i.i.d.) Lévy-stable random variables, X_i, with finite expected value. Choose the Lévy-stable family because it affords a continuum of power-law tails that is useful for exploring the effect of heavy tails on diversification, and require that the mean be finite so that the expected-severity component of the two-dimensional risk finance analysis is well defined. (In addition, it is reasonable to assume that, under ordinary circumstances, a firm would be unwilling to assume a loss portfolio with infinite mean.)

Generally, one would prefer to work with power-law severity distributions that, like the actual severities they model, are defined on only the nonnegative real numbers. Prominent examples are the Pareto I, Pareto II, and generalized Pareto distributions, which have been used extensively in the actuarial literature. The two-parameter Pareto I and Pareto II distributions are characterized by their respective probability density functions (PDFs), $f_I(x) = \alpha\theta^{\alpha}/x^{\alpha+1}, x > 0$ and $f_{II}(x) = \alpha\theta^{\alpha}/(x+\theta)^{\alpha+1}, x > 0$, where $\theta > 0$ and the means are finite for $\alpha > 0$. The generalized Pareto family possesses a third parameter that permits incorporation of not only the Pareto I and Pareto II distributions, but also the shifted and ordinary exponential distributions.

Pareto distributions have been found to provide good empirical models of large severities in a variety of insurance and reinsurance contexts, including fire claims (see, e.g., McNeil, 1997), medical claims (see, e.g., Cébrian et al., 2003), and automobile liability claims (see, e.g., Verlaak et al., 2009). This family of distributions also has been justified theoretically based upon the role of the generalized Pareto distribution as a limiting distribution for random variables exceeding a threshold (see, e.g., McNeil, 1997 and Embrechts et al., 2003) and the use of power laws to model natural events, such as earthquakes and hurricanes, underlying catastrophe severities (see, e.g., Ibragimov et al., 2008).

Unfortunately, sums of i.i.d. Pareto random variables do not offer analytically tractable convolution expressions. For this reason, the following research will work with a set of Lévy-stable distributions calibrated to be asymptotically equivalent to those arising from the Pareto II case (i.e., the simplest Pareto distribution whose sample space is the entire set of positive real numbers). Specifically, this paper selects the parameters of the Lévy-stable distribution so that the sums of the associated i.i.d. severities converge in distribution to the sums of corresponding i.i.d. Pareto II (a, θ) severities as the number of loss events, *n*, becomes large. Other families of heavy-tailed distributions, such as the noncentral *t* family and Tukey's *g* and *h* family (see, e.g., Martinez and Iglewicz, 1984), might be used in place of the Lévy-stable family. However, these other distributions are not easily justified as approximations to sums of i.i.d. Pareto random variables.

The Lévy-stable distribution is usually described by its characteristic function,

$$\psi_X(\omega) = \exp(-\gamma^\alpha \omega^\alpha) \exp(i[\delta\omega + \gamma^\alpha \omega^\alpha \beta \tan(\pi\alpha/2) \times (1 - \gamma^{1-\alpha}\omega^{1-\alpha})])$$

for $\omega > 0$. This particular parameterization, equivalent to $\mathrm{S}(\alpha;\ \beta;\ \gamma;\ \delta; 0)$ of Nolan (2008) may be interpreted as follows: $\alpha \in (0, 2]$ is the tail parameter (with smaller values of α implying heavier tails, and $\alpha = 2$ in the Gaussian case); $\beta \in [-1, 1]$ is the skewness parameter (with negative values implying negative skewness, positive values implying positive skewness, and $\beta = 0$ in the Gaussian case); $\gamma \in (0, \infty)$ is the dispersion parameter (which is proportional to the standard deviation in the Gaussian case—that is, $\gamma = SD[X]/\sqrt{2}$); and $\delta \in (-\infty, \infty)$ is the location parameter (which equals the median if $\beta = 0$, and also equals the mean if $\alpha \in (1, 2]$ and $\beta = 0$, as in the Gaussian case).

For $X_i \sim$ i.i.d. Lévy-stable $(\alpha, \beta, \gamma, \delta)$, the desired Pareto-calibrated parameters are:

$$\alpha = \begin{cases} a & \text{if } a \in (1, 2) \\ 2 & \text{if } a > 2 \end{cases} \quad [5] \tag{28}$$

$$\beta = \begin{cases} 1 & \text{if } a \in (1, 2) \\ 0 & \text{if } a > 2 \end{cases} \tag{29}$$

[5] The case of $a = 2$ is omitted because it involves a singularity at which X is Gaussian with infinite variance.

$$\gamma = \begin{cases} \theta & \text{if } a \in (1,2) \\ \theta/[2^{1/2}(a-1)(a-2)^{1/2}] & \text{if } a > 2 \end{cases} \tag{30}$$

$$\delta = \begin{cases} \theta/(a-1) + \theta\tan(\pi a/2) & \text{if } a \in (1,2) \\ \theta/(a-1) & \text{if } a > 2 \end{cases} \tag{31}$$

Given the convolution properties of the Lévy-stable distribution, (see, e.g., Nolan, 2008) the parameters indicated in Equations (28) through (31) then imply that:

L=$\sum_{i=1}^{n} X_i$ ~ Lévy-stable $(a, 1, n^{1/a}\theta,\ n\theta/(a-1) + n^{1/a}\theta\tan(\pi a/2))$ for $a \in$ (1, 2)

(32)

and

L=$\sum_{i=1}^{n} X_i$ ~ Lévy-stable $(2, 0, n^{1/2}\theta/[2^{1/2}(a-1)(a-2)^{1/2}],\ n\theta/(a-1))$ (33)

$\Leftrightarrow$ Gaussian $(n\theta/(a-1), n\theta^2[(a-1)^2(a-2)])$ for $a > 2,$

Although the choice of a nonstochastic frequency, *n*, is made to simplify the asymptotic analysis, this assumption can be relaxed to permit sufficiently well-behaved stochastic frequencies, *N*, whose means increase with *n*. For example, if X_i is modeled as a mixture of a Pareto II random variable and a single point mass at 0 (to capture the impact of an insurance deductible or reinsurance retention), then the sum L=$\sum_{i=1}^{n} X_i$ may be rewritten as L=$\sum_{j=1}^{n} X'_j$, where the sequence X'_j is formed of the positive X_i , and *N* ~ Binomial (*n*, Pr {X_i > 0}). This would require a recalibration of the parameters in Equations (28) through (31), but would not alter any asymptotic results because the effect of the point mass would vanish as $n \to \infty$. Furthermore, Chapter 3 shows via simulation that the results of the present article hold for *N* ~ Poisson (*n*), but caution that the same results are not valid if the coefficient of variation of *N* (i.e., *SD*[*N*]/*E*[*N*]) remains positive as $n \to \infty$.

2.2.2. A Simple Risk Measure

In analyzing a firm's decision whether to pool or transfer the loss portfolio L, it is assumed that its management compares the expected cost of this exposure to its expected benefit. As explained in the next section, the expected cost is considered to be proportional to some measure of the portfolio's risk, conceived as a type of "spread," and the expected benefit to be captured by a profit loading proportional to the portfolio's expected value.

For Lévy-stable severities with $\alpha = 2$ (i.e., the Gaussian case), risk (as spread) can be measured by the variance or standard deviation; however, for all Lévy-stable severities with $\alpha \in (1, 2)$, these quantities are infinite, and so it is necessary to select an alternative measure. Interestingly, despite the growth of a large literature on applications of Lévy-stable distributions to model financial data,[6] there has been little work on the development of risk measures appropriate for Lévy-stable data. Fama and Samuelson independently addressed the problem of risk-versus-return analyses when financial assets have Lévy-stable distributions with infinite variances, and both proposed using the Lévy-stable distribution's dispersion parameter, γ (which is proportional to the standard deviation in the Gaussian case), as a measure of risk. However, this choice was largely *ad hoc*, and motivated by a desire to provide a standard-deviation-like way of capturing the impact of portfolio diversification.

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[6] Mandelbrot and Fama were the first to argue that Lévy-stable distributions provide good models of financial asset returns. This early work spawned a vast literature on the estimation of Lévy-stable models for asset returns. Prominent examples from the last two decades include Jansen, 1991; McCulloch, 1997; Rachev & Mittnik, 2000; Gabaix et al. (2003).

In previous work, Powers & Powers, (2009) proposed the cosine-based standard deviation,

$$CBSD[X] = (1/\omega)\cos^{-1}(\mathrm{E}[\cos(\omega(\mathrm{X} - \tau))])$$

where τ satisfies $E[\sin(\omega(X - \tau))] = 0$, and $\omega > 0$ is a free parameter as a risk measure for heavy-tailed distributions. As $\omega \rightarrow 0^+$, this measure approaches the conventional standard deviation if it is finite, and the parameter ω may be chosen to maximize the marginal "information"[7] associated with the spectral density, $|\psi_X(\omega)|^2$, of the random variable X. Selecting ω in this way for the Lévy-stable family yields:

$$CBSD[X] = \cos^{-1}(\exp(-1/2))2^{1/\alpha}\gamma \approx (0.9191)2^{1/\alpha}\gamma \tag{34}$$

In a similar manner, it is possible to define the cosine-based variance as

$$CBVar[X] = (CBSD[X])^2$$

and then employ the same ω to solve for

$$CBVar[X] = (0.8448)(2^{1/\alpha}\gamma)^2 \tag{35}$$

in the Lévy-stable case.

The risk measures in Equations (34) and (35) are not only natural extensions of the ordinary standard deviation and variance, respectively, but also readily applicable to analytical work because of their simple expressions in terms of the parameters α and γ. For purposes of measuring the risk of a Lévy-stable portfolio of losses, this paper therefore will subsume these two quantities into a single risk measure, $R[X] = (2^{1/\alpha}\gamma)^p$, in which the parameter $p \geq 1$ is selected to capture the firm's intrinsic sensitivity to risk

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[7] In information theory, the quantity $-E[\ln(f(\omega))] = -\int_{\Omega} \ln(f(\omega))\, f(\omega) d\omega$ denotes the information associated with the PDF f (ω), and the marginal information afforded by the particular value ω is therefore given by $-\ln(f(\omega))\, f(\omega)$. Place the term "information" in quotation marks to indicate that the (unnormalized) spectral density $|\psi_X(\omega)|^2$ is not necessarily a proper PDF.

(i.e., a larger value of p indicates greater sensitivity to risk, and less ability to benefit from effects of diversification). Note that for a fixed value of α, this risk measure provides a simple generalization of the dispersion parameter used by Fama and Samuelson.

2.3. Firm Decision Making

2.3.1. The Pooling/Transfer Boundary

To construct the boundary between those exposures that a firm decides to pool and those that it decides to transfer, assume that the firm is willing to pool the loss portfolio L only if the expected cost of this exposure is less than its expected benefit. As mentioned above, treat the expected cost as proportional to the portfolio's risk, $R\,[L]$; this agrees with the intuition that the greater the risk of the portfolio, the more financial damage is done to the firm, on average. As also mentioned, treat the expected benefit as proportional to the portfolio's expected value, $E\,[L]$; this conforms to the notion that the firm's expected total losses grow in proportion to its total revenues, which generate a constant rate of profit, on average.

In short, the firm should find pooling acceptable only if the portfolios expected benefit through expected profit outweighs its expected cost through risk of financial harm. Otherwise, it should transfer the portfolio to an insurance company (whose decision making will be discussed later). Mathematically, this implies that the firm is willing to pool L only if

$$R\,[L] \,/\, E\,[L] < k \tag{36}$$

for some k > 0. For loss portfolios with finite variances, this is consistent with the common actuarial practice of setting the required premium equal to the expected total loss plus a positive constant times either the conventional standard deviation (for p = 1) or the conventional variance (for p = 2).

Applying inequality (36) to the portfolio distributions in Equations (32) and (33) yields:

$$\frac{(2^{1/a}n^{1/a}\theta)^p}{nE[X]} < k \Rightarrow \frac{[2^{1/a}n^{1/a}E[X](a-1)]^p}{nE[X]} < k \Rightarrow E[X]$$
$$< k^{-1/(1-p)}2^{p/a(1-p)}(a-1)^{p/(1-p)}n^{(p-a)/a(1-p)}$$

for $a \in (1, 2)$ (37)

and

$$\frac{(2^{1/2}n^{1/2}\theta/[2^{1/2}(a-1)(a-2)^{1/2}])^p}{nE[X]} < k \Rightarrow \frac{[n^{1/2}E[X]/(a-2)^{1/2}]^p}{nE[X]} < k \Rightarrow E[X]$$
$$< k^{-1/(1-p)}(a-2)^{-p/2(1-p)}n^{(p-2)/2(1-p)}$$

for $a > 2$ (38)

respectively. Then note that inequalities (37) and (38) both describe regions of the expected frequency-by-expected severity (i.e., $n \times E[X]$) coordinate plane whose boundary curves are given by power functions of the form:

$$E[X] = \varphi n^{\zeta} \quad (39)$$

for positive coefficients $\varphi = \varphi(k, a, p)$ and exponents

$$\zeta = \zeta(a,p) = \begin{cases} (p-a)/a(1-p) \text{ for } a \in (1,2) \\ (p-2)/2(1-p) \text{ for } a > 2 \end{cases}$$

A careful examination of the ζ values reveals that the pooling/transfer boundary curves must be of one of the following three morphologically distinct types:

(I) decreasing, concave-upward functions as long as

$p > a$ for $a \in (1, 2)$ or $p > 2$ for $a > 2$; (40)

(II) increasing, concave-downward functions as long as

$p \in (2a/(1 + a), a)$ for $a \in (1, 2)$ or $p \in (4/3, 2)$ for $a > 2$; and (41)

(III) increasing, concave-upward functions as long as

$p \in (1, 2a/(1 + a))$ for $a \in (1, 2)$ or $p \in (1, 4/3)$ for $a > 2$. (42)

These three types of boundary curves are illustrated and discussed below. Their regions of applicability in the (a, p) plane are considered in Section 2.4.

2.3.2. The Transfer/Avoidance Boundary

To explore the transfer/avoidance boundary requires a shift in perspective. Rather than considering the firm's risk finance decision making, one must consider the risk control step of its risk management process, in which the firm determines which exposures to avoid. Fortunately, this analysis can be made fairly straightforward by assuming: (1) that the firm will choose to transfer, rather than avoid, a given loss portfolio only if its insurance company finds it sufficiently profitable to provide coverage for the exposure by including it in the insurer's internal pool of risks;[8] and (2) that the insurance company's underwriting decision is conceptually identical to the original firm's pooling/transfer decision, with "providing coverage" taking the place of "pooling," and "rejecting coverage" taking the place of "transfer" (i.e., the insurance company

[8] This is equivalent to saying that the firm's selection between transfer and avoidance is dictated by the underwriting criteria of the insurance market.

should compare the expected cost of the loss portfolio to its expected benefit, and assume the portfolio only if the latter is greater than the former).[9]

For simplicity, assume further that all differences between the insurance company's risk preferences and those of the original firm are captured by the cost-to-benefit decision threshold, k, with p remaining unchanged, and that the insurer's k, to be denoted by k_I, is greater than that of the original firm (so that the insurance company is in some sense less "risk averse"). It then follows that the insurer's providing-coverage/rejecting-coverage boundary—equivalent to the original firm's transfer/avoidance boundary—will be above and parallel to the original firm's pooling/transfer boundary. For a fixed value of $a = 1.8$, and hypothetical choices of p, k, and k_I , these two boundaries are illustrated for the Type I, Type II, and Type III cases in Figs. 7 through 9, respectively.

These three risk finance paradigms form the basis for developing a conservative paradigm in the following section.

2.4. Analysis

2.4.1. Comparisons with the Conventional Paradigm

A close inspection of Figs. 7 through 9 reveals that the Type I risk finance paradigm, with avoidance in the upper-right corner and pooling along the bottom, is most similar to the conventional paradigm of Fig. 6. Apart from the replacement of the simple

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[9] Clearly, these assumptions do not permit the insurance company to provide coverage for the firm's loss portfolio and then transfer this exposure to one or more layers of reinsurance. Exclude this possibility because it does not enhance the analysis conceptually, but complicates matters by creating a marginal extension of the transfer region (at the expense of the avoidance region) for each layer of reinsurance employed.

rectangular boundaries of Fig. 6 with the decreasing, concave-upward curves of Fig. 7, the most noticeable differences between these two schemes are: (i) that the Type I paradigm provides a clearly distinct region for transfer on the right side (i.e., for high expected frequencies), between pooling and avoidance; and (ii) that avoidance ultimately dominates both transfer and pooling for arbitrarily high expected severities (in the upper-left corner) and arbitrarily high expected frequencies (in the lower-right corner).[10]

Instructively, observations (i) and (ii) directly address problems (1) and (2) of the introduction. This is because (ii) indicates the need for an avoidance region above transfer for low expected frequencies, and (i) and (ii) both show that the benefits of diversification begin to wane for higher expected frequencies, whether expected severities are high or low. The explanation for this weakening of diversification is the relatively smaller values of *a* and larger values of *p* specified by conditions (40) for Type I boundary curves.

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[10] Although the dominance of avoidance for arbitrarily high expected frequencies (in the lower-right corner) is not obvious from Fig. 7, it can be shown quite easily by applying conditions (40) to Equation (39).

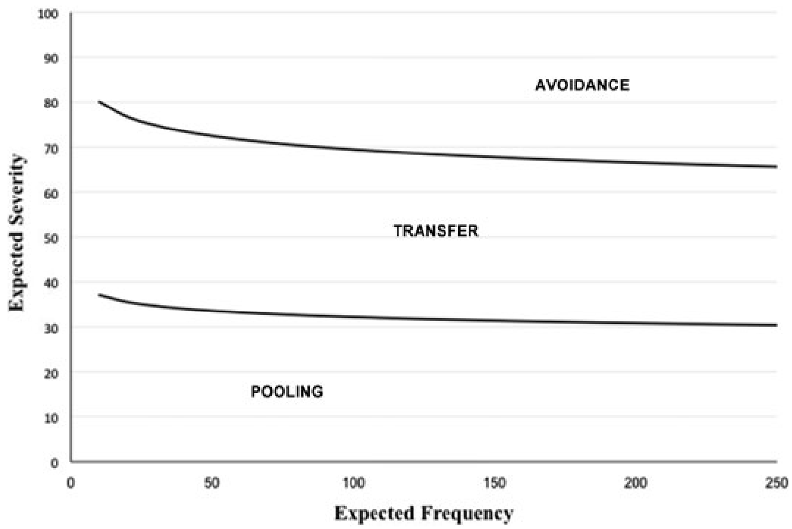

Figure 7. Type I risk finance paradigm.
Note: Type I regions based upon parameter values a = 1.8, p = 1.9, k = 20, 000, and k_I = 40,000.

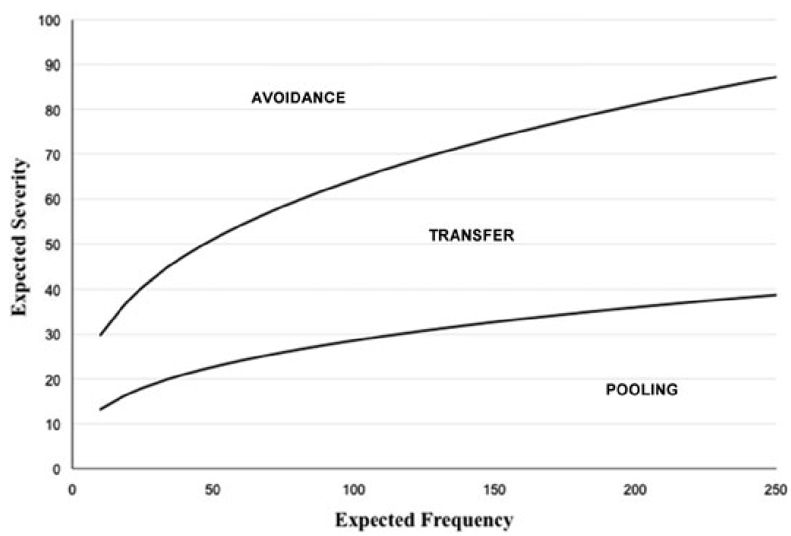

Figure 8. Type II risk finance paradigm.
Note: Type II regions based upon parameter values a = 1.8, p = 1.5, k = 100, and k_I = 150.

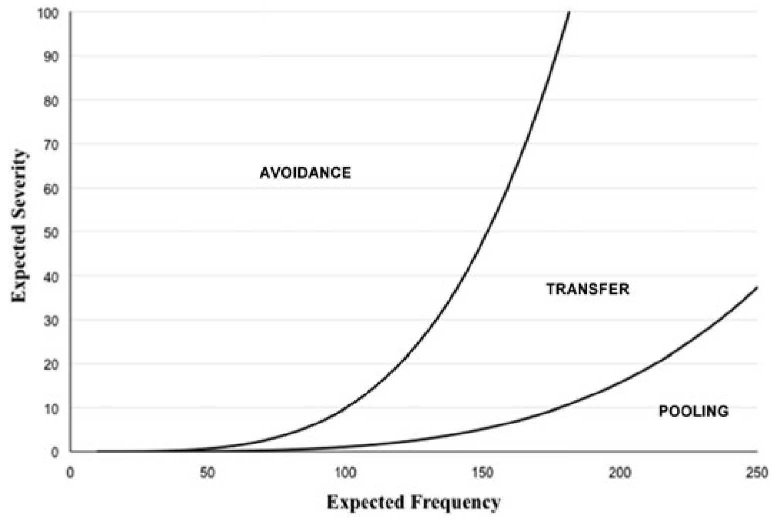

Figure 9. Type III risk finance paradigm.
Note: Type III regions based upon parameter values *a* = 1.8, *p* = 1.1, *k* = 0.4, and k_I = 0.5.

Fig. 10 summarizes the relationships among conditions (40)–(42) and the corresponding morphologically distinct types of boundary curves (Type I, Type II, and Type III, respectively). This figure shows that the Type I boundary curves of Fig. 7 are consistent with an apparently conservative set of assumptions in which: (1) the severity component of *L* possesses relatively heavier tails, as reflected in smaller values of a; and (2) the firm is particularly sensitive to the uncertainty of *L*, and so employs a relatively larger value of *p* in its risk measure. By conservative assumptions, this paper means suppositions that prepare the firm for worst-case scenarios with regard to the values of *a* and *p* by erring on the side of caution—in other words, favoring transfer over pooling and avoidance over transfer—if the parameters a and *p* are known imperfectly. In short, such assumptions would imply lower values for all points along each of the boundary curves in a paradigm of a given Type (I, II, or III), leading to what one will call a more conservative paradigm.

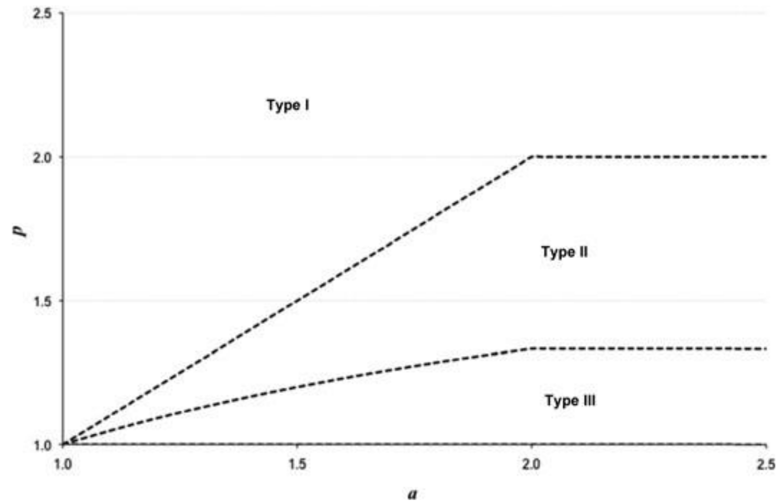

Figure 10. Regions for morphologically distinct boundary curves.

2.4.2. Toward a Conservative Paradigm

It is important to recognize that the conservative aspects of the Type I paradigm manifest themselves primarily on the right-hand side of Fig. 7. A comparison of the upper-left corner of Fig. 7 with the corresponding corners of Figs. 8 and 9 shows that the Type I paradigm actually could be the least conservative in that region because it entertains the possibility of both transfer and avoidance there, whereas the other paradigms specify avoidance alone.

Of course, the upper-left corner of the risk finance paradigm is precisely the focus of this article: the case of catastrophe losses. Therefore, it is imperative to confirm analytically which category of boundary curves is truly most conservative in that corner. At the same time, one would like to corroborate formally these observations about the rest of the paradigm. To this end, one must choose among three approaches to constructing a conservative paradigm: (1) addressing worst-case scenarios for *a* and *p* simultaneously; (2) addressing worst-case scenarios for *a* first (holding *p* fixed), and then exploring the effects of changes in *p*; or (3) addressing worst-case scenarios for *p* first (holding a fixed), and then exploring the effects of changes in a.

From a theoretical point of view, it might seem that approach (1) is best because it immediately considers the truly worst cases possible. However, from the perspective of a practicing enterprise risk manager likely to be transitioning from the conventional risk finance paradigm of Fig. 6, it is believed that approach (3) is most useful. This is because it addresses the rather subjective, difficult-to-measure, and difficult-to-explain risk-sensitivity parameter before moving on to the more easily estimated and better understood severity tail parameter.

Following this approach, compare the actual values of the ordinates of Equation (39) as the parameter *p* varies in Fig. 10, while holding *a*, *n*, and *k* fixed. This is accomplished by computing partial derivatives of $E[X]$ with respect to *p* along the curves described by Equation (39), and noting that:

(1) if $\partial E[X] / \partial p < 0$ for fixed *a*, *n*, and *k*, (43)

then the Type I region provides the most conservative (i.e., lowest) boundary curves as $p \to \infty$ because this region occupies the top of Fig. 10; and

(2) if $\partial E[X] / \partial p > 0$ for fixed *a*, *n*, and *k*, (44)

then the Type III region provides the most conservative (i.e., lowest) boundary curves as $p \to 1^+$ because this region occupies the bottom of Fig. 10.

As shown in the Appendix, if $a \in (1, 2)$, then condition (43) is true for all

$$n > n^* = [2^{1/a}(a-1)/k]^{a/(a-1)} \tag{45}$$

whereas condition (44) is true for all

$$n < n^* = [2^{1/a}(a-1)/k]^{a/(a-1)} \tag{46}$$

Similarly, if $a > 2$, then Equation (43) is true for all

$$n > n^* = 1/[k^2(a-2)] \quad (47)$$

whereas Equation (44) is true for all

$$n < n^* = 1/[k^2(a-2)] \quad (48)$$

Thus, Type III boundary curves are most conservative on the left-hand side of the paradigm (including the upper-left corner) as defined for numbers of loss events, n, less than some critical number of events, n^*. Alternatively, Type I boundary curves are most conservative on the right-hand side, for n greater than n^*.

These observations are illustrated conceptually in Fig. 11, a risk finance paradigm for fixed a that captures the conservative features of both the Type I and Type III curves by employing schematic piecewise linear functions in the appropriate regions. This paper emphasizes that this figure is not intended to yield precise quantitative information, but rather to provide a rough, qualitative description of the firm's risk finance behaviors in preparing itself for worst-case scenarios regarding p, for fixed a. In this sense, it is a suitable successor to the conventional paradigm of Fig. 6.

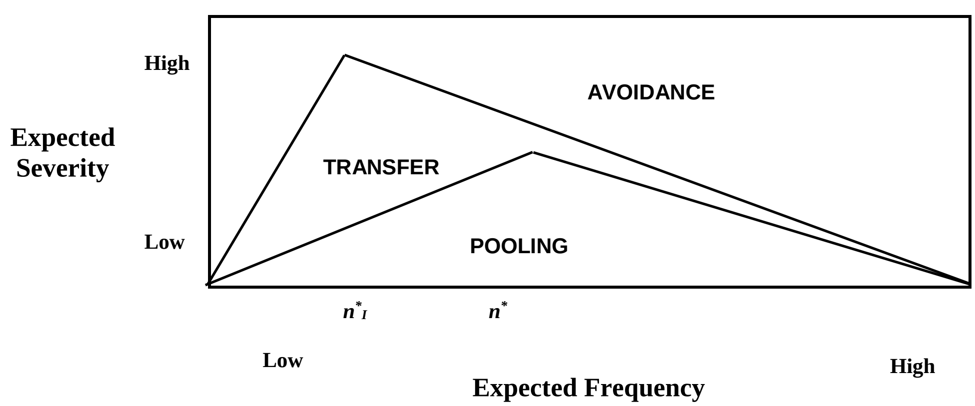

Figure 11. Conservative risk finance paradigm for fixed a.

As can be seen from this figure, the apex of the pooling/transfer boundary (which occurs at n^*, a function of a and k) must lie to the right of the apex of the transfer/avoidance boundary (which occurs at n^*_I, the same function of a and k_I). This is because the value of the firm's decision threshold, k, is smaller than the insurance company's decision threshold, k_I. Hence, the value of k_I is crucial to determining whether or not coverage will be available in a given catastrophe insurance market, and a sufficiently large value of this threshold, ceteris paribus, will cause the new paradigm of Fig. 11 to resemble Fig. 6 more closely.

To study the effects of changes in the tail parameter, a, first note from Equations (45) and (46) that, if $a \in (1, 2)$, then n^* is an increasing function of a whose slope is inversely related to k, and that $n^* \to 0^+$ as $a \to 1^+$. In other words, the heavier the tail of the severity distribution, the smaller n^* will be, and this change occurs faster for firms that are more "risk averse." Both of these effects are rather intuitive because one would expect the negative impact of heavy tails on diversification to push n^* to the left (i.e., to recognize that the decline in diversification benefits begins at a smaller value of n), and it seems quite natural that more "risk averse" firms would be more affected by any changes in tail heaviness.

From Equations (47) and (48) one can see that, if $a > 2$, then n^* is a decreasing function of a whose slope is positively related to k, and that $n^* \to \infty$ as a $\to 2^+$. In this case, the decline in n^* over a is somewhat surprising because one would not expect heavier severity tails to push n^* to the right; in particular, one would not expect n^* to become unbounded as $a \to 2^+$, while remaining finite as $a \to 2^-$. Interestingly, the disparate behavior on the two sides of $a = 2$ is an artifact of the Pareto calibration in which the individual severity X_i is Lévy-stable with bounded dispersion θ (but infinite standard deviation) as a $\rightarrow 2^{-}$, but Gaussian with unbounded standard deviation $\theta/[(a - 1)(a - 2)^{1/2}]$ as $a \rightarrow 2^{+}$. Consequently, the usefulness of the model for values of a in a small neighborhood of 2 requires further study.

2.5. Conclusions

For catastrophe losses, the conventional risk finance paradigm identifies transfer, as opposed to pooling or avoidance, as the preferred solution. However, using a loss portfolio characterized by non-stochastic frequency and a class of Pareto-calibrated Lévy-stable severity distributions, it is shown that the conventional analysis does not account for differences attributable to either the firm's intrinsic sensitivity to risk or the heaviness of the severity's tail. To address these shortcomings, a conservative risk finance paradigm is proposed that can be used to prepare the firm for worst-case scenarios with regard to both risk sensitivity and tail heaviness.

Naturally, the underlying model is subject to certain limitations. First, the assumption of nonstochastic frequency is realistic in only restricted settings, and second, the assumption of Pareto-calibrated Lévy-stable severities, while offering a mathematically tractable method of exploring the continuum of power-law tails, imposes a specific form on the shape of the severity distribution that is not appropriate in all cases.

As a continuous research, various relaxations of the frequency and severity assumptions can be placed.

This continuous work, as in the Chapter 3 of this thesis, requires extensive numerical computation, but offers the potential of greater insight into the general applicability of the proposed conservative risk finance paradigm.

CHAPTER 3

A RISK-BASED RISK FINANCE PARADIGM

3.1. Introduction

3.1.1. The Conventional Risk Finance Paradigm

In enterprise risk management (ERM), the portfolio of total losses to which a firm is exposed in a given time period may be expressed by the sum

$$L = X_1 + . . . + X_N,$$

where N denotes the frequency of loss events (taken as a random variable defined on the nonnegative integers), and the X_i are individual severities (taken as positive, real-valued random variables). Such portfolios are commonly analyzed within a two-dimensional space spanned by expected frequency ($E[N]$) and expected severity ($E[X_i]$) (see, e.g., Zuckerman, 2010), with the most appropriate method of risk finance indicated by the placement of L within the matrix of Figure 6. in Chapter 2 (see, e.g., Baranoff *et al*., 2009, p. 96).

For the case of loss portfolios with low expected severities, the logic behind the conventional paradigm is fairly straightforward: those portfolios with low expected frequencies are not very risky, and therefore can be financed as ordinary operational expenses; whereas those with high expected frequencies are best handled by pooling because they are amenable to risk reduction through diversification. For the case of loss portfolios with high expected severities, the rationale is somewhat different: those portfolios with low expected frequencies are too risky to be handled by informal or formal retention, and must be transferred through insurance or some other hedging mechanism; whereas those with high expected frequencies are simply too risky to be financed even by transfer, presumably because markets do not exist to accept such risks.

To enjoy the benefits of diversification, the random variable L must possess (at least) two salutary statistical properties: (1) a high expected frequency (i.e., a large value of $E[N]$); and (2) less than perfect positive correlations between individual severities (i.e., $Corr[X_i, X_j] < 1$ for all i not equal to j). For portfolios with low expected severities, the first of these properties is stated explicitly by the conventional paradigm, and the second is implied. What is left unexplained is why portfolios with high expected severities cannot benefit from diversification in the same way as those with low expected severities.

To resolve this issue, either by explaining the failure of diversification for portfolios of high expected severities, or by revising the conventional paradigm to correct this inconsistency, requires that one examine statistical properties of L beyond simply the expected frequency and expected severity. Chapter 2 describes a program by explicitly considering the impact of the severity distribution's tail behavior on the benefits of diversification. In the present Chapter, this work is extended by offering a more comprehensive analysis of L.

3.1.2. A Risk-Based Approach

One characteristic of L not explicitly addressed by the conventional paradigm already has been noted: the less than perfect positive correlations between individual severities. Ironically, a second unaddressed characteristic is the actual "risk" of L; that is, its dispersion or spread, as measured by a quantity such as the standard deviation, variance, coefficient of variation, inter-quartile range, etc.

In exploring the role of dense-tailed[11] severities in risk finance, the cosine-based standard deviation is employed here,

$$CBSD[L] = (1/\omega)\cos^{-1}(E[\cos(\omega(L - \tau))]),$$

where τ satisfies $E[\sin(\omega(L - \tau))] = 0$, and $\omega > 0$ is a free parameter chosen to optimize an information-theoretic criterion. This quantity extends the ordinary standard deviation to distributions for which the second moment is infinite, and for losses from the four-parameter Lévy-stable$(\alpha,\beta,\gamma,\delta)$ distribution[12] is written as

$$CBSD[L] = \cos^{-1}(\exp(-1/2))\, 2^{1/\alpha}\gamma \approx (0.9191)\, 2^{1/\alpha}\gamma,$$

which is increasing in the dispersion parameter, γ, and decreasing in the tail parameter, α.[13]

To identify regions of pooling, transfer, and avoidance in the expected frequency-by-expected severity plane, it is then computed an extension of the coefficient of variation – the ratio

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[11] By *dense-tailed*, it means that severities with infinite second moments (or equivalently, infinite standard deviations or variances). The term *heavy-tailed* is used for the same purpose, and other authors use terms such as *fat-tailed* or *long-tailed*. Dense-tailed is preferred here because it emphasizes the fact that a large amount of weight (probability) is compressed in the tail of the distribution, and does not suggest that some distributions embody more weight than others.

[12] The four parameters may be characterized as follows: $\alpha \in (0,2]$ is the tail parameter (with smaller values of α implying denser tails, and $\alpha = 2$ in the Gaussian case); $\beta \in [-1,1]$ is the skewness parameter (with negative [positive] values implying negative [positive] skewness, and $\beta = 0$ in the Gaussian case); $\gamma \in (0,\infty)$ is the dispersion parameter (which is proportional to the standard deviation in the Gaussian case – i.e., $\gamma = SD[L]/\sqrt{2}$); and $\delta \in (-\infty,\infty)$ is the location parameter (which equals the median if $\beta = 0$, and also equals the mean if $\alpha \in (1,2]$ and $\beta = 0$, as in the Gaussian case). This corresponds to the $\mathbf{S}(\alpha,\beta,\gamma,\delta;0)$ parameterization of Nolan (2008).

[13] Since this cosine-based standard deviation is affected by the tail parameter, α, tail density, along with dispersion/spread, will be incorporate into this concept of risk.

$$R[L] = (CBSD[L])^s/E[L],$$

where s is a parameter that reflects how sensitive the enterprise is to risk[14] – and proposed that pooling [transfer] be selected if $R[L] \le [>]\ k$ and transfer [avoidance] be selected if $R[L] \le [>]\ k_{\mathrm{I}}$, for some constants $k < k_{\mathrm{I}}$. Essentially, this means that enhanced benefits of diversification will cause firms to prefer transfer over avoidance in a manner analogous to preferring pooling over transfer, a principle that Chapter 2 supported by arguing that transfer can occur only if the firm's insurance company is able to pool the risk it assumes, and that the firm must choose avoidance otherwise. In other words, k_{I} captures the insurance company's risk appetite in the same way that k captures the firm's risk tolerance[15].

3.2. Dense-Tailed Severities

To assess the impact of the severity tail density on risk, the study in Chapter 2 is restricted to the case of total losses with non-stochastic frequency. In particular, it modeled the portfolio of total losses as a sum

$$L = X_1 + \ldots + X_n,$$

where n is a positive integer denoting a fixed number of loss events, and the X_i are Lévy-stable$(\alpha,\beta,\gamma,\delta)$ severities whose parameters are calibrated to be asymptotically

[14] Specifically, larger values of $s \ge 1$ indicate greater sensitivity to risk, so that $(CBSD[L])^2$ and $(CBSD[L])^1$ are analogous to the ordinary variance and ordinary standard deviation, respectively, with the former being an additive risk measure, and the latter a sub-additive risk measure.

[15] Clearly, the parameters s and k are related through the firm's underlying risk profile, which could be summarized in its entirety by a utility function in an expected-utility framework. The principal distinction between the two quantities is that the former identifies the aspect of L's probability distribution that the firm finds most relevant in assessing risk, whereas the latter measures the degree to which the firm is able to bear the risk assessed.

equivalent to those arising from a two-parameter Pareto distribution (see, e.g., Zaliapin et al., 2005). In this way, L can be treated as a Lévy-stable random variable for all positive integers n, and thus can be used to explore the impact of both severity tail density and increasing frequency on the benefits of diversification.

In the previous chapter, it is shown that the boundary between the regions of pooling and transfer – as well as that between the regions of transfer and avoidance – always takes on one of three distinct shapes in the expected frequency-by-expected severity plane: (I) decreasing and concave upward; (II) increasing and concave downward; and (III) increasing and concave upward. Furthermore, the shape of the boundary curve is determined entirely by the pair of parameters α and s, with: Type I for $s > \alpha$ and $\alpha \in (1,2)$ or $s > 2$ and $\alpha > 2$; Type II for $s \in (2\alpha/(1+\alpha),\alpha)$ and $\alpha \in (1,2)$ or $s \in (4/3,2)$ and $\alpha > 2$; and Type III for $s \in (1,2\alpha/(1+\alpha))$ and $\alpha \in (1,2)$ or $s \in (1,4/3)$ and $\alpha > 2$.

Roughly speaking, these results show that for a fixed level of s, the benefits of diversification tend to decrease as α decreases (i.e., as the severity tail becomes denser), whereas for a fixed level of α, the benefits of diversification tend to decrease as s increases (i.e., as the firm becomes more sensitive to risk). These results can be illustrated schematically using the diagram in Figure 11 in Chapter 2.

This paradigm is conservative in recognizing that (1) the benefits of diversification tend to increase over n, but that (2) for smaller values of α, these benefits actually can diminish over n if the value of s is sufficiently large. Since the expected frequency-by-expected severity plane is not sufficiently detailed to distinguish between smaller and larger values of α, it follows that a cautious firm should anticipate the possibility of more transfer and avoidance for very large values of n. Instructively, the apex of the pooling/transfer boundary (at $n = n^*$) must occur to the right of the apex of the transfer/avoidance boundary (at $n = n^*_{\mathrm{I}}$) because the value of the firm's parameter k is smaller than the insurance company's parameter k_{I}.

3.3. Erratic Frequencies

In this section, consider the risk associated with the stochastic frequency, N. Returning to the original model,

$$L = X_1 + . . . + X_N,$$

The following research studies the impact of the variability of N on the risk measure $R[L]$, and consequently on the pooling, transfer, and avoidance regions of the firm's risk finance paradigm.

A simple starting point is to let N be a Poisson random variable with mean $\lambda > 0$. Joining this assumption to Chapter 2's Pareto-calibrated Lévy-stable severities, it is possible to compute – by statistical simulation – pooling/transfer and transfer/avoidance boundary curves for values of the parameters α and s in each of the ranges associated with Type I, Type II, and Type III curves, respectively.

As shown in Figures 12, 13, and 14, these simulated boundaries are identical in shape (i.e., direction and curvature) to those computed analytically in Chapter 2 for non-stochastic n. Thus, although the actual numerical values of the simulated curves are somewhat lower than the corresponding values in the non-stochastic case (to account for the variability introduced by the random frequency), it is found that the Poisson distribution is sufficiently “well behaved” to maintain the same general paradigm in the expected frequency-by-expected severity plane.

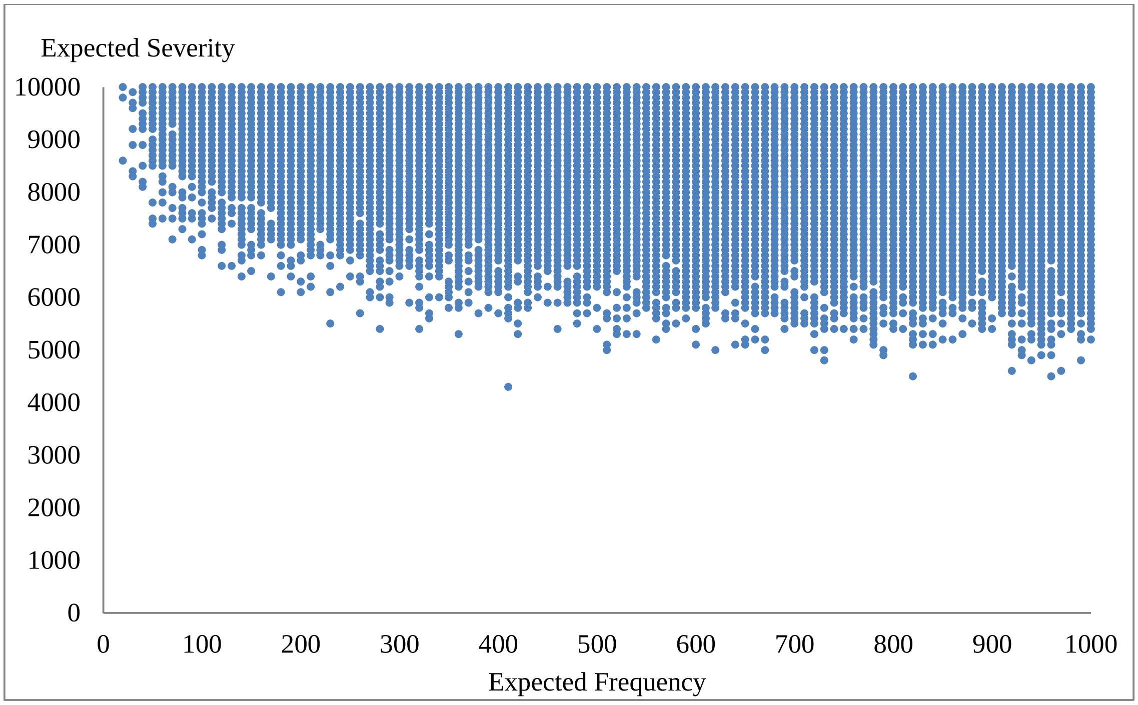
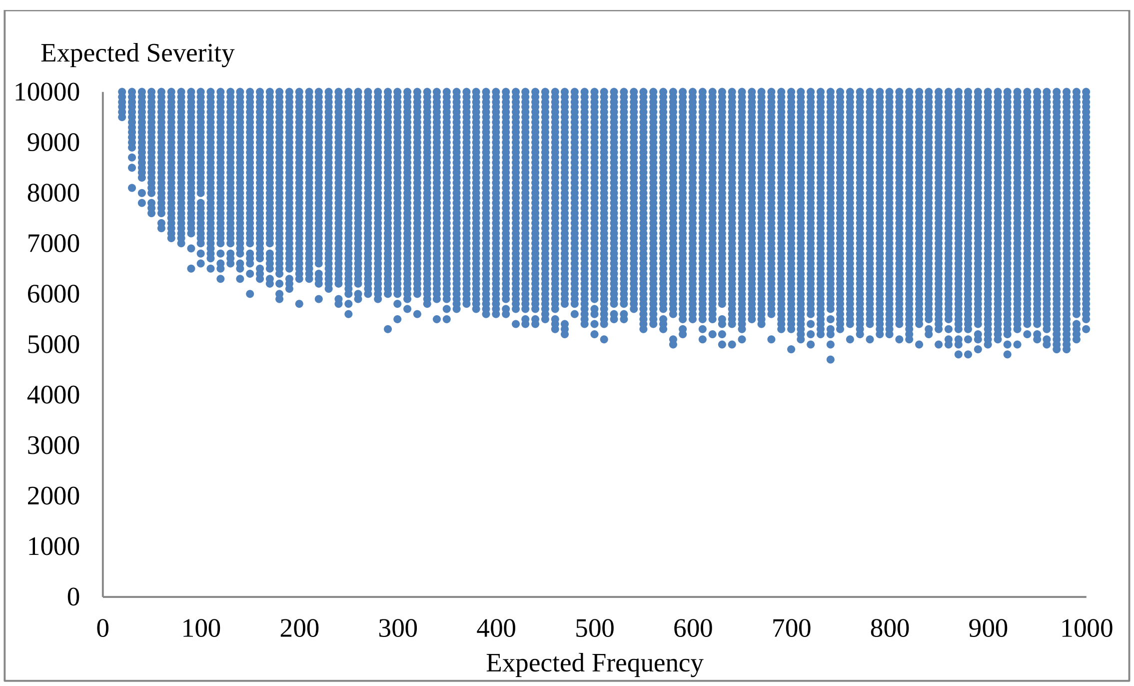


Figure 12. Type I Boundary Curves (Poisson N)

12a. Dense-Tailed Case
($\alpha = 1.5$, $s = 1.6$, $k = 215$)



12b. Light-Tailed Case
($\alpha = 2.5$, $s = 2.1$, $k = 30{,}000$)

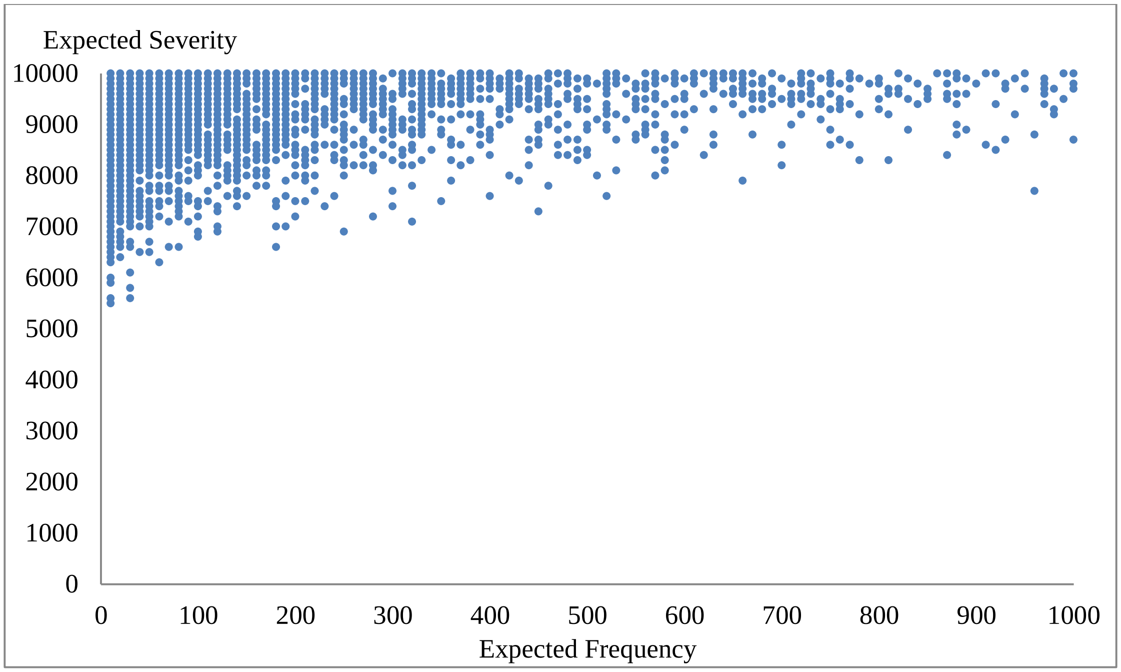
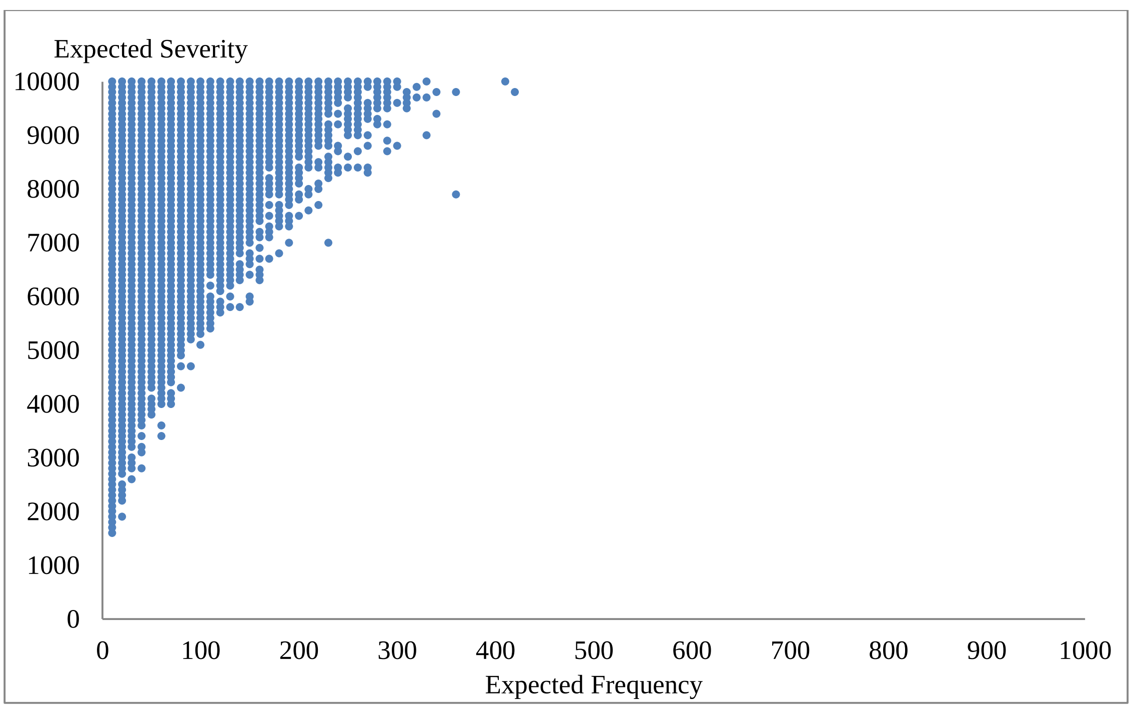


Figure 13. Type II Boundary Curves (Poisson N)

13a. Dense-Tailed Case
(α = 1.5, s = 1.4, k = 21)



13b. Light-Tailed Case
(α = 2.5, s = 1.4, k = 10)

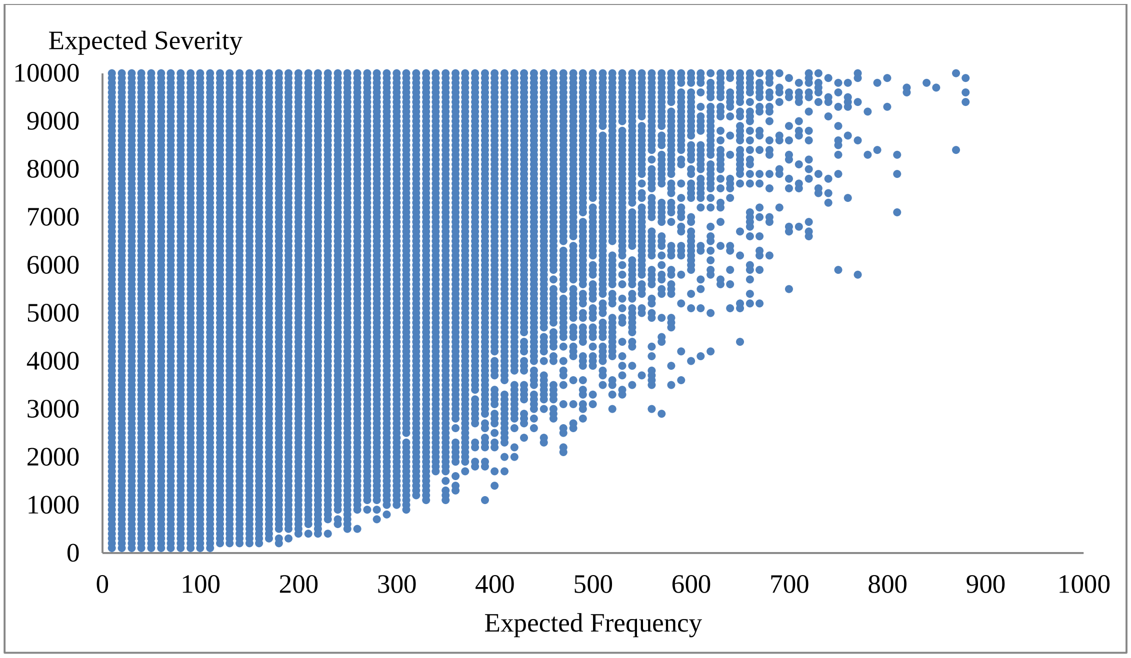
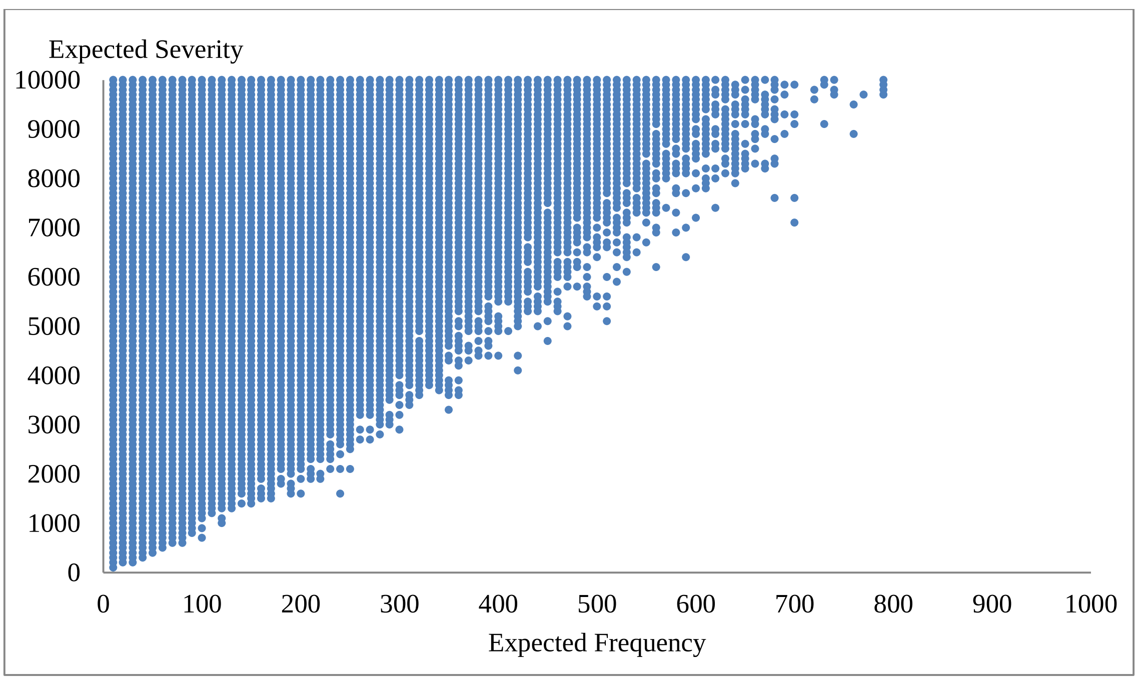


Figure 14. Type III Boundary Curves (Poisson N)

14a. Dense-Tailed Case
($\alpha = 1.5$, $s = 1.1$, $k = 0.38$)



14b. Light-Tailed Case
($\alpha = 2.5$, $s = 1.25$, $k = 1.2$)

This moderately surprising result leads to a fundamental question: Are there other, non-Poisson, frequency distributions that are sufficiently erratic to cause significant deviations in the shapes of the pooling/transfer and transfer/avoidance boundary curves? Interestingly, it is not difficult to show that this question may be answered in the affirmative, and that the coefficient of variation of N, $CV[N] = SD[N]/E[N]$, plays a critical role in determining just how "badly behaved" a particular distribution is.

Given that dense-tailed *frequency* distributions rarely appear in ERM applications, it is reasonable to assume that both the standard deviation and variance of N are finite. Making a similar assumption for the severities, X_i, $CBSD[L]$ can be replaced in the preceding expression for the risk measure $R[L]$, and work with the simpler quantity,

$$R[L] = (SD[L])^s/E[L].$$

Writing both $E[L]$ and $SD[L]$ in terms of the more fundamental parameters $E[N]$, $SD[N]$, $E[X_i]$, and $SD[X_i]$, it is fairly straightforward to show that, as the expected frequency, $E[N]$, increases to infinity, so does the quantity $R[L]$, *unless* the following three conditions hold: (1) $s \in (1,2]$; (2) $CV[N]$ converges to 0; and (3) $(SD[N])^s/E[N]$ converges to a finite number.

Having restricted attention to light-tailed severities, the three conditions in each of Figures 12b, 13b, and 14b can be checked, and verified that they are satisfied only in the last two (because $s = 2.1$ in 12b). Since a finite [infinite] value of $R[L]$ corresponds to an increasing [decreasing] boundary curve, it follows that the curves in 13b and 14b must be increasing, whereas the curve in 12b must be decreasing, exactly as plotted. Furthermore, it is trivial to show that conditions (1) through (3) hold for the parameter values of 13b and 14b, but not 12b, in the case of non-stochastic *n*, explaining why Poisson *N* and non-stochastic *n* yield the same results.

To identify a random frequency whose behavior deviates from the non-stochastic case requires that it is found that either $CV[N]$ converging to something greater than 0, or $(SD[N])^s/E[N]$ diverging to infinity. Since the first of these conditions implies the second, but the second does not imply the first, it follows that *N* must be chosen so that $CV[N] > 0$ in the limit. Given this level of erratic behavior, $R[L]$ must increase to infinity as does $E[N]$, and so there can be no Type II or Type III boundary curves.

3.4. Conclusions

From the preceding analyses, one can see that there are four distinct properties of a loss portfolio, *L*, that enhance the benefits of diversification: (1) a high expected frequency; and (2) less than perfect positive correlations between individual severities; (3) light-tailed severities; and (4) a predictable (i.e., non-erratic) frequency. A fifth consideration – the firm's sensitivity to risk, *s* – is also important, but is not a characteristic of the loss portfolio itself.

To capture all of these properties in a single risk finance paradigm, one must depart from the simple expected frequency-by-expected severity plane. As an alternative, the circular risk finance paradigm of Figure 15 is proposed, which divides the four separate properties above neatly into two frequency dimensions: Predictable $\rightarrow$ Erratic and High → Low; and two severity dimensions: Independent → Associated[16] and Light-tailed → Dense-tailed.

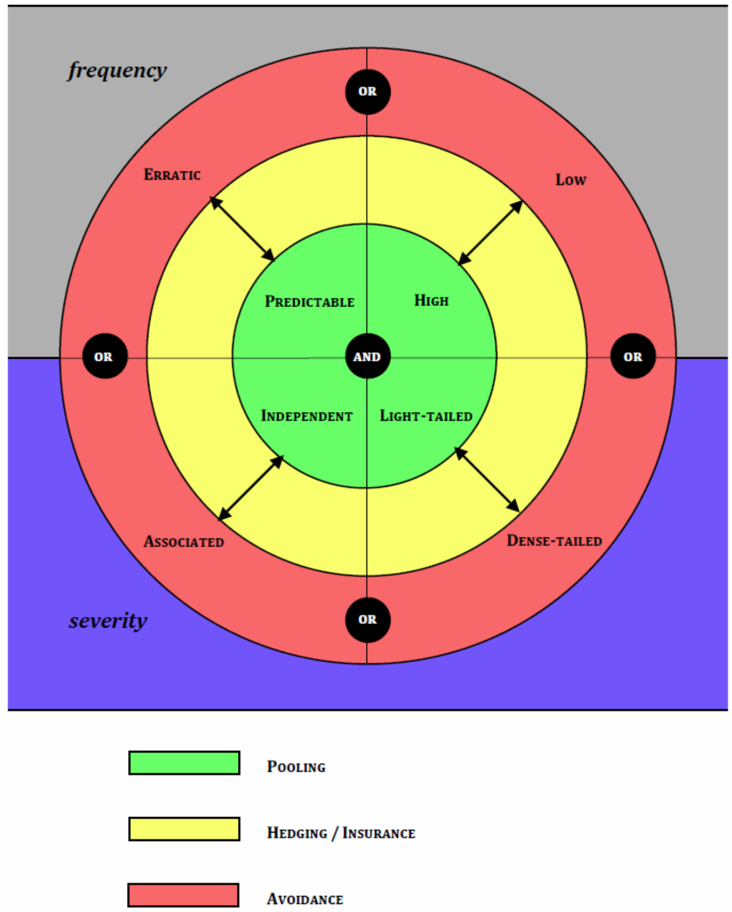


Figure 15. Risk-Based Risk Finance Paradigm

As can be seen from this figure, the loss portfolios that are most appropriately pooled are those with both predictable and high frequencies, as well as both independent and light-tailed severities, as indicated by the green zone at the center of this paradigm.

[16] *associated* rather than *positively correlated* is used as the antinome of *independent* because statistical correlations are not well defined for dense-tailed severities.

Any deviation from one or more of these four characteristics will make pooling less appropriate, and thus require hedging through insurance, as indicated by the yellow zone, or possibly even avoidance, as indicated by the worst-case red zone.

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APPENDICES

APPENDIX A

PROOF OF CHAPTER 1

Proof of Lemma 1

With compensation scheme:

$$\begin{cases} \max\limits_{\{q_L, q_H, \alpha\}} (1-\alpha)[\beta\rho_H(\lambda_H - \lambda_L)q_H + (\beta-1)\lambda_L(1-\rho_H)q_L + \rho_H(\beta\lambda_L - \lambda_H)] \\ q_L, q_H = \arg\max\limits_{\hat{q}_L, \hat{q}_H} \left\{ \alpha[\beta\rho_H(\lambda_H - \lambda_L)\hat{q}_H + (\beta-1)\lambda_L(1-\rho_H)\hat{q}_L + \rho_H(\beta\lambda_L - \lambda_H)] - c_L(1-\rho_H)^2\hat{q}_L{}^2 - c_H\rho_H{}^2\hat{q}_H{}^2 \right\} \\ \alpha[\beta\rho_H(\lambda_H - \lambda_L)\hat{q}_H + (\beta-1)\lambda_L(1-\rho_H)\hat{q}_L + \rho_H(\beta\lambda_L - \lambda_H)] - c_L(1-\rho_H)^2\hat{q}_L{}^2 - c_H\rho_H{}^2\hat{q}_H{}^2 \geq 0 \end{cases} \quad \text{(A1)}$$

Derive IC's by taking first order derivative of equation

$$q_L, q_H = \arg\max\limits_{\hat{q}_L, \hat{q}_H} \left\{ \alpha[\beta\rho_H(\lambda_H - \lambda_L)\hat{q}_H + (\beta-1)\lambda_L(1-\rho_H)\hat{q}_L + \rho_H(\beta\lambda_L - \lambda_H)] - c_L(1-\rho_H)^2\hat{q}_L{}^2 - c_H\rho_H{}^2\hat{q}_H{}^2 \right\} \quad \text{(A2)}$$

get

$$q_L = \begin{cases} \frac{1}{(1-\rho_H)}\left[\frac{\alpha(\beta-1)\lambda_L}{2c_L}\right] & \text{if} \quad \alpha < \frac{2c_L(1-\rho_H)}{\lambda_L(\beta-1)} \\ 1 & \text{if} \quad \alpha \geq \frac{2c_L(1-\rho_H)}{\lambda_L(\beta-1)} \end{cases} \quad \text{and} \quad q_H = \begin{cases} \frac{1}{\rho_H}\left[\frac{\alpha\beta(\lambda_H - \lambda_L)}{2c_H}\right] & \text{if} \quad \alpha < \frac{2c_H\rho_H}{\beta(\lambda_H - \lambda_L)} \\ 1 & \text{if} \quad \alpha \geq \frac{2c_H\rho_H}{\beta(\lambda_H - \lambda_L)} \end{cases} \quad \text{(A3)}$$

Taking second order derivative of q_L and q_H it can be obtained that:

$$\frac{\partial^2 IC}{\partial q_L^2} = -2c_L(1-\rho_H)^2 = A < 0; \quad \frac{\partial^2 IC}{\partial q_H^2} = -2c_H\rho_H{}^2 = C < 0; \quad \frac{\partial^2 IC}{\partial q_H \partial q_L} = B = 0$$

$$AC - B = 4c_L c_H \rho_H{}^2(1-\rho_H)^2 > 0 \quad \text{(A4)}$$

Therefore, $q_L = \frac{1}{(1-\rho_H)}\left[\frac{\alpha(\beta-1)\lambda_L}{2c_L}\right]$ and $q_H = \frac{1}{\rho_H}\left[\frac{\alpha\beta(\lambda_H - \lambda_L)}{2c_H}\right]$ are indeed maximum values.

Plug $q_L = \frac{1}{(1-\rho_H)}\left[\frac{\alpha(\beta-1)\lambda_L}{2c_L}\right]$ and $q_H = \frac{1}{\rho_H}\left[\frac{\alpha\beta(\lambda_H-\lambda_L)}{2c_H}\right]$ into equation

$$(1-\alpha)\left[\beta\rho_H(\lambda_H-\lambda_L)q_H + (\beta-1)\lambda_L(1-\rho_H)q_L + \rho_H(\beta\lambda_L-\lambda_H)\right] \tag{A5}$$

Virtual surplus can be described as the following maximization problem:

$$\max_{\{\alpha\}}(1-\alpha)\left[\beta^2(\lambda_H-\lambda_L)^2\frac{\alpha}{2c_H} + (\beta-1)^2\lambda_L{}^2\frac{\alpha}{2c_L} + \rho_H(\beta\lambda_L-\lambda_H)\right] \tag{A6}$$

Therefore equilibrium $\alpha*$ can be solve by taking first order derivative of the above equation as:

$$\alpha* = \frac{1}{2} + \frac{\rho_H(\lambda_H-\beta\lambda_L)}{[\beta(\lambda_H-\lambda_L)]^2\frac{1}{c_H} + [(\beta-1)\lambda_L]^2\frac{1}{c_L}} \tag{A7}$$

$\alpha*$ is indeed the maximum value since the second order derivative with respect to $\alpha*$ of equation

$$(1-\alpha)\left[\beta^2(\lambda_H-\lambda_L)^2\frac{\alpha}{2c_H} + (\beta-1)^2\lambda_L{}^2\frac{\alpha}{2c_L} + \rho_H(\beta\lambda_L-\lambda_H)\right]$$

(A8)

Is negative:

$$\frac{\partial^2}{\partial\alpha^2} = -\left[\beta^2(\lambda_H-\lambda_L)^2\frac{1}{c_H} + (\beta-1)^2\lambda_L{}^2\frac{1}{c_L}\right] \tag{A9}$$

Plug α* into $q_L = \frac{1}{(1-\rho_H)}\left[\frac{\alpha(\beta-1)\lambda_L}{2c_L}\right]$ and $\frac{1}{\rho_H}\left[\frac{\alpha\beta(\lambda_H-\lambda_L)}{2c_H}\right]$ equilibrium q_L^*, q_H^* can be obtained as:

$$q_L^* = \frac{(\beta-1)\beta^2\lambda_L(\lambda_H-\lambda_L)^2\frac{1}{c_H}+(\beta-1)^3\lambda_L{}^3\frac{1}{c_L}+2\rho_H(\beta-1)\lambda_L(\lambda_H-\beta\lambda_L)}{4c_L(1-\rho_H)\left[\beta^2(\lambda_H-\lambda_L)^2\frac{1}{c_H}+(\beta-1)^2\lambda_L{}^2\frac{1}{c_L}\right]} \quad \text{(A10)}$$

$$q_H^* = \frac{\beta^3(\lambda_H-\lambda_L)^3\frac{1}{c_H}+\beta(\beta-1)^2\lambda_L{}^2(\lambda_H-\lambda_L)\frac{1}{c_L}+2\rho_H\beta(\lambda_H-\beta\lambda_L)(\lambda_H-\lambda_L)}{4\rho_H c_H\left[\beta^2(\lambda_H-\lambda_L)^2\frac{1}{c_H}+(\beta-1)^2\lambda_L{}^2\frac{1}{c_L}\right]}$$

(A11)

Proof of proposition 1:

From equation (3) and equation (5), insurer's net profit when he is underwriting the business itself can be written as:

$$\beta\rho_H(\lambda_H-\lambda_L)q_H^I+(\beta-1)\lambda_L(1-\rho_H)q_L^I+\rho_H(\beta\lambda_L-\lambda_H)-c_L^I\left[(1-\rho_H)q_L^I\right]^2-c_H^I(\rho_H q_H^I)^2 \quad \text{(A12)}$$

So the insurer's problem becomes:

$$q_L^I, q_H^I = \underset{\hat{q}_L^I,\hat{q}_H^I}{\arg\max}\left\{\beta\rho_H(\lambda_H-\lambda_L)\hat{q}_H^I+(\beta-1)\lambda_L(1-\rho_H)\hat{q}_L^I+\rho_H(\beta\lambda_L-\lambda_H)-c_L^I\left[(1-\rho_H)\hat{q}_L^I\right]^2-c_H^I(\rho_H\hat{q}_H^I)^2\right\} \quad \text{(A13)}$$

Taking first order differentiation with respect to $\hat{q}_L^I$ and $\hat{q}_H^I$ the optimal value of q_L^I and q_H^I can be obtained as:

$$q_L^I = \frac{1}{(1-\rho_H)}\left[\frac{(\beta-1)\lambda_L}{2c_L^I}\right] \text{ and } q_H^I = \frac{1}{\rho_H}\left[\frac{\beta(\lambda_H-\lambda_L)}{2c_H^I}\right] \quad \text{(A14)}$$

It can be shown that q_H^I and q_L^I are indeed the maximum value, following same procedure as in Lemma 1.

Plug q_H^I and q_L^I back to (A12), insurer's net profit when he underwrites by himself can be obtained as:

$$\beta^2(\lambda_H-\lambda_L)^2\frac{1}{4c_H^I}+(\beta-1)^2\lambda_L{}^2\frac{1}{4c_L^I}-\rho_H(\lambda_H-\beta\lambda_L) \tag{A15}$$

Therefore, the condition when insurer should underwrite by himself and has positive net profit is:

$$\beta^2(\lambda_H-\lambda_L)^2\frac{1}{4c_H^I}+(\beta-1)^2\lambda_L{}^2\frac{1}{4c_L^I}-\rho_H(\lambda_H-\beta\lambda_L)\geq 0 \tag{A16}$$

Proof of Lemma 2:

Insurer's net profit when uses independent underwriter can be obtained by plugging equilibrium $\alpha *$ into equation (A6)

$$(1-\alpha)\left[\beta^2(\lambda_H-\lambda_L)^2\frac{\alpha}{2c_H}+(\beta-1)^2\lambda_L{}^2\frac{\alpha}{2c_L}+\rho_H(\beta\lambda_L-\lambda_H)\right] \text{ as:}$$

$$\frac{1}{2}\left\{\beta^2(\lambda_H-\lambda_L)^2\frac{1}{4c_H}+(\beta-1)^2\lambda_L{}^2\frac{1}{4c_L}-\rho_H(\lambda_H-\beta\lambda_L)+\frac{\rho_H{}^2(\lambda_H-\beta\lambda_L)^2}{\beta^2(\lambda_H-\lambda_L)^2\frac{1}{c_H}+(\beta-1)^2\lambda_L{}^2\frac{1}{c_L}}\right\} \tag{A17}$$

Therefore, the benefit from using independent underwriter can be represented by the difference between (A17) and (A15)

$$\begin{aligned}&\textit{Profit(using independent underwriter)-Profit(underwriting himself)} =\\&\frac{1}{2}\left\{\beta^2(\lambda_H-\lambda_L)^2\frac{1}{4c_H}+(\beta-1)^2\lambda_L{}^2\frac{1}{4c_L}-\rho_H(\lambda_H-\beta\lambda_L)+\frac{\rho_H{}^2(\lambda_H-\beta\lambda_L)^2}{\beta^2(\lambda_H-\lambda_L)^2\frac{1}{c_H}+(\beta-1)^2\lambda_L{}^2\frac{1}{c_L}}\right\}\\&-\left[\beta^2(\lambda_H-\lambda_L)^2\frac{1}{4c_H^I}+(\beta-1)^2\lambda_L{}^2\frac{1}{4c_L^I}-\rho_H(\lambda_H-\beta\lambda_L)\right]\end{aligned} \tag{A18}$$

Rewrite (A18), the Profit Margin can be written as a cleaner format:

$$\text{Profit Margin}=\frac{1}{4}\left\{\beta^2(\lambda_H-\lambda_L)^2(\frac{1}{2c_H}-\frac{1}{c_H^I})+(\beta-1)^2\lambda_L{}^2(\frac{1}{2c_L}-\frac{1}{c_L^I})+2\rho_H(\lambda_H-\beta\lambda_L)+\frac{\rho_H{}^2(\lambda_H-\beta\lambda_L)^2}{\beta^2(\lambda_H-\lambda_L)^2\frac{1}{2c_H}+(\beta-1)^2\lambda_L{}^2\frac{1}{2c_L}}\right\}$$

(A19)

Proof of Proposition 2

Take first order derivative with respect to c_H^I and c_L^I about profit margin (A19), it can be obtained that:

$$\frac{\partial \text{Profit Margin}}{\partial c_H^I} = \frac{1}{4}\beta^2(\lambda_H - \lambda_L)^2(\frac{1}{{c_H^I}^2}) > 0 \qquad \text{(A20)}$$

$$\frac{\partial \text{Profit Margin}}{\partial c_L^I} = \frac{1}{4}(\beta-1)^2 {\lambda_L}^2 \frac{1}{{c_L^I}^2} > 0 \qquad \text{(A21)}$$

Take first order derivative with respect to c_H about profit margin (A19), it can be obtained that:

$$\frac{\partial \text{Profit Margin}}{\partial c_H} = \frac{1}{4}\beta^2(\lambda_H - \lambda_L)^2 \frac{1}{2{c_H}^2}\left\{\frac{{\rho_H}^2(\lambda_H - \beta\lambda_L)^2 - [\beta^2(\lambda_H-\lambda_L)^2 \frac{1}{2c_H} + (\beta-1)^2{\lambda_L}^2\frac{1}{2c_L}]^2}{[\beta^2(\lambda_H-\lambda_L)^2 \frac{1}{2c_H} + (\beta-1)^2{\lambda_L}^2\frac{1}{2c_L}]^2}\right\} \qquad \text{(A22)}$$

Since the optimal commissions loading

$$\alpha^* = 0.5 + [\rho_H(\lambda_H - \beta\lambda_L)]/[\beta^2(\lambda_H-\lambda_L)^2/c_H + (\beta-1)^2{\lambda_L}^2/c_L] \qquad \text{(A7)}$$

It follows that $[\beta^2(\lambda_H-\lambda_L)^2 \frac{1}{2c_H} + (\beta-1)^2{\lambda_L}^2\frac{1}{2c_L}]^2 = \frac{{\rho_H}^2(\lambda_H-\beta\lambda_L)^2}{(2\alpha^*-1)^2}$ (A23)

Therefore, substituting $[\beta^2(\lambda_H-\lambda_L)^2 \frac{1}{2c_H} + (\beta-1)^2{\lambda_L}^2\frac{1}{2c_L}]^2$ with $\frac{{\rho_H}^2(\lambda_H-\beta\lambda_L)^2}{(2\alpha^*-1)^2}$,

$$\frac{\partial \text{Profit Margin}}{\partial c_H} = \frac{\frac{1}{4}\beta^2(\lambda_H-\lambda_L)^2 \frac{1}{2{c_H}^2}{\rho_H}^2(\lambda_H-\beta\lambda_L)^2}{[\beta^2(\lambda_H-\lambda_L)^2 \frac{1}{2c_H} + (\beta-1)^2{\lambda_L}^2\frac{1}{2c_L}]^2}\frac{4\alpha^*(\alpha^*-1)}{(2\alpha^*-1)^2} \qquad \text{(A24)}$$

All terms are non-negative except for $4\alpha^*(\alpha^*-1)$. Since the commissions loading α^* is strictly less than 1, $4\alpha^*(\alpha^*-1)$ should be strictly negative.

Therefore, $\frac{\partial \text{Profit Margin}}{\partial c_H} < 0$

Proof of $\frac{\partial \text{Profit Margin}}{\partial c_L} < 0$ follows the same procedure.

Proof of Proposition 3

Take first order derivative with respect to ρ_H of equation (A19):

$$\frac{\partial \text{Profit Margin}}{\partial \rho_H} = \frac{1}{2}(\lambda_H - \beta\lambda_L) + \frac{\rho_H(\lambda_H - \beta\lambda_L)^2}{\beta^2(\lambda_H - \lambda_L)^2 \frac{1}{c_H} + (\beta-1)^2 \lambda_L{}^2 \frac{1}{c_L}} \tag{A25}$$

Since the optimal commissions loading

$$\alpha^* = 0.5 + [\rho_H(\lambda_H - \beta\lambda_L)]/[\beta^2(\lambda_H - \lambda_L)^2 / c_H + (\beta-1)^2 \lambda_L{}^2 / c_L] \tag{A7}$$

It follows that $$\beta^2(\lambda_H - \lambda_L)^2 \frac{1}{c_H} + (\beta-1)^2 \lambda_L{}^2 \frac{1}{c_L} = \frac{\rho_H(\lambda_H - \beta\lambda_L)}{(\alpha^* - \frac{1}{2})} \tag{A26}$$

Therefore, substituting $\beta^2(\lambda_H - \lambda_L)^2 \frac{1}{c_H} + (\beta-1)^2 \lambda_L{}^2 \frac{1}{c_L}$ with $\frac{\rho_H(\lambda_H - \beta\lambda_L)}{(\alpha^* - \frac{1}{2})}$,

$\frac{\partial \text{Profit Margin}}{\partial \rho_H} = \alpha^*(\lambda_H - \beta\lambda_L) > 0$ when $\lambda_H - \beta\lambda_L > 0$

Proof of Proposition 4

Take first order derivative with respect to $(\lambda_H - \beta\lambda_L)$ on (A19) get:

$$\frac{\partial \text{Profit Margin}}{\partial(\lambda_H - \beta\lambda_L)} = \frac{1}{2}\rho_H + \frac{\rho_H{}^2(\lambda_H - \beta\lambda_L)}{\beta^2(\lambda_H - \lambda_L)^2 \frac{1}{c_H} + (\beta-1)^2 \lambda_L{}^2 \frac{1}{c_L}} \tag{A27}$$

Plug in (A26), it follows that $\frac{\partial \text{Profit Margin}}{\partial(\lambda_H - \beta\lambda_L)} = \alpha^* \rho_H$

Proof of Proposition 5

Assume insurer has the same cost function c_H and c_L as the independent underwriter, then the profit margin (A19) can be rewritten as:

$$\text{Profit Margin} = \frac{1}{4}\left\{ -\beta^2(\lambda_H - \lambda_L)^2 \frac{1}{2c_H} - (\beta-1)^2 \lambda_L{}^2 \frac{1}{2c_L} + 2\rho_H(\lambda_H - \beta\lambda_L) + \frac{\rho_H{}^2(\lambda_H - \beta\lambda_L)^2}{\beta^2(\lambda_H - \lambda_L)^2 \frac{1}{c_H} + (\beta-1)^2 \lambda_L{}^2 \frac{1}{c_L}} \right\} \quad \text{(A28)}$$

Following proposition 1, if $\rho_H(\lambda_H - \beta\lambda_L) > \beta^2(\lambda_H - \lambda_L)^2 / 4c_H + (\beta-1)^2 \lambda_L{}^2 / 4c_L$, it is unprofitable for insurer to underwrite by himself.

Therefore, plug $\rho_H(\lambda_H - \beta\lambda_L) = \beta^2(\lambda_H - \lambda_L)^2 / 4c_H + (\beta-1)^2 \lambda_L{}^2 / 4c_L$ into the profit margin given by (A28), after some algebra, the lower boundary of the profit margin can be written as:

$$\text{Profit Margin} > \frac{1}{32}\left\{ \beta^2(\lambda_H - \lambda_L)^2 \frac{1}{c_H} + (\beta-1)^2 \lambda_L{}^2 \frac{1}{c_L} \right\} > 0 \quad \text{(A29)}$$

Proof of proposition 6

If the profit margin of using independent underwriter (assuming the insurer and the independent underwriter have the same underwriting costs) is negative, then the insurer will not use independent underwriter. Otherwise, the insurer will choose to use independent underwriter.

Substituting $\beta^2(\lambda_H - \lambda_L)^2 / c_H + (\beta-1)^2 \lambda_L{}^2 / c_L$ with $\rho_H(\lambda_H - \beta\lambda_L)/(\alpha^* - 0.5)$ in (A28), it follows that

$$\begin{aligned} \text{Profit Margin} = \frac{1}{4}\rho_H(\lambda_H - \beta\lambda_L)\frac{\left[2(\alpha^* - 0.5) - 1\right]^2}{(\alpha^* - 0.5)} &> 0 \quad \textit{iff} \quad (\lambda_H - \beta\lambda_L)(\alpha^* - 0.5) > 0 \\ &< 0 \quad \textit{iff} \quad (\lambda_H - \beta\lambda_L)(\alpha^* - 0.5) < 0 \end{aligned} \quad \text{(A30)}$$

Proof of Proposition 7

[IR] constraint is:

$$\alpha\left[\beta\rho_H(\lambda_H - \lambda_L)\hat{q}_H + (\beta-1)\lambda_L(1-\rho_H)\hat{q}_L + \rho_H(\beta\lambda_L - \lambda_H)\right] - c_L(1-\rho_H)^2 \hat{q}_L{}^2 - c_H \rho_H{}^2 \hat{q}_H{}^2 \geq 0 \quad \text{(A31)}$$

Plug in $q_L = [\alpha(\beta-1)\lambda_L]/[2c_L(1-\rho_H)]$, and $q_H = [\alpha\beta(\lambda_H - \lambda_L)]/[2c_H / \rho_H]$, [$IR$] constraint can be re-written as:

$$[IR] = \frac{1}{4}\alpha^2\left[\beta^2(\lambda_H - \lambda_L)^2 \frac{1}{c_H} + (\beta-1)^2 \lambda_L{}^2 \frac{1}{c_L}\right] - \alpha\rho_H(\lambda_H - \beta\lambda_L) \geq 0 \qquad \text{(A32)}$$

Replacing relative terms with (A26), [*IR*] constraint can be further simplified as:

$$[IR] = \alpha\rho_H(\lambda_H - \beta\lambda_L)\left(\frac{2-3\alpha}{4\alpha-2}\right) \geq 0 \qquad \text{(A33)}$$

It then follows that if risk factor $(\lambda_H - \beta\lambda_L) \geq 0$, then $[IR] \geq 0$ iff $(2-3\alpha)(4\alpha-2) \geq 0$; if risk factor $(\lambda_H - \beta\lambda_L) < 0$, then $[IR] \geq 0$ iff $(2-3\alpha)(4\alpha-2) \leq 0$.

Therefore, [*IR*] constraint can be written as:

$$\begin{cases} (\lambda_H - \beta\lambda_L) \geq 0 \\ 1/2 \leq \alpha \leq 2/3 \end{cases} \text{or} \quad \begin{cases} (\lambda_H - \beta\lambda_L) < 0 \\ \alpha < 1/2 \quad or \quad \alpha > 2/3 \end{cases}$$

(A34)

Proof of Proposition 8

It follows directly by taking differentiation of q_L^* (A10) and q_H^* (A11), with respect to $\rho_H(\lambda_H - \beta\lambda_L)$.

Proof of Proposition 9

It follows directly from equation (A7)

Proof of Proposition 10

It follows directly by differentiating α^* at (A7) with respect to $(\lambda_H - \beta\lambda_L)$.

Proof of Proposition 11

$$\frac{\partial\alpha^*}{\partial c_H} = \rho_H(\lambda_H - \beta\lambda_L)\frac{\beta^2(\lambda_H - \lambda_L)^2 \frac{1}{c_H{}^2}}{\left\{[\beta(\lambda_H - \lambda_L)]^2 \frac{1}{c_H} + [(\beta-1)\lambda_L]^2 \frac{1}{c_L}\right\}^2} \qquad \text{(A35)}$$

Therefore, when $\lambda_H - \beta\lambda_L \geq 0$, $\partial\alpha^* / \partial c_H > 0$; otherwise $\partial\alpha^* / \partial c_H < 0$

$$\frac{\partial \alpha^*}{\partial c_L} = \rho_H(\lambda_H - \beta\lambda_L)\frac{(\beta-1)^2 \lambda_L{}^2 \dfrac{1}{c_L{}^2}}{\left\{[\beta(\lambda_H - \lambda_L)]^2 \dfrac{1}{c_H} + [(\beta-1)\lambda_L]^2 \dfrac{1}{c_L}\right\}^2} \qquad \text{(A36)}$$

Therefore, when $\lambda_H - \beta\lambda_L > 0$, $\partial\alpha^* / \partial c_L > 0$; otherwise, $\partial\alpha^* / \partial c_L < 0$

Proof of Proposition 12

Insurer's expected payoff=Pr (win)×(1- α)×Profit+Pr (loss)×0= α= α(1 −α)×Profit (A37)

Take first order derivative with respect to α:

FOC (α): 1-2 α =0; (A38)

SOC (α): -2< 0; so α =1/2 is indeed maximizes the function. (A39)

Therefore, insurer's best response bid will be α =1/2.

Proof of Lemma 3

For each period t>=2, independent underwriter's optimum underwriting accuracy complies with (α=1/2):

$$q_{Lt}, q_{Ht} = \underset{\hat{q}_{Lt},\hat{q}_{Ht}}{\arg\max}\left\{\frac{1}{2}[\beta\rho_H(\lambda_H - \lambda_L)\hat{q}_{Ht} + (\beta-1)\lambda_L(1-\rho_H)\hat{q}_{Lt} + \rho_H(\beta\lambda_L - \lambda_H)] - c_{Lt}(1-\rho_H)^2\hat{q}_{Lt}{}^2 - c_{Ht}\rho_H{}^2\hat{q}_{Ht}{}^2\right\}$$

(A40)

Take first order derivative with respect to $\hat{q}_{Lt}$ and $\hat{q}_{Ht}$ it can be obtained that:

$$\hat{q}_{Lt} = \frac{(\beta-1)\lambda_L}{4c_{Lt}(1-\rho_H)}; \qquad \text{and} \qquad \hat{q}_{Ht} = \frac{\beta(\lambda_H - \lambda_L)}{4\rho_H c_{Ht}}$$

(A41)

Take second order derivative with respect to $\hat{q}_{Lt}$ and $\hat{q}_{Ht}$ get:

$$\frac{\partial^2 IC}{\partial \hat{q}_{Lt}^2} = -2c_{Lt}(1-\rho_H)^2 = A < 0 \qquad \frac{\partial^2 IC}{\partial \hat{q}_{Ht}^2} = -2c_{Ht}\rho_H{}^2 = C < 0 \qquad \frac{\partial^2 IC}{\partial \hat{q}_{Ht}\partial \hat{q}_{Lt}} = B = 0 \qquad \text{(A42)}$$

$$AC - B = 4c_{Lt}c_{Ht}\rho_H{}^2(1-\rho_H)^2 > 0 \qquad \text{(A43)}$$

Therefore, $\hat{q}_{Lt}$ and $\hat{q}_{Ht}$ indeed maximize (A40).

Plug $\hat{q}_{Lt}$ and $\hat{q}_{Ht}$ into insurer's payoff and following some algebra, insurer's payoff for each period t>=2 can be written as:

$$inusrer's\ payoff_t = \frac{1}{2}[\beta^2(\lambda_H-\lambda_L)^2\frac{1}{4c_{tH}}+(\beta-1)^2\lambda_L{}^2\frac{1}{4c_{tL}}-\rho_H(\lambda_H-\beta\lambda_L)] \qquad \text{(A44)}$$

Therefore, insurer's total payoff when he uses independent underwriter is

$$Insurer's\ Total\ Profit_{use\ independent\ underwriter}$$
$$=\frac{1}{2}\sum_{t=1}^{T}\{(\frac{1}{1+i})^{t-1}[\beta^2(\lambda_H-\lambda_L)^2\frac{1}{4c_{Ht}}+(\beta-1)^2\lambda_L{}^2\frac{1}{4c_{Lt}}-\rho_H(\lambda_H-\beta\lambda_L)]\}+\frac{1}{2}\frac{\rho_H{}^2(\lambda_H-\beta\lambda_L)^2}{\beta^2(\lambda_H-\lambda_L)^2\frac{1}{c_{H1}}+(\beta-1)^2\lambda_L{}^2\frac{1}{c_{L1}}}$$

(A45)

Insurer's total payoff when he underwriters by himself can be written as:

$$Insurer's\ Total\ Profit_{use\,direct\,writing}=\sum_{t=1}^{T}(\frac{1}{1+i})^{t-1}[\beta^2(\lambda_H-\lambda_L)^2\frac{1}{4c_{Ht}^I}+(\beta-1)^2\lambda_L{}^2\frac{1}{4c_{Lt}^I}-\rho_H(\lambda_H-\beta\lambda_L)] \qquad \text{(A46)}$$

Subtracting insurer's total profit from underwriting by himself from insurer's total profit from using independent underwriter, total profit margin from using independent underwriter can be written as:

$$Total\ Profit\ Margin = Total\ Profit_{use\,independent\,underwriter} - Total\ Profit_{use\,direct\,writing}$$
$$=\frac{1}{4}\sum_{t=1}^{T}(\frac{1}{1+i})^{t-1}[\beta^2(\lambda_H-\lambda_L)^2(\frac{1}{2c_{Ht}}-\frac{1}{c_{Ht}^I})+(\beta-1)^2\lambda_L{}^2(\frac{1}{2c_{Lt}}-\frac{1}{c_{Lt}^I})+2\rho_H(\lambda_H-\beta\lambda_L)]+\frac{\rho_H{}^2(\lambda_H-\beta\lambda_L)^2}{\beta^2(\lambda_H-\lambda_L)^2\frac{2}{c_{H1}}+(\beta-1)^2\lambda_L{}^2\frac{2}{c_{L1}}}$$

(A47)

Proof of Proposition 13

$$\frac{\partial\ Total\ Profit\ Margin}{\partial t}=\frac{1}{4}\ln(\frac{1}{1+i})\sum_{t=1}^{T}(\frac{1}{1+i})^{t-1}[\beta^2(\lambda_H-\lambda_L)^2(\frac{1}{2c_{Ht}}-\frac{1}{c_{Ht}^I})+(\beta-1)^2\lambda_L{}^2(\frac{1}{2c_{Lt}}-\frac{1}{c_{Lt}^I})+2\rho_H(\lambda_H-\beta\lambda_L)] \qquad \text{(A48)}$$

Total profit margin will decrease with contract period t if and only if when the following term is positive:

$$\sum_{t=1}^{T}(\frac{1}{1+i})^{t-1}[\beta^2(\lambda_H-\lambda_L)^2(\frac{1}{2c_{Ht}}-\frac{1}{c_{Ht}^I})+(\beta-1)^2\lambda_L{}^2(\frac{1}{2c_{Lt}}-\frac{1}{c_{Lt}^I})+2\rho_H(\lambda_H-\beta\lambda_L)] \quad \text{(A49)}$$

Since the total profit margin (A47) can be rewritten as:

$$(A49)+\frac{1}{2}\frac{\rho_H{}^2(\lambda_H-\beta\lambda_L)^2}{\beta^2(\lambda_H-\lambda_L)^2\frac{1}{c_{H1}}+(\beta-1)^2\lambda_L{}^2\frac{1}{c_{L1}}} \quad \text{(A50)}$$

The second term of (A50) comes from period 1, so the following substitution can be made:

$$\beta^2(\lambda_H-\lambda_L)^2\frac{1}{c_{H1}}+(\beta-1)^2\lambda_L{}^2\frac{1}{c_{L1}}=\frac{\rho_H(\lambda_H-\beta\lambda_L)}{(\alpha^*-\frac{1}{2})} \quad \text{(A26)}$$

It then follows that:

$$\begin{aligned}&\textit{Total Profit Margin}=(A49)+\frac{1}{2}\rho_H(\lambda_H-\beta\lambda_L)(\alpha^*-\frac{1}{2})>0\\&\Leftrightarrow(A49)>-\frac{1}{2}\rho_H(\lambda_H-\beta\lambda_L)(\alpha^*-\frac{1}{2})\end{aligned} \quad \text{(A51)}$$

From [*IR*] (A34):

$$\begin{cases}(\lambda_H-\beta\lambda_L)>0\\1/2<\alpha\le 2/3\end{cases}\text{ or }\begin{cases}(\lambda_H-\beta\lambda_L)<0\\\alpha<1/2\quad or\quad \alpha\ge 2/3\end{cases} \quad \text{(A34)}$$

So when $\lambda_H-\beta\lambda_L>0$, [*IR*] requires:

$$1/2<\alpha\le 2/3\Leftrightarrow-\frac{1}{12}\rho_H(\lambda_H-\beta\lambda_L)\le-\frac{1}{2}\rho_H(\lambda_H-\beta\lambda_L)(\alpha^*-\frac{1}{2})<0 \quad \text{(A52)}$$

So for (A51) to hold, it requires (A49)>0

When $\lambda_H-\beta\lambda_L<0$, [*IR*] requires:

$$\begin{cases}\alpha<1/2\\or\\\alpha\ge 2/3\end{cases}\Leftrightarrow\begin{cases}-1/2\rho_H(\lambda_H-\beta\lambda_L)(\alpha^*-1/2)<0\\or\\-1/2\rho_H(\lambda_H-\beta\lambda_L)(\alpha^*-1/2)\ge-1/12\rho_H(\lambda_H-\beta\lambda_L)\end{cases} \quad \text{(A53)}$$

Since $\lambda_H - \beta\lambda_L < 0$ in this case, $-1/12\rho_H(\lambda_H - \beta\lambda_L) > 0$.

For (A51) to hold, it requires $(A49) > -1/12\rho_H(\lambda_H - \beta\lambda_L) > 0$

For both cases, (A49) need to be positive. Therefore, (A48) should be negative.

Therefore, total profit margin should be negatively related with the length of contract period t.

Proof of Proposition 14

$$\frac{\partial Total\ Profit\ Margin}{\partial(1/1+i)} = \frac{1}{4}\sum_{t=2}^{T}(\frac{1}{1+i})^{t-2}(t-1)[\beta^2(\lambda_H - \lambda_L)^2(\frac{1}{2c_{Ht}} - \frac{1}{c_{Ht}^I}) + (\beta-1)^2\lambda_L{}^2(\frac{1}{2c_{Lt}} - \frac{1}{c_{Lt}^I}) + 2\rho_H(\lambda_H - \beta\lambda_L)] \quad \text{(A54)}$$

For multi-period contract, $(t-1) \geq 0 \quad because \quad t \geq 1$

Therefore, $\frac{\partial Total\ Profit\ Margin}{\partial(1/1+i)} < 0$ because the term (A49) is positive.

APPENDIX B

PROOF OF CHAPTER 2

PART 1: DERIVATION OF CONDITIONS (45) AND (46):

From Equation (39), set $E[X] = k^{-1/(1-p)}2^{p/a(1-p)}(a-1)^{p/(1-p)}n^{(p-a)/a(1-p)}$ for $a \in (1, 2)$. It then follows that:

$$\frac{\partial}{\partial p}[k^{-1/(1-p)}2^{p/a(1-p)}(a-1)^{p/(1-p)}n^{(p-a)/a(1-p)}]$$

$$= \frac{\partial}{\partial p}\{k^{-1/(1-p)}[2^{1/a}(a-1)]^{p/(1-p)}n^{(p-a)/a(1-p)}\}$$

$$= \frac{\partial}{\partial p}[k^{-1/(1-p)}][2^{1/a}(a-1)]^{p/(1-p)}n^{(p-a)/a(1-p)}$$

$$+k^{-1/(1-p)}\frac{\partial}{\partial p}\{[2^{1/a}(a-1)]^{p/(1-p)}\}n^{(p-a)/a(1-p)}$$

$$+k^{-1/(1-p)}[2^{1/a}(a-1)]^{p/(1-p)}\frac{\partial}{\partial p}[n^{(p-a)/a(1-p)}]$$

$$= k^{-1/(1-p)}\left[-\frac{ln(k)}{(1-p)^2}\right] \times [2^{1/a}(a-1)]^{p/(1-p)}n^{(p-a)/a(1-p)}$$

$$+k^{-1/(1-p)}[2^{1/a}(a-1)]^{p/(1-p)} \times \left[\frac{ln\left(2^{1/a}(a-1)\right)}{(1-p)^2}\right]n^{(p-a)/a(1-p)}$$

$$+k^{-1/(1-p)}[2^{1/a}(a-1)]^{p/(1-p)}n^{(p-a)/a(1-p)} \times \left[-\frac{ln(n)(a-1))}{(1-p)^2 a}\right]$$

$$= \left[\frac{k^{-1/(1-p)}}{(1-p)^2}\right][2^{1/a}(a-1)]^{p/(1-p)}n^{(p-a)/a(1-p)}$$

$\times\, ln\left(2^{1/a}(a-1)/[kn^{(a-1)/a}]\right),$

which is negative for $n > [2^{1/a}(a-1)/k]^{a/(a-1)}$ and positive for $n <$ $[2^{1/a}(a-1)/k]^{a/(a-1)}$

PART 2: DERIVATION OF CONDITIONS (47) AND (48):

From Equation (39), set $E[X] = k^{-1/(1-p)}(a-2)^{-p/2(1-p)}n^{(p-2)/2(1-p)}$ for $a >$

2.

It then follows that:

$$\frac{\partial}{\partial p}[k^{-1/(1-p)}(a-2)^{-p/2(1-p)}n^{(p-2)/2(1-p)}]$$

$$= \frac{\partial}{\partial p}[k^{-1/(1-p)}](a-2)^{-p/2(1-p)}n^{(p-2)/2(1-p)}$$

$$+k^{-1/(1-p)}\frac{\partial}{\partial p}[(a-2)^{-p/2(1-p)}]n^{(p-2)/2(1-p)}$$

$$+k^{-1/(1-p)}(a-2)^{-p/2(1-p)}\frac{\partial}{\partial p}[n^{(p-2)/2(1-p)}]$$

$$= k^{-1/(1-p)}\left[-\frac{ln(k)}{(1-p)^2}\right](a-2)^{-p/2(1-p)}n^{(p-2)/2(1-p)}$$

$$+k^{-1/(1-p)}(a-2)^{-p/2(1-p)}\left[-\frac{ln(a-2)}{2(1-p)^2}\right]n^{(p-2)/2(1-p)}$$

$$+k^{-1/(1-p)}(a-2)^{-p/2(1-p)}n^{(p-2)/2(1-p)}\left[-\frac{ln(n)}{2(1-p)^2}\right]$$

$$= \left[\frac{k^{-1/(1-p)}}{(1-p)^2}\right](a-2)^{-p/2(1-p)}n^{(p-2)/2(1-p)}$$

$$\times\, ln\left(1/[k(a-2)^{1/2}n^{1/2}]\right)$$

which is negative for $n > 1/[k^2(a-2)]$ and positive for $n < 1/[k^2(a-2)]$