CLASSROOM PEER EFFECTS, EFFORT, AND RACE

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by
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ABSTRACT

CLASSROOM PEER EFFECTS, EFFORT, AND RACE

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DOCTOR OF PHILOSOPHY

Temple University, August, 2010

Professor Dimitrios Diamantaras, Chair

This dissertation develops a theoretical model of educational peer effects and then empirically tests whether or not they exist. In the theoretical model, each student selects an effort level to maximize utility; this effort choice depends on his peer group's effort and race. The students' equilibrium effort expression results in hypotheses that can be directly investigated empirically, a definition of the social multiplier, and conditions under which a social multiplier exists. The empirical model uses student-level data with observations on complete classrooms and two measures of effort, self-assessed effort and time spent studying, to investigate whether or not peer effects exist. The estimation results of the empirical model, interpreted using a simulation-based technique, find a positive relationship between the amount of time a student spends studying and time spent studying by peers who share his race; for self-assessed effort, the results are ambiguous. Simulations of policy experiments show that effort is higher in more racially homogeneous classrooms and that a social multiplier exists for both a reduction in the time a student spends working at a part time job and an increase in the student's socioeconomic status.

I dedicate this dissertation to my wife, Jenner. I could not have finished this endeavor without her unwavering love, support, confidence, encouragement, and endless sacrifices so that I could finish. Thank you.

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Professor Roslyn Mickelson of the University of North Carolina at Charlotte deserves much acknowledgement and thanks as well. She generously provided me with her Charlotte Mecklenburg School data set, without which this study of peer effects would not have been possible. Professor Mickelson always quickly and thoroughly answered any questions that I had about the data set and provided many suggestions that improved this dissertation.

Temple University's Economics Department faculty and graduate students have also contributed significantly to my completion of this dissertation. Professors Andrew Buck and George Lady provided extremely generous and useful advice on empirical issues. Professors Fyodor Kushnirsky and William Stull, by giving me financial support for my studies and data, enabled the completion of this project. My peers in the graduate program gave me great recommendations for my dissertation work.

Finally and most importantly, my family has been there for me every step of the way. Jenner, Henry, and Liesl were most directly affected by my Ph.D. work, and their unwavering understanding and support was crucial to my completion of the degree. My parents, Mike and Jane Edelman, supported this endeavor from the very beginning; by teaching me that I could do anything I put my mind to, they made the achievement of this goal possible.

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CHAPTER 1

Introduction

Do educational peer effects exist? If so, how exactly does a student's achievement depend on his peers' characteristics and actions?[1] The answers to these questions have implications for virtually any education policy issue that affects classroom composition, including the ability-tracking versus mainstreaming debate, policies that promote racial and economic desegregation such as busing and other rules that govern student assignment, and school choice programs including charter schools and vouchers. Beyond that, if peer effects do exist, then the magnitude of the impact of any educational intervention designed to target student achievement — paying students to work harder, paying teachers for higher test scores — is in doubt.

This dissertation presents a theory and empirical estimation of educational peer effects. The theoretical model provides a behavioral foundation that explains how a student's effort choice might depend on her peer group's actions and characteristics. The empirical model then tests the predictions of the theoretical model using student-level data from the Charlotte-Mecklenburg School district.

Educational peer effects — if they exist — impact several different branches of economics. Economics of education studies isolate determinants of student achievement and then maximize achievement subject to resource constraints;

[1] I switch randomly between male and female pronouns.

without understanding how achievement depends on the interactions between students, these models potentially reach misleading conclusions. Studies of public goods provision model how household mobility depends on local factors such as the level of public goods, tax rates, and housing market conditions; since parents care deeply about who their children go to school with, these models need to account for educational peer effects to avoid overlooking a factor key to the household's decision making. Development economics studies assess how poor countries can increase living standards through raising human capital levels; if one person's human capital accumulation depends on his interactions with his peers, then peer effects need to play a role in these models as well.

The first part of this dissertation develops a theoretical framework with which to examine how educational peer effects might take place. This framework employs findings from the sociology and social interactions literatures to motivate how the student behaves. It then develops a behavioral model that specifies how the peer effect takes place by focusing on how a student's effort choice depends on the actions and characteristics of her peer group. Specifically, the student feels pressure to conform to the behavior of the peer group, and the more the student interacts with a group of students, the stronger this pressure becomes. Since the student tends to interact more with students of her own race, the peer effect is stronger within the racial group. The theoretical model's equilibrium effort expression results in hypotheses relating peer effects to the model's exogenous variables that can be tested empirically. Finally, the theoretical model demonstrates how the social multiplier takes place, the conditions in which it exists, and the magnitude of the social multiplier in terms of the theoretical model's exogenous parameters.

Next, this dissertation describes and executes an empirical strategy to determine whether or not the theoretical model's peer effects exists. The empirical approach uses student-level data from the Charlotte-Mecklenburg School (CMS) district. The CMS data set is unique for several reasons. For one, it has two variables that describe how much effort a student puts forth in the classroom, allowing me to directly estimate peer effects that take place through effort. Second, the CMS data has observations for *complete classrooms* of students; this feature of the CMS data enables me to determine precisely how a student's effort choice depends on other students in the same classroom. Finally, the CMS data contains sufficient student-, classroom-, and school-level controls to address identification challenges that have plagued peer effects studies.

The nature of the CMS data — specifically, the fact that the effort measures are ordered discrete variables — also presents challenges to estimation. To construct peer measures of the effort variables without compromising the ordered discrete nature of the effort variables, I use a unique approach that calculates the proportion of the classroom that puts forth each possible level of effort. I then use a simulation-based technique to determine how the student's effort depends on the peer measures of effort and whether the strength of the peer effect depends on the racial composition of the classroom.

This dissertation builds upon previous studies of peer effects in the following ways. First, it develops a theoretical model that (a) casts the student as the utility-maximizing agent and (b) defines the choice variable — effort — as a factor that is directly under the student's control. Second, it provides a framework within which to examine how race, an exogenously-determined characteristic, potentially influences how the peer effect takes place; furthermore, this framework has the flexibility to incorporate other exogenous characteristics that might impact the peer effect, including ability and socioeconomic status. Third, it provides a theoretical demonstration of the social multiplier that results directly from the theoretical model itself. Fourth, it estimates peer effects using effort directly rather than proxying for effort with another variable such as achievement that, while observable, might not be the precise variable through which the peer effect takes place. Fifth, it provides estimates of the social multiplier for two policy interventions, reducing how much a student works at a part-time job and raising the student's socioeconomic status.

This dissertation is organized as follows. Chapter 2 describes the literature on effort and peer effects. Chapter 3 presents a theoretical model of peer effects, hypotheses that result from this model, and a definition of the social multiplier. Chapter 4 describes how the peer effects modeled in Chapter 3 are estimated empirically and estimates the social multiplier. Chapter 5 concludes the dissertation and discusses ways in which this dissertation's study of peer effects could be extended in future work.

CHAPTER 2

Literature Review

2.1 Introduction

This chapter reviews studies central to analysis of peer effects. First, it examines how the student's effort affects achievement and the factors that determine how much effort a student puts forth. Next, it reviews how peer effects studies attempt to determine whether or not peer effects exist, and if so, how they take place. These studies look at two distinct types of peer effects, exogenous peer effects and endogenous peer effects. It then discusses factors that complicate the estimation of peer effects. Next, it examines the literature on student behavior and interactions to determine potential channels though which peer effects take place. It concludes with a discussion of the social multiplier, which most likely exists if endogenous peer effects are present.

2.2 Education Production and Effort

If a student exerts more effort, her academic performance almost certainly improves. Sociology and education studies have long acknowledged that student effort is an important input into the education production process. One model of the causal mechanisms that produce learning identifies effort — along with ability and opportunities for learning — as a factor that explains much of the variation in learning across students Sorenson & Hallinan (1977). Another approach measures how background characteristics affect educational outcomes through their impact on student effort Natriello & McDill (1986). And a recent study of 10th-graders finds that the effect that student effort has on achievement dwarfs school-specific attributes Stewart (2008).

Education production studies in economics, on the other hand, seldom include effort as an input. Rather, these studies focus on family factors (parental education, income, and family size), teacher characteristics (education level and experience), school organization (class sizes and facilities), and expenditure levels.[1] A comprehensive review of education production studies does not mention one study in which student effort is an important input Hanushek (1986). One reason that these studies do not use effort is that they choose inputs based on the availability of data and student effort has not until recently been a commonly-captured variable. Another reason for the omission of effort is that most of these studies take an "input-output" or "cost-quality" approach that attempts to determine the combination of inputs that would yield a targeted output at minimum cost. In such an approach, a variable like student effort that is difficult to quantify and even harder to price does not play an obvious role. A third reason that effort has not been used as an input is that effort is under the control of the student and the student has rarely been cast as the primary decision maker in models of education production.

With such little doubt about the role that effort plays in achievement, it is surprising that economics studies have modeled the student's effort choice so rarely. A review of economics of education studies identifies the view of the student as the decision maker as a major theme that has been overlooked by the field Akerlof & Kranton (2002); while inputs into education have received much attention, student agency has not. What makes this omission even more surprising is that the theory of the firm has long embraced worker's effort

[1] One exception, a study in which student motivation is proxied by unexcused absences and lateness, finds that student motivation is a strong predictor of student learning Summers & Wolfe (1977).

as a central determinant of output since Shapiro and Stiglitz's 1984 seminal contribution on the topic.[2]

Recently, the economics literature has started to include student effort as an input into the education production process. One study of education production casts the student as the optimizing agent who chooses an effort level to maximize utility Cooley (2010). Here, the effort variable captures how hard a student works on assignments, attentiveness, and cooperation with other students. Effort affects utility in two ways. It increases utility indirectly through its impact on achievement. Since effort is costly, the student also derives disutility from effort. Because effort is not observable in Cooley's data, however, student achievement becomes the dependent variable of interest in the empirical estimation of the model. Using similar approaches, studies that empirically test the relationship between effort and achievement find that student effort has a positive effect on achievement Stinebrickner & Stinebrickner (2008); Mihaljevic (2008); De Fraja et al. (forthcoming).

Many of the determinants of effort that are student-specific are relatively well understood. Several exogenous characteristics increase student effort, including higher ability, being female, higher parental educational and occupational status, and higher teacher's standards for learning Natriello & McDill (1986). Other characteristics that have been shown to increase the student's effort on average include a higher student's birth weight, having a parent who reads to his child more frequently, and higher household income De Fraja et al. (forthcoming).

One factor that appears to affect the student's effort but is not yet well understood is the student's peer group's actions and characteristics. One way that peers can affect the student's effort is through that standards that the peer group sets for achievement Natriello & McDill (1986). Having a peer group with higher socioeconomic status may also increase the student's effort

[2] De Fraja, Oliveira, and Zanchi forthcoming point out this contrast.

De Fraja et al. (forthcoming). These types of effects are studied in detail by the peer effects literature.

2.3 Peer Effects

Studies of peer effects attempt to explicitly model and measure how the student's educational outcome depends on the actions and characteristics of the student's peer group. Specifically, these studies attempt to capture whether or not an externality results from interactions between students. The influences that peer effects studies examine can be broken down into two types: *exogenous* peer effects and *endogenous* peer effects. Exogenous peer effects capture the extent to which the group's characteristics affect the individual's decision-making. An exogenous peer effect exists if a student's achievement depends on her peer group's racial composition, for example. Endogenous peer effects capture how the individual's actions depend on other individuals' actions. An endogenous peer effect occurs if a student's effort depends on the effort that her peer group puts forth.

2.3.1 Exogenous Peer Effects

Most peer effects studies measure exogenous peer effects. These studies focus primarily on two exogenous characteristics, ability and race. Studies on ability propose that peers' ability levels affect a student's achievement; these studies are considered to measure exogenous effects because ability is an attribute of the student, not a choice. Studies on race measure the effect that peer group racial composition has on a student's achievement.

Studies of peer effects that operate through peer ability generally find that a student's achievement increases with peer ability. Estimates of education production functions in the presence of peer effects conclude that an increase the student's classmates' ability improves the student's achievement Arnott & Rowse (1987); Hoxby (2000); Sacerdote (2001); Zimmerman (2003); this effect weakens, however, at higher levels of peer ability Henderson et al. (1978). Peer ability, through its effect on educational outcomes, also potentially explains how parents select their residence and a school for their children to attend De Bartolome (1990); Benabou (1996); Nechyba (1999); Calabrese et al. (2006).

While the student may benefit from the presence of high-ability peers, she can also experience adverse effects from being among high-ability students through relative status effects. Relative status effects occur when a student is concerned with her ranking with the peer group. Students in special gifted classes can experience increased negative self-perceptions through relative status effects; these depressed self-perceptions, in turn, lead to worse school performance for students who experience these relative status effects Zeidner & Schleyer (1998).[3]

The effect of race and racial diversity is even less clear-cut than the effect of peer ability. Higher proportions of a student's classmates that are minority can have a negative effect on educational outcomes, especially for blacks and Hispanics Hoxby (2000); Angrist & Lang (2004); Card & Rothstein (2007); Hanushek et al. (2009). Academic performance at the university level, on the other hand, peaks at medium levels of diversity and drops again at high levels of diversity Terenzini et al. (2001). And both black and white students who attend more diverse elementary schools benefit academically Mickelson (2001, 2003).

2.3.2 Endogenous Peer Effects

While studies of exogenous peer effects take a step towards showing that peer effects do exist and describing the factors that potentially cause these effects, they do not address *how* the peer effect actually takes place. Studies of endogenous peer effects, on the other hand, seek to demonstrate the channel through which the student's action depends on the actions of others in his peer

[3]Damiano, Li, and Suen 2010 provide a theoretical model of relative status effects.

group.[4] The advantage of examining endogenous peer effects is that it gives us insight into how a student incorporates peers' actions into his decision making process.

The literature on endogenous peer effects in the classroom is limited. One way that the student's effort can be affected by that of his peer group is through peer pressure: if the student's effort choice deviates from that of the peer group, then the student experiences lower utility Akerlof & Kranton (2002); Cooley (2010). A related way in which the peer group's effort choice can affect the student's effort choice is through strategic complementarities; in this case, the student enjoys higher utility if he responds to higher peer effort by selecting a higher effort level himself Calvó-Armengol et al. (2009).[5]

2.3.3 Are Peer Effects Exogenous, Endogenous, or Both?

Both exogenous and endogenous peer effects studies provide evidence that peer effects exists. The findings of these studies raise more questions. Can both exogenous and endogenous effects take place at the same time? How do the exogenous characteristics of the peer group affect the endogenous peer effect?

Three studies, Akerlof and Kranton 2002, Fryer and Torelli 2010, and Cooley 2010, present frameworks that come the closest to addressing how the peer group's exogenous characteristics affect the student's effort choice. In Akerlof and Kranton 2002, the student's utility depends on the degree to which her effort choice matches that of her social category; the social category is determined by race, gender, and social status-related designations such as "jocks" and "nerds."

[4]Studies outside of the educational peer effects literature have also modeled endogenous effects. Examples include the effect that neighbors' criminal activities have on an individual's decision to commit a crime Glaeser et al. (1996), how a teenager's smoking behavior depends on friends' decisions to smoke or not Nakajima (2007); Cutler & Glaeser (2010), and whether or not an unemployed person's decision to apply for unemployment benefits depends on how many other workers apply for such benefits Kroft (2008).

[5]Brock and Durlauf 2001 present generalized cases of both the conformity and the strategic complementarity cases.

Fryer and Torelli 2010 present a two-audience signaling model in which the student must choose between allocating effort to school work and to gaining acceptance by the peer group, which is defined by race. This model predicts that there will be racial differences between social status and academic achievement and that these differences will be exacerbated when there is more interracial contact; both predictions fit Fryer and Torelli's empirical findings.

Cooley's 2010 empirical approach allows the size of the peer effect to vary across different race-based reference groups within the classroom. This study finds that a white student's achievement depends only on that of white peers and a nonwhite student's achievement depends only on that of nonwhite peers. In other words, the achievement of nonwhite peers does not affect a white student's achievement, and vice-versa. While this study shows how a student's achievement depends on that of different racial groups, it does not identify how effort depends on the peer group's exogenous characteristics.

To my knowledge, no studies address whether or not the endogenous peer effect depends on the exogenous characteristics of the peer group. And yet, according to the findings of the rapidly growing literature on peer effects, both types of effects exist. My study attempts to fill this void. Specifically, it shows how the strength of the endogenous peer effect depends on the distribution of an exogenous characteristic within the classroom. The endogenous peer effect operates through conformity, an influence employed by most existing endogenous peer effects studies that seems to be prevalent in the classroom. The exogenous characteristic that I use to illustrate this approach is race. I choose race because it affects how students interact with each other, as described in Section 2.4.

2.4 Conformity, Interactions, and Race

At the heart of the study of peer effects is understanding what causes a student to react to peer actions and characteristics. In this section, I describe the literature on conformity, a pressure faced by students that is used in several endogenous peer effects studies. I then describe literature that presents one way in which a student might interact within a group of students. I use these findings to develop the theoretical model presented in Chapter 3.

Conformism is a strong social influence through which social groups penalize individuals who deviate from social norms.[6] Peer conformity is an especially strong influence in the classroom, affecting activities ranging from substance abuse to school performance Santor et al. (2000). When it comes to effort, the student faces a tradeoff between the higher achievement that comes from more effort and the social acceptance that comes from conforming to the behavior of his peers Bishop et al. (2004); Bishop (2006).

One challenge to determining how conformity affects the student's effort choice is defining the group to which that student is attempting to conform. One way to identify the student's social group is to assign the student to a particular "type" that is determined by the student's physical attributes and behaviors Akerlof & Kranton (2002). A second method is to define the student's reference group as those students with whom the student spends the most time — in most cases, the students who share a classroom Cooley (2010). A third way is to allow the student to interact within a larger macro group, but to define the strength of interactions with particular sub-groups through an exogenous characteristic such as race or economic status Weinberg (2006).

Implicitly, each of these three approaches attempts to identify the group within which the student interacts the most. A large sociology literature on homophily finds that an individual prefers to interact with others who are similar to himself in some observable way.[7] Homophily can operate through numerous exogenous characteristics, including race, gender, socioeconomic status, age, religion, and educational attainment.

Race serves as one of the strongest observable dimensions along which homophily operates. School-aged children in particular tend to form early child-

[6] Akerlof 1980 and Bernheim 1994 present overviews of the literature on conformity.

[7] McPherson, Smith-Lovin, and Cook 2001 provide an overview of the homophily literature.

hood friendships on the basis of racial homophily McPherson et al. (2001). The friendship networks of high school students also tend to form on the basis of racial homophily Weinberg (2006); Currarini et al. (2009a). Furthermore, a student tends to form same-race friendships at a rate that exceeds that student's racial group's share of the overall population, a tendency known as inbreeding homophily Weinberg (2006); Currarini et al. (2009a). I use these findings to inform the theoretical model presented in Chapter 3.

2.5 Identification Challenges

The empirical estimation of peer effects presents just as significant of challenges as those involved in formulating a theoretical model. These issues include the reflection problem, correlated effects, and endogenous peer group selection, which is a special case of correlated effects. In this section, I discuss these challenges and how this paper's empirical approach attempts to overcome these challenges.

The reflection problem, formulated by Manski 1993, occurs when the researcher cannot answer the following question: does group behavior affect individual behavior or is group behavior just an aggregation of individual behaviors? This problem arises because the student and peer group effort choices are determined simultaneously. In the linear-in-means model with endogenous and exogenous effects, this simultaneity induces nonidentification because the estimation cannot distinguish between the endogenous effects and the exogenous effects Moffitt (2001).

Correlated effects arise if the group's average behavior is correlated with an unobserved exogenous group characteristic Manski (1993). If students within a classroom tend to achieve at a similar level because they have the same teacher or share an family background characteristic such as parents with a college education, for example, then correlated effects exist Hoxby (2000). Any estimate that attributes individual achievement to peer group achievement would overstate the effect in the presence of correlated effects.

Endogenous peer group selection is a special case of correlated effects that also makes estimation of peer effects difficult. This problem, also known as selection bias, arises when the peer group itself is determined endogenously. In other words, the individual selects her peer group; this selection can generate correlation between unobserved variables that, in turn, biases estimates of peer effects. Consider a student whose parents highly value his education and this high valuation of education results in a better outcome for the student. If these parents seek out a classroom with students whose parents are similar to themselves and the researcher does not observe the parental valuation of education, then this selection process will bias the estimation of peer effects. In other words, an unobserved characteristic that plays a role in peer group selection and is correlated across the classroom generates an effect on the group's average behavior. A study of teenage pregnancy and dropout behavior finds that once the endogenous nature of peer group selection is controlled for, the effect of peer actions on the individual's action disappears Evans et al. (1992). Here, the fact that parents of similar socioeconomic status tend to select similar schools for their children generates selection bias because socioeconomic status also plays an important role in whether or not a teenager becomes pregnant.

2.6 The Social Multiplier

If it can be shown that peer effects exist, then a *social multiplier* could be present for any public policy change that affects student effort. The social multiplier is the name for the effect that creates a disparity between the sum of direct individual impacts of a change to an exogenous variable and the aggregate impact of this change.[8] A social multiplier exists if interactions between individuals amplify a change in incentives faced by an individual beyond

[8] Glaeser, Sacerdote, and Scheinkman 2003 provide an overview of the social multiplier and discuss applications.

those that the individual would face in the absence of interactions Durlauf & Cohen-Cole (2004).

Exogenous and endogenous peer effects have quite different implications for the social multiplier. There is generally no social multiplier in the presence of exogenous peer effects. For exogenous peer effects, a change to the distribution of exogenous characteristics within a peer group can generate a net positive effect on that group. Overall, however, there is no net improvement in aggregate welfare. This is because any change in the distribution of an exogenous characteristic is necessarily offset by an equal but opposite change in another group. For example, if a student's achievement depends on peers' abilities, then regrouping students helps some students at the expense of others; aggregate welfare, however, does not change.[9]

Endogenous peer effects, on the other hand, almost always result in a social multiplier. Consider the situation in which a person's decision to smoke depends on her peers' decisions regarding smoking Nakajima (2007); Cutler & Glaeser (2010). If an increase in anti-smoking advertising causes one person to quit smoking, the presence of endogenous effects may cause one or two additional people to quit smoking. That is, the aggregate impact of the policy shift is larger than the sum of the direct individual impacts. In the presence of endogenous peer effects, a policy change has a multiplier effect.

Empirical estimates of the social multiplier find evidence of a social multiplier in diverse public policy settings. The birth control pill, by allowing women to delay marriage, creates a social multiplier through the increase in marriage match quality that it facilitates: because more potential spouses remain in the marriage market longer, one woman's decision to delay marriage encourages other women to do the same Goldin & Katz (2002). For every 10 workers that

[9]An exception is if the exogenous peer effects model is non-linear in the peer group measure. Consider the "boutique" model in which a student's achievement improves if peer achievement is more dispersed Hoxby & Weingarth (2006); in this case, reallocation of students from classrooms stratified by ability to classrooms that include a range of abilities increases aggregate welfare. Even in this case, reallocation is a one-time improvement that cannot be repeated.

are induced to accept unemployment insurance from an increase in benefits, an additional 10 workers accept unemployment insurance that would not have in the absence of social interactions Kroft (2008). An increase in the mean ability of a kindergarten student's peer group by 1% results in a 1.9% increase in mathematics achievement across the group Graham (2008). A study of the effect of smoking bans on an individual's propensity to smoke finds a social multiplier of 4 at the metropolitan area level and 12 at the state level Cutler & Glaeser (2010)

Fewer studies describe the theoretical underpinnings of the social multiplier. One definition, "the estimated ratio of aggregate coefficients to individual coefficients" (Glaeser et al., 2003, p. 346), is useful for empirical estimation but not necessarily for theoretical formulation. Scheinkman's 2008 description of the social multiplier, "the ratio of the effect on the average action caused by a change in a parameter to the effect on the average action that would occur if individual agents ignored the change in actions of their peers," serves as arguably the clearest basis for the theoretical formulation of the social multiplier.[10] Central to this definition is understanding how the individual's action depends on the actions of the peers — in other words, a clear formulation of the endogenous peer effect.

2.7 Chapter Summary

This chapter describes the existing literature on the role of effort in educational production, endogenous and exogenous peer effects, and how peer effects might actually take place in the classroom. It concludes that while endogenous peer effects appear to exist, they most likely depend at least in part on the exogenous characteristics of the student's peer group. Race is one exogenous characteristic that consistently plays a strong role in determining how students interact with one another. If these types of peer effects do exist, they

[10] This definition describes in words what Glaeser and Scheinkman 2002 describe in symbols in Equation (24) on p. 21, which is too involved to present here.

have implications for the assessment of education policy changes due to the presence of a social multiplier.

CHAPTER 3

Theoretical Model of Peer Effects

3.1 Introduction

Does a student's effort depend on the actions and characteristics of students with whom she interacts? If so, how? This chapter develops a theoretical framework with which to examine these questions. This framework provides a behavioral foundation that explains how a student's effort choice might depend on his peer group's effort and racial composition. It results in hypotheses that can be tested empirically. Finally, it uses the comparative statics results to define and quantify the social multiplier.

3.2 Assumptions

First, here are the assumptions about how the student behaves.

Assumption 1: Peer Pressure. *The student feels pressure to conform to the behavior of students with whom he interacts. If other students put forth more effort, then the student will feel pressure to increase effort. If other students put forth less effort, then the student will feel pressure to decrease effort.*

This assumption comes from the literature on conformity and social norms Bernheim (1994); Santor et al. (2000); Akerlof & Kranton (2002); Weinberg (2006).

Assumption 2: Interactions with Peers. *The student prefers to interact with students who are similar to himself.*

This preference, known as *homophily* in sociology,[1] operates through sociodemographic characteristics such as age, race, gender, and socioeconomic status.

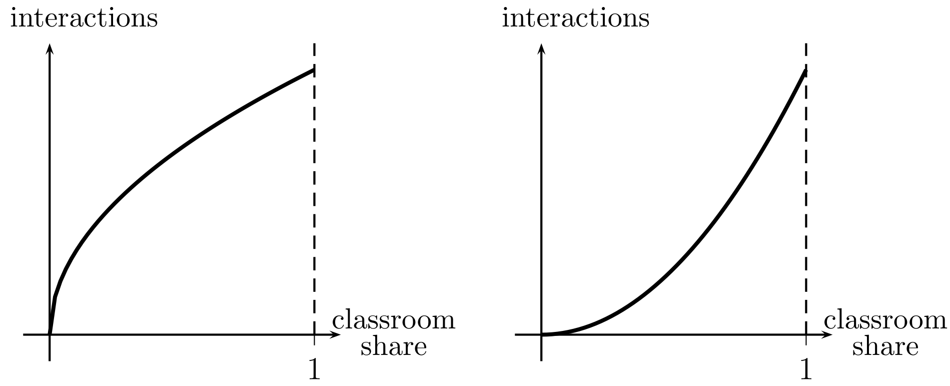
Assumption 3: Race and Interactions. *The student prefers to interact with students of the same race as himself.*

This assumption is based on findings that the homophilic preferences discussed in Assumption 2 operate most strongly through the student's race McPherson et al. (2001); Currarini et al. (2009a).[2] Furthermore, racial homophily is not a linear process; rather, the tendency to interact within a race exceeds that race's relative fraction of the group, a preference known as *inbreeding homophily* in the literature Weinberg (2006); Currarini et al. (2009a,b). This means that interactions within one's own racial group are increasing and concave in the racial group's share of the classroom. Interactions with students of another race, on the other hand, are increasing and convex in that racial group's share of the classroom. Figure 3.1 presents a visual interpretation of this assumption.

Consider the implications of Assumption 3. Interactions of white students with white students are increasing and concave in the share of the classroom that is white. This means that once a small number of white students is introduced into a group, a white student will have more whites with whom to associate. Interactions of black students with white students are increasing and convex in the share of the classroom that is white. In other words, black students will not interact heavily with white students until most of the class-

[1]See McPherson, Smith-Lovin, and Cook 2001 for an overview of the research on homophily.

[2]Cooley 2010 also finds evidence that students form race-based reference groups within the classroom.



(a) Own racial group (b) Other racial group

Figure 3.1: Interactions with own and other racial group

room is made up of white students. Interactions of Hispanic students with white students are increasing and convex in the share of the classroom that is white; just like black students, Hispanic students will not interact heavily with white students until most of the classroom is made up of white students. And white students will not interact heavily with black students until most of the classroom is made up of black students.

While this study focuses on race, the theoretical framework allows the researcher to replace this assumption with an assumption regarding other exogenous characteristics that generate homophilic behavior such as age, sex, ability, or socioeconomic status. The framework is also flexible enough to incorporate multiple characteristics simultaneously.

Assumption 4: Classroom as Peer Group. *The classroom defines the group of students with whom the student interacts.*

While it is certain that the student also interacts with other students outside of the classroom, it is a useful and reasonable approximation to assume that most of the student's interactions that are relevant to the effort choice take place within the classroom. The student spends most of the school day in the classroom with the same group of students. And effort, the choice variable, is largely determined and observed inside the classroom.

Assumption 5: Simultaneous Decision Making. *Students select effort levels simultaneously.*

For the most part, students enter a classroom at the same time, are presented with a task at the same time, and then work on that task at the same time. While some sort of sequential decision making might make sense in certain cases, it is simplest to start with simultaneous decision making and then relax this assumption to allow for sequential decision making in future work.

Assumption 6: Perfect Information. *Students have complete knowledge about how much effort other students exert and their characteristics.*

Much of effort is openly observable in the classroom. For those components that are not observable, such as time spent on homework at home, students can often infer peer effort through peer performance in the classroom, which is observable. If evidence emerges that students systematically misrepresent effort to other students, then future work could relax this assumption to allow for incomplete information.

Based on these six assumptions, then, the peer effect takes place in the following way. When the student decides how much effort to put forth, she takes into account how much effort her peers put forth. This happens because of peer pressure. The strength of this peer pressure effect depends on how much the student interacts with other groups of students. If she interacts a lot with a certain group of students, then she will feel more pressure to conform to that group's effort choice. If she does not interact much with a certain group, then she will not feel much pressure to match their effort choice. And interactions depend on students' races.

3.3 Game Setup

3.3.1 Variables and Parameters

Players: a classroom of $C \in \mathbb{N}$ students in a particular time period, where $i \in \{1, \ldots, C\}$ indexes the students and where $-i \in \{1, \ldots, i-1, i+1, \ldots, C\}$

indexes all students other than student i. I cast the student as the individual decision maker in the classroom. By definition, a peer effect is the impact that the student's peer group's actions or characteristics have on the student's action or outcome. For this reason, I view the problem from the student's perspective.

Choice variable: effort $e_i \in [0, \hat{e}]$, where $\hat{e} \in \mathbb{R}_{++}$. Effort captures how hard a student tries in the classroom, how attentive the student is during class, and how much time a student spends on homework, among other things. My decision to use effort as the choice variable is reasonable because effort is a major determinant of educational achievement. It is also a variable that is directly under the student's control. While other factors — parents, teachers, and other students — can influence a student's effort, it is ultimately up to the student to respond to these influences and decide how hard to work in the classroom.

As indicated by $e_i \in [0, \hat{e}]$, the set from which the student chooses effort is bounded by 0 and $\hat{e}$. Because a student has only a finite amount of time to devote to effort, I put an upper bound on effort ($\hat{e}$). Effort has a lower bound because *no* effort is the least effort one can put forth; in other words, negative effort is not possible.[3] One implication of putting a lower and an upper bound on effort is that student i will be maximizing utility subject to the following inequality constraints: (1) $e_i \geq 0$; (2) $\hat{e} - e_i \geq 0$.

Race: Let $J \in (0, C]$ and let $E = \{1, 2, \ldots, J\}$ represent the set of all races. Let $j : C \rightarrow E$ assign to each student i a race from E. Let k index the races other than $j(i)$.

Let $p_{j(i)}$ represent the number of students who belong to race $j(i)$. Let the vector $\mathbf{p} = (p_1, p_2, \ldots, p_G)$ represent the number of students in each racial group. Similarly, let the vector $\mathbf{p_{-j(i)}} = \big(p_1 \ldots, p_{j(i)-1}, p_{j(i)+1}, \ldots, p_G\big)$ describe the class's population, but with the number of students in student i's racial

[3] One type of peer effect that could be considered negative effort is student disruption and misbehavior Lazear (2001); Figlio (2007); while this study does not include disruption in the effort variable, it could be a possible extension.

group $p_{j(i)}$ removed. Finally, let $s_{j(i)} \equiv \frac{p_{j(i)}}{C}$ represent the proportion of the classroom that is of race $j(i)$ and let $s_k \equiv \frac{p_k}{C}$ represent the proportion of the classroom that is of race k for $k \neq j(i)$.

Student-level exogenous characteristics: Let $n \in \mathbb{N}$. $\mathbf{S}_i \in \mathbb{R}^n$ represents predetermined student-level characteristics that potentially affect student i's effort. These characteristics include ability, sex, educational and occupational aspirations, and parents' socioeconomic status.

Classroom-level exogenous characteristics: Let $m \in \mathbb{N}$. $\mathbf{T} \in \mathbb{R}^m$ represents predetermined characteristics that are common to all C students in the classroom. These characteristics include differences in teachers' abilities, the school's policies regarding rewards for high effort (public recognition and scholarships), and penalties for low effort (detention and grade retention).

Taste for effort function: Let $\theta_i : \mathbb{R}^{m+n} \to \mathbb{R}$ assign the effect that exogenous characteristics $\mathbf{S_i}$ and $\mathbf{T}$ have on student i's utility. The student's taste for effort can depend on student-level exogenous characteristics $\mathbf{S_i}$ such as ability, sex, educational and occupational aspirations, and parents' socioeconomic status Natriello & McDill (1986); De Fraja et al. (forthcoming). It can also depend on characteristics $\mathbf{T}$ common to all C students such as rewards for high effort, penalties for low effort, and teacher heterogeneity. Because θ_i depends on $\mathbf{S_i}$, it allows for heterogeneity in the student's taste for effort.

Peer effort: Let $e_{-i,j(i)}$ represent the effort put forth by a student other than student i who is of race $j(i)$. Let $e_{-i,k}$ represent the effort put forth by a student other than student i who is of race k. Let

$$\bar{e}_{j(i)} = \begin{cases} \frac{\sum_{-i} e_{-i,j(i)}}{p_{j(i)}-1}, & \text{if} \quad p_{j(i)} > 1 \\ 0, & \text{otherwise} \end{cases} \tag{3.1}$$

Table 3.1: Theoretical Model Variables

Variable	**Description**
C	number of students in the classroom
i	student who is maximizing utility
e_i	effort choice of student i, bounded by 0 and $\hat{e}$
$j(i)$	student i's race
$p_{j(i)}$	number of student's who are of race $j(i)$
k	index of races other than $j(i)$
$s_{j(i)}$	proportion of classroom that is of race j
s_k	proportion of classroom that is of race k
$\mathbf{S_i}$	vector of student i's exogenous characteristics
$\mathbf{T}$	vector of classroom-level characteristics
θ_i	student's taste for effort function
$e_{-i,j(i)}$	effort choice of student other than i who is of race $j(i)$
$e_{-i,k}$	effort choice of student other than i who is of race k
$\bar{e}_{j(i)}$	mean peer effort put forth by peers of race $j(i)$
$\bar{e}_k$	mean peer effort put forth by peers of race k
$\sigma_{j(i)}$	strength of effect that racial group $j(i)$ has on i's effort choice
σ_k	strength of effect that racial group k has on i's effort choice

represent the mean peer effort put forth by peers of race $j(i)$. And let

$$\bar{e}_k = \begin{cases} \frac{\sum_{-i} e_{-i,k}}{p_k}, & \text{if} \quad p_k > 0 \\ 0, & \text{otherwise} \end{cases} \tag{3.2}$$

represent the mean peer effort put forth by peers of race k.

Strength of peer effect functions: Let $\sigma_{j(i)} : [0,1] \to \mathbb{R}_+$ assign the strength of the effect that student i's racial peer group's effort choice has on his own effort choice. Assumption 3 implies that $\sigma_{j(i)}$ is increasing and concave in $s_{j(i)}$.

For race k, let $\sigma_k : [0,1] \to \mathbb{R}_+$ assign the strength of the effect that racial peer group k has on student i's effort choice. Assumption 3 implies that σ_k is increasing and convex in s_k.

Table 3.1 presents a list and brief summary of the theoretical model's variables.

3.3.2 Optimization Problem

Let $v : \mathbb{R}^{2J+m+n+1} \to \mathbb{R}$ represent student i's utility function. Student i selects e_i to solve the following optimization problem:

$$\begin{aligned} \max_{e_i} \quad & v(e_i, \bar{e}_{j(i)}, \bar{\mathbf{e}}_{\mathbf{k}}; \mathbf{S}_{\mathbf{i}}, \mathbf{T}, s_{j(i)}, \mathbf{s}_{\mathbf{k}}) \\ \text{such that} \quad & e_i \geq 0 \\ & \hat{e} - e_i \geq 0. \end{aligned} \tag{3.3}$$

Each student i simultaneously selects e_i to solve the optimization problem defined in (3.3).

3.3.3 Student i's Utility

To see how different variables affect student i's utility, I rewrite the utility function in additively separable components using variables and functions introduced above. First, let $u : \mathbb{R}^{m+n+1} \to \mathbb{R}$ and $h : \mathbb{R}_{+}^{2J+1} \to \mathbb{R}$. Then student i's utility function is

$$v(\cdot) = u(e_i, \theta_i(\mathbf{S}_{\mathbf{i}}, \mathbf{T})) + h(e_i, \bar{e}_{j(i)}, \bar{\mathbf{e}}_{\mathbf{k}}; \sigma_{j(i)}(s_{j(i)}), \sigma_k(\mathbf{s}_{\mathbf{k}})) \tag{3.4}$$

Here, $u(e_i, \theta_i(\mathbf{S}_{\mathbf{i}}, \mathbf{T}))$ can be thought of as the *private* utility that student i derives from her effort choice. Effort generates private utility for the student by increasing her achievement, where higher achievement yields higher utility. It can also increase utility through a work ethic channel; working hard, in and of itself, can make the student feel better about herself. The utility derived from effort can vary by student. One student might have parents who believe strongly in the value of education and therefore encourage and reward effort more strongly than parents who do not value education. $\theta_i(\mathbf{S}_{\mathbf{i}}, \mathbf{T})$ captures this heterogeneity.

Effort also has costs. If a student is spending time on homework or spending energy in class, he is sacrificing time and energy for other activities that he might enjoy more than working hard such as playing sports or watching TV. Similar to the returns to effort, the costs of effort also vary by student. Family income, for example, could affect the cost of effort. A student from a poorer family might have to hold a part-time job, which would imply that this student has less time available to devote to homework or studying. For this student, the cost of putting forth more effort is higher than for a student who does not have such time constraints.

The *social* utility that student i derives from his effort choice is represented by $h(\cdot)$. Here, social utility captures the utility derived from student i's effort choice that depends on the actions and characteristics of students other than student i. As discussed in Assumption 1, student i's utility depends in part on how closely his effort choice conforms to the effort choices of the students with whom he interacts. And intensity of interactions depends in part on race, as outlined in Assumption 3; this effect takes place through $\sigma_{j(i)}(s_{j(i)})$ and $\sigma_k(\mathbf{s_k})$.

3.3.4 Solution to Optimization Problem

Next, I present the necessary conditions for a maximum to the optimization problem in (3.3) to exist.[4] Let $e_i^* \in [0, \hat{e}]$ solve the optimization problem presented above and name the inequality constraints:

$$g_1(e_i) \equiv e_i \geq 0 \tag{3.5}$$

$$g_2(e_i, \hat{e}) \equiv \hat{e} - e_i \geq 0. \tag{3.6}$$

The following theorem presents the necessary conditions for a maximum for student i's optimization problem in (3.3):

Karush-Kuhn-Tucker (KKT) Theorem *Let e_i^* solve the optimization problem in (3.3). Let the inequality constraints g_1, g_2 satisfy the constraint*

[4] I follow Chapter 27 of Varian 1992 for this section.

qualification.[5] *Then there exists a set of KKT multipliers* $\boldsymbol{\lambda_i} \in \mathbb{R}^2_+$ *such that* $(e_i^*, \boldsymbol{\lambda_i})$ *satisfy*

$$Dv(e^*) + \lambda_{i,1} e_i^* + \lambda_{i,2}(\hat{e} - e_i^*) = 0 \tag{3.7}$$

$$e_i^* \geq 0, \quad \lambda_{i,1} e_i^* = 0 \tag{3.8}$$

$$\hat{e} - e_i^* \geq 0, \quad \lambda_{i,2}(\hat{e} - e_i^*) = 0 \tag{3.9}$$

In other words, if $(e_i^*, \boldsymbol{\lambda_i})$ maximize (3.3), then they must fulfill the conditions put forth in the KKT Theorem. A second theorem presents the sufficient conditions for a global maximum.

KKT Sufficiency Theorem *If* $v(\cdot)$ *is a concave function and* g_1*,* g_2 *are convex functions, then the KKT conditions are both necessary and sufficient for a maximum.*

3.3.5 Equilibrium

Since all C students simultaneously select an effort level to solve the optimization problem and each student has perfect information about each other student's actions and characteristics, an appropriate equilibrium concept is a pure strategy Nash equilibrium (PSNE). The vector $\mathbf{e}^*$ is a PSNE if, for all i, $(e_i^*, \boldsymbol{\lambda_i})$ satisfy the KKT Theorem and $v(\cdot)$ and g_1, g_2 satisfy KKT Sufficiency.

3.4 Utility Function Specification

Next, I introduce a utility function specification with two components, private utility, $u(\cdot)$, and social utility, $h(\cdot)$.

[5]The constraint qualification holds if gradients of the binding constraints are linearly independent. In the context of this problem, the constraint qualification is satisfied since, by the nature of the constraints, I have at most one binding inequality constraint at a time.

3.4.1 Private Utility

The first component of student i's utility function is private utility, where $\beta \in \mathbb{R}_{++}$:

$$u(e_i, \theta_i(\mathbf{S_i}, \mathbf{T})) = \theta_i(\mathbf{S_i}, \mathbf{T})e_i - \frac{\beta e_i^2}{2}. \tag{3.10}$$

Effort generates utility for the student by increasing her academic achievement, represented by the function $\theta_i(\mathbf{S_i}, \mathbf{T})$ below, where $\boldsymbol{\alpha} \in \mathbb{R}^N_{++}$, $\boldsymbol{\gamma} \in \mathbb{R}^P_{++}$, n indexes the α values, and m indexes the γ values:

$$\theta_i(\mathbf{S_i}, \mathbf{T}) = \sum_n (\alpha_n S_{i,n}) + \sum_m (\gamma_m T_m). \tag{3.11}$$

This specification of $\theta_i(\mathbf{S_i}, \mathbf{T})$ demonstrates that the degree to which effort increases private utility varies across students, as described in the definition of the taste for effort function.

Effort also provides disutility for the student, captured by the term $\left(-\frac{\beta e_i^2}{2}\right)$, through the opportunity cost of time spent on effort. This term also allows for diminishing returns to effort; even if a student prefers effort to any other activity, the returns to marginal effort through achievement or work ethic are decreasing. This is a standard private utility specification for the peer effects literature; see Akerlof and Kranton 2002, Calvó-Armengol, Patacchini, and Zenou 2009, and Cooley 2009 for examples.

3.4.2 Social Utility

The second component of student i's utility function is social utility:

$$h(e_i, \bar{e}_{j(i)}, \bar{\mathbf{e}}_\mathbf{k}, s_{j(i)}, \mathbf{s_k}) = -\frac{\sigma_{j(i)}(s_{j(i)})}{2}(e_i - \bar{e}_{j(i)})^2 - \sum_k \frac{\sigma_k(s_k)}{2}(e_i - \bar{e}_k)^2. \tag{3.12}$$

This parameterization of the social utility function is called the *conformity* parameterization. Developed in Bernheim 1994 and used in Brock and Durlauf 2001 and Akerlof and Kranton 2002, conformity penalizes deviations further from the mean relatively more strongly. In other words, if a student puts forth an effort level that is either lower or higher than that of the peer group, she receives disutility because she is viewed as an outcast by the rest of the group. I use this specification because the conformity concept matches the type of peer influence that much of the literature supports Natriello & McDill (1986); Bishop et al. (2004); Falk & Ichino (2006); Mas & Moretti (2006).

In this specification, student i considers both the effort choice of the racial peer group, $\bar{e}_{j(i)}$, and the effort choices of each of the other racial groups, $\bar{\mathbf{e}}_{\mathbf{k}}$, when making his effort choice. First, by Assumption 3, $\sigma_{j(i)}$ is an increasing, concave function of his racial peer group's share of the classroom, $s_{j(i)}$; let

$$\sigma_{j(i)}(s_{j(i)}) = s_{j(i)}^{\frac{1}{2}}. \tag{3.13}$$

Also by Assumption 3, σ_k is an increasing, convex function of each other racial peer group's classroom share, s_k; let

$$\sigma_k(s_k) = s_k^2. \tag{3.14}$$

By (3.13) and (3.14), (3.12) becomes

$$h(e_i, \bar{e}_{j(i)}, \bar{\mathbf{e}}_{\mathbf{k}}, s_{j(i)}, \mathbf{s}_{\mathbf{k}}) = -\frac{s_{j(i)}^{\frac{1}{2}}}{2}\left(e_i - \bar{e}_{j(i)}\right)^2 - \sum_k \frac{s_k^2}{2}(e_i - \bar{e}_k)^2. \tag{3.15}$$

3.4.3 Optimization Problem

Student i selects e_i to solve the following optimization problem:

$$\begin{aligned} \max_{e_i} \quad & U_i(\cdot) = \theta_i(\mathbf{S_i}, \mathbf{T})e_i - \frac{\beta e_i^2}{2} - \frac{s_{j(i)}^{\frac{1}{2}}}{2}(e_i - \bar{e}_{j(i)})^2 - \sum_k \frac{s_k^2}{2}(e_i - \bar{e}_k)^2 \\ \text{such that} \quad & e_i \geq 0 \\ & \hat{e} - e_i \geq 0 \end{aligned} \tag{3.16}$$

The Lagrangian for (3.16) is

$$\mathcal{L}(e_i, \bar{e}_{j(i)}, \bar{\mathbf{e}}_{\mathbf{k}}, \theta_i, \sigma_i, \boldsymbol{\lambda_i}) = U_i(\cdot) + \lambda_{i,1} e_i + \lambda_{i,2}(\hat{e} - e_i) \tag{3.17}$$

The first order conditions are

$$\mathcal{L}_{e_i} = \theta_i - \beta e_i - s_{j(i)}^{\frac{1}{2}} e_i + s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} - \sum_k \left(s_k^2 \left(e_i - \bar{e}_k \right) \right) + \lambda_{i,1} - \lambda_{i,2} = 0 \tag{3.18}$$

$$\mathcal{L}_{\lambda_{i,1}} = e_i \geq 0, \quad \lambda_{i,1} \geq 0, \quad \mathcal{L}_{\lambda_{i,1}} \lambda_{i,1} = 0 \tag{3.19}$$

$$\mathcal{L}_{\lambda_{i,2}} = \hat{e} - e_i \geq 0, \quad \lambda_{i,2} \geq 0, \quad \mathcal{L}_{\lambda_{i,2}} \lambda_{i,2} = 0 \tag{3.20}$$

3.4.4 Equilibria

The following propositions provide characterizations of the various forms that the PSNE of the optimization problem described in (3.16) take. Normally, the propositions would be in terms of exogenous characteristics only. I present the propositions in terms of $\bar{e}_{j(i)}$ and $\bar{e}_k$ rather than in terms of only exogenous variables for two reasons. First, the conditions in terms of only exogenous variables can be found, but are tedious and difficult to interpret. Second, the information gained from putting the propositions in terms of only exogenous variables is limited because doing so abstracts from how the student's effort choice depends on peer effort; keeping the peer effort variables in the propositions, on the other hand, allows the propositions to retain the insight of the peer effect approach.

Propositions 1-3 characterize all of the potential equilibria of the effort choice game by finding the equilibria for all possible values of θ_i. Let

$$\mathrm{A} \equiv -\left(s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k \right) \tag{3.21}$$

$$\mathrm{B} \equiv \beta \hat{e} + s_{j(i)}^{\frac{1}{2}} \left(\hat{e} - \bar{e}_{j(i)} \right) + \sum_k s_k^2 \left(\hat{e} - \bar{e}_k \right). \tag{3.22}$$

Proposition 1 (Interior PSNE) *If all C students have θ_i such that $A \leq \theta_i \leq B$, then a PSNE to the effort choice game is $\mathbf{e}^*$ such that*

$$e_i^* = \frac{\theta_i + s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2} \tag{3.23}$$

for all i.

Proof. Since $U_i(\cdot)$ is concave in e_i and both constraints are convex functions of e_i, the KKT Sufficiency Theorem states that conditions (3.18)-(3.20) are both necessary and sufficient for a maximum to (3.16) By condition (3.18),

$$\theta_i = \beta e_i + s_{j(i)}^{\frac{1}{2}} \left(e_i - \bar{e}_{j(i)}\right) + \sum_k \left(s_k^2 \left(e_i - \bar{e}_k\right)\right) - \lambda_{i,1} + \lambda_{i,2}. \tag{3.24}$$

Consider $\theta_i = \text{A} = -\left(s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k\right)$, the smallest value that θ_i can take. From (3.18),

$$0 = \beta e_i + s_{j(i)}^{\frac{1}{2}} e_i + \sum_k s_k^2 e_i - \lambda_{i,1} + \lambda_{i,2}. \tag{3.25}$$

If $\lambda_{i,1} > 0$, then, by (3.19), $e_i^* = 0$, which implies that $0 = -\lambda_{i,1} + \lambda_{i,2}$. $0 = -\lambda_{i,1} + \lambda_{i,2}$ requires that $\lambda_{i,2} > 0$. By (3.20), however, $\lambda_{i,2} > 0$ requires that $e_i^* = \hat{e}$, which is impossible since (3.19) requires that $e_i^* = 0$ and $\hat{e} > 0$ by definition. It follows that $\lambda_{i,1} = 0$.

If $\lambda_{i,2} > 0$, then (3.20) implies that $e_i^* = \hat{e}$. For (3.25) to hold, $\lambda_{i,1} > 0$, which is impossible since $\lambda_{i,2} > 0$ requires that $e_i^* = \hat{e}$. It follows that $\lambda_{i,1} = \lambda_{i,2} = 0$, which implies that

$$\mathcal{L}_{e_i} = \theta_i - \beta e_i - s_{j(i)}^{\frac{1}{2}} e_i + s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} - \sum_k \left(s_k^2 e_i - s_k^2 \bar{e}_k\right) = 0. \tag{3.26}$$

Finally, (3.26) implies that

$$e_i^* = \frac{\theta_i + s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2}. \tag{3.27}$$

For A $< \theta_i <$ B, I can use the same logic to show that $\lambda_{i,1} = \lambda_{i,2} = 0$ and

$$e_i^* = \frac{\theta_i + s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2}. \tag{3.28}$$

Next, consider $\theta_i = \text{B} = \beta \hat{e} + s_{j(i)}^{\frac{1}{2}} \left(\hat{e} - \bar{e}_{j(i)}\right) + \sum_k s_k^2 \left(\hat{e} - \bar{e}_k\right)$, the largest value that θ_i can take. From (3.18),

$$0 = -\lambda_{i,1} + \lambda_{i,2}. \tag{3.29}$$

If $\lambda_{i,1} > 0$, then it is necessary that $\lambda_{i,2} > 0$ for the equality to hold; as demonstrated above, it is impossible for both $\lambda_{i,1} > 0$ and $\lambda_{i,2} > 0$ at the same time. It follows that $\lambda_{i,1} = \lambda_{i,2} = 0$ and

$$e_i^* = \frac{\theta_i + s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2}. \quad \blacksquare \tag{3.30}$$

Proposition 1 describes the conditions in which all students choose an effort level that is an interior solution to the student's maximization problem given in (3.16). Because e_i^* is a function of $\bar{e}_{j(i)}$ and $\bar{e}_k$ for all i, each student's effort choice depends in part on the effort choices of his peers.

Proposition 2 (Mixed PSNE) *If at least one student has θ_i such that $A \leq \theta_i \leq B$ and at least one student has θ_i such that* either $\theta_i <$ A or $\theta_i >$ B, *then a PSNE to the effort choice game is $\mathbf{e}^*$ such that the equilibrium effort choices* *are*

$$e_i^* = \begin{cases} \frac{\theta_i + s_{j(i)}^{\frac{1}{2}}\bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2}, & \text{if A} \leq \theta_i \leq \text{B} \\ 0, & \text{if } \theta_i < \text{A} \\ \hat{e}, & \text{if } \theta_i > \text{B} \end{cases} \tag{3.31}$$

Proof. As in the proof to Proposition 1, by the KKT Sufficiency Theorem, conditions (3.18)-(3.20) are both necessary and sufficient for a maximum to (3.16). For A $\leq \theta_i \leq$ B,

$$e_i^* = \frac{\theta_i + s_{j(i)}^{\frac{1}{2}}\bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2} \tag{3.32}$$

by the arguments in the proof to Proposition 1.

For $\theta_i <$ A, (3.18) becomes

$$0 > \beta e_i + s_{j(i)}^{\frac{1}{2}} e_i + \sum_k s_k^2 e_i - \lambda_{i,1} + \lambda_{i,2}. \tag{3.33}$$

For the inequality to hold, either $\lambda_{i,2} < 0$, which violates (3.20), or

$$\lambda_{i,1} > \beta e_i + s_{j(i)}^{\frac{1}{2}} e_i + \sum_k s_k^2 e_i + \lambda_{i,2} \tag{3.34}$$

For (3.34) to hold, it must be that $\lambda_{i,1} > 0$. By (3.19), then, $e_i^* = 0$.

For $\theta_i >$ B, (3.18) becomes

$$0 < -\lambda_{i,1} + \lambda_{i,2}. \tag{3.35}$$

Since $\lambda_{i,1} < 0$ violates (3.19) and $\lambda_{i,1}$ and $\lambda_{i,2}$ cannot simultaneously be greater than 0, it must be that $\lambda_{i,1} = 0$ and $\lambda_{i,2} > 0$. By (3.20), then, $e_i^* = \hat{e}$. ■

Proposition 2 demonstrates how the theoretical approach captures multiple types of peer effects discussed in the peer effects literature. Consider students 1 and 2, where $j(1) = j(2)$.[6]

First, consider the case when $\theta_1 < \mathrm{A}$, which means that $e_1^* = 0$. Next, assume that student 2 has a θ_2 value such that $\mathrm{A} \leq \theta_2 \leq \mathrm{B}$. By Proposition 1,

$$e_2^* = \frac{\theta_i + s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2}. \tag{3.36}$$

In this situation, student 1 selects $e_1^* = 0$ even though student 2 selects $e_2^* > 0$. If student 1 behaved according to the peer effects theory, then e_1^* would depend in part on e_2^*. But because $\theta_1 < \mathrm{A}$, student 1's effort choice is not affected by student 2's effort choice. Student 2, on the other hand, still considers student 1's effort choice when deciding how much effort to put forth; this is because student 1's effort choice depends in part on $\bar{e}_{j(i)}$ and $\bar{e}_{j(i)}$ is a function of e_1^*. In other words, student 1's low effort choice has a negative effect on student k's effort choice, but student 2's higher effort choice does not have a positive effect — or any effect at all — on student 1's effort choice.[7] It follows that this is a case in which peer effects are *one-way*.[8]

This finding is of interest because it captures an additional type of peer effect, the "bad apple" effect, in which the presence of a student with a low outcome has a negative effect on the rest of her peers Hoxby & Weingarth (2006). It is also of interest because it could help explain a situation in which a particular sub-group of students ends up at a low-effort equilibrium while students of other sub-groups end up at higher-effort equilibria. This situa-

[6] All results also hold for $j(1) \neq j(2)$.

[7] This situation and that given below are analogous to Akerlof and Kranton's 2002 case in which the student only cares about the private benefits and costs of effort. In Akerlof and Kranton 2002, the parameter that determines when this case arises is determined exogenously, whereas in this approach the parameter results from students' decisions.

[8] As far as I know, this is the first theoretical model to generate one-way peer effects endogenously.

tion would help explain the persistence of a black-white achievement gap, for example Hanushek et al. (2009).

Next, consider the case when $\theta_1 > \mathrm{B}$ and $\mathrm{A} \leq \theta_2 \leq \mathrm{B}$. By Proposition 2, $e_1^* = \hat{e}$ while

$$e_2^* = \frac{\theta_i + s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2}. \tag{3.37}$$

In this situation, student 1 selects $e_1^* = \hat{e}$ even though student 2 selects $e_2^* < \hat{e}$. Again, student 1's effort choice does not depend on student 2's effort choice, whereas student 2's effort choice is a function of $\bar{e}_{j(i)}$ and $\bar{e}_{j(i)}$ is a function of e_1^*. This example demonstrates another type of peer effect that is one-way. The peer effects literature refers to this type of peer effect as the "shining light" effect; here, the presence of a student with a high outcome has a positive effect on the rest of her peers Hoxby & Weingarth (2006). Furthermore, just like the bad apple effect, the shining light effect could result in divergence between the high effort sub-group and the lower-effort sub-groups.

Proposition 3 (Corner PSNE) *If all C students have θ_i such that either $\theta_i < A$ or $\theta_i > B$, then a PSNE to the effort choice game is $\mathbf{e}^*$ such that the equilibrium effort choices are*

$$e_i^* = \begin{cases} 0, & \text{if } \theta_i < \mathrm{A} \\ \hat{e}, & \text{if } \theta_i > \mathrm{B} \end{cases} \tag{3.38}$$

Proof. The proof for Proposition 3 follows directly from the proof for Proposition 2 above.

For the values of θ_i given in Proposition 3, *none* of the students' effort choices depends on that of his peers. In other words, there are no peer effects at all for the given values of θ_i.

Figure 3.2: Values of θ_i and Solution Types

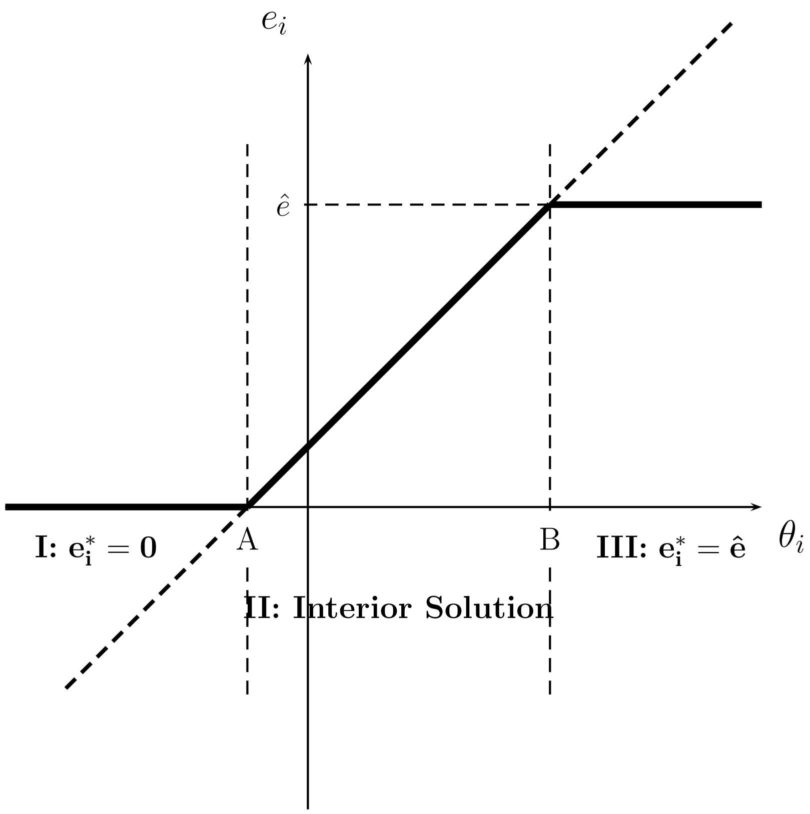


Figure 3.2 shows how e_i^* depends on the value of θ_i. For θ_i in Region I, $e_i^* = 0$. For θ_i in Region II,

$$e_i^* = \frac{\theta_i + s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2}. \tag{3.39}$$

And for Region III, $e_i^* = \hat{e}$.

In a classroom with students who have θ_i values that fall only in Region II, each student will be subject to the effort choices of of the peer group. Such a θ distribution results in most dramatic changes in equilibrium effort in response to changes in exogenous characteristics. A classroom with θ_i values in both Regions I and II will exhibit one-way bad apple peer effects. A classroom with θ_i values in both Regions II and III will exhibit one-way shining light peer effects. And a classroom with θ_i values in all three regions will exhibit both bad apple and shining light peer effects.

3.5 Hypotheses

In this section, I formulate hypotheses related to equilibrium effort. The first five hypotheses relate to the Interior equilibrium effort choice,

$$e_i^* = \frac{\theta + s_{j(i)}^{\frac{1}{2}} \bar{e}_{j(i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2}. \tag{3.40}$$

The last two hypotheses relate to the Mixed equilibrium effort choices, $e_i^* = 0$ and $e_i^* = \hat{e}$.

3.5.1 Own Racial Peer Group Effort

The first relationship of interest is how e_i^* responds to a change in $\bar{e}_{j(i)}$, average effort put forth by peers who are of the same race as student i:

$$\frac{\partial e_i^*}{\partial \bar{e}_{j(i)}} = \frac{s_{j(i)}^{\frac{1}{2}}}{\beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2}. \tag{3.41}$$

To simplify this result and future results, let $D_i \equiv \beta + s_{j(i)}^{\frac{1}{2}} + \sum_k s_k^2$. It follows that

$$\frac{\partial e_i^*}{\partial \bar{e}_{j(i)}} = \frac{s_{j(i)}^{\frac{1}{2}}}{D_i}. \tag{3.42}$$

To determine the sign of (3.42),

$$s_{j(i)} > 0, \beta > 0, s_k \geq 0 \Rightarrow \frac{\partial e_i^*}{\partial \bar{e}_{j(i)}} > 0 \tag{3.43}$$

In other words, if the effort level of the student's racial peer group increases, the student's effort will increase as well. This partial derivative provides a hypothesis to be tested in Chapter 4.

- $H_{1,0}$: *An increase in the student's own racial peer group's effort either does not affect effort or decreases effort.*
- $H_{1,A}$: *An increase in the student's own racial peer group's effort increases effort.*

3.5.2 Other Racial Peer Group Effort

Another relationship to examine is how e_i^* responds to a change in $\bar{e}_k$, effort put forth by a peer group of a race other than student i's:

$$\frac{\partial e_i^*}{\partial \bar{e}_k} = \frac{s_k^2}{D_i}. \tag{3.44}$$

Since $\bar{e}_k$ must be greater than zero, it follows that $s_k > 0$ as well. This means that $\frac{\partial e_i^*}{\partial \bar{e}_k}$ is also positive. This partial derivative provides a second hypothesis to be tested in Chapter 4.

- $H_{2,0}$: *An increase in the mean effort level of another racial group either does not affect effort or decreases effort.*
- $H_{2,A}$: *An increase in the mean effort level of another racial group increases effort.*

3.5.3 Strength of Peer Effect by Racial Group

A third result of interest is how the magnitude of $\frac{\partial e_i^*}{\partial \bar{e}_{j(i)}}$ compares to that of $\frac{\partial e_i^*}{\partial \bar{e}_k}$. By inspection, for a given $\bar{s} \in (0, 0.5]$,

$$\frac{\partial e_i^*}{\partial \bar{e}_{j(i)}} = \frac{\bar{s}^{\frac{1}{2}}}{D_i} > \frac{\partial e_i^*}{\partial \bar{e}_k} = \frac{\bar{s}^2}{D_i} \tag{3.45}$$

And for any $s_{j(i)} > s_k$, the same inequality given in 3.45 holds. This result indicates that for a given classroom racial share, student i's effort choice is more influenced by students of the same race as student i than by students of different races. This result provides a third hypothesis to be tested empirically:

- $H_{3,0}$: *For a given racial group share, peer effort of the student's own racial group has the same or weaker effect on effort than does the peer effort of each other racial group.*
- $H_{3,A}$: *For a given racial group share, peer effort of the student's own racial group has a stronger effect on effort than does the peer effort of each other racial group.*

3.5.4 Strength of Peer Effect and Own Racial Group Share

A fourth result of interest is how the strength of the peer effect changes as the student's own racial group share changes:

$$\frac{\partial e_i^{*2}}{\partial \bar{e}_{j(i)} \partial s_{j(i)}} = \frac{1}{2} s_j^{-\frac{1}{2}} \left(\beta + s_j^{\frac{1}{2}} + \sum_k s_k^2\right)^{-1} \left[1 - s_j^{\frac{1}{2}} \left(\beta + s_j^{\frac{1}{2}} + \sum_k s_k^2\right)^{-1}\right] \tag{3.46}$$

Since the first two terms are positive, the difference inside the square brackets determines the sign of $\frac{\partial e_i^{*2}}{\partial \bar{e}_{j(i)} \partial s_{j(i)}}$. $\beta > 0$ implies that $\left(\beta + s_j^{\frac{1}{2}} + \sum_k s_k^2\right) > s_j^{\frac{1}{2}}$ for all s_j, s_k, which means that the difference inside the square brackets is greater than zero. It follows that $\frac{\partial e_i^{*2}}{\partial \bar{e}_{j(i)} \partial s_{j(i)}} > 0$. This result means that the strength of the peer effect from the student's own racial group increases as the student's own racial group's classroom share increases. A fourth hypothesis to be tested empirically is the following:

- $H_{4,0}$: *As the student's own racial group's classroom share increases, peer effort of the student's own racial group has the same or weaker effect on effort.*

- $H_{4,A}$: *As the student's own racial group's classroom share increases, peer effort of the student's own racial group has a stronger effect on effort.*

3.5.5 Strength of Peer Effect and Other Racial Group Share

A fifth result of interest is how the strength of the peer effect changes as each other racial group share changes:

$$\frac{\partial e_i^{*2}}{\partial \bar{e}_k \partial s_k} = 2 s_k \left(\beta + s_j^{\frac{1}{2}} + \sum_k s_k^2\right)^{-1} \left[1 - s_k^2 \left(\beta + s_j^{\frac{1}{2}} + \sum_k s_k^2\right)^{-1}\right]. \quad (3.47)$$

As in the case for 3.46, the term inside the square brackets is greater than zero for all β, s_j, s_k, which means that $\frac{\partial e_i^{*2}}{\partial \bar{e}_k \partial s_k} > 0$. This result means that the strength of the peer effect from a racial group other than the student's own increases as that racial group's classroom share increases. A fifth hypothesis to be tested empirically is the following:

- $H_{5,0}$: *As the classroom share of a racial group other than the student's own increases, peer effort of that racial group has the same or weaker effect on effort.*

- $H_{5,A}$: *As the classroom share of a racial group other than the student's own increases, peer effort of that racial group has a stronger effect on effort.*

3.5.6 Bad Apple Effect

For the next two hypotheses, I examine the Mixed equilibrium effort choices. First, consider $\theta_i < A$ for at least one student i and $A \leq \theta_{-i} \leq B$ for at least one student $-i$. By Proposition 2, this distribution of θ values results in a Mixed PSNE in which $e_i^* = 0$ and

$$e_{-i}^* = \frac{\theta_{-i} + s_{j(-i)}^{\frac{1}{2}} \bar{e}_{j(-i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(-i)}^{\frac{1}{2}} + \sum_k s_k^2}. \tag{3.48}$$

In this situation, student i inflicts a one-way bad apple peer effect on student $-i$; in other words, student i's effort choice has a negative effect on student $-i$'s effort choice, but student $-i$'s effort choice does not affect student i's effort choice at all. A sixth hypothesis is the following:

- $H_{6,0}$: *The strength of the peer effect on effort is the same as or greater for student i with $\theta_i < A$ than it is for student $-i$ with $A \leq \theta_{-i} \leq B$.*

- $H_{6,A}$: *The strength of the peer effect on effort is less for student i with* $\theta_i < A$ *than it is for student* $-i$ *with* $A \leq \theta_{-i} \leq B$.

3.5.7 Shining Light Effect

Next, consider $\theta_i > B$ for at least one student i and $A \leq \theta_{-i} \leq B$ for at least one student $-i$. By Proposition 2, this distribution of θ values results in a Mixed PSNE in which $e_i^* = \hat{e}$ and

$$e_{-i}^* = \frac{\theta_{-i} + s_{j(-i)}^{\frac{1}{2}} \bar{e}_{j(-i)} + \sum_k s_k^2 \bar{e}_k}{\beta + s_{j(-i)}^{\frac{1}{2}} + \sum_k s_k^2}. \tag{3.49}$$

In this situation, student i inflicts a one-way shining light peer effect on student $-i$; in other words, student i's effort choice has a positive effect on student $-i$'s effort choice, but student $-i$'s effort choice does not affect student i's effort choice at all. A seventh hypothesis is the following:

- $H_{7,0}$: *The strength of the peer effect on effort is the same as or greater for student i with* $\theta_i > B$ *than it is for student* $-i$ *with* $A \leq \theta_{-i} \leq B$.

- $H_{7,A}$: *The strength of the peer effect on effort is less for student i with* $\theta_i > B$ *than it is for student* $-i$ *with* $A \leq \theta_{-i} \leq B$.

3.6 Comparative Statics

Next, I examine how Interior equilibrium effort changes in response to changes in exogenous characteristics. Each of these comparative statics results indicates that a change in an exogenous characteristic, be it student-level or classroom-level, has an effect the reverberates beyond the individual.

3.6.1 Peer Exogenous Characteristic: S_{-i}

The first comparative static result of interest is how e_i^* responds to a change in $S_{-i,n} \in \mathbf{S_{-i}}$, an exogenous characteristic specific to student $-i$. This comparative statics result helps us understand the effect that a change in the composition of student i's peer group has on student i's effort.[9]

Consider what happens to student i's effort when exogenous characteristic n of classmate $-i$ changes. First, let $j(i) = j(-i)$. in other words, students i and $-i$ are of the same race:

$$\frac{\partial e_i^*}{\partial S_{-i,n}} = \left(\frac{\partial e_{-i}^*}{\partial S_{-i,n}}\right)\left(\frac{\partial \bar{e}_j^*}{\partial e_{-i}^*}\right)\left(\frac{\partial e_i^*}{\partial \bar{e}_j^*}\right) \tag{3.50}$$

The first term in (3.50), $\frac{\partial e_{-i}^*}{\partial S_{-i,n}}$, represents the effect that the change in student $-i$'s exogenous characteristic has on student $-i$'s effort. For student i,

$$\frac{\partial e_i^*}{\partial S_{i,n}} = \frac{\alpha_n}{D_i}. \tag{3.51}$$

By (3.51),

$$\frac{\partial e_{-i}^*}{\partial S_{-i,n}} = \frac{\alpha_n}{D_{-i}}. \tag{3.52}$$

The second term, $\left(\frac{\partial \bar{e}_j^*}{\partial e_{-i}^*}\right)$, captures how a change in student $-i$'s optimal effort affects the different measures of peer effort. Since $j(i) = j(-i)$,

$$\frac{\partial \bar{e}_{j(i)}^*}{\partial e_{-i}^*} = \frac{1}{p_{j(i)} - 1}. \tag{3.53}$$

[9] Boston's Metco program, a desegregation program that bussed students from Boston's schools to more affluent suburbs, is an example of a program that changes the composition of the peer group. See Angrist and Lang 2004 for details.

By (3.42) and (3.53), then,

$$\frac{\partial e_i^*}{\partial S_{-i,n}}_{\{j(i)=j(-i)\}} = \left(\frac{\alpha_n}{D_{-i}}\right)\left(\frac{1}{p_{j(i)}-1}\right)\left(\frac{s_j^{\frac{1}{2}}}{D_i}\right) \tag{3.54}$$

Next, let $j(i) \neq j(-i)$, which means that student $-i$ is not of the same race as student i. In this case,

$$\frac{\partial \bar{e}_k^*}{\partial e_{-i}^*} = \frac{1}{p_k}. \tag{3.55}$$

By (3.44) and (3.55),

$$\frac{\partial e_i^*}{\partial S_{-i,n}}_{\{j(i)\neq j(-i)\}} = \left(\frac{\alpha_n}{D_i}\right)\left(\frac{1}{p_k}\right)\left(\frac{s_k^2}{D_i}\right). \tag{3.56}$$

What do (3.54) and (3.56) tell us? In the absence of peer effects, both terms would equal zero. After all, in the game in which student i's utility is purely private — and no peer effects existed — there would be no reason to expect a change in student $-i$'s exogenous characteristic to affect student i's effort. Since $\frac{\partial \theta_{-i}}{\partial S_{-i,n}} = \alpha_n$, however, (3.54) and (3.56) tell us that student i's peers do matter and that a change in one of student $-i$'s exogenous characteristics can have an impact beyond just student $-i$'s effort level. This result is the first indication that a *social multiplier* might exist; I examine this issue in detail in Section 3.7.

3.6.2 Change in Racial Composition of the Classroom

What happens to student i's optimal effort choice if the racial composition of the classroom changes? Consider what happens when there are two racial groups in the classroom ($J = 2$). Many classrooms fit this description and starting with such a case makes the results easier to interpret.

Consider students $1, 2 \in C$ and $w \notin C$; in other words, student w is not currently in the classroom. Let $j(1) = 1$ and let all exogenous characteristics $\mathbf{S}$ be the same for students 2 and w except for race: $j(2) = 2$ and $j(w) = 1$. And let $e_2^* = e_w^*$. This means that the only thing that differentiates student 2 from student w is race. For some undisclosed reason, student 2 leaves the classroom permanently and is replaced by student w. As a result, the classroom shares of racial groups 1 and 2 change. What effect does this change in classroom racial composition have on student 1's effort, if any at all?

First, rewrite e_i^* for student 1 and the case of two racial groups, using the result that $s_2 = 1 - s_1$ and equations (3.13) and (3.14):

$$e_1^* = \frac{\theta + (s_1)^{\frac{1}{2}}\,\bar{e}_1 + (1 - s_1)^2\,\bar{e}_2}{\beta + (s_1)^{\frac{1}{2}} + (1 - s_1)^2}. \tag{3.57}$$

Next, perform the comparative statics:

$$\frac{\partial e_1^*}{\partial s_1} = \left(\beta + s_1^{\frac{1}{2}} + (1 - s_1)^2\right)^{-1} \times \left[\frac{1}{2}(s_1)^{-\frac{1}{2}}\,\bar{e}_1 + (2s_1 - 2)\,\bar{e}_2 - \frac{\left(\theta + (s_1)^{\frac{1}{2}}\,\bar{e}_1 + (1 - s_1)^2\,\bar{e}_2\right)\left(\frac{1}{2}(s_1)^{-\frac{1}{2}} - 2 + 2s_1\right)}{\left(\beta + s_1^{\frac{1}{2}} + (1 - s_1)^2\right)}\right] \tag{3.58}$$

Since the first term of (3.58) is greater than zero, it is the difference inside the square brackets that determines the sign of $\frac{\partial e_1^*}{\partial s_1}$. The sign of this difference depends on the size of s_1 and the relative magnitudes of $\bar{e}_1$ and $\bar{e}_2$. This makes sense since this comparative statics result examines how student 1's effort changes in response to a change in the classroom's racial balance. Whether or not her effort will increase depends on her racial group's classroom share, how high her own racial peer group's effort is, and how high the other racial peer group's effort is. What is of note, however, is that $\frac{\partial e_1^*}{\partial s_1} = 0$ only under very particular circumstances; in other words, a change in the classroom's racial composition will almost always have an effect on the student's effort.

3.6.3 Exogenous Characteristic Common to all C students

Next, consider a change to T_m, an exogenous characteristic common to all C students. This comparative statics result illustrates what happens when a class gets a different teacher, the school institutes a policy to reward effort, or the school decides to punish low effort, for example.

This comparative static result can be split into three parts, the *private* effect, the effect of student i's *own racial group*, and the effect of *other racial groups*:

$$\frac{\partial e_i^*}{\partial T_m} = \underbrace{\frac{\gamma_m}{D_i}}_{\text{private}} + \underbrace{\frac{s_j^{\frac{1}{2}} \frac{\partial \bar{e}_j^*}{\partial T_m}}{D_i}}_{\text{own racial group}} + \underbrace{\frac{\sum_k s_k^2 \frac{\partial \bar{e}_k^*}{\partial T_m}}{D_i}}_{\text{other racial groups}}. \tag{3.59}$$

The private effect captures how much student i's optimal effort choice changes without any regard for how other students react to the change. This term measures how student i would react to a change in T_m in the *absence of* peer effects.

The own racial group effect tells us how much student i's optimal effort choice changes in response to the change in his own racial group's effort choice induced by the change to T_m. By (3.42), $\frac{s_j^{\frac{1}{2}}}{D_i} = \frac{\partial e_i^*}{\partial \bar{e}_j^*}$, which makes sense. But the remaining term of the own racial group effect, $\frac{\partial \bar{e}_j^*}{\partial T_m}$, requires further investigation because each of the students in student i's racial group is also responding to the change in T_m. Specifically,

$$\frac{\partial \bar{e}_j^*}{\partial T_m} = \frac{\partial \bar{e}_j}{\partial e_{-i}} \sum_{-i_{j(i)=j(-i)}} \frac{\partial e_{-i}}{\partial T_m} \tag{3.60}$$

Here, the first term represents the effect that a change in $-i$'s effort has on the peer effort measure $\bar{e}_j$. The summation aggregates the private effect that the change to T_m has on each $-i$. Since each of the other $p_{j(i)} - 1$ students in student i's racial group also responds to the same change in T_m,

$$\sum_{-i_{j(i)=j(-i)}} \frac{\partial e_{-i}}{\partial T_m} = \left(p_{j(i)} - 1\right) \gamma_m \tag{3.61}$$

By (3.53) and (3.61), then,

$$\frac{\partial \bar{e}_j^*}{\partial T_m} = \left(\frac{1}{p_{j(i)} - 1}\right) \left(p_{j(i)} - 1\right) \gamma_m = \gamma_m. \tag{3.62}$$

It follows that the own racial group effect is

$$\frac{s_j^{\frac{1}{2}} \gamma_m}{D_i}. \tag{3.63}$$

The last term in (3.60), the effect of other racial groups, is similar to that for the own racial group except that there is a summation across the other racial groups:

$$\sum_k \frac{s_k^2 \gamma_m}{D_i} \tag{3.64}$$

Finally, the complete effect of a change to T_m on student i's optimal effort is

$$\frac{\partial e_i^*}{\partial T_m} = \underbrace{\frac{\gamma_m}{D_i}}_{\text{private}} + \underbrace{\frac{s_j^{\frac{1}{2}}\gamma_m}{D_i}}_{\text{own racial group}} + \underbrace{\sum_k \frac{s_k^2\gamma_m}{D_i}}_{\text{other racial groups}} \tag{3.65}$$

This comparative static result also indicates that this model generates a social multiplier. After all, in the absence of peer effects, $\frac{\partial e_i^*}{\partial T_m}$ would simply be equal to the private effect of a change to T_m on e_i^*. But (3.65) tells us that the effect of a change to T is potentially much larger than just the private effect. I formally demonstrate this in Section 3.7.

3.7 The Social Multiplier

The three comparative statics results above indicate that a change in a student- or classroom-level exogenous characteristic has an effect beyond just the individual for whom the characteristic changes. These findings provide preliminary evidence that a *social multiplier* exists. The social multiplier is the idea that interactions between individuals amplify a change in incentives faced by an individual. From a macro perspective, a social multiplier exists if the aggregate impact of the policy shift is larger than the sum of the direct individual impacts. This section formally defines the social multiplier and discusses how the model and comparative statics results demonstrate the conditions in which a social multiplier exists.

3.7.1 Definition of the Social Multiplier

Scheinkman's 2008 description serves as the basis for the definition of the social multiplier:

> The *social multiplier* measures the ratio of the effect on the average action caused by a change in a parameter to the effect on the average action that would occur if individual agents ignored the change in actions of their peers.

Let $\mathbf{W} \in \mathbb{R}_+^{m+n}$ represent all exogenous characteristics, both those that are student-specific ($\mathbf{S_i}$) and those that are common to all students ($\mathbf{T}$). Let $w = 1, \ldots, (m+n)$ index the components of $\mathbf{W}$. And let $\tilde{e}_i \in [0, \hat{e}]$.

Define the social multiplier as

$$M_{W_w} \equiv \frac{\frac{1}{C}\sum_i \frac{\partial e_i^*}{\partial W_w}}{\frac{1}{C}\sum_i \frac{\partial e_i^*}{\partial W_w}_{\{e_{-i}=\tilde{e}_{-i}\forall -i\}}}. \tag{3.66}$$

The numerator of (3.66) represents the average of the changes to e_i^* caused by a change to exogenous characteristic W_w. The denominator represents the average of the changes e_i^* caused by a change to an exogenous characteristic *when each student's effort other than student i's effort is restricted to be constant.* By holding effort constant for all peers, the denominator captures how much student i's equilibrium effort would change if student i ignored the changes in effort of his peers. As a result, M_{W_w} measures the degree to which the effect that a change in an exogenous variable generates is *multiplied* across the reference group.

$M_{W_w} > 1$ indicates that a positive social multiplier is present; in this situation, the effect of a change in an exogenous characteristic is amplified across the population. $M_{W_w} < 1$ indicates that the social multiplier is negative;

here, the effect of a change in an exogenous characteristic is made weaker across the population. And $M_{W_w} = 1$ indicates that a social multiplier does not exist.

3.7.2 Quantifying the Social Multiplier

Next, I use the comparative statics results from above to examine whether or not a social multiplier exists for changes to $S_{i,n}$ and T_m.

First, consider a change to $S_{i,n}$, exogenous characteristic n of student i. Results (3.54) and (3.56) from Section 3.6.1 indicate that this comparative statics result could be evidence of a social multiplier. By (3.54) and (3.56), the social multiplier for a change to $S_{i,n}$ is

$$M_{S_i} = \frac{\frac{1}{C}\left[\frac{\alpha_n}{D_i} + \sum_{-i_{\,j(-i)=j(i)}} \left(\frac{\alpha_n}{D_i}\right)\left(\frac{1}{p_{j(-i)}-1}\right)\left(\frac{s^{\frac{1}{2}}_{j(-i)}}{D_{-i}}\right)\right]}{\frac{1}{C}\left(0+0+\cdots+0+\frac{\alpha_n}{D_i}+0+\cdots+0+0\right)} + \tag{3.67}$$

$$\frac{\frac{1}{C}\sum_{-i_{\,j(-i)\neq j(i)}} \left(\frac{\alpha_n}{D_i}\right)\left(\frac{1}{p_{j(-i)}}\right)\left(\frac{s^{2}_{j(-i)}}{D_{-i}}\right)}{\frac{1}{C}\left(0+0+\cdots+0+\frac{\alpha_n}{D_i}+0+\cdots+0+0\right)}. \tag{3.68}$$

After simplification,

$$M_{S_i} = 1 + \sum_{-i_{\{j(-i)=j(i)\}}} \left(\frac{1}{p_{j(-i)}-1}\right)\left(\frac{s^{\frac{1}{2}}_{j(-i)}}{D_{-i}}\right) + \sum_{-i_{\{j(-i)\neq j(i)\}}} \left(\frac{1}{p_{j(-i)}}\right)\left(\frac{s^{2}_{j(-i)}}{D_{-i}}\right). \tag{3.69}$$

Since $s_{j(i)}$ is the same for all i such that $j(-i) = j(i)$ and $s_{k(i)}$ is the same for all i such that $j(-i) = j(i)$,

$$M_{S_i} = 1 + \frac{s_{j(i)}^{\frac{1}{2}}}{D_i} + \sum_k \frac{s_k^2}{D_k}. \tag{3.70}$$

Since the sum of the second two terms is always greater than 0, $M_{S_i} > 1$, which indicates that there is a positive social multiplier when there is a change to one of student i's exogenous characteristics. So the magnitude of M_{S_i} depends on β and the racial group shares.

Next, consider a change to T_m, an exogenous classroom- or school-level characteristic that is common to all C students. As indicated by result (3.65) in Section 3.6.3, this comparative statics result could also provide evidence of a social multiplier. By (3.65), the social multiplier for a change to T_m is

$$M_{T_m} = \frac{\left(\frac{\gamma_m}{D_1} + \frac{\gamma_m s_{j(1)}^{\frac{1}{2}}}{D_1} + \sum_k \frac{\gamma_m s_{k(1)}^2}{D_1}\right) + \cdots + \left(\frac{\gamma_m}{D_C} + \frac{\gamma_m s_{j(C)}^{\frac{1}{2}}}{D_C} + \sum_k \frac{\gamma_m s_{k(C)}^2}{D_C}\right)}{\frac{\gamma_m}{D_1} + \cdots + \frac{\gamma_m}{D_C}} \tag{3.71}$$

Rearranging terms leads to

$$M_{T_m} = \frac{\frac{\gamma_m}{D_1}\left(1 + s_{j(1)}^{\frac{1}{2}} + \sum_k s_{k(1)}^2\right) + \cdots + \frac{\gamma_m}{D_C}\left(1 + s_{j(C)}^{\frac{1}{2}} + \sum_k s_{k(C)}^2\right)}{\frac{\gamma_m}{D_1} + \cdots + \frac{\gamma_m}{D_C}} \tag{3.72}$$

Since $s_{j(i)}^{\frac{1}{2}} + \sum_k s_{k(i)}^2 > 0$ for all i, $M_{T_m} > 1$, meaning that there is a positive social multiplier in this case as well.

3.8 Chapter Summary

The theoretical model of peer effects presented in this chapter improves upon the existing peer effects literature in the following ways. It uses findings from the sociology and social interactions literatures as assumptions that motivate how the student behaves. It then develops a behavioral model that specifies how the peer effect takes place by focusing on how a student's effort choice depends on the actions and characteristics of her peer group. Specifically, it shows how classroom racial composition affects effort, the endogenous variable. As a result, the model provides a possible explanation of how race and racial diversity affect the student's educational outcome. Furthermore, it has the flexibility to explain how other exogenous characteristics such as gender, income, and ability impact the student through peer influence.

The model results in hypotheses relating the student's effort to peer effort and classroom racial composition that can be directly tested empirically. The comparative statics results describe how peer effects depend on the model's exogenous variables. These comparative statics results provide evidence that a change to an exogenous variable reverberates beyond the individual student — a potential social multiplier. Finally, the theoretical model demonstrates how the social multiplier takes place, the conditions in which it exists, and the magnitude of the social multiplier in terms of the theoretical model's exogenous parameters.

CHAPTER 4

Empirical Estimation of Peer Effects

4.1 Introduction

This chapter empirically investigates how the student's effort depends on peer effort. First, it describes the dataset used to test the theoretical model's hypotheses; this dataset results from a combination of survey and administrative data from the Charlotte Mecklenburg School (CMS) district in North Carolina. Next, it presents the empirical model and describes how it addresses challenges to the estimation of peer effects including the reflection problem, correlated effects, and selection bias. It then interprets the empirical model's estimation results in terms of the hypotheses from Chapter 3. Finally, it calculates the implications that peer effects have for three policy interventions: a change in classroom racial composition, a reduction in the time a student spends working at a part time job, and an improvement in socioeconomic status. For the last two interventions, a social multiplier is estimated.

4.2 Data

4.2.1 Overview of the CMS District

The history of the Charlotte Mecklenburg School district makes it an interesting place to study peer effects, especially peer effects that depend on the racial mix of the classroom. This is because CMS has played a pivotal role in the history of desegregation in the United States.[1] In the landmark 1971 *Swann v. Charlotte-Mecklenburg Schools* 1971 decision, the Supreme Court upheld the use of mandatory within-district busing as a way to overcome a history of segregated education in the CMS district. Mandatory busing continued for over twenty years until 1992, when the district ended the program in favor of a system of controlled choice among magnet schools. While controlled choice allotted students through an application lottery process, it still mandated a 60%/40% white/black balance for each school. As a result of these desegregation measures, CMS classrooms should be more racially balanced than they would have been in the absence of these policies.

Recent history also makes the CMS district an important place to study peer effects. In 1997 — the year in which the students were surveyed for the CMS data set — the parent of a white student sued the CMS district because his daughter was denied entrance to a magnet school on the basis of race. This lawsuit set in motion a series of hearings on CMS desegregation that lasted until September 2001, when, after nearly three years in the courts, the Fourth Circuit Court of Appeals affirmed an earlier ruling that the CMS district had achieved "unitary" status; this ruling meant that the district was sufficiently

[1] Unless otherwise noted, the following description of the history of the CMS district is based on Charlotte Mecklenburg Schools 2010.

desegregated and mandated that CMS end race-based desegregation policies. Since that ruling, student assignment has been governed by a neighborhood school-based assignment program with an option to enroll in a magnet school in which race is not a factor.[2] With the rules that govern how parents select a school for their children in such flux, it is even more important to attempt to understand how peer effects take place in the classroom.

4.2.2 Sample

The CMS data set results from a survey conducted by Roslyn Mickelson in 1997.[3] The survey randomly sampled 8th grade English classrooms from every middle school (MS) and 12th grade English classrooms from every high schools (HS) in the CMS district.[4] *Each* student in each selected classroom was surveyed; on average, 90% of the selected students participated in the survey, resulting in observations for 4,454 students, 2,658 in middle school and 1,796 in high school.[5] The fact that the CMS data has observations for complete classrooms of students allows me to estimate peer effects exactly as they are formulated in Chapter 3, which is at the classroom level.[6]

[2]For a more detailed discussion of CMS's role in the history of desegregation, see Mickelson 2001; 2003.

[3]Mickelson describes the CMS data and her findings in Mickelson 2001; 2003

[4]English classrooms were chosen because English is the only subject that all students must take each year.

[5]Missing observations for EFFORT and STUDY, the two dependent variables discussed in Section 4.2.3, reduce the sample size by 72 for MS and 37 for HS.

[6]While students do not necessarily spend the entire school day together, then spend at least one class period together and likely more, since classrooms are stratified by track level.

4.2.3 Variables

The CMS data contains two variables that measure effort, the choice variable in the theoretical model in Chapter 3. The first, EFFORT, results from the question, "*How much effort do you put into your schoolwork*?" with responses

1. Don't try at all.
2. Just enough to get by.
3. Average amount of effort.
4. Try pretty hard but not as hard as I can.
5. As hard as I can.

The second measure of effort, STUDY, answers the question, "*When you are not at school or at a paid job, how many hours per day do you spend doing homework or studying*?" with responses

1. less than 1 hour
2. 1 hour
3. 2 hours
4. 3 hours
5. 4 or more hours

Other background and school-level characteristics include gender, ability, race, socioeconomic status (SES), hours spent working at a part-time job, whether or not the student lives with both parents, track, percent black in school, percentage of school's teachers with tenure, and percentage of school's teachers who are fully licensed. Section A.1 in Appendix A provides a detailed description of these variables; Table A.3 in Appendix A presents summary statistics.

4.3 Estimation

4.3.1 Student's Effort

There is little consensus on the best way to empirically measure a student's effort on schoolwork. One group of studies proposes that time spent on homework, STUDY, is the most representative measure of a student's effort. For one, because time spent on homework is a reported amount of time, it is more likely to be free of individual biases and interpretations than alternative measures of effort Carbonaro (2005). Furthermore, it is a relatively clear measure of how hard a student works because it is unconstrained by the scheduling practices of the school Natriello & McDill (1986): how much time a student chooses to spend on homework is for the most part up to the student.

Another group of studies favors the use of more subjective measures of student effort to assess how hard a student works. These studies argue that, since homework results from assignments determined by the teacher, time spent on homework captures the teacher's influence rather than the student's decision making Trautwein & Köller (2003). Subjective measures such as the EFFORT variable in the CMS data, on the other hand, directly elicit the student's opinion of how hard she works on school work. This type of measure of effort is important because it reflects how hard a student thinks he works relative to other students.[7]

This study's empirical approach uses two separate estimation specifications, one with EFFORT as the dependent variable and one with STUDY as the dependent variable. An advantage of this approach is that hypothesis testing will be robust to both measures of effort, not just one or the other. Furthermore, since EFFORT is a relative measure of effort while STUDY is more absolute, using both measures of effort presents a more complete picture of how hard a student works on schoolwork without overemphasizing or neglecting one measure or the other. Finally, when using multiple measures of effort, the limitations of any one measure of effort become less important.

4.3.2 Peer Measures of Effort

The explanatory variables central to this dissertation are the peer group measures of EFFORT and STUDY. Chapter 3 relates the student's effort to the mean of the student's own racial peer group's effort and the mean of other racial peer group effort levels. A natural way to examine this relationship empirically would be to take the mean of own and other racial peer group EFFORT and STUDY levels and investigate whether these measures had an effect on the student's own EFFORT and STUDY choices.

The ordinal natures of EFFORT and STUDY, however, complicate this approach because one cannot take the mean of an ordinal variable. The response options for EFFORT make EFFORT ordinal because the distances between options are not well defined and the responses themselves are not

[7]This more subjective measure of effort is used in Akerlof and Kranton 2002, De Fraja, Oliveira, and Zanchi forthcoming, and Fryer and Torelli 2010, among others.

comparable across students. And while it could be argued that STUDY is more cardinal than ordinal, the response options are discrete approximations of time spent studying rather than precise measures. Furthermore, the lower and upper bounds of the STUDY responses, “less than 1 hour” and “more than 4 hours,” are not well defined.

I propose the *proportions of* students that fall into particular EFFORT or STUDY categories, constructed by racial group, as an alternative central measure of peer effort.[8] These measures allow me to relate the student’s own effort choice to those of her peer groups without relying on the assumption that EFFORT and STUDY are cardinal variables.

These measures relate the student’s own effort to peer effort in the following way. If an increase in the proportion of students who select a particular effort level results in a higher probability that the student himself selects that effort level, then one explanation is that the student seeks to conform to the behavior of the peer group. If an increase in the peer proportion leads to a lower probability that the student selects that particular effort level or no change at all, then this finding would be evidence that conformity peer effects do not exist.

I calculate the proportion of students that select each level of effort for both the student’s own and other racial group. If there are 11 black students in the classroom, for example, and 3 choose EFFORT=4, then the value for E4OWN, the proportion of the student’s own racial group that selects EFFORT=4, for a black student who chooses EFFORT=3 is $\frac{3}{10} = 0.3$; for one of the black students who selects EFFORT=4, E4OWN would be $\frac{2}{10} = 0.2$. Section A.2

[8] I am not aware of any other studies that use this approach.

in Appendix A provides a detailed description of the construction of these variables.

I also include linear, squared, and cubed interaction terms with the classroom shares of the student's own and other racial peer groups for each of the E-OWN and E-OTHER variables. In the model of peer effects presented in Chapter 3, the strength of the peer effect increases as the racial group's classroom share increases. Including interaction terms allows the strength of the peer effects to depend on classroom share. The squared and cubed terms capture the any nonlinearities in how students interact with each other.

A final set of variables related to peer effort, EOWNVAR, EOTHERVAR, SOWNVAR, and SOTHERVAR, measure the variance across the each of the E-OWN, E-OTHER, S-OWN, and S-OTHER groups of variables.[9] These measures provide insight into whether or not the student's EFFORT and STUDY choices depend on how dispersed the peers' EFFORT and STUDY choices are. If low variance predicts higher effort, then it might be that a student does better in a classroom where the peers behave similarly to each other, regardless of how much effort they put forth. Table 4.1 provides a summary of the key explanatory variables using EFFORT=4 to illustrate.

4.3.3 Model Specification

I use ordered probit (OP) models with clustered standard errors to estimate how the student's effort depends on peer measures of effort and control variables. I estimate one OP model for EFFORT and the peer EFFORT variables

[9] I generate these measures of variance using an index of qualitative variation (IQV) that calculates the ratio of observed variation to the maximum expected variation.

Table 4.1: Explanatory Variables Related to EFFORT/STUDY=4

Variable	Explanation
POWN	share of the classroom is of the student's race
E4OWN	proportion of student's own racial group that selects EFFORT=4
E4OWNP	E4OWN × POWN
E4OWNPSQ	E4OWN × POWN^2
E4OWNPCU	E4OWN × POWN^3
S4OWN	proportion of student's own racial group that selects STUDY=4
S4OWNP	S4OWN × POWN
S4OWNPSQ	S4OWN × POWN^2
S4OWNPCU	S4OWN × POWN^3
POTHER	largest racial group classroom share that is not the student's own
E4OTHER	proportion of student's own racial group that selects EFFORT=4
E4OTHERP	E4OTHER × POTHER
E4OTHERPSQ	E4OTHER × POTHER^2
E4OTHERPCU	E4OTHER × POTHER^3
S4OTHER	proportion of student's own racial group that selects STUDY=4
S4OTHERP	S4OTHER × POTHER
S4OTHERPSQ	S4OTHER × POTHER^2
S4OTHERPCU	S4OTHER × POTHER^3

and one OP model for STUDY and the peer STUDY variables.[10] OP estimation accounts for the ordered, discrete natures of EFFORT and STUDY.[11]

Let $e_i^* \in \mathbb{R}_{++}$ represent student i's optimal effort choice and $s_i^* \in \mathbb{R}_{++}$ represent student i's optimal studying choice. Assume that the two latent variables, e_i^* and s_i^*, are determined by

$$e_i^* = \mathbf{x}_{1i}^{'}\beta_1 + \epsilon_{g1i} \tag{4.1}$$

$$s_i^* = \mathbf{x}_{2i}^{'}\beta_2 + \epsilon_{g2i} \tag{4.2}$$

[10]While the related nature of EFFORT and STUDY might lend itself to bivariate ordered probit estimation (BOP), BOP severely complicates inference, hypothesis testing, and interpretation of coefficients because one must individually examine each possible combination of EFFORT and STUDY outcomes. For this reason, I use separate ordered probit models. While BOP results in a small improvement in estimation efficiency, OP enables clearer interpretation of the estimation results in terms of the theoretical model's hypotheses. I include results for BOP estimation in Table A.9 in Appendix A for comparison.

[11]Section 21.3 of Greene 2003 explains how probit and logit models improve upon the linear probability model.

where $\mathbf{x}_{1i}$ and $\mathbf{x}_{2i}$ are vectors of explanatory variables, β_1 and β_2 are vectors of unknown parameters not containing intercepts, and ϵ_{g1i} and ϵ_{g2i} are the error terms.

In the data, we observe EFFORT and STUDY as ordered discrete variables. The relationship between e_i^* and EFFORT is then[12]

$$\text{EFFORT} = \begin{cases} 1 & \text{if} \quad 0 \leq e_i^* < \mu_{11} \\ 2 & \text{if} \quad \mu_{11} \leq e_i^* < \mu_{12} \\ 3 & \text{if} \quad \mu_{12} \leq e_i^* < \mu_{13} \\ 4 & \text{if} \quad \mu_{13} \leq e_i^* < \mu_{14} \\ 5 & \text{if} \quad \mu_{14} \leq e_i^* < \infty \end{cases} \tag{4.3}$$

where the threshold parameters satisfy the condition that $\mu_{11} < \mu_{12} < \mu_{13} < \mu_{14}$.

The probabilities for EFFORT are

$$\begin{aligned}
\text{Prob}\,(\text{EFFORT} = 1|\ \mathbf{x}_{1i}) &= \Phi\left(\mu_{11} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1}\right) \\
\text{Prob}\,(\text{EFFORT} = 2|\ \mathbf{x}_{1i}) &= \Phi\left(\mu_{12} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1}\right) - \Phi\left(\mu_{11} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1}\right) \\
\text{Prob}\,(\text{EFFORT} = 3|\ \mathbf{x}_{1i}) &= \Phi\left(\mu_{13} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1}\right) - \Phi\left(\mu_{12} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1}\right) \\
\text{Prob}\,(\text{EFFORT} = 4|\ \mathbf{x}_{1i}) &= \Phi\left(\mu_{14} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1}\right) - \Phi\left(\mu_{13} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1}\right) \\
\text{Prob}\,(\text{EFFORT} = 5|\ \mathbf{x}_{1i}) &= 1 - \Phi\left(\mu_{14} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1}\right).
\end{aligned}$$

[12]The details for STUDY are analogous to those for EFFORT.

The logarithmic likelihood of observation i for EFFORT is

$$\log\, L_i = \sum_{j=1}^{5} \log\left[\text{Prob}\left(\text{EFFORT} = j\right)\right], \tag{4.4}$$

which means that the log likelihood for the entire sample N is

$$\log\, L = \sum_{i=1}^{N} \log L_i. \tag{4.5}$$

Clustered standard errors allow the error terms to be correlated at the classroom level.[13] Failure to correct for this correlation normally results in standard errors that are too small, leading to incorrect inference in most cases Wooldridge (2003, 2006).[14] Errors could be correlated within the classroom because of the influence of the teacher, unobserved classroom dynamics, or external shocks that affect each student within a classroom similarly.

Explanatory variables for the main specification include the interaction terms described in Section 4.3.2 and control variables. Student-level control variables for the main specification include ability, race, sex, SES status, whether or not a student lives with both parents, and time spent at a part-time job. Classroom- and school-level controls include track, percent black in school, percentage of teachers tenured and licensed, whether or not the school is a magnet, and school fixed effects.

[13]Multilevel modeling, also known as hierarchical linear modeling, is an alternative to clustered standard errors. It estimates how much each level of analysis — individual, classroom, track, and school — contributes to the explanation of the model and how much each level contributes to the explanation of the error term Primo et al. (2007); Gelman (2007).

[14]In a study of mean wages using the Panel Study of Income Dynamics, Pepper 2002 demonstrates how failing to account for clustering at the union status and years of schooling levels result in a nearly 15% understatement of standard errors.

4.4 Identification

Any attempt to estimate peer effects confronts three challenges: the reflection problem, correlated effects, and endogenous peer group selection. The reflection problem arises because the student and peer group effort choices are determined simultaneously, and because of this simultaneity, the researcher has difficulty determining whether group behavior affects individual behavior or group behavior is simply an aggregation of individual behaviors.

In this study's formulation of peer effects, simultaneity is a concern because peer measures of EFFORT and STUDY, which are determined at the same time as the student's EFFORT and STUDY choices, appear on the right-hand side of the estimating equation. One characteristic of the empirical approach, the discrete choice nature of OP, addresses simultaneity problem because the discrete choice framework estimates the parameter coefficients by maximizing the log-likelihood, which converges even in the presence of simultaneity (Greene, 2003, pp. 715-716).[15]

Correlated effects arise if the group's average behavior is correlated with an unobserved exogenous group characteristic Manski (1993). If students within a classroom tend to put forth similar levels of effort because they have the same teacher or share an family background characteristic such as parents with a college education, for example, then correlated effects exist. Any estimate that attributes individual effort to peer group effort would overstate the peer effect in the presence of correlated effects.

[15]Brock and Durlauf 2001; 2006 illustrate in detail how the discrete choice framework overcomes the reflection problem.

The empirical approach attempts to overcome correlated effects in two ways. First, clustered standard errors allow for the possibility that observations for EFFORT and STUDY within the classroom are correlated in some way that other independent variables do not capture.[16] Second, estimation includes controls that account for characteristics that could be correlated at the group level; these controls include a track indicator, school fixed effects, the percentage of teachers tenured and licensed, and whether or not the school is a magnet school.

Selection bias is a special case of correlated effects that further complicates the estimation of the effect that the peer group has on an individual's actions or outcomes. This problem arises when the peer group itself is determined endogenously. If the individual selects his peer group, this selection can generate correlation between unobserved variables that, in turn, biases estimates of peer effects.

To address selection bias, I include school fixed effects in the estimation. School fixed effects account for similar parents' tendencies to select the same neighborhoods and schools for their children. I also include a magnet school indicator to account for selection bias that arises from similar parents' attempts to lottery into magnet schools. Even if parents' attempts to lottery into magnet schools generate selection bias, the fact that CMS still mandated a 60%/40% white/black balance at the time the data was collected mitigates the extent to which selection bias affects peer effects estimates.

Finally, I include the percentage of the school's student body that is black; this variable controls for selection bias by accounting for the effect that the

[16] While clustered standard errors do nothing to address any bias due to correlated effects, they do improve the efficiency of estimation.

school's racial composition has on parents' school choice decision, which also has the potential to generate selection bias. School racial composition is a major criterion on which parents base decisions related to school choice Glazerman (1997); Eckes (2006); Lankford & Wyckoff (2006). If parents of the same race select the same school based on school racial composition, then failing to control for the effect of school racial composition on the student's effort choice would risk ignoring correlation between variables that generate an effect on the choice variable.

4.5 Results

Interpreting estimated coefficients of an ordered probit model is complicated because neither the sign nor the magnitude of the coefficient gives us much information about how the dependent variable responds to a change in the explanatory variable. This difficulty arises for two reasons. First, in a discrete choice model, there is no conditional mean function to analyze; rather, one needs to calculate the probabilities themselves. Second, for the ordered choice model, even the partial effects cannot be interpreted decisively because there are more than two possible outcomes Greene & Hensher (2009). The sum of all changes due to a change in an independent variable must sum to zero because the probabilities of the effort choices must still sum to 1. This means that for a given explanatory variable with explanatory power, marginal effects will be negative for some outcomes and positive for others.

The problems with inference are even more severe for interaction terms. In linear models, the coefficient of an interaction term has a straightforward interpretation. In a nonlinear model such as OP, however, the marginal effect represented by the coefficient of the interaction term does not equal the true interaction effect of a change to the interaction term Ai & Norton (2003).[17] The true magnitude of the interaction effect and related confidence interval depend on the values of all of the independent variables, the magnitude of the change in the independent variable of interest, and even the independent variable's starting value Zelner (2009). Inference based on the reported marginal effect of an interaction variable will therefore be misleading.[18]

I take the following steps to interpret the estimation results of the OP model. First, I use marginal effects to determine how non-interaction terms affect EFFORT and STUDY. Second, I describe and implement a simulation-based technique that enables interpretation of the interaction terms.[19]

Significance for control variables can be determined directly from ordered probit estimation results; these results are presented in Tables A.4-A.9 in Appendix A.[20] Since one cannot interpret direction or magnitude of the effects of these variables directly from the OP estimates, I compute the marginal effects of significant control variables for each possible EFFORT and STUDY outcome and present these results in Tables A.10 and A.11 in Appendix A.[21]

[17] Ai and Norton 2003 reviewed 13 economics journals on JSTOR that used interaction terms in nonlinear models. Of the 72 articles published between 1980 and 1999, none of them interpreted the coefficient of the interaction term correctly.

[18] See Mallick 2009 for a detailed presentation of the difficulties of interpreting the effects of interaction terms in OP models.

[19] As robustness checks, I also estimate the ordered probit models for alternate specifications. These specifications include estimation without the cubed interaction terms, estimation without school fixed effects, and separate estimation for MS and HS students. I report these estimation results alongside the results for the main specification in Tables A.4-A.9 in Appendix A. As none of the main OP estimation results change, I proceed with estimation interpretation and simulations using the MAIN specification since it uses information from the most students. I also report robustness checks for the simulations in Sections 4.6.1 and 4.6.2.

[20] Estimates are for the most part consistent across the alternate specifications outlined in footnote 19.

[21] Marginal effects are computed at the mean values of the explanatory variables.

For the paid job, track, SES, and school dummy variables series, I perform joint significance tests to determine whether or not these variables affect the student's EFFORT and STUDY choices; the results are reported in the footnotes of Tables A.4, A.5, A.7, and A.8 in Appendix A. Paid job and school fixed effects are both jointly significant at the 5% level for both EFFORT and STUDY, while SES is significant for STUDY only. The significance of the school fixed effects indicates that individual schools do have an effect on how hard a student works. This effect could be due to school-specific variables such as principals, facilities, or funding. It could also be evidence of selection bias, in which parents with similar characteristics tend to select similar schools for their children. I revisit selection bias in Section 4.6.1.

Table 4.2 summarizes the direction of influence of individual significant non-interaction variables. Here, a "+" indicates that the variable increases the probabilities of higher EFFORT and STUDY outcomes. A "–" means that the variable has a negative effect on the probabilities of higher EFFORT and STUDY variables.[22] For both EFFORT and STUDY, being female has a positive effect on the probabilities of higher effort choices. Being white has a negative effect, as does working a paid job for 20+ hours. Both being in an upper SES category (60-79 percentile) and living with both parents increase the probabilities of higher effort choices. For just EFFORT, being in the 40-59 SES percentile and being in a magnet school have a positive effect on effort, whereas being in high school has a negative effect. For STUDY, working a paid job 6-10 hours and being in the top SES category have a positive effect on effort; ability, not having an ability measure, having increased variance

[22]Direction of influence is only noted if the variable is significant at the 5% level for the given estimating equation.

Table 4.2: Control Variables Significant at 5% level

Variable	**EFFORT**	**STUDY**
Female	+	+
White	−	−
Paid job, 6-10 hrs		+
Paid job, 20+ hrs	−	−
Ability		−
Missing Ability		−
SES, 40-59 percentile	+	
SES, 60-79 percentile	+	+
SES, 80+ percentile		+
Live with both parents	+	+
High School	−	
SOWNVAR		−
% teachers fully licensed		−
Magnet school	+	

across STUDY responses, and having a higher percentage of licensed teachers all decrease effort.

These marginal effects results are generally in line with the findings of Natriello and McDill 1986 and De Fraja, Oliveira, and Zanchi forthcoming discussed in Chapter 2. One exception, high school's negative effect on EFFORT, could be evidence that high school students feel less comfortable reporting high EFFORT levels. Ability's negative effect on STUDY could mean that a smarter student has to spend less time studying than a student with a lower ability measure. Having a higher percentage of licensed teachers could decrease STUDY through licensed teachers enhanced ability to promote learning during class time rather than outside of class time.

The result that SOWNVAR has a significant negative effect on higher STUDY choices is of particular interest. It means that increased variance across the own racial group's STUDY choices decreases STUDY. So as the classroom becomes more disordered with respect to the own racial group's STUDY choice, the student studies less. In other words, a student works harder in a classroom where peers behave similarly to each other, regardless of how much time they spend studying. While it merits further investigation, this finding could be evidence that grouping students who behave in a similar manner could lead to overall improvements in educational outcomes.

4.6 Simulations and Inference for Interaction Terms

I adapt a simulation-based technique developed in Zelner 2009 to interpret the estimation results of the interaction terms. This approach works as follows. First, it calculates the effect that discrete changes in independent variables that make up the interaction terms have on the predicted probabilities of EFFORT and STUDY outcomes. Second, it uses a technique developed by King, Tomz, and Wittenberg 2000 to determine the degree of significance of the effects generated by these discrete changes.

This approach overcomes the problems of inference for interaction terms by simulating actual changes in the independent variables that make up the interaction terms and how these changes affect the predicted probabilities of the different dependent variable outcomes for set of specified values of the explanatory variables; as discussed in Section 4.5, interpretation of OP estimated coefficients do not account for the complete impact of a change in an independent variable, but rather just the marginal impact of a change to the specified combination of independent variables that generate the interaction term.[23]

This simulation technique consists of the following steps:

1. Estimate the OP model using maximum likelihood to get the estimated coefficient vector $\hat{\beta}$ and the estimated variance-covariance matrix $\hat{V}(\hat{\beta})$.

2. Take $M = 1000$ draws from the multivariate normal distribution with mean $\hat{\beta}$ and variance $\hat{V}(\hat{\beta})$, which result in M simulated coefficient vectors.[24]

3. Use these M simulated coefficient vectors to calculate the simulated predicted probabilities, changes in predicted probabilities that result from discrete changes to independent variables, and the associated confidence intervals.

In the first set of simulations, presented in Sections 4.6.1 and 4.6.2, I estimate how moving one of the student's peers from one EFFORT level to another affects the probability that the student selects a particular EFFORT level.[25] I do this both for a peer who shares the student's own race and for a peer who is of a race other than the student at hand. In order to simulate the move of one peer student from one effort level to another, I decrease E2OWN by 10 percentage points and increase E5OWN by 10 percentage points. A 10

[23]The delta method used by Ai and Norton 2004 and Mallick 2009, which is based on a first-order Taylor approximation, works well for linear interaction terms, but provides a poor approximation for nonlinear functions, which I include in my model.

[24]I implement these simulations using the CLARIFY suite of Stata commands Tomz et al. (2001). I include Stata code for one series of simulations in Appendix B.

[25]I use the same process for STUDY.

percentage point change fits with the average sizes of the own racial group (12 students) and other racial group (8 students). The results of these simulations are displayed in Figures 4.1-4.8.

For all simulations, I report the 95% confidence intervals using dashed lines for the lower and upper bounds of the confidence interval. If the 95% confidence interval includes zero, then I conclude that the estimate is not statistically significant. If the 95% confidence interval lies either completely above or completely below zero, then I conclude that the estimate is statistically significant.

4.6.1 Own Racial Group Effort

The first set of simulations demonstrates the effect that moving one peer from one EFFORT level to another has on the student's own EFFORT choice. Unless otherwise specified, all simulations are performed for the "most common" student in the CMS data. This student is

- female;
- white;
- in a regular track;
- in middle school;
- of average ability (CAT score of 726);
- working 1-5 hours per week at a paid job; and
- living with both parents.

I set the E-OTHER variables equal to their sample means. And I set POWN at 0.6 and POTHER at 0.3, which are their sample means rounded to the nearest tenth.

The first type of peer effect to be investigated is the effect that the student's own racial group's EFFORT has on her own EFFORT. The first step of this analysis examines how the probability that the student chooses a particular EFFORT level changes as the proportion of students that select that EFFORT level increases. The second step investigates how the *change in* the probability that the student selects a particular EFFORT level varies as the proportion of the own racial group that selects that EFFORT level increases.

I use DC's Capital Gains, a current education policy intervention, to structure the simulation of the change in peer effort. Capital Gains pays students to attend class more regularly and on time, work harder on homework, and behave well in the classroom *Capital Gains* (2010). To simulate a successful Capital Gains intervention, I move one peer student of the student's own race from EFFORT=2 to EFFORT=5 and then evaluate the effect that this move has on the student's own EFFORT. This change in peer EFFORT decreases E2OWN by 0.1 and increases E5OWN by 0.1.

For each incremental change, I calculate the simulated predicted probability that a student selects EFFORT=5 (Pr(E=5)), along with its standard deviation and 95% confidence interval. Each incremental change represents one peer student of the student's own racial group changing his EFFORT choice from 2 to 5. Each time E2OWN and E5OWN change, I recalculate each of the interaction terms that contain E2OWN or E5OWN (E2OWNP, E2OWNPSQ, E2OWNPCU, E5OWNP, E5OWNPSQ, E5OWNPCU). By changing each of

Table 4.3: Steps for Peer Effort Simulation

			Simulation Results	
ACTION	E2OWN	E5OWN	(a)	(b)
1. Set initial values	0.9	0.1	Pr(E=5) & C.I.	—
2. Peer 1 increases EFFORT	0.8	0.2	Pr(E=5) & C.I.	dPr(E=5) & C.I.
3. Peer 2 increases EFFORT	0.7	0.3	Pr(E=5) & C.I.	dPr(E=5) & C.I.
⋮ ⋮	⋮	⋮	⋮	⋮
9. Peer 8 increases EFFORT	0.1	0.9	Pr(E=5) & C.I.	dPr(E=5) & C.I.
10. Peer 9 increases EFFORT	0.0	1.0	Pr(E=5) & C.I.	dPr(E=5) & C.I.

these interaction terms, this discrete change approach captures the entire affect that a change in an independent variable has on the predicted probabilities.

Each time that I change the values of E2OWN and E5OWN, I also estimate how these increases in E5OWN affect the *change in* the probability that a student select EFFORT=5 (dPr(E=5)). This part of the exercise works as follows. First, I set the parameter values to same values as those detailed above. Second, I set E2OWN at 0.1, E5OWN at 0.9, and calculate the related interaction terms based on these values. Then, I estimate the change in the simulated predicted probability of EFFORT=5 and its standard deviation and 95% confidence interval due to the changes in the interaction terms. Table 4.3 demonstrates how this process works for both parts of the simulation.

I use the same process to determine how the effort choices of a racial group other than the student's own affects the student's effort, if at all. Specifically, I set all of the variables to the same values as those listed above, except that I set E2OTHER = 0.9, E5OTHER = 0.1, and then proceed with the incremental changes outlined in Table 4.3.

As robustness checks, I perform the simulations for each of the alternate estimation specifications outlined in Section 4.5, as well as for a black, female student, a black, male student, and a white, male student. As the results of these simulations presented in Figures A.3-A.26 in Appendix A show, there is little change in the patterns of the simulations results.

The simulation results for the effect that the student's own racial group's EFFORT has on the probability that the student selects EFFORT=5 are displayed in Figures 4.1 and 4.2. The solid black line in Figure 4.1 represents the relationship between E5OWN — the proportion of the student's own racial group that selects EFFORT=5 — and the probability that the student himself selects EFFORT=5. The upper dashed line represents the upper bound of the 95% confidence interval and the lower dashed line represents the lower bound of the 95% confidence interval.

In Figure 4.2, the solid black line represents the relationship between E5OWN and the *change in* the probability that the student selects EFFORT=5 for each discrete change in E5OWN; these are the marginal effects of changes in E5OWN. Again, the upper and lower dashed lines represent the upper and lower bounds of the 95% confidence interval. The thicker solid black horizontal line at dPr(EFFORT=5)= 0 represents *no change* in the probability that the student selects EFFORT=5 in response to a change in E5OWN. In this case, dPr(EFFORT=5) itself is slightly greater than zero, but since the 95% confidence interval includes 0, the effect of the student's own racial group's effort level on the student's own effort level is not statistically different from 0 at the 5% level. The simulations for STUDY, shown in Figures 4.3 and 4.4, find a positive, significant effect of S5OWN on the probability that the student selects STUDY=5.

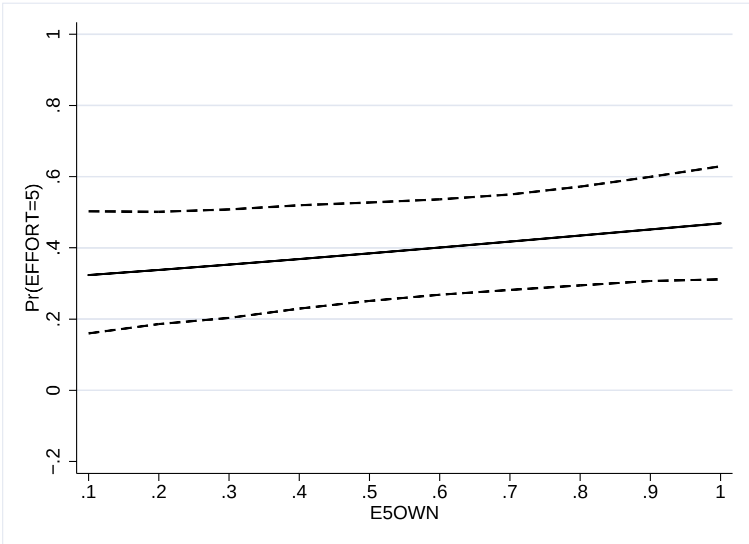


Figure 4.1: Effect of E5OWN on Pr(EFFORT=5)

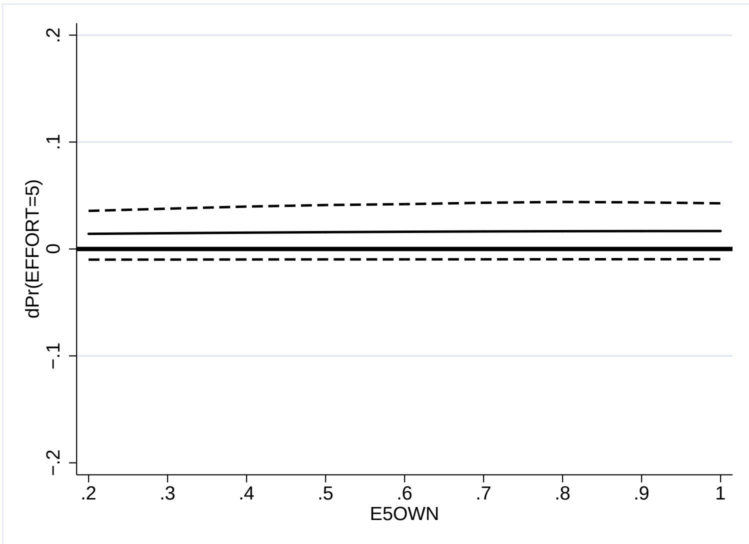


Figure 4.2: Effect of E5OWN on Change in Pr(EFFORT=5)

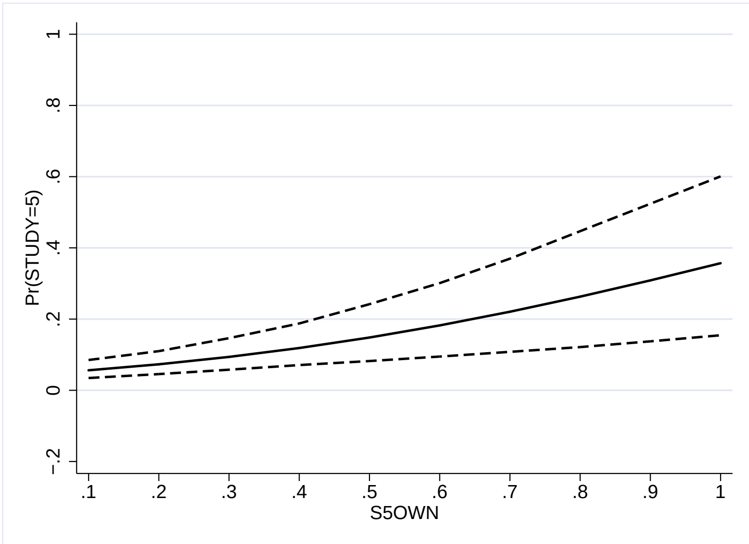


Figure 4.3: Effect of S5OWN on Pr(STUDY=5)

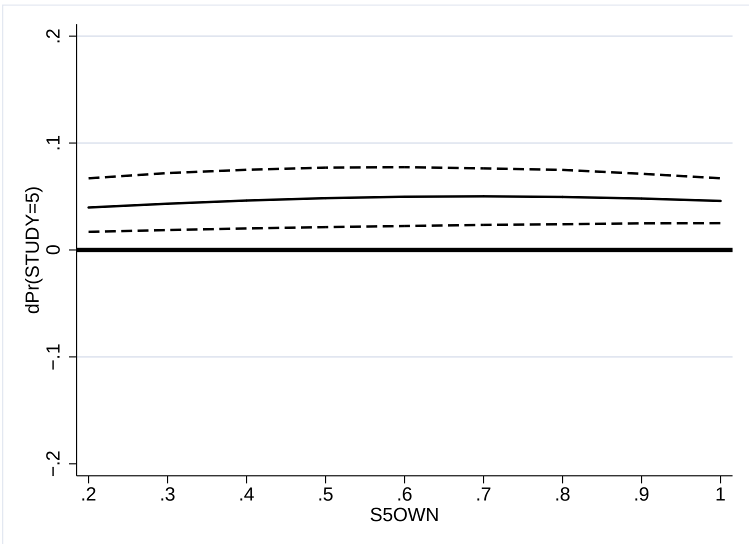


Figure 4.4: Effect of S5OWN on Change in Pr(STUDY=5)

But what do these simulation results mean in terms of the hypotheses from Chapter 3? The null and alternate hypotheses relating own effort to own racial group effort are

- $H_{1,0}$: *An increase in the student's own racial peer group's effort either does not affect effort or decreases effort.*
- $H_{1,A}$: *An increase in the student's own racial peer group's effort increases effort.*

The simulation results for the effect of the student's own racial group's effort on her own effort choice fail to reject the null hypothesis for EFFORT. For STUDY, however, the simulations provide evidence to reject the null hypothesis in favor of the alternate hypothesis: an increase in own racial peer group's effort increases the probability that the student exerts more effort. And because the estimation includes the interaction terms described in Section 4.3.2, this result is robust to the classroom share of the own racial group.

As robustness checks, I also report the simulation results for alternate specifications in Figures A.43-A.66 in Appendix A. Of particular interest are the results for the specification that omits the controls for school heterogeneity, reported in Figures A.51-A.54. These figures show that the peer effect is positive and significant for EFFORT and slightly more positive for STUDY than when these controls are included. These results, coupled with the OP estimation results that the school fixed effects are significant at the 5% level, provide evidence that selection bias does affect the estimation of peer effects. Parents seek out schools with children who are similar to their own; these similarities tend to overstate the size of the peer effect if selection is not controlled for. Sorting in the presence of peer effects, a topic directly related to school choice and public finance literatures, is one that should be investigated more thoroughly in future work.[26]

4.6.2 Other Racial Group Effort

Figures 4.5 and 4.6 show the results for the simulations of the effect of E5OTHER on the probability that the student selects EFFORT=5. These figures indicate that for EFFORT, the other racial group's effort choice has a significant, negative effect on the probability that the student selects EFFORT=5.

Figures 4.7 and 4.8 show the results for the simulations of the effect that S5OTHER has on the probability that a student selects STUDY=5. In contrast to EFFORT, the proportion of the other racial group that reports STUDY=5 has a positive, significant effect on the student's own STUDY choice.

[26] Results for separate black and white estimation, reported in Figures A.27-A.42 in Appendix A, are also of interest. These results show that the patterns for separate estimation are similar to those for pooled estimation, with the exception that separate estimation results lose some significance. Identifying how the peer effect differs for black and white students will be further explored in future work.

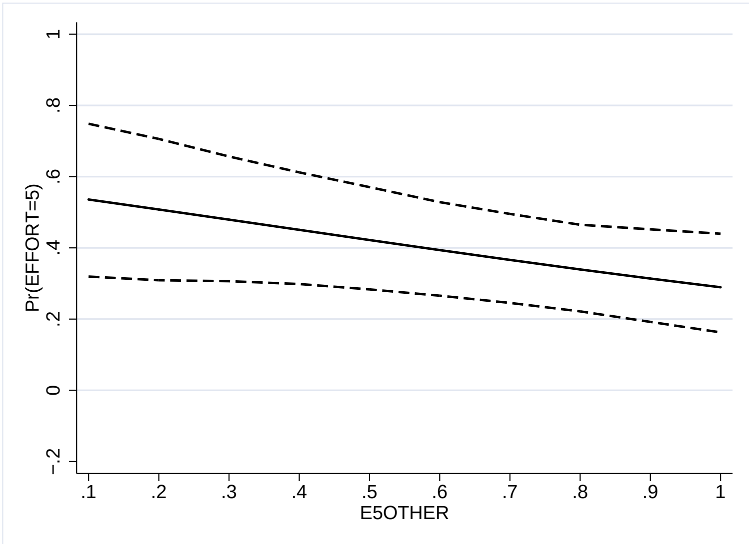


Figure 4.5: Effect of E5OTHER on Pr(EFFORT=5)

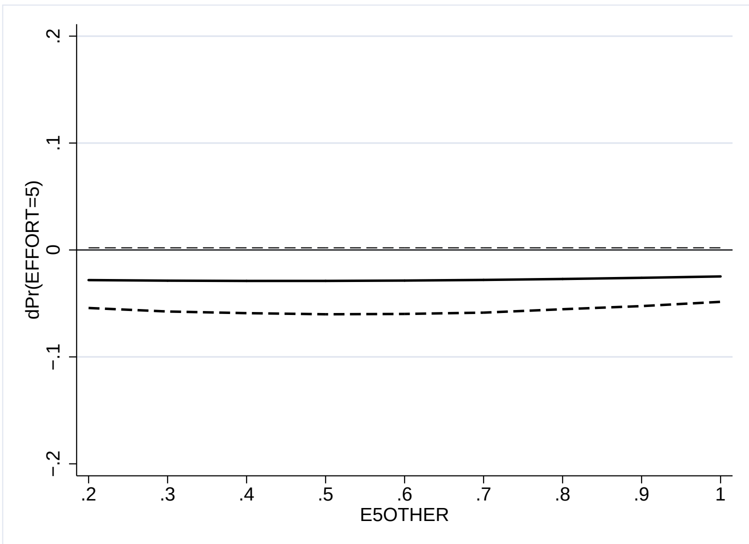


Figure 4.6: Effect of E5OTHER on Change in Pr(EFFORT=5)

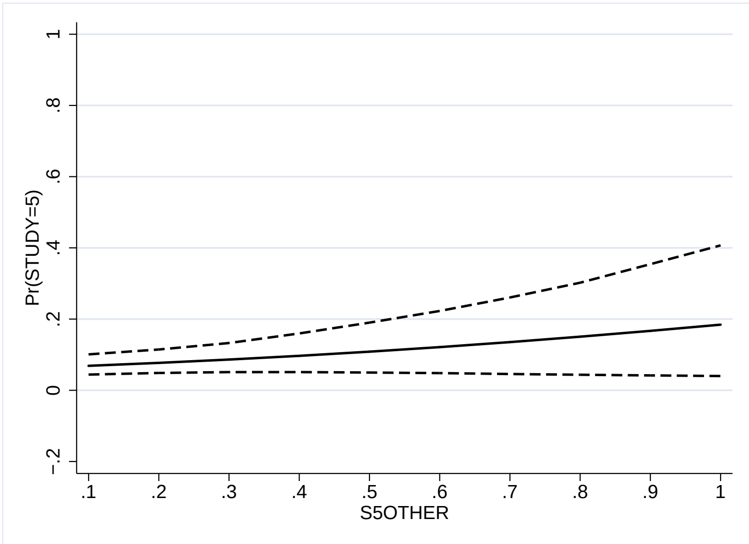


Figure 4.7: Effect of S5OTHER on Pr(STUDY=5)

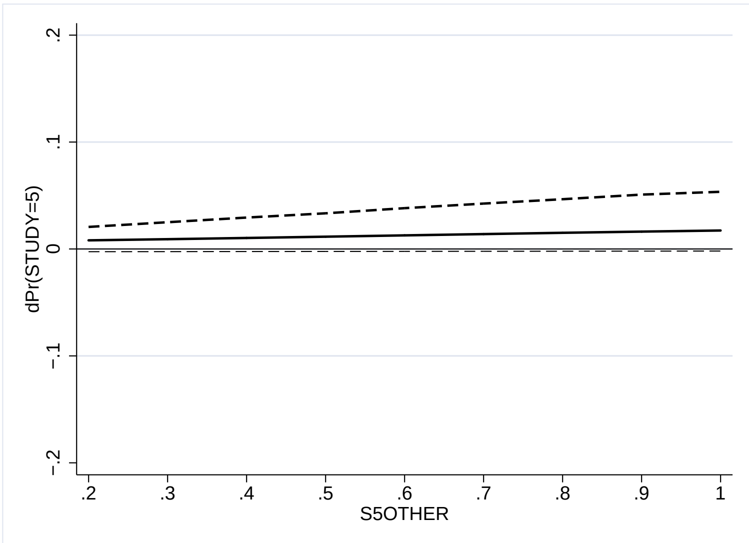


Figure 4.8: Effect of S5OTHER on Change in Pr(STUDY=5)

The null and alternate hypotheses relating own effort to other racial group effort from Chapter 3 are

- $H_{2,0}$: *An increase in the mean effort level of another racial group either does not affect effort or decreases effort.*
- $H_{2,A}$: *An increase in the mean effort level of another racial group increases effort.*

Because the effect is negative and insignificant for EFFORT, the simulation results fail to reject the null hypothesis for EFFORT.[27] For STUDY, the effect is positive but the 95% confidence interval appears to include zero, which means that there is not evidence to reject the null hypothesis in favor of the alternate hypothesis.

A third set of hypotheses from Chapter 3 relates the strength of the own racial group effect to that of the other racial group:

- $H_{3,0}$: *For a given racial group share, peer effort of the student's own racial group has the same or weaker effect on effort than does the peer effort of each other racial group.*
- $H_{3,A}$: *For a given racial group share, peer effort of the student's own racial group has a stronger effect on effort than does the peer effort of each other racial group.*

I cannot conclusively reject the null hypothesis in favor of the alternate hypothesis for EFFORT because the effect of own effort is not significant. For

[27]Since this negative relationship holds across all race/gender combinations (see Figures A.8, A.16, A.24, A.28, and A.36 in Appendix A), it appears that the "acting white" effect found in Fryer and Torelli 2010 is not driving this relationship.

STUDY, there is also no conclusive evidence that would allow me to reject the null hypothesis relating racial group share to the strength of the peer effect. Comparing Figures 4.4 and 4.8, however, one can infer that the effect of the own racial group is probably stronger than that of the other racial group.[28]

Overall, the results for STUDY confirm the theoretical model's finding that an increase in own racial group effort increases the student's own effort, at least when moving a peer student from a low effort level to a high effort level; for other racial group effort, the effect is positive but not significant at the 95% level. The results for EFFORT, on the other hand, are less convincing. This series of findings could reflect the suitability of EFFORT as a measure of how hard the student works on schoolwork. It could also arise because of the relative nature of the EFFORT responses.[29]

4.7 Implications for Policy

In this section, I use simulations to calculate how different policy experiments impact effort. The first policy experiment estimates how changes to classroom racial composition affects effort. In the second and third experiments, I use a novel approach to estimate the social multiplier triggered by a decrease in the amount of time a student spends working at a paid job and an improvement in his socioeconomic status.

[28]Future work could devise simulations that would allow me to determine the relative sizes of the two effects.

[29]The final four sets of hypotheses from Chapter 3 are not able to be tested using the simulation-based technique described in Section 4.6. Those hypotheses that relate the strength of peer effect to the racial group's classroom share, while not directly testable, are partially validated since estimation controls for own racial group share by using a series of interaction terms between peer effort and racial group classroom share. The final two sets of hypotheses derived in Chapter 3, those related to the bad apple effect and the shining light effect, require more detailed data.

4.7.1 Change in Classroom Racial Composition

One factor of interest to policy makers and parents alike is classroom racial composition. This set of simulations explores how racial composition affects effort. I use the same initial values as in the simulations described in Section 4.6.1, except that I set POWN=0.1, POTHER=0.9, and the peer EFFORT and STUDY variables to their sample mean values.

I simulate what happens when I remove one peer student who is of the other racial group from the classroom and replace him with a student of the student's own race but exactly the same to the student removed in all other ways. I do this by increasing POWN by from 0.1 to 0.9 by increments of 0.1, simultaneously decreasing POTHER from 0.9 to 0.1 by increments of 0.1, and estimating how these changes in POWN and POTHER affect the probability that a student selects EFFORT=5 or STUDY=5 and the changes in these probabilities. Each time I change the values of POWN and POTHER, I recalculate each of the interaction terms to match the current POWN and POTHER values.

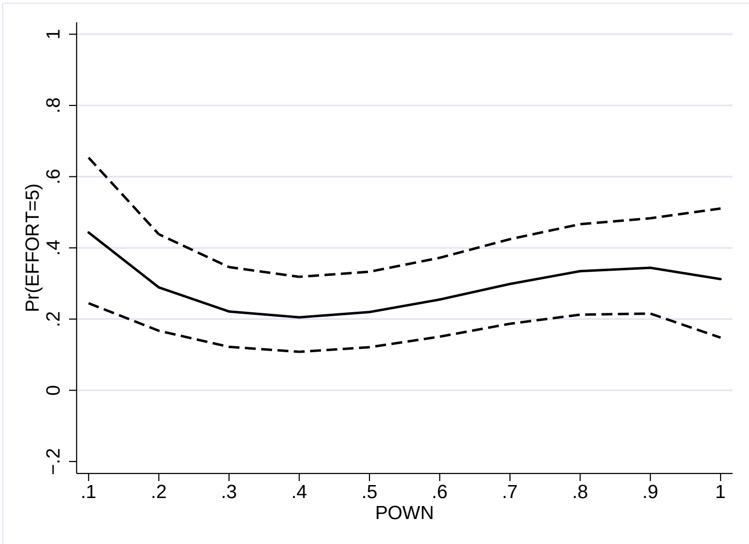


Figure 4.9: Effect of POWN on Pr(EFFORT=5)

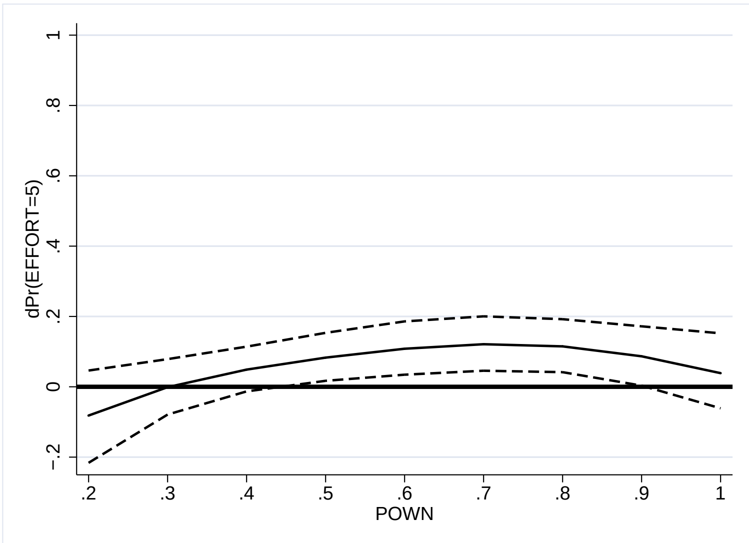


Figure 4.10: Effect of POWN on Change in Pr(EFFORT=5)

Figure 4.9 indicates that the highest probabilities of EFFORT=5 are for low and high values of POWN. If such a relationship held for other values of EFFORT, then these simulations could be interpreted as further evidence of peer effects. This is because when a student is in the racial majority and that majority is large, she potentially feels more pressure to conform to the behavior of the peer group; when a student is in the small racial minority, the student still feels pressure to work hard if the other racial group — which is large — is also working hard. Simulations run for other EFFORT levels, however, show no effect at all of classroom composition on EFFORT.[30] This makes it unlikely that peer effects are driving the relationship between Pr(EFFORT=5) and POWN.

Another possible explanation is that higher levels of racial homogeneity promote more effort. The probability of EFFORT=5 is highest for low and high levels of POWN. Since both of these cases are for relatively homogeneous classrooms, students put forth more effort in more racially homogeneous classrooms. At medium levels of POWN, which correspond to more racially heterogeneous classrooms, the probability that a student puts forth EFFORT=5 is lower. If this is the case, then this simulation result supports others' findings that racial diversity can lead to worse educational outcomes for students Hoxby (2000); Angrist & Lang (2004); Card & Rothstein (2007); Hanushek et al. (2009).[31]

[30]The simulations for STUDY indicate very little change in STUDY resulting from a change in POWN at all levels of STUDY.

[31]I also include the simulation results for separate black and white estimation in Figures A.67 and A.68 in Appendix A. These results show that the relationship for EFFORT is more pronounced for black students than it is for white students. These results could mean that the effect of classroom racial composition is race-dependent. This topic will be further explored in future work.

4.7.2 The Social Multiplier

The simulation results for own racial group STUDY and other racial group STUDY in Sections 4.6.1 and 4.6.2 provide evidence that educational peer effects do exist. As discussed in Chapter 3, if peer effects exist and they are endogenous — as peer effects that operate through effort are — then a social multiplier should result. When a social multiplier exists, the aggregate impact of a change to an exogenous variable is larger than the sum of the direct individual impacts. In this section, I first explain how I use simulations to estimate the social multiplier. I then estimate the social multiplier for a change to two exogenous variables that affect effort, time spent working at a paid job and socioeconomic status.

As in Chapter 3, I use Scheinkman's 2008 definition as the basis for the empirical formulation of the social multiplier:[32]

> The *social multiplier* measures the ratio of the effect on the average action caused by a change in a parameter to the effect on the average action that would occur if individual agents ignored the change in actions of their peers.

The key to implementing this definition empirically is to identify what it means for the student to "ignore" the change in actions of her peers in the context of the ordered probit model. The following steps credibly isolate estimation when a student ignores her peers and, in doing so, provide a method to estimate the social multiplier:[33]

[32] A related definition is "the estimated ratio of aggregate coefficients to individual coefficients" (Glaeser et al., 2003, p. 346).

[33] I am not aware of any other studies that estimate of the social multiplier using the method described above. The merits and drawbacks of this approach warrant further investigation in future work.

1. Estimate the OP equation as specified in Section 4.3.3; this specification includes all of the peer measures of effort, interaction terms, and school controls.

2. Estimate the change in the probability that a student selects EFFORT=5 due to a change in an exogenous variable and call this change (A).

3. Re-estimate the OP equation, but this time exclude the peer measures of effort as explanatory variables.

4. Re-estimate the change in the probability that a student selects EFFORT=5 due to the same change in an exogenous variable and call this change (B).

5. Calculate $\frac{(B)}{(A)}$. This ratio is the estimate of the social multiplier.

To understand the logic of $\frac{(B)}{(A)}$ as an estimate of the social multiplier, recall the theoretical definition of the social multiplier derived in Chapter 3: the social multiplier is the ratio of the average effect on effort caused by a change in an exogenous variable to the average effect on effort caused by the exact same change in an exogenous variable *when each other student's effort is held constant.*

Here, (B) represents the change to effort caused by a change to an exogenous variable when peer effects are not accounted for in estimation. By not accounting for peer effects in estimation, I am allowing peer effects to affect effort indirectly through the change to the exogenous variable. In this way, (B) captures the *total* effect that the change to the exogenous variable has on effort. This total effect includes the both the student's private response to the change in the exogenous variable and the student's response to any change in his peers' effort levels — the peer effect.

Conversely, (A) separately estimates the private effect and the peer effect by specifically accounting for the change that the exogenous variable has on effort through peer effects. This is because, by including the peer variables in estimation, estimation captures the variation in effort due to the change in the exogenous variable that is explained by peer effort. In effect, (A) answers the question, "Controlling for peer effort, how does a change in an exogenous variable affect effort?" This is because by including the peer variables in estimation, the peer effect is explicitly modeled by the estimating specification. In this way, it is analogous to the denominator in the theoretical definition of the social multiplier that asks, "When each other student's effort is restricted to be constant, how much does the student's own effort change in response to a change in an exogenous variable?" If peer effects are self-reinforcing, then (A) should be smaller than (B).

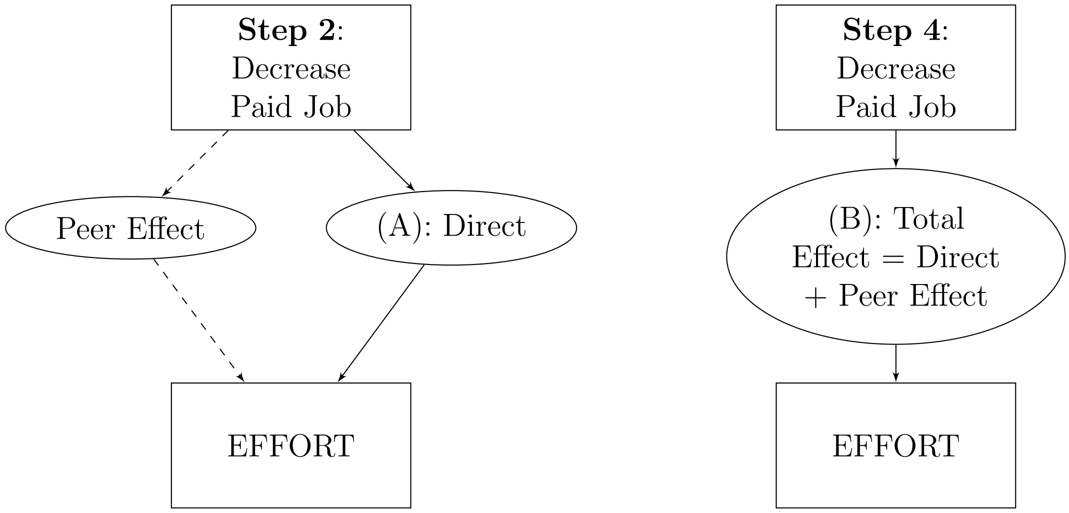
As a result, $\frac{(B)}{(A)}$ mirrors Scheinkman's definition: it is the ratio of the effect on the average effort choice caused by a change to an exogenous variable to the effect on the average effort choice that would occur if the student ignored her peers. Figure 4.11 provides a visual illustration of how this technique detects the social multiplier.

For the first set of social multiplier simulations, I analyze whether or not a reduction in time spent working at a paid job generates a social multiplier. A policy experiment could attempt to reduce how many hours a student spent working at a paid job by compensating the student for this reduction in hours spent working. In theory, this extra time would allow the student to put forth

Figure 4.11: Detecting the Social Multiplier

Step 1: Estimate OP *including* peer variables

Step 3: Estimate OP *excluding* peer variables



more effort on school work, since the marginal effects for “Paid job, 20+ hrs” are negative and significant for both higher EFFORT and higher STUDY choices.

Next, I calculate the social multiplier for a change to parents’ socioeconomic status by increasing SES from the 20-39th percentile (SES2) to the 60-79th percentile (SES4).[34] This experiment simulates how a change to a classroom’s socioeconomic composition affects effort and whether or not this effect reverberates beyond the individual student. Since the socioeconomic composition of a student’s classmates are determined by how students are assigned to classrooms, this policy experiment provides insight into how school choice and student assignment policies affect aggregate effort. Wake County’s

[34] I choose these percentiles because SES2 is the lowest possible percentile in the estimation and SES4 is significant for both EFFORT and STUDY.

recent decision to end its income-based busing program is one recent example Wake County Public School System (2010).

Table 4.4 describes the mechanics of how the social multiplier is calculated for the effect of a reduction in paid job hours on EFFORT. The "Pr(EFFORT)" column gives the probability that the student selects that particular EFFORT level *before* the change to paid job hours; since all of the possible EFFORT choices are represented, each "Pr(EFFORT)" column must sum to one. "Δ(Pr)" represents the change in Pr(EFFORT) due to the change in paid job hours; these columns sum to zero. Finally, the "Social Multiplier" column calculates $\frac{B}{A}$.

Table 4.4: Mechanics of Social Multiplier— Paid Job's Effect on EFFORT

	(A)		**(B)**		**Social**
Level	Pr(EFFORT)	Δ(Pr)	Pr(EFFORT)	Δ(Pr)	**Multiplier**
1	0.0042	-0.0011	0.0052	-0.0015	**1.34**
2	0.0571	-0.0105	0.0630	-0.0133	**1.27**
3	0.1807	-0.0194	0.1861	-0.0221	**1.14**
4	0.5035	-0.0029	0.4997	-0.0021	**0.72**
5	0.2555	0.0332	0.2453	0.0392	**1.18**

Estimates for the social multiplier range from 1.34 for EFFORT=1 to 0.72 for EFFORT=4. For EFFORT=1, the social multiplier estimate of 1.34 means that that the aggregate impact on the probability that a student chooses EFFORT=1 is 34% larger than the sum of the direct individual impacts. For EFFORT=4, the aggregate impact is actually 28% smaller; this could mean that the peer effects are actually negating the change in effort that would take place in the absence of peer effects; relative status effects, where the student's utility depends on his relative class ranking, is one type of peer effect that could create this type of multiplier Zeidner & Schleyer (1998); Damiano et al.

Table 4.5: Social Multiplier Estimates

	Paid Job		SES	
LEVEL	EFFORT	STUDY	EFFORT	STUDY
1	1.34 (−)	1.15 (−)	1.38 (−)	1.11 (−)
2	1.27 (−)	0.88 (−)	1.21 (−)	0.90 (−)
3	1.14 (−)	1.12 (+)	1.14 (−)	1.03 (+)
4	0.72 (+)	1.05 (+)	0.95 (+)	1.05 (+)
5	1.18 (+)	1.08 (+)	1.14 (+)	1.10 (+)

"+" indicates a positive effect.
"–" indicates a negative effect.

(2010). Social multiplier estimates for both policy experiments are reported in Table 4.5.

Reducing paid job hours increases the probability of the highest EFFORT and STUDY levels, and the aggregate impact is 18% larger than the sum of the direct individual impacts for EFFORT and 8% larger for STUDY. The negative effect on the lowest EFFORT choice is 34% larger than in the absence of peer effects; for STUDY, it is 15% larger. These results show that positive effect of compensating a student so that he can afford to work fewer hours — through a need-based grant, for example — extends beyond the student's own effort to that of her peers.

Raising the student's socioeconomic status increases the probabilities of the highest EFFORT and STUDY choices, and the aggregate effect is 14% larger for EFFORT and 10% larger for STUDY. For the lowest EFFORT and STUDY choices, the increase in SES decreases the probabilities and these decreases are 38% larger for EFFORT and 11% larger for STUDY in the presence of peer effects. In the case of Wake County, if the end to income-based busing results in schools that are more stratified by income, then schools with a more affluent student body will experience a larger than expected increase in effort due to peer effects. Effort at schools with poorer student bodies, on the other hand, will drop by more than expected as a result of these same peer effects.[35]

The fact that the magnitude of the social multiplier does not depend on the policy intervention is further evidence that this approach captures the social multiplier. This is because it is not the magnitude of the effect of the exogenous change that matters, but rather how much that change reinforces the effect between students. In comparing the EFFORT estimates for Paid Job and SES in Table 4.5, there are only minor variations in the magnitudes of the social multiplier estimates for a given EFFORT level; STUDY provides the same pattern of findings.

These policy experiments provide preliminary evidence that a social multiplier does exist; future work will further explore this method of estimating the social multiplier. Compared to other social multiplier studies, however, these estimates are more modest than most; both Kroft 2008 and Graham 2008 find social multiplier estimates of close to 2, while Cutler and Glaeser's 2010 estimate a social multiplier of 4. One reason that this study's estimates are smaller is that the interactions are confined to within the classroom, which limits the extent to which the multiplier can grow, whereas the aforementioned studies estimate a multiplier over significantly larger populations. Another potential explanation is that while the effects of own racial group and other racial group peer measures of STUDY are positive and significant, they are relatively small.[36]

[35]Future work will explore how social multiplier estimates can be sharpened through the presentation of confidence intervals.

[36]The low social multiplier estimates could also be due to the presence of "bad apple" and "shining light" students, for whom peer effects do not affect the effort choice, thereby reducing the size of the multiplier effect.

4.8 Chapter Summary

This chapter describes how the peer effects described in Chapter 3 are estimated empirically. The CMS district, because of its unique place in the history of desegregation in the United States, makes for fitting school district in which to investigate how a student's effort depends on peer effort and race. Furthermore, the CMS data set itself has two advantages over most other data sets used in studies of peer effects: it has complete classrooms of observations and it contains two distinct measures of student effort, EFFORT and STUDY.

Next, this chapter presents a novel way to transform EFFORT and STUDY, both ordered discrete variables, into central measures of peer effort: using classroom proportions of each EFFORT and STUDY response as explanatory variables. Because the peer effects being estimated depend on the student's own and other racial groups' classroom shares, the empirical strategy adapts a simulation-based technique to determine degree of significance and direction of influence of the interaction terms containing peer effort and classroom shares to facilitate the testing of Chapter 3's hypotheses.

These simulations fail to reject the null hypotheses for both own and other racial group EFFORT effects. This failure could be due to the relative nature of EFFORT. They do provide evidence, however, that an increase in the student's own racial peer group's STUDY choice increases the student's own STUDY choice. Furthermore, because the interaction terms are included in the estimation, this finding is robust to the classroom share of the student's own racial group. Simulation results also find that an increase in the peer STUDY measure of the other racial group increases the student's own STUDY time, but this relationship is not significant at the 5% level.

This chapter concludes with three policy experiments. In the first experiment, more racially homogeneous classrooms are shown to result in increased effort. The second experiment shows that a decrease in the number of hours a student spends working at a paid job increases high EFFORT and STUDY choices and results in a social multiplier of 1.18 for EFFORT and 1.08 for STUDY, which means that the aggregate effect is 18% larger for EFFORT and 8% larger for STUDY than the sum of the direct individual effects. In the third experiment, an increase in SES generates a social multiplier of 1.14 for EFFORT and 1.10 for STUDY.

CHAPTER 5

Conclusion, Extensions, and Future Work

This dissertation develops a theoretical model of peer effects that are directly investigated empirically. These peer effects take place through the student's effort choice, where the student attempts to conform to the behavior of the peer group and this tendency to conform increases the more the student interacts with a particular group of students. Chapter 2 describes the existing literature on the role of effort in educational production, endogenous and exogenous peer effects, and how peer effects take place in the classroom. This analysis concludes that while endogenous peer effects appear to exist, they most likely depend at least in part on the exogenous characteristics of the student's peer group. Race is one exogenous characteristic that consistently plays a strong role in determining how students interact with one another.

Chapter 3 presents a theoretical model in which the behavior of the student depends both on the peer group's behavior and exogenous characteristics. This model shows how a static, simultaneous-move game results in an expression of equilibrium effort that, in turn, demonstrates how multiple types of peer effects take place, including conformity effects, bad apple effects, and shining light effects. The equilibrium effort expression yields hypotheses that can be directed evaluated using data. The comparative statics of the theoretical model indicate that a social multiplier exists for changes to the model's exogenous characteristics. The theoretical model is then extended to define and quantify the social multiplier in terms of the model's parameters and outline the conditions in which the social multiplier exists.

The estimation of peer effects in Chapter 4 uses data from the Charlotte-Mecklenburg School (CMS) district, a data set that is interesting because of CMS's history of desegregation, because it contains two measures of effort, self-reported effort (EFFORT) and time spent studying (STUDY), and because it has complete classrooms of observations. Since EFFORT and STUDY are ordered discrete variables, the empirical model presents a novel way to transform the ordered discrete variables into central measures of peer effort: using classroom proportions of each EFFORT and STUDY response as explanatory variables. And since the peer effects being estimated depend on the student's own and other racial groups' classroom shares, the empirical strategy adapts a simulation-based technique to determine degree of significance and direction of influence of the interaction terms containing peer effort and classroom shares to facilitate the testing of Chapter 3's hypotheses.

These simulations provide evidence that an increase in the student's own racial peer group's STUDY choice increases the student's own STUDY choice; an increase in peer STUDY measure of the other racial group increases the student's own STUDY choice, but this effect is not significant at the 5% level.

Simulations also show that changes to hours spent working at a paid job and the socioeconomic status generate social multipliers; due to peer effects, the aggregate impact on high EFFORT and STUDY choices are up to 18% larger than the sum of direct individual impacts for these changes.

Future work will extend this model of educational peer effects to address several topics central to education policy. Promising avenues include the following:

- Extend the theoretical model to allow students' interactions to depend on multiple exogenous characteristics simultaneously, including gender, socioeconomic background, and ability. This line of research will provide insight into how coeducation, school choice, and other policies that determine how students are allocated across schools affect student effort and, therefore, aggregate educational outcomes.

- Convert the one-time, simultaneous-move game of the theoretical model into a repeated or sequential-move game. Both of these extensions would more closely simulate how students actually interact in the classroom and have the potential to show how a dynamic setting generates a social multiplier endogenously through trust.

- Apply the empirical approach to a panel data set with an intervention that targets effort; an example is DC's Capital Gains *Capital Gains* (2010). This extension would enable more precise estimation of peer effects, provide better arguments for causality, and allow for direct estimation of the social multiplier.

A recent development in Wake County, NC, 150 miles east of Charlotte, illustrates how central peer effects are to educational outcomes, residential decisions, and local politics. In the fall of 2009, four new education board members were elected on the promise to end Wake County's nationally-recognized income-based busing program started in 2000; on March 23, 2010, they fulfilled their promise by voting to end it Wake County Public School System (2010). The imminent end to the busing policy caused the county superintendent to resign, the N.A.A.C.P. to threaten legal action, parents who in the past chose private school for their children to reconsider public school, and parents without the means for private school to ponder the long-term consequences of a more racially and economically segregated educational experience for their children Brown (2010). Educational peer effects play a role in all of these dynamics.

This dissertation's model of educational peer effects gives us insight into how peer effects that result from a policy shift such as Wake County's decision to end busing affect individual students' outcomes. First, if the end to busing results in more racially homogeneous classrooms, then aggregate effort could increase. Due to the social multiplier, higher levels of socioeconomic stratification that will likely result from the Wake County decision could result in larger than expected improvements in educational outcomes for schools with more affluent student bodies; conversely, schools with poorer families could experience larger than expected declines in educational outcomes. Future work will refine the assumptions that govern how students interact with one another, capturing the effects of racial, economic, and ability-based diversity simultaneously. As illustrated by Wake County, these issues affect any parent selecting a school for her child and any policy maker deciding how best to allocate students across classrooms.

REFERENCES

Ai, C., & Norton, E. C. (2003). Interaction terms in logit and probit models. *Economics Letters*, *80*, 123-129.

Akerlof, G. A. (1980). A theory of social custom, of which unemployment may be one consequence. *Quarterly Journal of Economics*, *94*(4), 749-775.

Akerlof, G. A., & Kranton, R. E. (2002, December). Identity and schooling: Some lessons for the economics of education. *Journal of Economic Literature*, *XL*, 1167-1201.

Angrist, J. D., & Lang, K. (2004). Does school integration generate peer effects? Evidence from Boston's Metco program. *American Economic Review*, *94*(5), 1613-1634.

Arnott, R., & Rowse, J. (1987). Peer group effects and educational attainment. *Journal of Public Economics*, *32*(3), 287-305.

Benabou, R. (1996). Equity and efficiency in human capital investment: The local connection. *Review of Economic Studies*, *63*, 237-264.

Bernheim, B. D. (1994). A theory of conformity. *Journal of Political Economy*, *102*(5), 841-877.

Bishop, J. H. (2006). Drinking from the fountain of knowledge: Student incentive to study and learn-externalities, information problems and peer pressure. In E. Hanushek & F. Welch (Eds.), *Handbook on the economics of education* (Vol. 2). North Holland.

Bishop, J. H., Bishop, M., Bishop, M., Gelbwasser, L., Green, S., Peterson, E., et al. (2004). Why we harrass nerds and freaks: A formal theory of student culture and norms. *Journal of School Health*, *74*(7), 235-251.

Brock, W. A., & Durlauf, S. N. (2001). Discrete choice with social interactions. *Review of Economic Studies*, *68*(1), 235-260.

Brock, W. A., & Durlauf, S. N. (2006). Multinomial choice with social interactions. In L. E. Blume & S. N. Durlauf (Eds.), *The economy as an evolving complex system, III* (p. 175-206). Oxford University Press.

Brown, R. (2010, February 27, 2010). District may end N.C. economic diversity program. *The New York Times*, A18.

Butler, J., & Chatterjee, P. (1995). Pet econometrics: Ownership of cats and dogs. (Vanderbilt University Working Paper)

Calabrese, S., Epple, D., Romer, T., & Sieg, H. (2006). Local public good provision: Voting, peer effects, and mobility. *Journal of Public Economics*, *90*(6-7), 959-981.

Calhoun, C. A. (1991). Desired and excess fertility in Europe and the United States: Indirect estimates from world fertility survey data. *European Journal of Population*, *7*(1), 29-57.

Calvó-Armengol, A., Patacchini, E., & Zenou, Y. (2009). Peer effects and social networks in education. *Review of Economic Studies*, *76*(4), 1239-1267.

Capital Gains (Tech. Rep.). (2010). The Education Innovation Laboratory at Harvard University. (`http://www.edlabs.harvard.edu/`)

Carbonaro, W. (2005). Tracking, students' effort, and academic achievement. *Sociology of Education*, *78*(1), 27-49.

Card, D., & Rothstein, J. (2007). Racial segregation and the black-white test score gap. *Journal of Public Economics*, *91*(11-12), 2158-2184.

Charlotte Mecklenburg Schools. (2010). (`http://www.cms.k12.nc.us`)

Cooley, J. (2009, August). *Can achievement peer effect estimates inform policy? A view from inside the black box.* Department of Economics, University of Wisconsin-Madison. (`http://www.ssc.wisc.edu/~jcooley/exogpeer.pdf`)

Cooley, J. (2010, February). *Desegregation and the achievement gap: Do diverse peers help?* Department of Economics, University of Wisconsin-Madison. (`http://www.ssc.wisc.edu/~jcooley/CooleyDeseg.pdf`)

Currarini, S., Jackson, M. O., & Pin, P. (2009a). An economic model of friendship: Homophily, minorities, and segregation. *Econometrica*, *77*(4), 1003-1045.

Currarini, S., Jackson, M. O., & Pin, P. (2009b, June). *Identifying sources of racial homophily in high school friendship networks.* Department of Economics, Stanford University. (`http://www.stanford.edu/~jacksonm/racialhomophily.pdf`)

Cutler, D. M., & Glaeser, E. L. (2010, February). *Social interactions and smoking.* (http://www.nber.org/chapters/c8196)

Damiano, E., Li, H., & Suen, W. (2010). First in village or second in Rome? *International Economic Review*, *51*, 263-288.

De Bartolome, C. A. M. (1990). Equilibrium and inefficiency in a community model with peer group effects. *Journal of Political Economy*, *98*(1), 110-133.

De Fraja, G., Oliveira, T., & Zanchi, L. (forthcoming). Must try harder. Evaluating the role of effort in educational attainment. *The Review of Economics and Statistics.* (http://www.le.ac.uk/economics/gdf4/effort%20paper.pdf)

Durlauf, S. N., & Cohen-Cole, E. (2004). Social interaction models. In K. Kempf-Leonard (Ed.), *Encyclopedia of social measurement.* Academic Press.

Eckes, S. E. (2006). Barriers to integration in the Mississippi Delta: Could charter schools be the new vehicle for desegregation? *Analyses of Social Issues and Public Policy*, *6*(1), 15-30.

Evans, W. N., Oates, W. E., & Schwab, R. M. (1992). Measuring peer group effects: A study of teenage behavior. *The Journal of Political Economy*, *100*(5), 966-991.

Falk, A., & Ichino, A. (2006). Clean evidence on peer effects. *Journal of Labor Economics*, *24*(1), 39-57.

Figlio, D. N. (2007). Boys named Sue: Disruptive children and their peers. *Education Finance and Policy*, *2*(4).

Fryer, R. G., & Torelli, P. (2010). An empirical analysis of 'acting white'. *Journal of Public Economics*, *94*(5-6), 380-396.

Gelman, A. (2007, November). *Clustered standard errors vs. multilevel modeling.* Department of Statistics, Columbia University. (`http://www.stat.columbia.edu/~cook/movabletype/archives/2007/11/clustered_stand.html`)

Glaeser, E. L., Sacerdote, B., & Scheinkman, J. A. (1996). Crime and social interactions. *Quarterly Journal of Economics*, *111*(2), 507-548.

Glaeser, E. L., Sacerdote, B., & Scheinkman, J. A. (2003). The social multiplier. *Journal of the European Economic Association*, *1*(2-3), 345-353.

Glaeser, E. L., & Scheinkman, J. A. (2002). Non-market interactions. In M. Dewatripont, L. P. Hansen, & S. Turnovsky (Eds.), *Advances in economics and econometrics: Theory and applications, eighth world congress.* Cambridge, UK: Cambridge University Press.

Glazerman, S. (1997). *Determinants and consequences of parental school choice.* Unpublished doctoral dissertation, University of Chicago, Harris School of Public Policy.

Goldin, C., & Katz, L. F. (2002). The power of the pill: Oral contraception and women's career and marriage decisions. *Journal of Political Economy*, *110*(4), 730-770.

Graham, B. S. (2008). Identifying social interactions through conditional variance restrictions. *Econometrica*, *76*(3), 643-660.

Greene, W. H. (2003). *Econometric analysis.* Delhi: Pearson Education.

Greene, W. H., & Hensher, D. A. (2009, June). *Modeling ordered choices: A primer.* Stern School of Business, New York University. (http://pages.stern.nyu.edu/~wgreene/)

Hanushek, E. A. (1986, September). The economics of schooling: Production and efficiency in public schools. *Journal of Economic Literature*, *24*, 1141-1177.

Hanushek, E. A., Kain, J. F., & Rivkin, S. G. (2009). New evidence about Brown v. Board of Education: The complex effects of school racial composition on achievement. *Journal of Labor Economics*, *27*(3), 349-383.

Henderson, V., Mieszkowski, P., & Sauvageau, Y. (1978). Peer group effects and educational production functions. *Journal of Public Economics*, *10*(1), 97-106.

Hoxby, C. M. (2000, August). Peer effects in the classroom: Learning from gender and race variation. (NBER Working Paper 7867)

Hoxby, C. M., & Weingarth, G. (2006). *Taking race out of the equation: School reassignment and the structure of peer effects.* Department of Economics, Harvard University. (http://www.hks.harvard.edu/inequality/Seminar/Papers/Hoxby06.pdf)

King, G., Tomz, M., & Wittenberg, J. (2000). Making the most of statistical analyses: Improving interpretation and presentation. *American Journal of Political Science*, *44*(2), 341-355.

Kroft, K. (2008). Takeup, social multipliers, and optimal social insurance. *Journal of Public Economics*, *92*(3-4), 722-737.

Lankford, H., & Wyckoff, J. (2006). The effect of school choice and residential location on the racial segregation of students. In T. J. Gronberg & D. W. Jansen (Eds.), *Advances in applied microeconomics, volume 14, improving school accountability: Check-ups or choice* (p. 185-240). Elsevier.

Lazear, E. P. (2001). Educational production. *Quarterly Journal of Economics*, *116*(3), 777-803.

Mallick, D. (2009). Marginal and interaction effects in ordered response models. *Economics and Econometrics Research Institute Research Paper Series*, *22*, 1-9. (http://www.eeri.eu/documents/wp/EERI_RP_2009_22.pdf)

Manski, C. F. (1993). Identification of endogenous social effects: The reflection problem. *The Review of Economic Studies*, *60*(3), 531-542.

Mas, A., & Moretti, E. (2006, September). *Peers at work.* Institute for the Study of Labor (IZA) Discussion Paper Series No. 2292.

McPherson, M., Smith-Lovin, L., & Cook, J. M. (2001). Birds of a feather: Homophily and social networks. *Annual Review of Sociology*, *27*, 415-444.

Mickelson, R. A. (2001). Subverting Swann: First- and second-generation segregation in Charlotte-Mecklenburg schools. *American Educational Research Journal*, *38*(2), 215-252.

Mickelson, R. A. (2003). The academic consequences of desegregation and segregation: Evidence from the Charlotte-Mecklenburg schools. *North Carolina Law Review*, *81*, 1513-1562.

Mihaljevic, M. (2008). Examining the efficiency of Croatian higher education. *Eastern European Economics*, *46*(1), 74-88.

Moffitt, R. A. (2001). Policy interventions, low-level equilibria, and social interactions. In S. N. Durlauf & H. P. Young (Eds.), *Social dynamics* (p. 45-82). The MIT Press.

Nakajima, R. (2007). Measuring peer effects in youth smoking behavior. *Review of Economic Studies*, *74*(3), 897-935.

Natriello, G., & McDill, E. L. (1986). Performance standards, student effort on homework, and academic achievement. *Sociology of Education*, *59*(1), 18-31.

Nechyba, T. J. (1999). School finance induced migration and stratification patterns: The impact of private school vouchers. *Journal of Public Economic Theory*, *1*(1), 5-50.

Norton, E. C., Wang, H., & Ai, C. (2004). Computing interaction effects and standard errors in logit and probit models. *The Stata Journal*, *4*(2), 154-167.

Pepper, J. V. (2002). Robust inferences from random clustered samples: an application using data from the panel study of income dynamics. *Economics Letters*, *75*, 341-345.

Primo, D. M., Jacobsmeier, M. L., & Milyo, J. (2007). Estimating the impact of state policies and institutions with mixed-level data. *State Politics and Policy Quarterly*, *7*(4), 446-459.

Sacerdote, B. (2001). Peer effects with random assignment: Results for Dartmouth students. *Quarterly Journal of Economics*, *116*(2), 681-704.

Sajaia, Z. (forthcoming). Maximum likelihood estimation of a bivariate ordered probit model: implementation and Monte Carlo simulations. *The Stata Journal of Applied Econometrics.* (http://adeptanalytics.org/download/ado/bioprobit/bioprobit.pdf)

Santor, D. A., Messervey, D., & Kusumakar, V. (2000). Measuring peer pressure, popularity, and conformity in adolescent boys and girls: Predicting school performance, sexual attitudes, and substance abuse. *Journal of Youth and Adolescence*, *29*(2), 163-182.

Scheinkman, J. A. (2008). Social interactions (theory). In S. N. Durlauf & L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics Online.* Basingstoke: Palgrave Macmillan. (http://www.dictionaryofeconomics.com.libproxy.temple.edu/article?id=pde2008_T000200)

Scotti, C. (2006). A bivariate model of Fed and ECB main policy rates. *International Finance Discussion Papers: Board of Governors of the Federal Reserve*(875), 1-43.

Shapiro, C., & Stiglitz, J. E. (1984). Equilibrium unemployment as a worker discipline device. *American Economic Review*, *74*(3), 433-444.

Sorenson, A. B., & Hallinan, M. T. (1977). A reconceptualization of school effects. *Sociology of Education*, *50*(4), 273-289.

Stewart, E. B. (2008). School structural characteristics, student effort, peer associations, and parental involvement: The influence of school- and individual-level factors on academic achievement. *Education and Urban Society*, *40*(2), 179-204.

Stinebrickner, T. R., & Stinebrickner, R. (2008). The causal effect of studying on academic performance. *The B.E. Journal of Economic Analysis and Policy*, *8*(1, Article 14), 1-34.

Summers, A. A., & Wolfe, B. L. (1977). Do schools make a difference? *The American Economic Review*, *67*(4), 639-652.

Swann v. Charlotte-Mecklenburg schools (Vol. 402 U.S. 1) (No. 15). (1971).

Terenzini, P. T., Cabrera, A. F., Colbeck, C. L., Bjorklund, S. A., & Parente, J. M. (2001). Racial and ethnic diversity in the classroom: Does it promote student learning? *Journal of Higher Education*, *72*(5), 509-531.

Tomz, M., Wittenberg, J., & King, G. (2001). *Clarify: Software for interpreting and presenting statistical results. Version 2.0.* Harvard University: Cambridge, MA.

Trautwein, U., & Köller, O. (2003). The relationship between homework and achievement–still much of a mystery. *Educational Psychology Review*, *15*(2), 115.

Varian, H. R. (1992). *Microeconomic analysis (third edition).* W. W. Norton & Company.

Wake County Public School System. (2010, March). *Board of education approves resolution on community based assignment.* (http://www.wcpss.net/news/2010_march24_community_assignment/)

Weinberg, B. A. (2006, October). *Social interactions and endogenous associations.* Department of Economics, Ohio State University. (http://economics.sbs.ohio-state.edu/weinberg/socint1.pdf)

Wooldridge, J. M. (2003). Cluster-sample methods in applied econometrics. *The American Economic Review*, *93*(2), 133-138.

Wooldridge, J. M. (2006, June). *Cluster-sample methods in applied econometrics: An extended analysis.* Department of Economics, Michigan State University. (`http://www.econ.msu.edu/faculty/wooldridge/current%20research/clus1aea.pdf`)

Zeidner, M., & Schleyer, E. J. (1998). The big-fish-little-pond effect for academic self-concept, test anxiety, and school grades in gifted children. *Contemporary Educational Psychology*, *24*, 305-329.

Zelner, B. A. (2009). Using simulation to interpret results from logit, probit, and other nonlinear models. *Strategic Management Journal*, *30*, 1335-1348.

Zimmerman, D. J. (2003). Peer effects in academic outcomes: Evidence from a natural experiment. *Review of Economics and Statistics*, *85*(1), 9-23.

APPENDIX A

Data Description and Estimation Results

A.1 Variable Descriptions

This section presents a list and brief description of each variable used in estimation.

- **White:** Binary dummy variable that takes a value of 1 if the student is white and 0 otherwise.
- **Female:** Binary dummy variable that takes a value of 1 if a student is female and 0 if male.
- **Paid job:** Ordered discrete variable that measures how many hours per week a student works in a paid job, ranging from "none" (1) to "20 or more hours" (5).

- **Ability:** Continuous variable that measures how a student performed on a past California Achievement Test (CAT) Total Language Battery. For MS students, grade 2 CAT scores are used; for HS students, grade 6 CAT scores are used. Students who entered the CMS district after the CAT was administered having missing values for ability.
- **Missing ability:** Binary dummy variable that takes a value of 1 if a student has a missing value for ability.
- **SES:** Continuous variable that measures the student's parents' socioeconomic status split into quantiles for estimation.
- **Live with both parents:** Binary dummy variable that takes a value of 1 if a student has lived most of his life with two parents and 0 otherwise.
- **High School:** Binary dummy variable that takes a value of 1 if the student is in high school and 0 otherwise.
- **Track:** Ordered discrete variable that takes a value of 1 for regular, 2 academic, and 3 for AP/IB.
- **EOWNVAR:** Measure the variance across the each of the E-OWN variables constructed using an index of qualitative variation (IQV) that calculates the ratio of observed variation to the maximum expected variation.
- **EOTHERVAR:** Measure the variance across the each of the E-OTHER variables constructed using an IQV.
- **SOWNVAR:** Measure the variance across the each of the S-OWN variables constructed using an IQV.

- **SOTHERVAR:** Measure the variance across the each of the S-OTHER variables constructed using an IQV.
- **Percent black in school:** Percentage of the school's student body that is black.
- **% of school's teachers w/ tenure:** Percentage of the student's school's teachers who are tenured.
- **% of school's teachers fully licensed:** Percentage of the student's school's teachers who are fully licensed.
- **Magnet:** Binary dummy variable that takes a value of 1 if the student's school is a magnet school and 0 otherwise.
- **School dummies:** Series of dummy variables that take a value of 1 if the student is in that particular school and 0 otherwise.

A.2 Construction of Peer Measures of EFFORT and STUDY

The peer measures of EFFORT and STUDY are constructed as follows. First, I calculate the proportion of the student's own racial peer group that selects EFFORT = 1 and call this variable E1OWN. I do the same for EFFORT = 2, ..., EFFORT = 5 to calculate E2OWN, ..., E5OWN;[1] I refer

[1]For students who are the only ones of their own race within the classroom (34 students in MS and 41 students in HS), I estimate two specifications, one in which the student's "own" racial peer group is the largest racial group in the classroom and one in which the student's "own" racial peer group is the second largest racial group in the classroom. The estimation results do not change.

Table A.1: Calculation of Peer Variables

EFFORT level	Students/Total	E-OWN	Students/Total	E-OTHER
1	1/11	E1OWN = 9.1%	1/8	E1OTHER = 12.5%
2	2/11	E2OWN = 18.2%	1/8	E2OTHER = 12.5%
3	(3-1)/11	E3OWN = 18.2%	2/8	E3OTHER = 25.0%
4	3/11	E4OWN = 27.3%	2/8	E4OTHER = 25.0%
5	3/11	E5OWN = 27.3%	2/8	E5OTHER = 25.0%

to this set of variables, E1OWN, . . . , E5OWN, as E-OWN. I then calculate the same proportion for the largest racial group in the classroom that is not the student's own and call these variables E1OTHER, . . . , E5OTHER and this group of variables E-OTHER. I do the same for STUDY, resulting in S1OWN, . . . , S5OWN (S-OWN) and S1OTHER, . . . , S5OTHER (S-OTHER). These variables allow me to determine how the student's EFFORT and STUDY choices depend on the decision-making of the other students in the classroom. A strong positive relationship between the student's EFFORT and STUDY choices and these measures of peer effort could be evidence that the student takes into account what his own racial peer group and other racial groups do when he decides how hard to work.

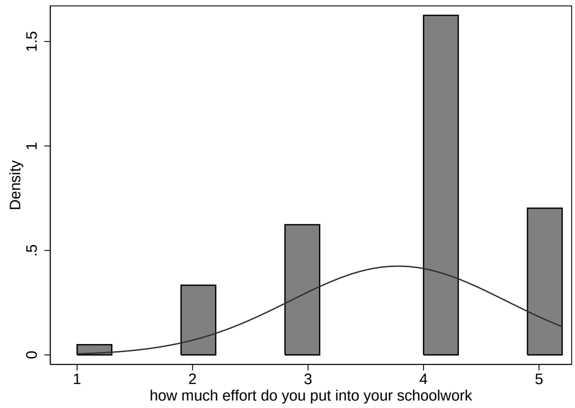
The following hypothetical classroom provides an example of how the explanatory variables described above are constructed. Consider a classroom with 20 students, 12 white and 8 black. Of the white students, 1 chooses forth EFFORT=1, 2 choose EFFORT=2, 3 choose EFFORT=3, 3 choose EFFORT=4, and 3 choose EFFORT=5. For the black students, 1 chooses EFFORT=1, 1 chooses EFFORT=2, 2 choose EFFORT=3, 2 choose EFFORT=4, and 2 choose EFFORT=5. Table A.1 shows how the peer variables are calculated for this classroom when the student of interest is white with EFFORT=3.

A.3 Construction of Interaction Variables

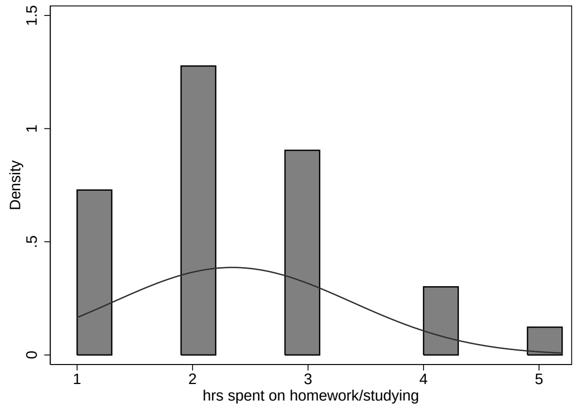
For the linear terms, I multiply each of the E-OWN and E-OTHER variables by the appropriate racial group's classroom share and call these variables E1POWN, . . . , E5POWN and E1POTHER, . . . , E5POTHER. For the squared terms, I multiply each of the E-OWN and E-OTHER variables by the square of the appropriate racial group's classroom share and call these variables E1POWNSQ, . . . , E5POWNSQ and E1POTHERSQ, . . . , E5POTHERSQ. For the cubed terms, I multiply each of the E-OWN and E-OTHER variables by the cube of the appropriate racial group's classroom share and call these variables E1POWNCU, . . . , E5POWNCU and E1POTHERCU, . . . , E5POTHERCU. I generate analogous interaction terms for STUDY as well.

A.4 Summary Statistics

Figures A.1 and A.2 and Table A.2 present summary statistics for EFFORT and STUDY. Table A.3 presents summary statistics for explanatory variables.

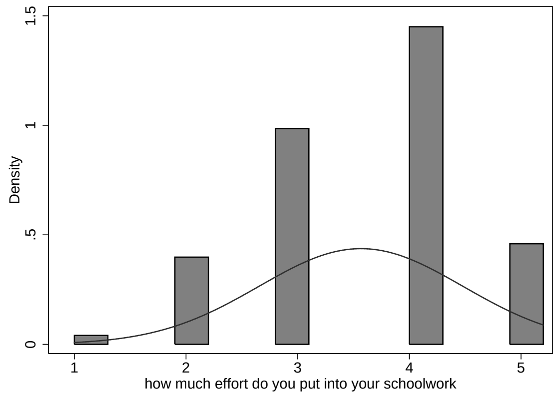


(a) Effort

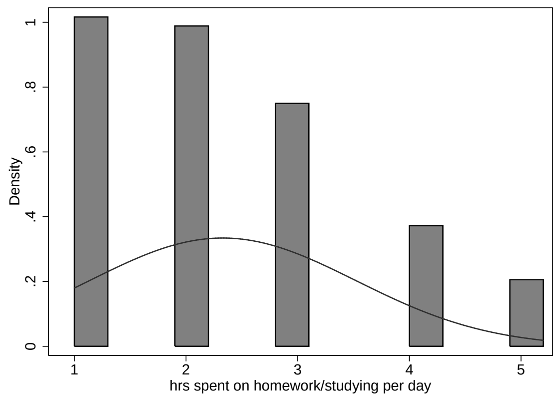


(b) Study

Figure A.1: Frequencies of EFFORT and STUDY for MS



(a) Effort



(b) Study

Figure A.2: Frequencies of EFFORT and STUDY for HS

Table A.2: Correlation between EFFORT and STUDY

	MS	HS
Overall	0.33*	0.49*
By Race		
White	0.34*	0.49*
Black	0.31*	0.44*
Asian/Pacific	0.25*	0.66*
Hispanic	0.58*	0.54*
American Indian	0.50	0.50
Multi-racial	—	—
By Gender		
Female	0.32*	0.46*
Male	0.30*	0.48*
By Track		
Regular	0.35*	0.44*
Academic	—	0.48*
AP/IB/PreIB	0.33*	0.52*

* $p < 0.05$

Table A.3: Summary Statistics for Explanatory Variables

			By School level				**By Race**			
	Overall		**MS**		**HS**		**White**		**Non-white**	
Variable	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
Female	0.52	0.5	0.51	0.5	0.53	0.5	0.5	0.5	0.54	0.5
White	0.59	0.49	0.57	0.5	0.61	0.49				
Black	0.35	0.48	0.37	0.48	0.33	0.47				
Other	0.06	0.24	0.07	0.25	0.05	0.23				
Paid job, 0 hrs	0.44	0.5	0.57	0.49	0.23	0.42	0.41	0.49	0.17	0.37
Paid job, 1-5 hrs	0.18	0.39	0.27	0.44	0.06	0.24	0.2	0.4	0.17	0.37
Paid job, 6-10 hrs	0.1	0.29	0.09	0.29	0.1	0.31	0.1	0.3	0.09	0.28
Paid job, 11-19 hrs	0.12	0.32	0.03	0.18	0.24	0.43	0.13	0.34	0.09	0.29
Paid job, 20+ hrs	0.16	0.37	0.03	0.16	0.35	0.48	0.15	0.36	0.17	0.37
Ability	726.13	49.07	702.18	45.59	758.81	31.75	741.98	40.41	703.60	51.44
Missing ability	1253	—	811	—	442	—	729	—	524	—
Live with both parents	0.66	0.47	0.66	0.48	0.67	0.47	0.78	0.42	0.5	0.5
Track: regular	0.55	0.5	0.68	0.47	0.37	0.48	0.43	0.49	0.73	0.44
Track: academic	0.3	0.46	0.26	0.44	0.34	0.48	0.38	0.49	0.17	0.38
Track: AP/IB	0.24	0.28	0.06	0.24	0.28	0.33	0.1	0.3	0.06	0.24
Magnet school	0.37	0.48	4454	0.58	0.49	0.35	0.48	0.41	0.49	
% black in school	38.58	16.16	39.94	15.73	36.56	16.56	33.3	15.19	46.03	14.46
% of school's teachers w/ tenure	60.19	10.83	53.88	8.42	69.52	6.35	61.38	10	58.5	11.71
% of school's teachers fully licensed	89.42	6.51	86.86	6.92	93.22	3.21	89.97	6.29	88.65	6.75
N	4454		2658		1796		2608		1846	

A.5 Ordered Probit Estimation Results

Tables A.4-A.9 provide the ordered probit estimation results. Estimation results are presented separately for EFFORT and STUDY. For each dependent variable, estimation results are split into three tables: control variables, school dummy variables, and interaction terms. Variables significant at the 5% level are starred. When appropriate, joint significance is noted in the footnote of the table.

Table A.4: Estimation Results for EFFORT: Control Variables

	SPECIFICATIONS									
	Main		**No Cubed Terms**		**No School Dummies**		**MS**		**HS**	
Variable	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.
Female	0.4565*	12.13	0.4553*	12.17	0.4476*	11.86	0.4226*	9.38	0.5141*	7.86
White	-0.1017*	-2.66	-0.1090*	-2.90	-0.0998*	-2.68	-0.1121*	-2.25	-0.0943	-1.36
Paid job, 1-5 hrs	0.0399	0.85	0.0399	0.84	0.0440	0.93	0.0248	0.49	0.0288	0.21
Paid job, 6-10 hrs	-0.0809	-1.37	-0.0789	-1.34	-0.0806	-1.37	-0.1083	-1.46	-0.0291	-0.30
Paid job, 11-19 hrs	-0.0991	-1.85	-0.0973	-1.83	-0.1029	-1.94	-0.0560	-0.50	-0.1082	-1.45
Paid job, 20+ hrs	-0.1846*	-3.22	-0.1863*	-3.26	-0.1828*	-3.21	-0.1455	-0.87	-0.1998*	-2.73
Ability	-0.0002	-0.40	-0.0002	-0.40	-0.0003	-0.63	-0.0007	-1.00	0.0007	0.55
Missing Ability	-0.1465	-0.38	-0.1471	-0.38	-0.2299	-0.62	-0.4867	-0.99	0.5690	0.57
SES, 20-39 percentile	0.0341	0.60	0.0365	0.64	0.0370	0.64	0.0274	0.34	0.0478	0.69
SES, 40-59 percentile	0.1191*	2.07	0.1224*	2.13	0.1118	1.95	0.1810*	2.19	0.0531	0.75
SES, 60-79 percentile	0.1216*	2.31	0.1242*	2.36	0.1138*	2.16	0.1220	1.65	0.1247	1.93
SES, 80+ percentile	0.0773	1.49	0.0798	1.53	0.0667	1.29	0.1175	1.65	0.0022	0.03
Live with both parents	0.1918*	5.18	0.1932*	5.23	0.1919*	5.29	0.2543*	5.35	0.1013	1.67
High School	-0.4428*	-2.76	-0.4327*	-2.65	-0.1136	-1.81				
Track: academic	0.0007	0.02	-0.0025	-0.07	-0.0032	-0.09	0.0868	1.25	-0.0469	-0.88
Track: AP/IB	-0.1149	-1.86	-0.1219*	-2.09	-0.0699	-1.39	-0.2253	-1.84	-0.0598	-0.77
EOWNVAR	0.1897	1.17	0.1876	1.18	0.1741	1.19	0.3193	1.45	0.0739	0.28
EOTHERVAR	0.0980	1.03	0.1112	1.28	0.0830	0.90	-0.0162	-0.12	0.2051	1.34
Percent black in school	0.0015	0.71	0.0017	0.82	0.0042*	3.96	0.0072	1.69	0.0005	0.10
% of school's teachers w/ tenure	0.0120	1.56	0.0115	1.45	0.0000	0.02	0.0103	1.90	-0.0070	-1.32
% of school's teachers fully licensed	-0.0104	-0.84	-0.0100	-0.82	0.0036	1.20	-0.0009	-0.07	-0.0041	-0.33
Magnet School	0.2112*	2.79	0.2155*	2.87	0.0148	0.42	-0.1981	-1.34	-0.0153	-0.14
N	4454		4454		4454		2658		1796	

* $p < 0.05$

Paid job dummmy variables are jointly significant at the 5% level for the Main specification (Prob> $\chi^2 = 0.01$)

Track dummmy variables are not jointly significant at the 5% level for the Main specification (Prob> $\chi^2 = 0.15$)

SES dummmy variables are not jointly significant at the 5% level for the Main specification (Prob> $\chi^2 = 0.09$)

Table A.5: Estimation Results for EFFORT: School Dummies

	SPECIFICATIONS									
	Main		**No Cubed Terms**		**No School Dummies**		**MS**		**HS**	
Variable	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.
School 1	-0.2214	-1.47	-0.1925	-1.30			-0.0593	-0.33		
School 2	-0.1873*	-2.10	-0.1606	-1.74			-0.2285*	-2.29		
School 4	0.0290	0.25	0.0401	0.34			0.0829	0.72		
School 6	-0.2641	-1.35	-0.2368	-1.29			0.3411	1.23		
School 7	0.1949	1.49	0.1901	1.50			0.1326	0.72		
School 8	0.0751	0.71	0.0960	0.93			-0.0274	-0.16		
School 9	-0.4586*	-2.01	-0.4412*	-1.98						
School 10	0.1287	1.01	0.1319	0.97			0.0974	0.58		
School 11	-0.0057	-0.05	-0.0005	-0.00			-0.0369	-0.22		
School 12	-0.0756	-0.93	-0.0684	-0.84			-0.0738	-0.96		
School 13	-0.2394	-1.83	-0.2299	-1.74			-0.2007	-1.41		
School 14	-0.3656*	-2.53	-0.3641*	-2.49			0.0014	0.01		
School 15	-0.1120	-1.04	-0.0912	-0.85			-0.0428	-0.36		
School 16	-0.1888	-1.48	-0.1651	-1.25			-0.0576	-0.40		
School 18	0.0494	0.26	0.0428	0.23			0.5413*	2.65		
School 19	-0.2891	-1.70	-0.2937	-1.76						
School 20	-0.0982	-0.71	-0.0836	-0.60						
School 21	0.0611	0.43	0.0676	0.48			0.4398*	3.12		
School 22	-0.4314*	-2.21	-0.4176*	-2.13			0.1037	0.41		
School 23	0.2580	1.49	0.2399	1.41			0.0747	0.32		
School 32	0.0440	0.33	0.0528	0.41					0.0320	0.25
School 33	-0.0617	-0.49	-0.0636	-0.50					0.0071	0.05
School 34	-0.0698	-0.47	-0.0798	-0.54						
School 35	-0.0388	-0.32	-0.0312	-0.25					0.0354	0.21
School 36	0.3369*	2.31	0.3385*	2.41					0.1291	1.38
School 37	0.0382	0.27	0.0469	0.34					-0.0439	-0.28
School 38	-0.1097	-1.27	-0.0794	-0.91						
School 39	-0.1324	-1.20	-0.1381	-1.25					0.1417	1.21
School 40	0.3953	1.81	0.3617	1.66						
N	4454		4454		4454		2658		1796	

* $p < 0.05$

School dummmy variables are jointly significant at the 5% level for the Main specification (Prob> $\chi^2 = 0.01$)

Table A.6: Estimation Results for EFFORT: Interaction Terms

	SPECIFICATIONS									
	Main		**No Cubed Terms**		**No School Dummies**		**MS**		**HS**	
Variable	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.
POWN	-2.1700*	-2.39	-2.0264*	-2.26	-2.1162*	-2.48	-3.2561*	-2.61	-0.7202	-0.47
E2OWN	0.8024	0.72	0.2813	0.35	0.8911	0.80	0.3143	0.23	1.7474	0.89
E2POWN	-3.4921	-0.47	1.3940	0.44	-4.2622	-0.58	-3.1258	-0.32	-8.0877	-0.61
E2POWNSQ	12.4010	0.79	0.3424	0.13	13.3413	0.88	19.2015	0.86	13.4472	0.53
E2POWNCU	-8.0917	-0.81			-8.2208	-0.85	-14.8219	-1.01	-6.1120	-0.41
E3OWN	0.2841	0.42	0.8084	1.44	0.2476	0.38	-0.4425	-0.44	1.2449	1.28
E3POWN	3.8427	0.96	-1.2182	-0.66	4.8137	1.23	5.8738	0.86	-0.8914	-0.16
E3POWNSQ	-9.1236	-1.13	2.5600	1.82	-11.3424	-1.42	-8.9350	-0.62	-5.4949	-0.49
E3POWNCU	7.6335	1.52			8.8925	1.80	7.3002	0.80	6.5547	0.93
E4OWN	0.3846	0.90	0.2498	0.70	0.4368	1.03	0.3951	0.72	0.2529	0.37
E4POWN	-0.4520	-0.19	1.2425	1.01	-0.2080	-0.09	-3.8131	-1.22	1.7899	0.40
E4POWNSQ	5.4654	1.13	1.2894	1.49	4.6594	1.03	14.8390*	2.33	-0.6158	-0.07
E4POWNCU	-2.5898	-0.86			-2.0201	-0.72	-7.8104	-1.94	-0.1943	-0.04
E5OWN	0.4444	0.53	0.5880	0.92	0.6339	0.76	-0.5380	-0.57	0.8588	0.51
E5POWN	3.6570	0.62	2.9158	1.29	3.0910	0.53	8.3409	1.20	1.6868	0.15
E5POWNSQ	-2.6470	-0.21	-1.4566	-0.83	-1.4944	-0.12	-9.6320	-0.59	-3.6852	-0.16
E5POWNCU	0.7024	0.09			0.1670	0.02	4.2486	0.40	2.7056	0.19
POTHER	-0.0680	-0.25	-0.0613	-0.23	-0.0038	-0.01	0.1453	0.36	-0.3787	-1.05
E2OTHER	0.2952	0.84	-0.1356	-0.51	0.3198	1.13	0.7174	1.81	-0.0010	-0.00
E2POTHER	-5.1195	-1.29	0.7275	0.43	-4.4557	-1.29	-9.7961	-1.68	-1.8905	-0.31
E2POTHERSQ	19.1309	1.59	0.2361	0.10	15.5254	1.47	40.0526*	2.02	5.2965	0.30
E2POTHERCU	-15.7568	-1.53			-12.3015	-1.33	-38.4598*	-2.14	-1.3089	-0.09
E3OTHER	0.0424	0.26	0.2053	1.50	0.0389	0.28	0.0799	0.19	0.0119	0.08
E3POTHER	1.2118	0.51	-1.6044	-1.60	1.4468	0.68	1.6829	0.35	1.4482	0.48
E3POTHERSQ	-6.8820	-0.98	1.8452	1.49	-6.9618	-1.09	-8.9050	-0.66	-7.1760	-0.77
E3POTHERCU	6.9995	1.26			6.6754	1.31	10.4841	0.98	6.5075	0.92
E4OTHER	-0.0131	-0.10	0.0610	0.65	0.0095	0.08	-0.0643	-0.26	0.0747	0.42
E4POTHER	0.4093	0.29	-0.5711	-0.86	0.4636	0.35	0.9667	0.37	-1.2580	-0.57
E4POTHERSQ	-2.5743	-0.63	0.3838	0.53	-2.5065	-0.65	-3.6273	-0.54	2.8066	0.40
E4POTHERCU	2.4224	0.74			2.1689	0.70	2.5852	0.53	-1.7004	-0.29
E5OTHER	-0.2255	-1.52	-0.2038	-1.51	-0.2137	-1.83	-0.5644	-1.45	-0.2708	-1.65
E5POTHER	0.7670	0.32	0.4905	0.42	1.1456	0.54	4.5338	0.98	2.7260	0.84
E5POTHERSQ	-0.9547	-0.13	-0.5361	-0.35	-1.2789	-0.19	-14.8812	-1.07	-2.1731	-0.21
E5POTHERCU	0.2008	0.03			0.2480	0.04	14.2801	1.19	-1.6457	-0.20
N	4454		4454		4454		2658		1796	

* $p < 0.05$

Table A.7: Estimation Results for STUDY: Control Variables

	SPECIFICATIONS									
	Main		**No Cubed Terms**		**No School Dummies**		**MS**		**HS**	
Variable	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.
Female	0.3447*	10.43	0.3438*	10.34	0.3383*	10.20	0.3793*	9.30	0.3190*	5.72
White	-0.2556*	-6.75	-0.2610*	-6.96	-0.2599*	-7.19	-0.2218*	-4.57	-0.4120*	-5.42
Paid job, 1-5 hrs	0.0711	1.68	0.0713	1.68	0.0727	1.73	0.1186*	2.52	-0.1349	-0.98
Paid job, 6-10 hrs	0.1766*	2.98	0.1743*	2.94	0.1774*	3.00	0.2387*	3.16	0.0640	0.63
Paid job, 11-19 hrs	-0.0411	-0.65	-0.0402	-0.63	-0.0337	-0.54	0.1780	1.39	-0.1604	-1.94
Paid job, 20+ hrs	-0.2015*	-3.06	-0.2016*	-3.09	-0.1889*	-2.93	0.0375	0.21	-0.2810*	-3.42
Ability	-0.0016*	-3.26	-0.0016*	-3.25	-0.0017*	-3.42	-0.0015*	-2.16	-0.0028*	-2.21
Missing Ability	-1.1450*	-3.14	-1.1448*	-3.13	-1.1699*	-3.29	-1.0494*	-2.12	-2.0189*	-2.09
SES, 20-39 percentile	-0.0505	-0.93	-0.0477	-0.87	-0.0445	-0.83	-0.0666	-0.91	-0.0202	-0.24
SES, 40-59 percentile	0.0830	1.38	0.0852	1.41	0.0907	1.56	0.0643	0.91	0.1181	1.23
SES, 60-79 percentile	0.1139*	2.03	0.1146*	2.05	0.1276*	2.34	0.1133	1.63	0.1130	1.31
SES, 80+ percentile	0.1302*	2.27	0.1318*	2.27	0.1449*	2.60	0.1360*	2.01	0.1757	1.72
High School	0.1604	0.81	0.1843	0.94	0.0438	0.78				
Track: academic	-0.0139	-0.36	-0.0144	-0.38	0.0189	0.53	-0.0110	-0.17	-0.0134	-0.21
Track: AP/IB	0.0048	0.08	0.0047	0.08	0.0528	1.00	0.1040	0.75	0.0321	0.44
Live with both parents	0.1366*	3.48	0.1367*	3.48	0.1354*	3.46	0.1893*	3.82	0.0471	0.81
SOWNVAR	-0.5409*	-3.65	-0.5496*	-3.90	-0.5328*	-3.68	-0.5375*	-3.04	-0.6768*	-2.45
SOTHERVAR	-0.0398	-0.51	-0.0142	-0.18	-0.0480	-0.67	-0.0585	-0.41	-0.0090	-0.06
Percent black in school	-0.0032	-1.20	-0.0033	-1.25	-0.0018	-1.66	-0.0113*	-2.60	0.0035	0.47
% of school's teachers w/ tenure	0.0024	0.22	0.0014	0.14	0.0068*	3.41	0.0086	1.78	0.0073	1.03
% of school's teachers fully licensed	-0.0065	-0.37	-0.0053	-0.32	-0.0048*	-2.15	-0.0256*	-3.02	0.0165	1.28
Magnet School	-0.0258	-0.28	-0.0198	-0.23	0.0245	0.83	0.2697*	2.08	-0.0212	-0.27
N	4454		4454		4454		2658		1796	

* $p < 0.05$

Paid job dummy variables are jointly significant at the 5% level for the Main specification (Prob> $\chi^2 = 0.00$)

Track dummmy variables are not jointly significant at the 5% level for the Main specification (Prob> $\chi^2 = 0.92$)

SES dummmy variables are jointly significant at the 5% level for the Main specification (Prob> $\chi^2 = 0.00$)

Table A.8: Estimation Results for STUDY: School Dummies

	SPECIFICATIONS									
	Main		**No Cubed Terms**		**No School Dummies**		**MS**		**HS**	
Variable	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.
School 1	-0.2916	-1.61	-0.2695	-1.51			-0.8755*	-5.07		
School 2	0.0914	1.22	0.1017	1.37			0.0970	0.86		
School 4	0.0942	0.62	0.0969	0.64			0.0137	0.11		
School 6	0.2015	0.86	0.2156	0.95			-0.2828	-1.03		
School 7	-0.1140	-0.68	-0.1359	-0.87			0.0760	0.41		
School 8	0.0254	0.21	0.0353	0.32			0.0166	0.13		
School 9	0.2766	1.15	0.2786	1.20						
School 10	0.1018	0.58	0.1223	0.71			0.2740	1.01		
School 11	0.1458	1.26	0.1536	1.51			0.2656	1.48		
School 12	-0.1708	-1.51	-0.1552	-1.42			-0.3710*	-2.88		
School 13	0.0037	0.02	0.0136	0.09			-0.1754	-1.67		
School 14	0.0921	0.49	0.0846	0.47			-0.2564	-1.34		
School 15	0.0769	0.73	0.0838	0.80			0.0266	0.24		
School 16	-0.1445	-0.96	-0.1383	-0.93			-0.5121*	-4.37		
School 18	0.1884	1.37	0.1961	1.49			0.0013	0.01		
School 19	0.0268	0.15	0.0218	0.13						
School 20	0.0866	0.88	0.0946	0.98						
School 21	0.0951	0.68	0.0819	0.64			-0.2213*	-2.06		
School 22	0.2549	0.92	0.2574	0.96			-0.1861	-1.22		
School 23	-0.1072	-0.44	-0.1264	-0.54			0.1694	0.85		
School 32	-0.0301	-0.17	-0.0605	-0.36					-0.0156	-0.13
School 33	0.0381	0.24	0.0152	0.10					0.0257	0.21
School 34	0.1509	0.98	0.1268	0.86						
School 35	-0.1556	-1.11	-0.1720	-1.29					0.0197	0.10
School 36	0.0090	0.06	0.0028	0.02					-0.0553	-0.59
School 37	0.0572	0.35	0.0093	0.06					0.1405	0.75
School 38	0.1376	1.22	0.1420	1.25						
School 39	0.0148	0.14	0.0225	0.20					-0.1126	-0.79
School 40	-0.0202	-0.07	-0.0327	-0.12						
N	4454		4454		4454		2658		1796	

* $p < 0.05$

School dummmy variables are jointly significant at the 5% level for the Main specification (Prob> $\chi^2 = 0.00$)

Table A.9: Estimation Results for STUDY: Interaction Terms

	SPECIFICATIONS									
	Main		**No Cubed Terms**		**No School Dummies**		**MS**		**HS**	
Variable	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.	Coeff.	z stat.
POWN	-0.8509*	-2.21	-0.8270*	-2.19	-0.9250*	-2.50	0.1687	0.33	-1.5154*	-2.89
S2OWN	0.0652	0.16	0.2832	0.97	0.0948	0.24	0.6686	1.31	-0.4280	-0.50
S2POWN	1.8143	0.59	-0.3738	-0.33	1.4851	0.53	-2.5832	-0.73	7.5963	1.11
S2POWNSQ	-4.0187	-0.58	1.2218	1.24	-2.8061	-0.46	4.1560	0.53	-18.6736	-1.20
S2POWNCU	3.4992	0.78			2.3915	0.62	-1.8897	-0.37	14.6696	1.42
S3OWN	0.5795	1.40	0.1401	0.50	0.6300	1.54	0.7425	1.35	1.0289	1.36
S3POWN	-3.9246	-1.12	1.1504	1.01	-3.7434	-1.10	-4.1628	-0.89	-10.7586	-1.71
S3POWNSQ	12.2217	1.59	-0.1027	-0.09	12.2510	1.65	9.7246	0.96	28.3784	1.89
S3POWNCU	-8.1044	-1.66			-8.0216	-1.70	-6.5377	-1.03	-18.0936	-1.76
S4OWN	-0.7437	-0.88	-0.0898	-0.17	-0.4998	-0.64	-2.0768*	-1.96	-0.1940	-0.13
S4POWN	10.5532	1.58	3.8179	1.84	9.6061	1.67	21.7582*	2.47	4.9813	0.43
S4POWNSQ	-18.2005	-1.19	-2.0226	-1.05	-17.0724	-1.35	-41.4656*	-2.04	-10.8149	-0.40
S4POWNCU	10.5749	1.06			11.1127	1.38	21.8627	1.70	10.0512	0.56
S5OWN	2.0995	1.35	0.8649	0.88	2.1889	1.43	1.0143	0.42	1.7747	0.80
S5POWN	-10.3776	-0.96	1.4868	0.39	-10.1425	-0.98	-9.5111	-0.60	-10.6621	-0.65
S5POWNSQ	27.2100	1.20	-0.1394	-0.04	26.1536	1.24	32.7824	0.93	30.4284	0.88
S5POWNCU	-17.8607	-1.27			-16.5777	-1.28	-27.9706	-1.20	-21.2299	-0.99
POTHER	-0.1482	-0.63	-0.1378	-0.57	-0.1589	-0.74	-0.2809	-0.76	-0.2293	-0.60
S2OTHER	-0.1730	-1.45	-0.1651	-1.88	-0.0862	-0.98	-0.0535	-0.18	-0.0381	-0.29
S2POTHER	0.7074	0.47	0.9468	1.49	-0.0312	-0.03	-0.9978	-0.35	0.1230	0.05
S2POTHERSQ	0.0851	0.02	-1.0991	-1.34	2.1663	0.59	6.3802	0.87	-0.8077	-0.10
S2POTHERCU	-0.9609	-0.28			-2.6013	-0.86	-5.4538	-1.04	0.0504	0.01
S3OTHER	-0.0242	-0.24	0.0847	1.05	-0.0459	-0.60	0.0542	0.38	-0.0802	-0.39
S3POTHER	0.8506	0.47	-0.7493	-0.92	1.5759	1.06	1.2438	0.51	1.1107	0.27
S3POTHERSQ	-3.9686	-0.66	0.4502	0.44	-4.5649	-0.89	-8.0105	-1.07	1.4644	0.11
S3POTHERCU	3.2089	0.65			3.1454	0.74	8.1561	1.41	-4.7421	-0.48
S4OTHER	-0.0484	-0.25	0.1628	0.78	0.0050	0.03	0.1406	0.38	0.1141	0.20
S4POTHER	2.7582	0.85	-0.1849	-0.10	2.6026	0.88	-3.2039	-0.58	9.3808	1.56
S4POTHERSQ	-9.1142	-0.87	-0.6171	-0.22	-8.3379	-0.84	8.0331	0.51	-34.3234*	-1.97
S4POTHERCU	6.6291	0.73			6.2315	0.70	-8.5137	-0.73	31.0902*	2.17
S5OTHER	-0.0006	-0.00	0.2212	1.37	-0.0417	-0.20	3.5169	1.63	-0.0833	-0.36
S5POTHER	3.5203	0.82	-0.5172	-0.30	2.8454	0.79	-30.5325	-1.65	8.7018*	2.13
S5POTHERSQ	-10.3595	-0.73	2.4108	1.01	-6.0846	-0.51	83.4887	1.78	-25.0517	-1.79
S5POTHERCU	10.5602	0.89			6.3927	0.62	-66.9975	-1.88	20.4048	1.71
N	4454		4454		4454		2658		1796	

* $p < 0.05$

A.6 Marginal Effects for Control Variables

Tables A.10 and A.11 present the marginal effects estimates for the control variables.

Table A.10: Marginal Effects for EFFORT

	Outcomes for EFFORT				
	1	2	3	4	5
Female	-0.0123	-0.0726	-0.0826	0.0542	0.1133
White	0.0025	0.0159	0.0189	-0.0117	-0.0256
Paid job, 20+ hrs	0.0055	0.0311	0.0330	-0.0262	-0.0433
SES, 40-59 percentile	-0.0028	-0.0181	-0.0223	0.0124	0.0308
SES, 60-79 percentile	-0.0028	-0.0184	-0.0228	0.0126	0.0315
Live with both parents	-0.0053	-0.0313	-0.0348	0.0249	0.0465
High School	0.0129	0.0730	0.0787	-0.0585	-0.1061
Magnet School	-0.0051	-0.0324	-0.0394	0.0227	0.0542
School 2	0.0059	0.0324	0.0329	-0.0283	-0.0428
School 9	0.0197	0.0885	0.0703	-0.0881	-0.0905
School 14	0.0141	0.0681	0.0593	-0.0653	-0.0762
School 22	0.0177	0.0820	0.0678	-0.0803	-0.0872
School 36	-0.0060	-0.0443	-0.0644	0.0179	0.0968

Table A.11: Marginal Effects for STUDY

	Outcomes for STUDY				
	1	2	3	4	5
Female	-0.1067	-0.0247	0.0596	0.0444	0.0274
White	0.0777	0.0207	-0.0435	-0.0337	-0.0212
Paid job, 6-10 hrs	-0.0518	-0.0171	0.0289	0.0241	0.0160
Paid job, 20+ hrs	0.0652	0.0103	-0.0366	-0.0248	-0.0142
Ability	0.0005	0.0001	-0.0003	-0.0002	-0.0001
Missing Ability	0.3933	-0.0114	-0.1989	-0.1172	-0.0658
SES, 60-79 percentile	-0.0343	-0.0098	0.0193	0.0152	0.0096
SES, 80+ percentile	-0.0392	-0.0112	0.0220	0.0174	0.0111
Live with both parents	-0.0429	-0.0092	0.0241	0.0175	0.0105
SOWNVAR	0.1673	0.0403	-0.0942	-0.0703	-0.0430
% of teachers licensed	0.0052	0.0013	-0.0029	-0.0022	-0.0013
School 1	0.0803	0.0088	-0.0447	-0.0286	-0.0157
School 20	-0.0787	-0.0318	0.0430	0.0395	0.0280

A.7 Alternative Specifications for Simulations

Figures A.3-A.66 present the simulation results for alternative starting values and estimation specifications.

A.7.1 Simulation Results for Other Gender/Race Combinations

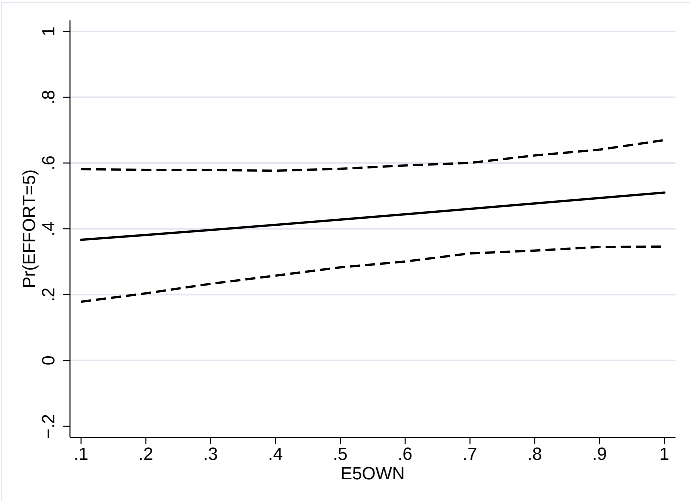


Figure A.3: Effect of E5OWN on Pr(EFFORT=5), Black Female

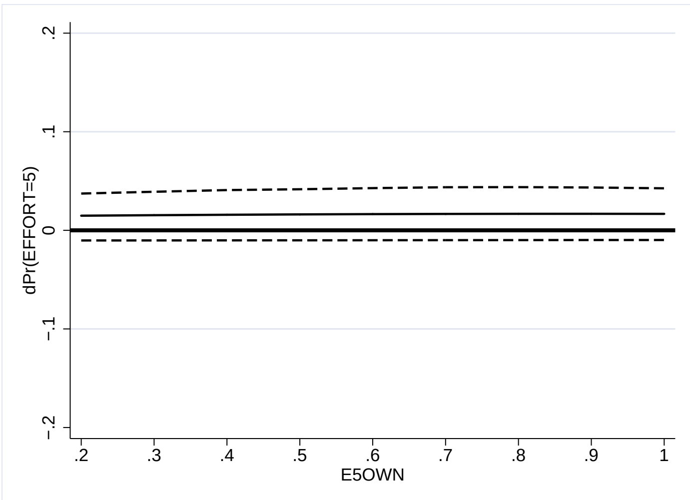


Figure A.4: Effect of E5OWN on dPr(EFFORT=5), Black Female

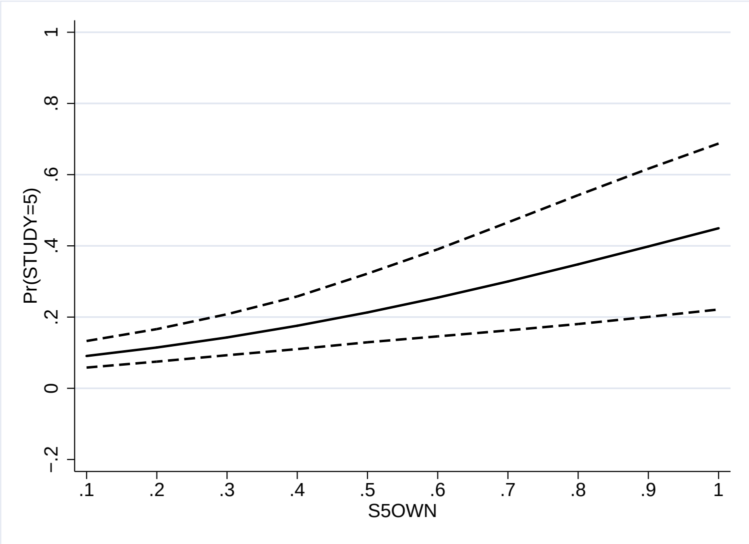


Figure A.5: Effect of S5OWN on Pr(STUDY=5), Black Female

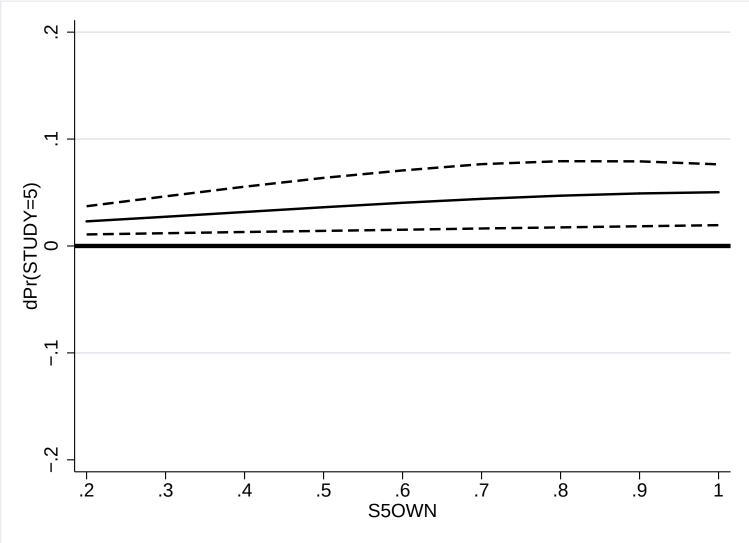


Figure A.6: Effect of S5OWN on dPr(STUDY=5), Black Female

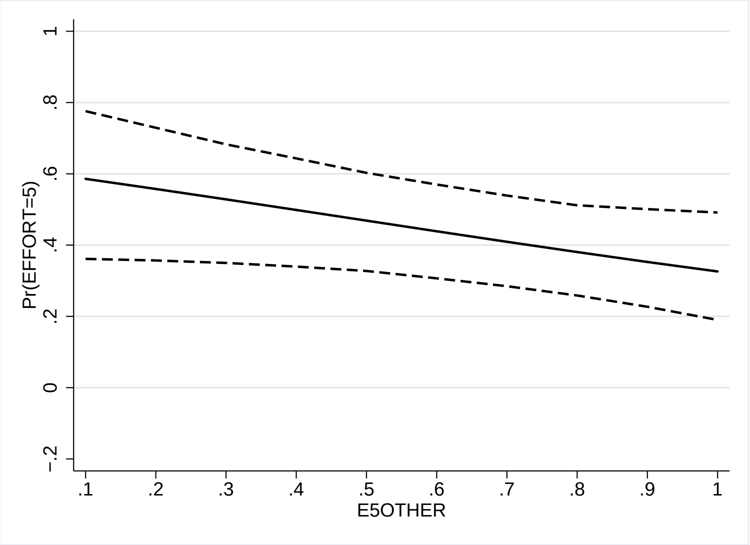


Figure A.7: Effect of E5OTHER on Pr(EFFORT=5), Black Female

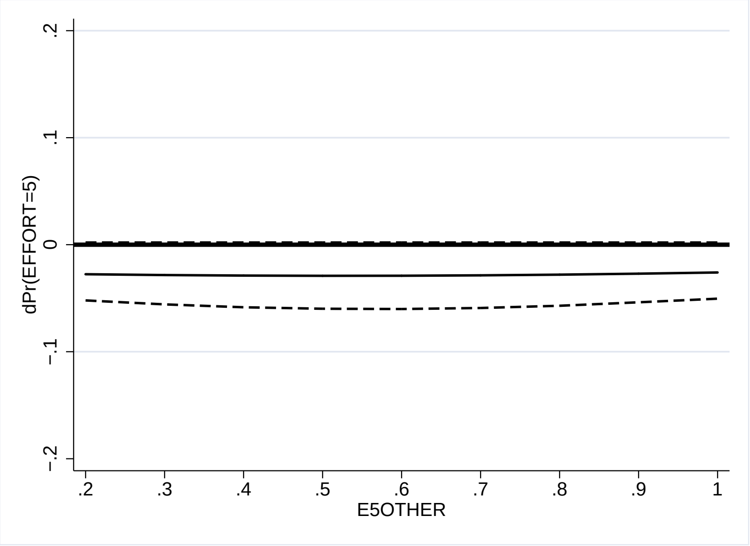


Figure A.8: Effect of E5OTHER on dPr(EFFORT=5), Black Female

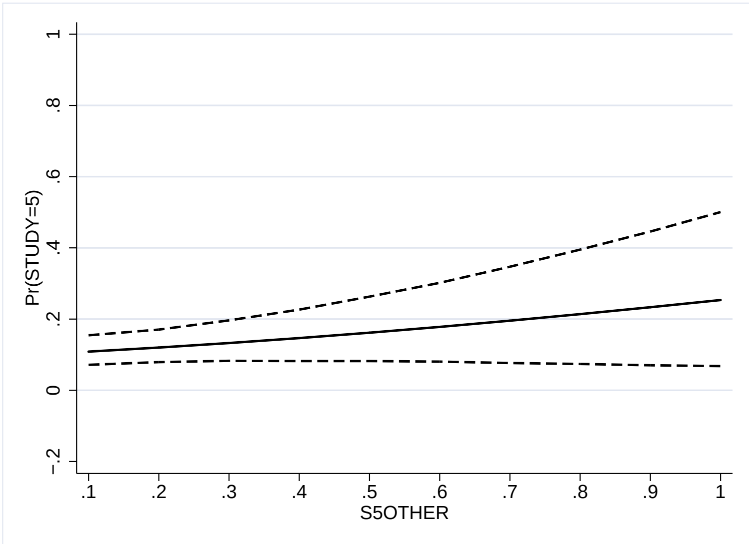


Figure A.9: Effect of S5OTHER on Pr(STUDY=5), Black Female

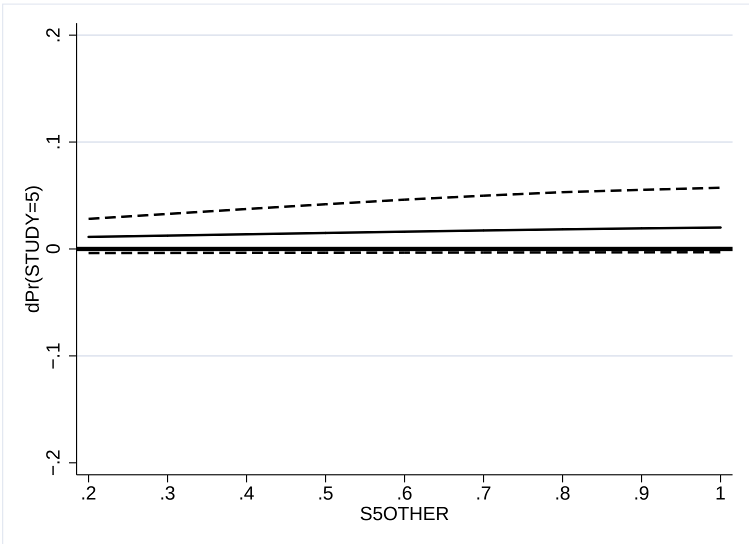


Figure A.10: Effect of S5OTHER on dPr(STUDY=5), Black Female

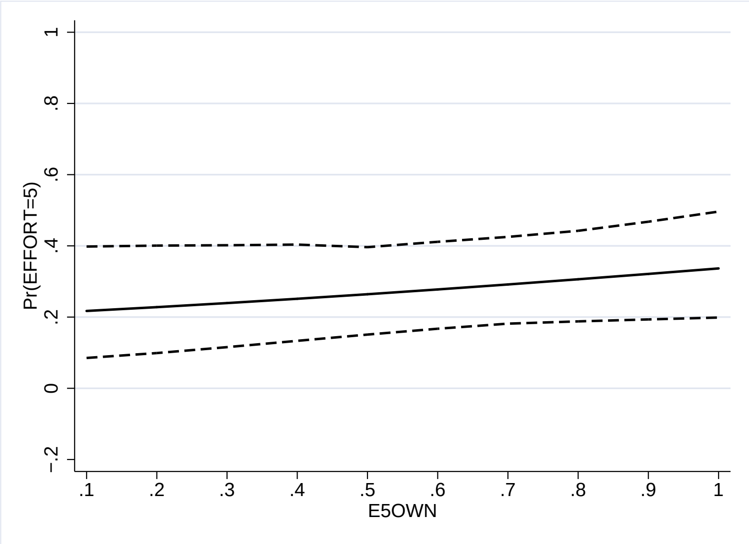


Figure A.11: Effect of E5OWN on Pr(EFFORT=5), Black Male

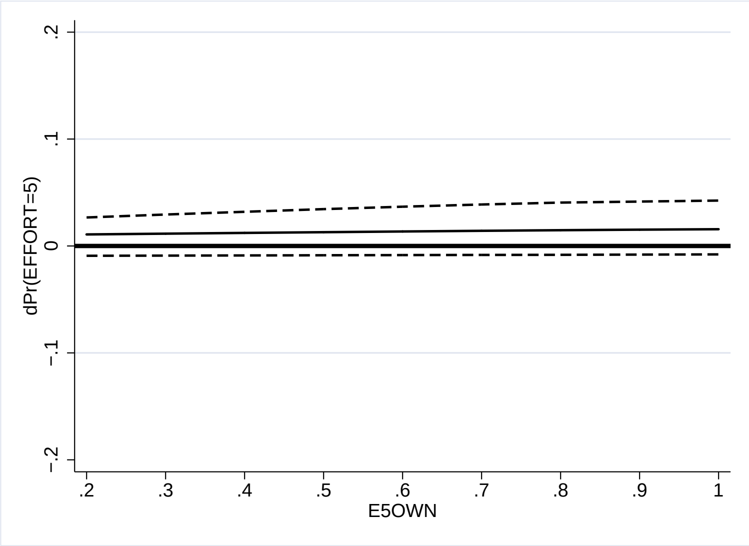


Figure A.12: Effect of E5OWN on dPr(EFFORT=5), Black Male

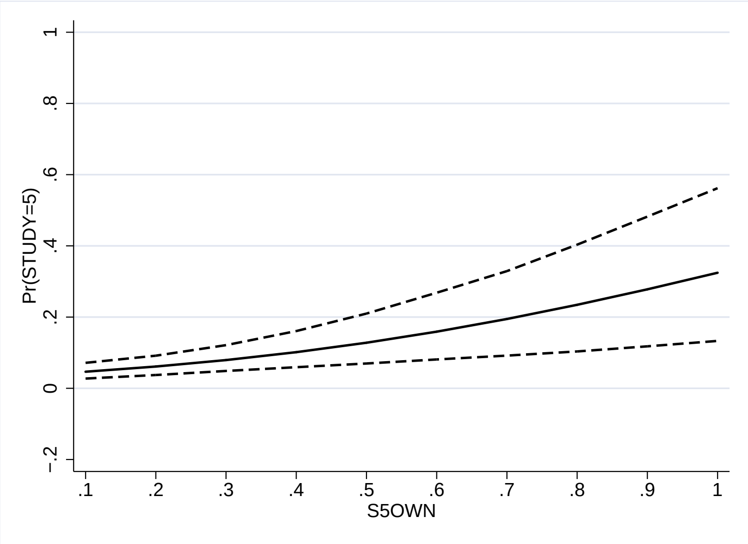


Figure A.13: Effect of S5OWN on Pr(STUDY=5), Black Male

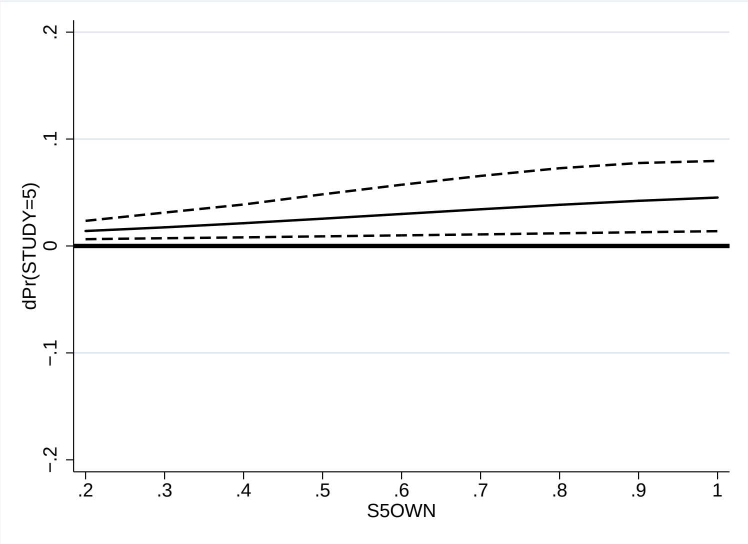


Figure A.14: Effect of S5OWN on dPr(STUDY=5), Black Male

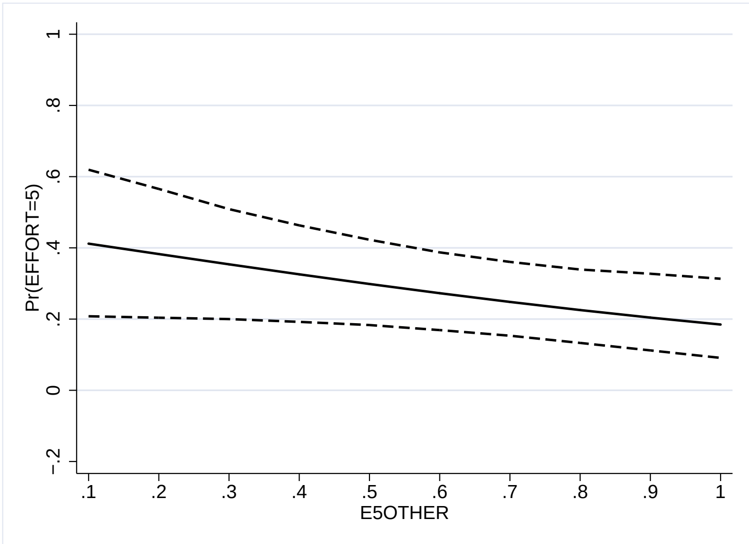


Figure A.15: Effect of E5OTHER on Pr(EFFORT=5), Black Male

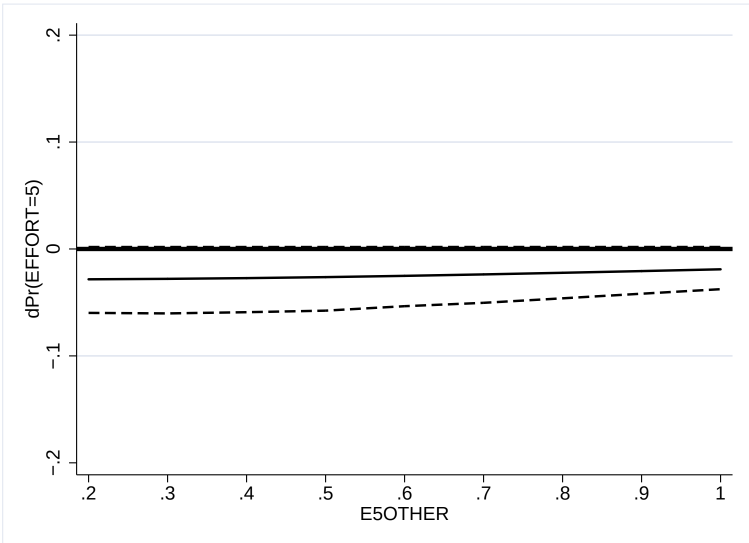


Figure A.16: Effect of E5OTHER on dPr(EFFORT=5), Black Male

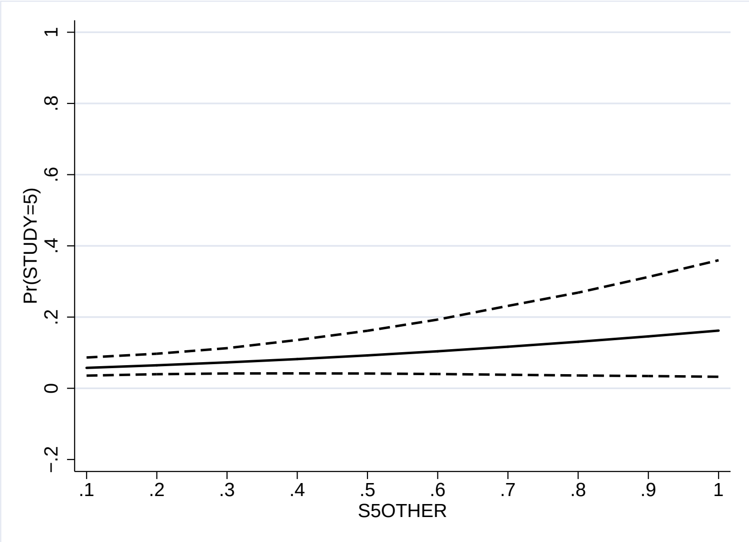


Figure A.17: Effect of S5OTHER on Pr(STUDY=5), Black Male

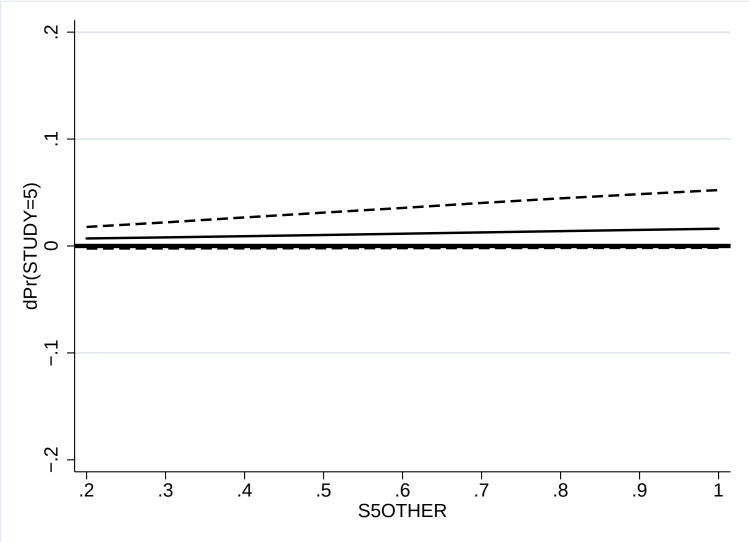


Figure A.18: Effect of S5OTHER on dPr(STUDY=5), Black Male

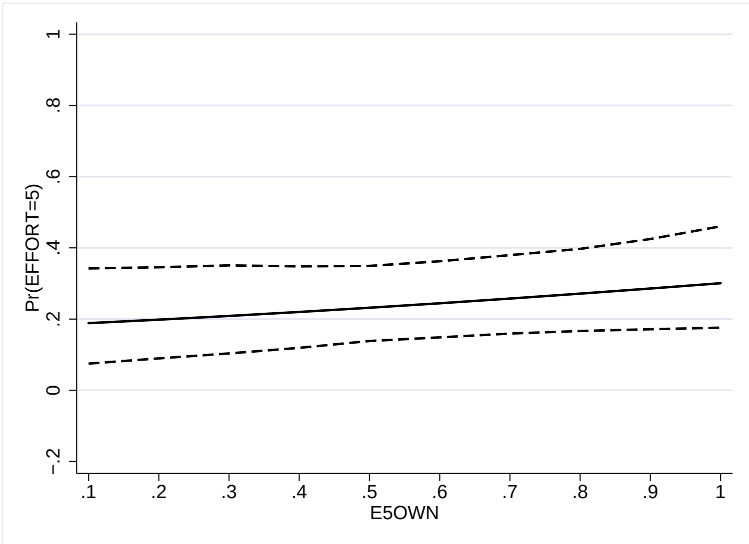


Figure A.19: Effect of E5OWN on Pr(EFFORT=5), White Male

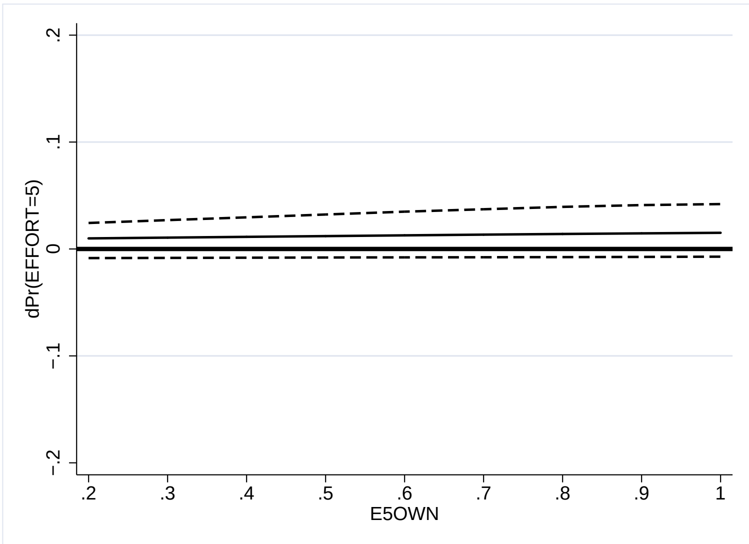


Figure A.20: Effect of E5OWN on dPr(EFFORT=5), White Male

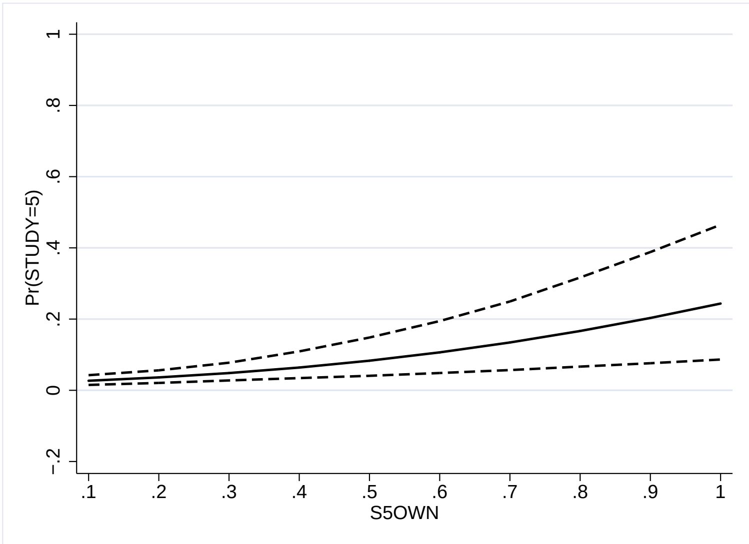


Figure A.21: Effect of S5OWN on Pr(STUDY=5), White Male

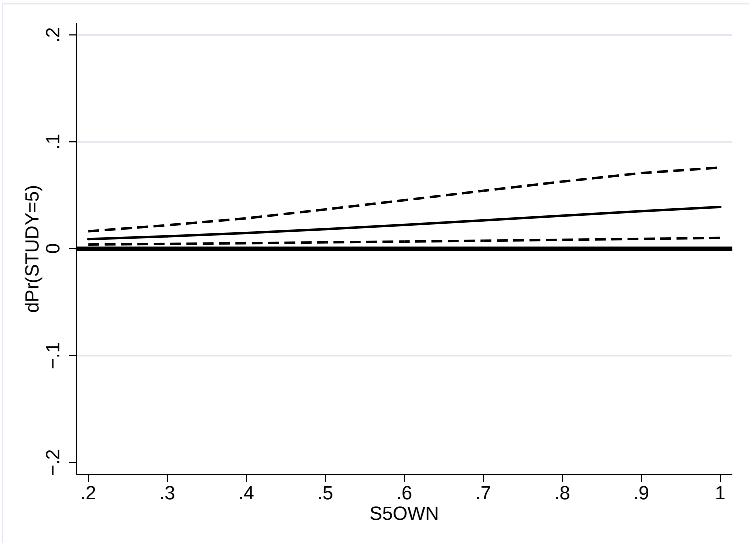


Figure A.22: Effect of S5OWN on dPr(STUDY=5), White Male

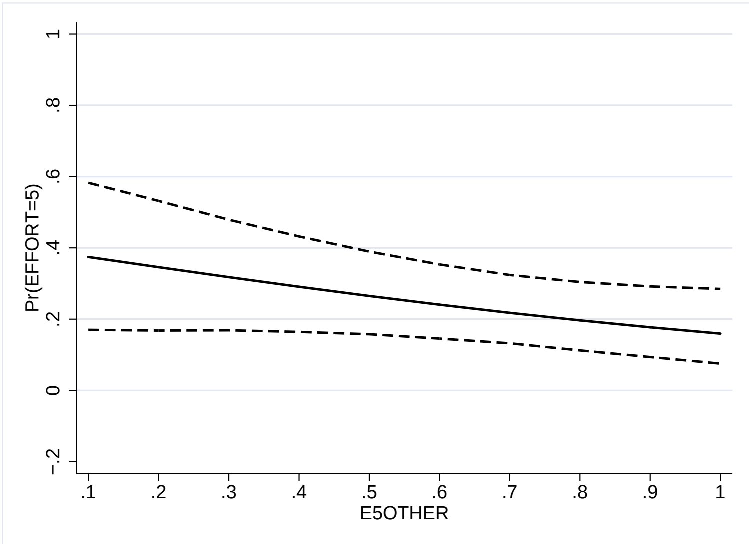


Figure A.23: Effect of E5OTHER on Pr(EFFORT=5), White Male

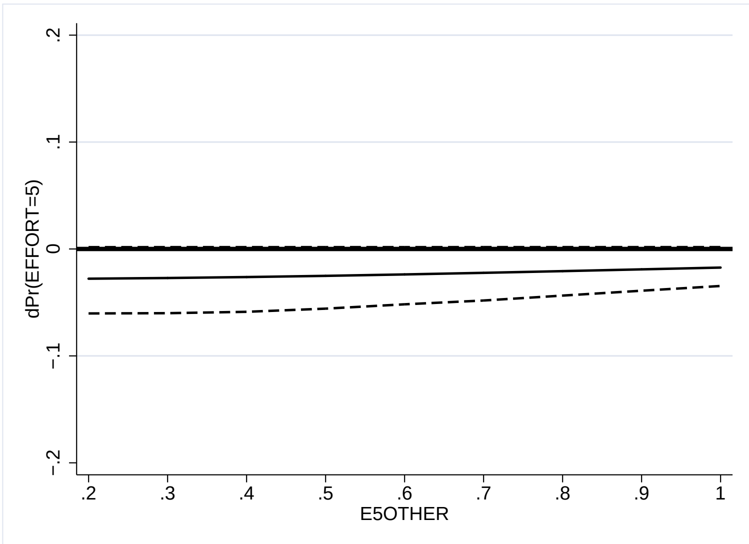


Figure A.24: Effect of E5OTHER on dPr(EFFORT=5), White Male

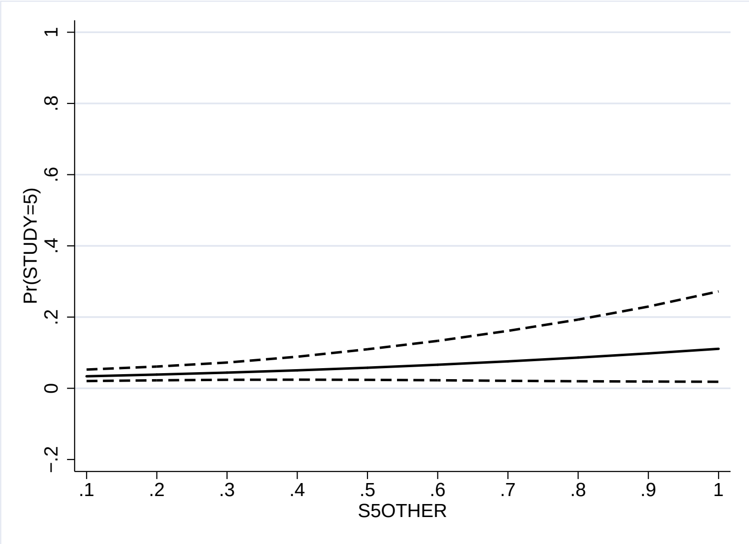


Figure A.25: Effect of S5OTHER on Pr(STUDY=5), White Male

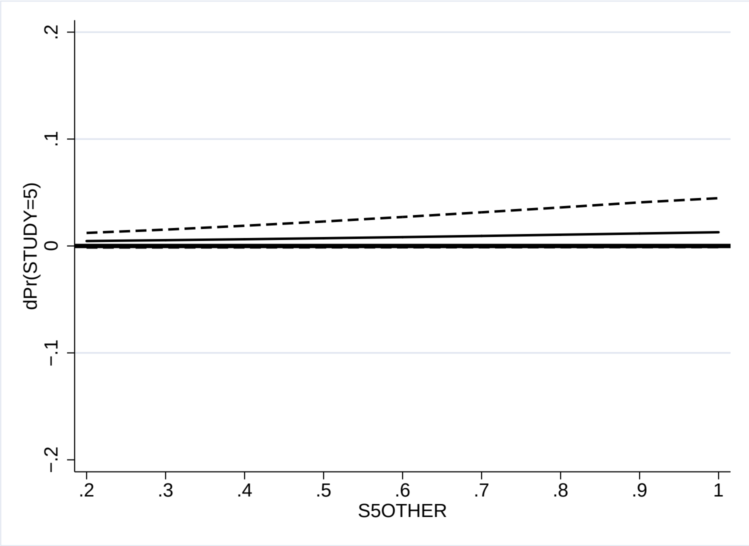


Figure A.26: Effect of S5OTHER on dPr(STUDY=5), White Male

A.7.2 Separate Black and White Estimation

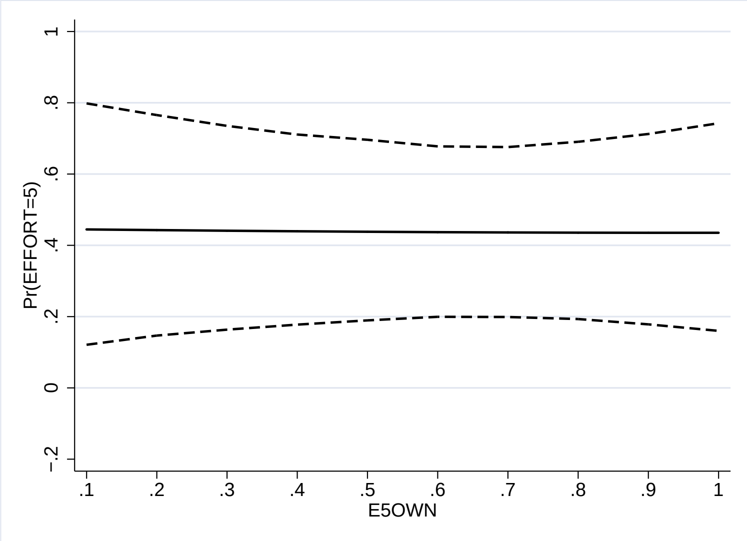


Figure A.27: Effect of E5OWN on Pr(EFFORT=5), Black Only

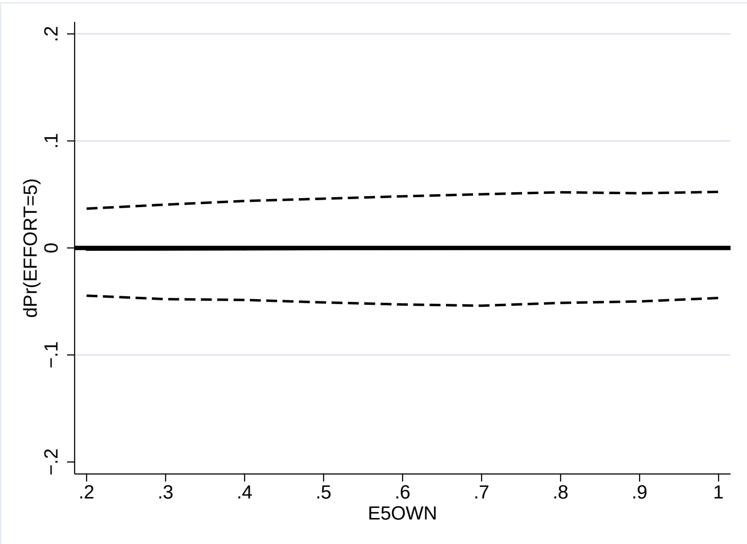


Figure A.28: Effect of E5OWN on dPr(EFFORT=5), Black Only

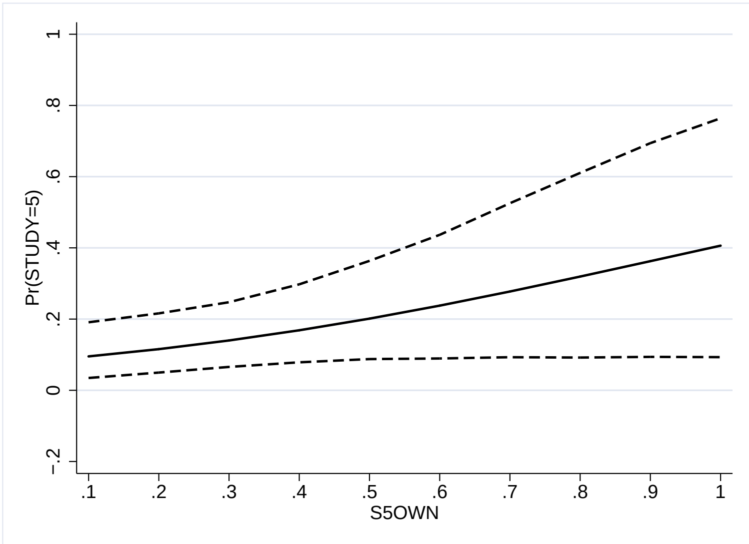


Figure A.29: Effect of S5OWN on Pr(STUDY=5), Black Only

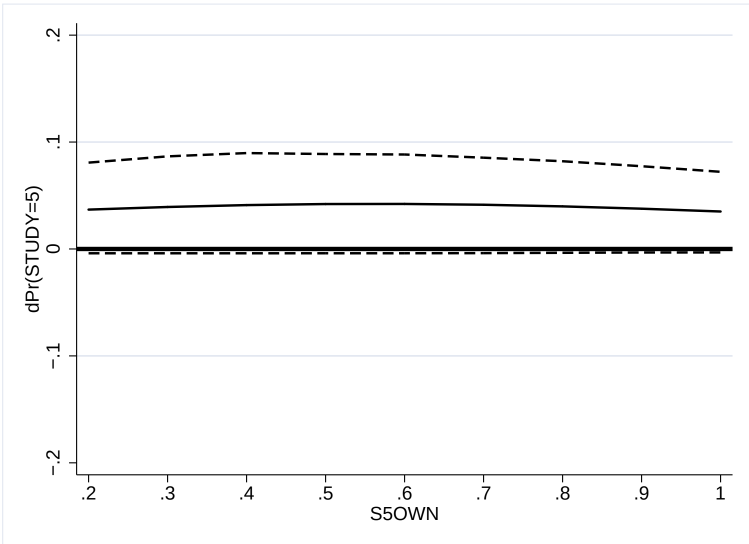


Figure A.30: Effect of S5OWN on dPr(STUDY=5), Black Only

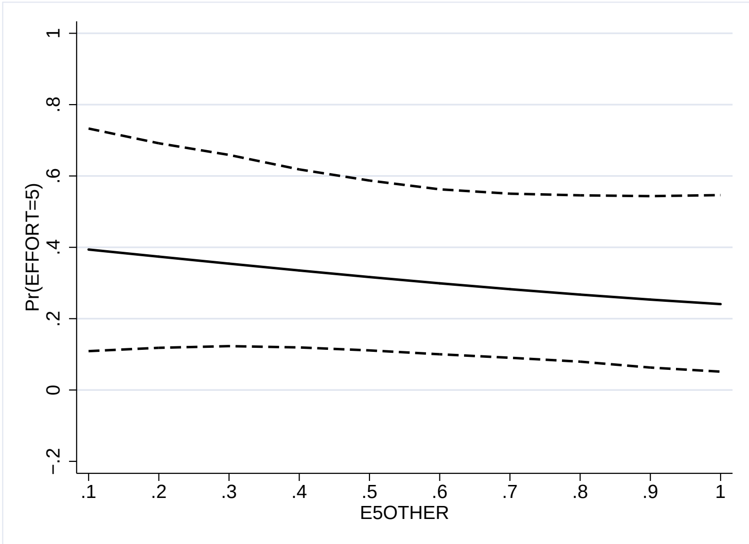


Figure A.31: Effect of E5OTHER on Pr(EFFORT=5), Black Only

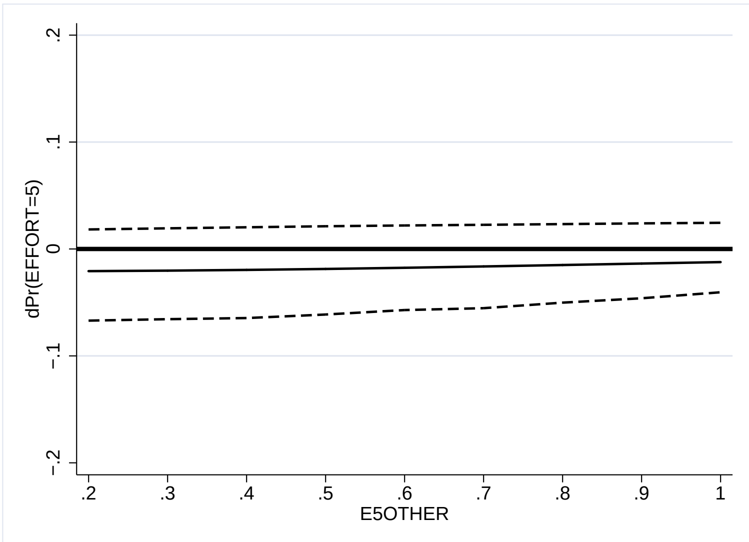


Figure A.32: Effect of E5OTHER on dPr(EFFORT=5), Black Only

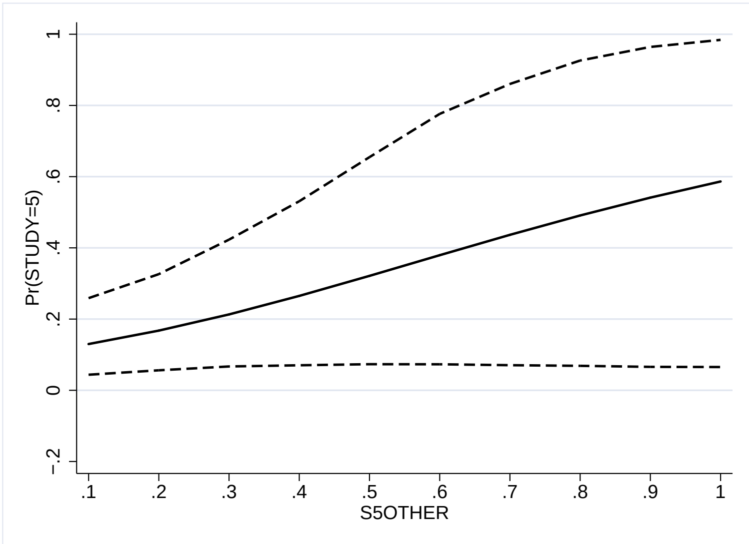


Figure A.33: Effect of S5OTHER on Pr(STUDY=5), Black Only

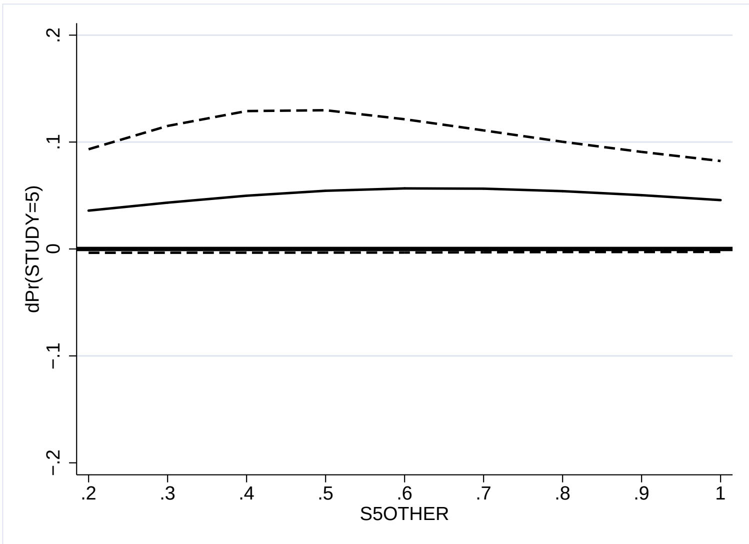


Figure A.34: Effect of S5OTHER on dPr(STUDY=5), Black Only

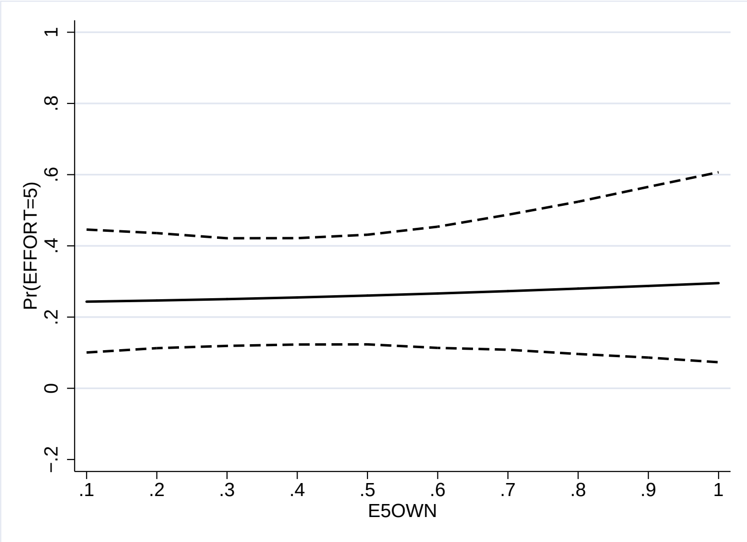


Figure A.35: Effect of E5OWN on Pr(EFFORT=5), White Only

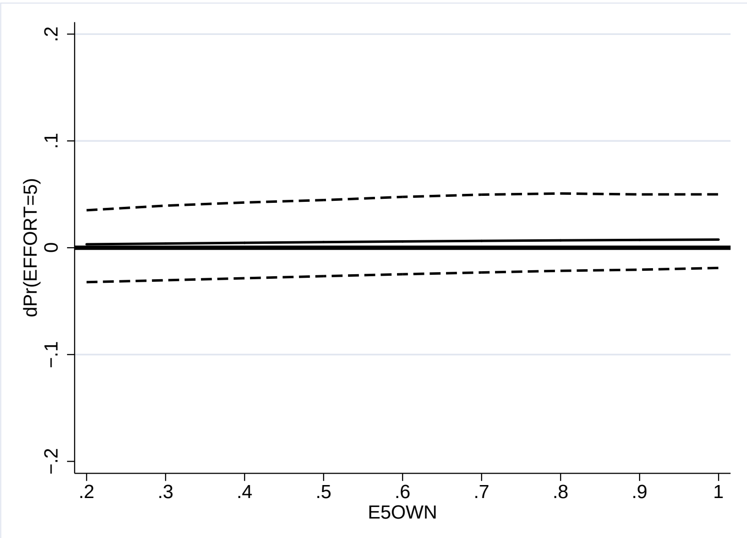


Figure A.36: Effect of E5OWN on dPr(EFFORT=5), White Only

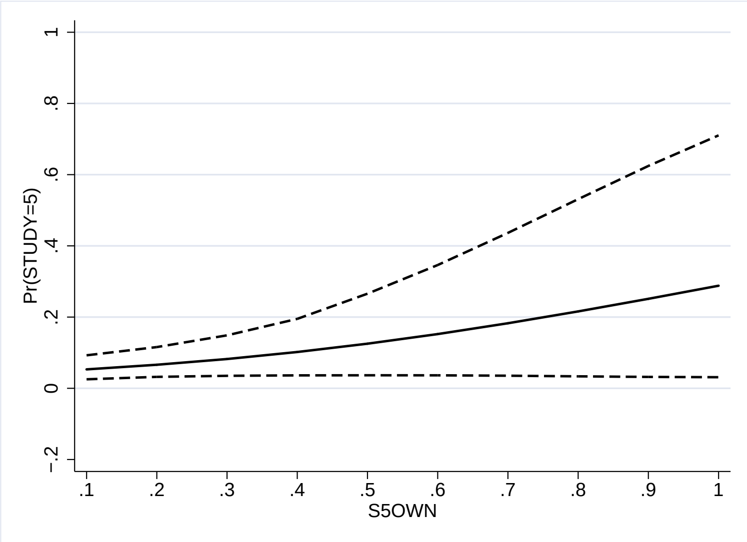


Figure A.37: Effect of S5OWN on Pr(STUDY=5), White Only

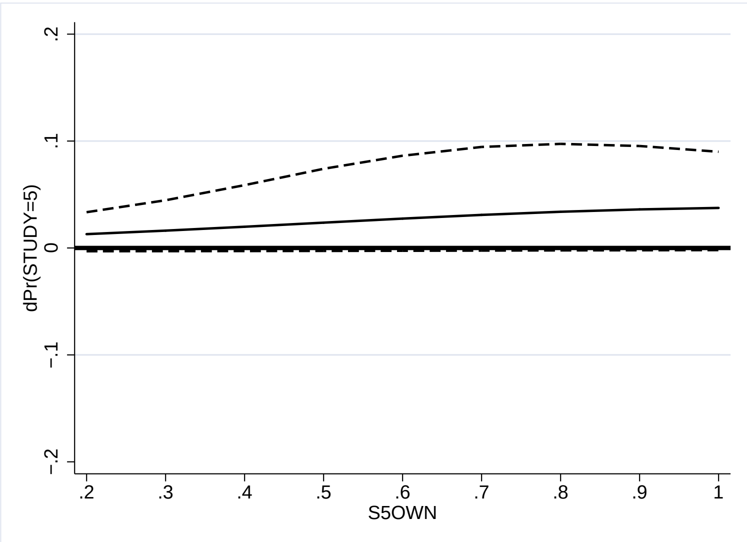


Figure A.38: Effect of S5OWN on dPr(STUDY=5), White Only

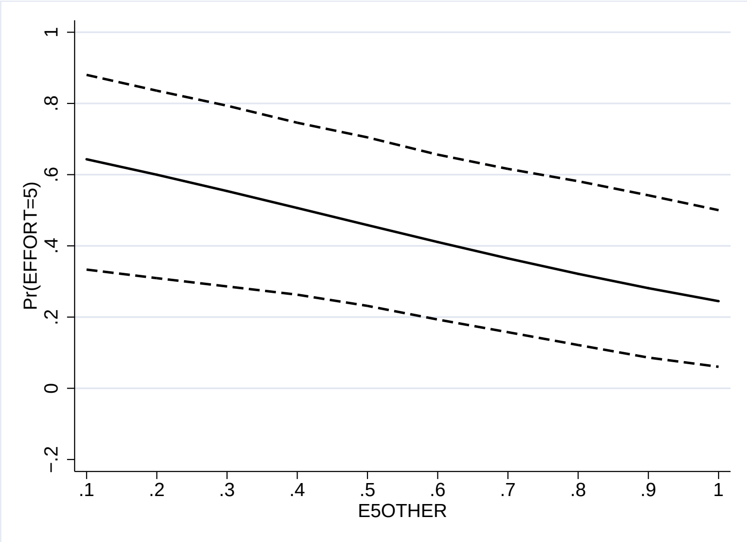


Figure A.39: Effect of E5OTHER on Pr(EFFORT=5), White Only

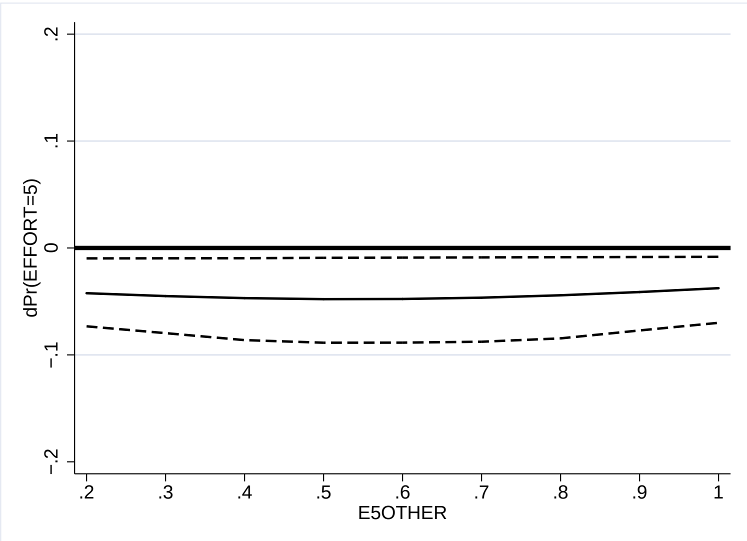


Figure A.40: Effect of E5OTHER on dPr(EFFORT=5), White Only

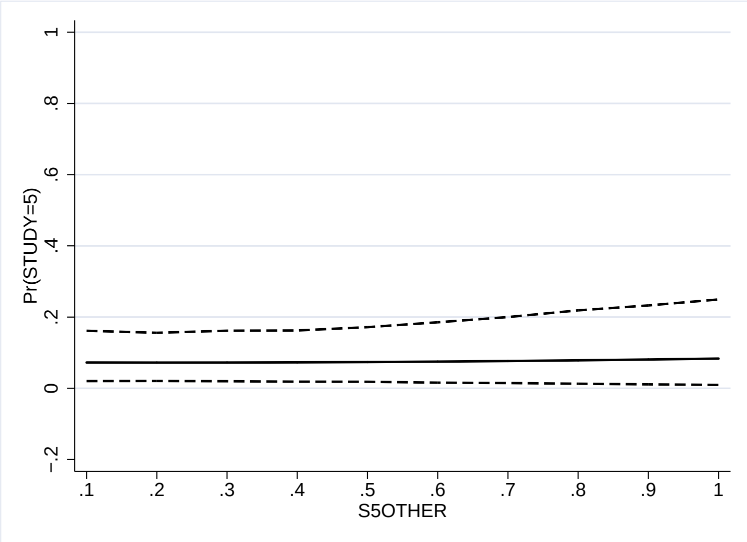


Figure A.41: Effect of S5OTHER on Pr(STUDY=5), White Only

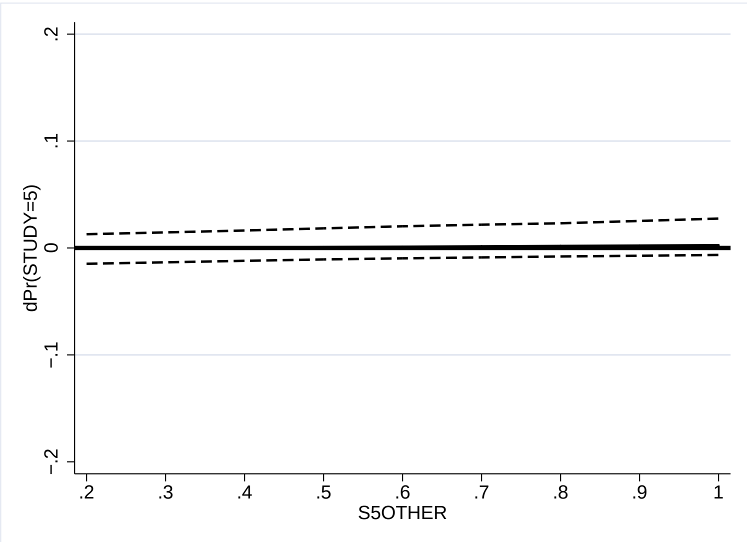


Figure A.42: Effect of S5OTHER on dPr(STUDY=5), White Only

A.7.3 Estimation without Cubed Interaction Terms

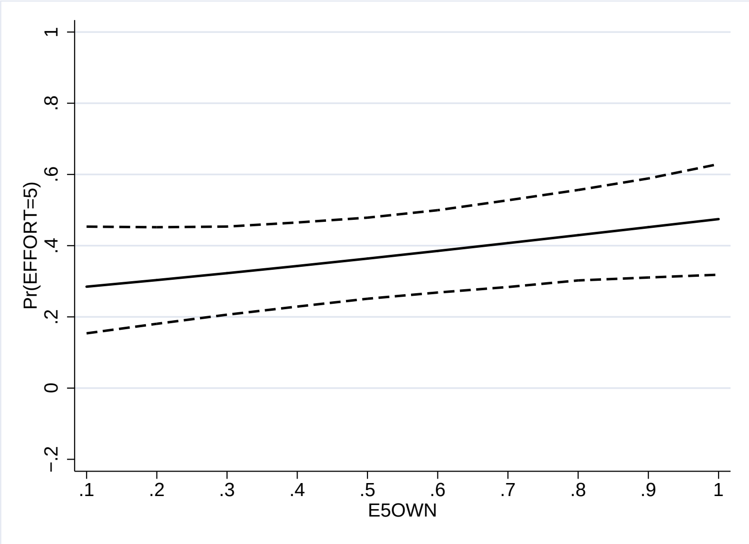


Figure A.43: Effect of E5OWN on Pr(EFFORT=5), No Cubed Terms

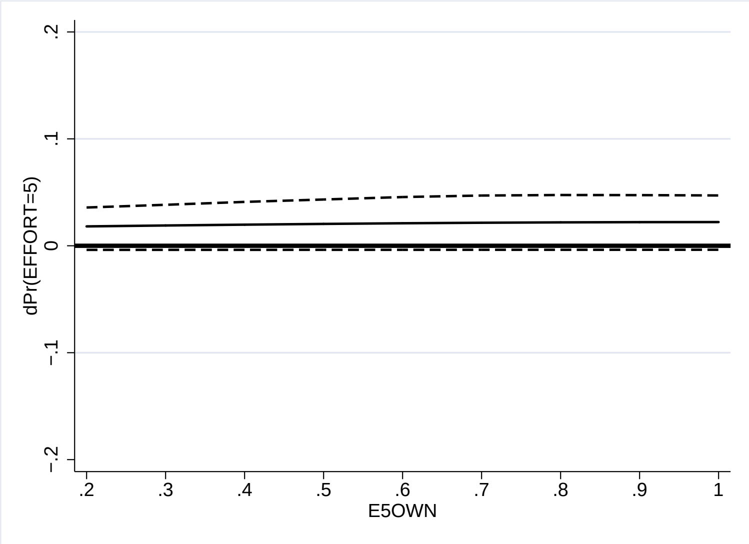


Figure A.44: Effect of E5OWN on dPr(EFFORT=5), No Cubed Terms

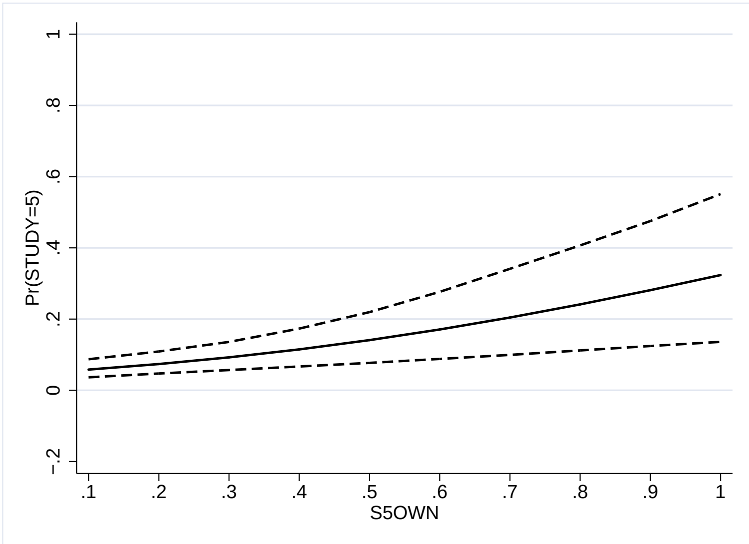


Figure A.45: Effect of S5OWN on Pr(STUDY=5), No Cubed Terms

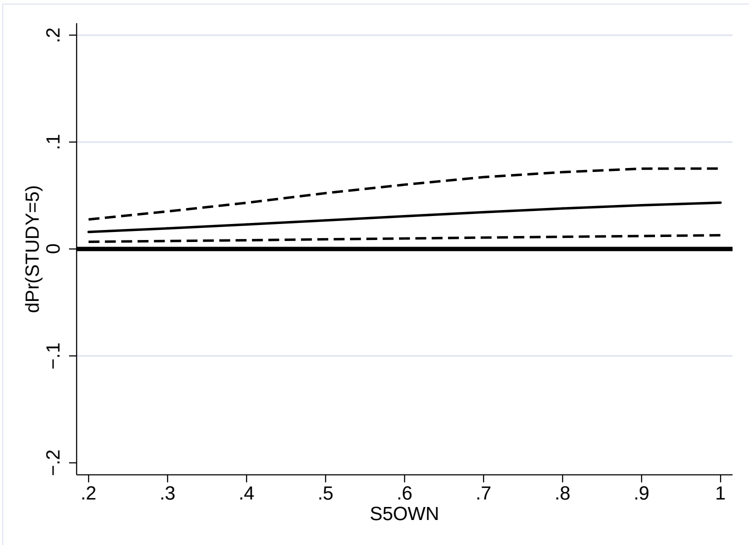


Figure A.46: Effect of S5OWN on dPr(STUDY=5), No Cubed Terms

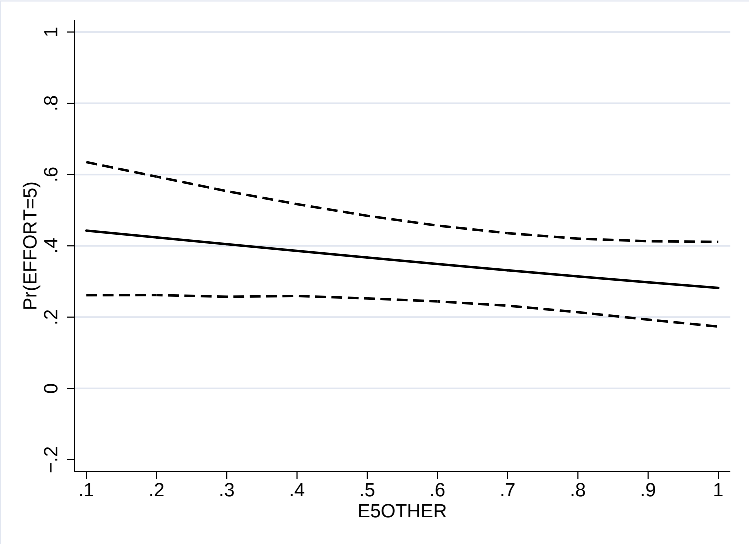


Figure A.47: Effect of E5OTHER on Pr(EFFORT=5), No Cubed Terms

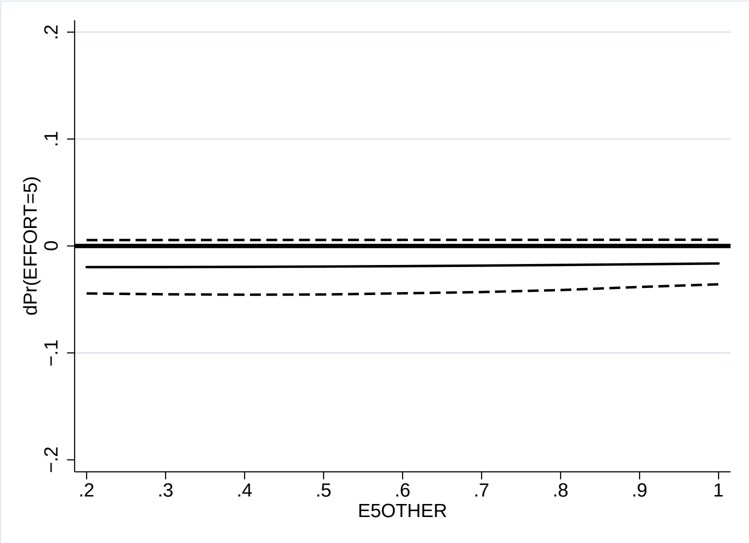


Figure A.48: Effect of E5OTHER on dPr(EFFORT=5), No Cubed Terms

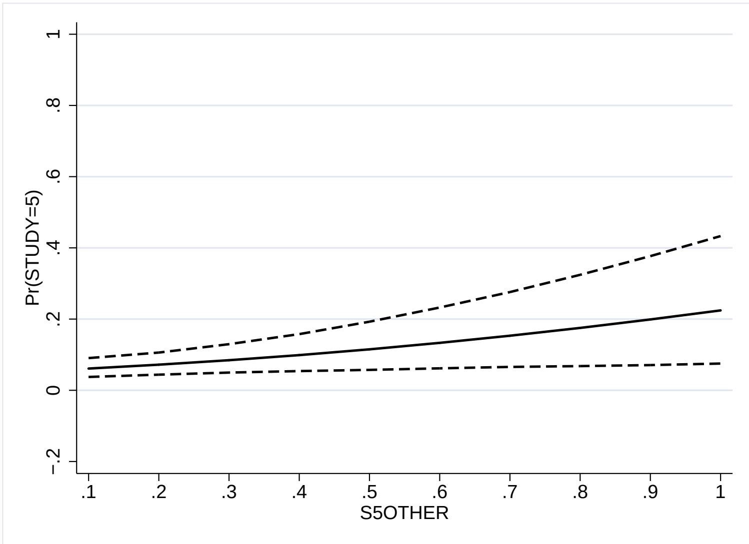


Figure A.49: Effect of S5OTHER on Pr(STUDY=5), No Cubed Terms

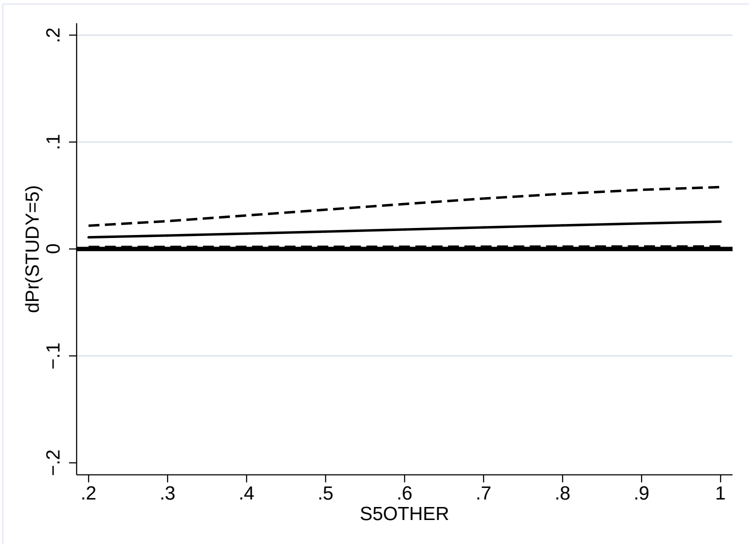


Figure A.50: Effect of S5OTHER on dPr(STUDY=5), No Cubed Terms

A.7.4 Estimation without School Dummies

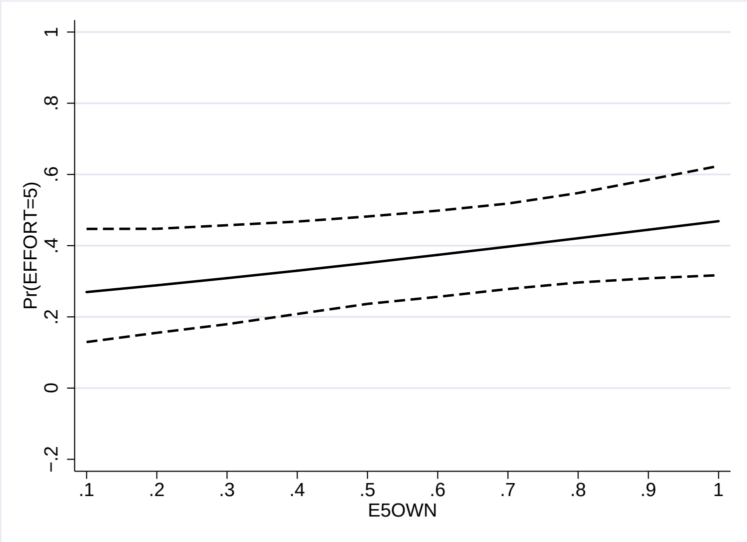


Figure A.51: Effect of E5OWN on Pr(EFFORT=5), No School Dummies

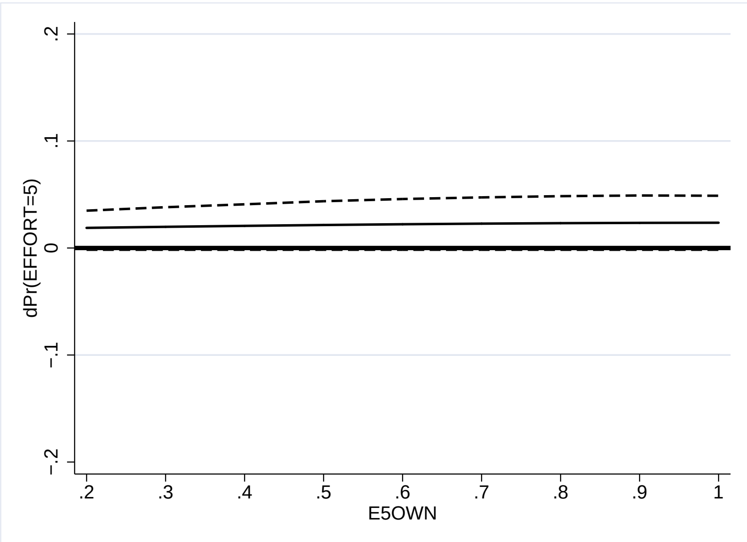


Figure A.52: Effect of E5OWN on dPr(EFFORT=5), No School Dummies

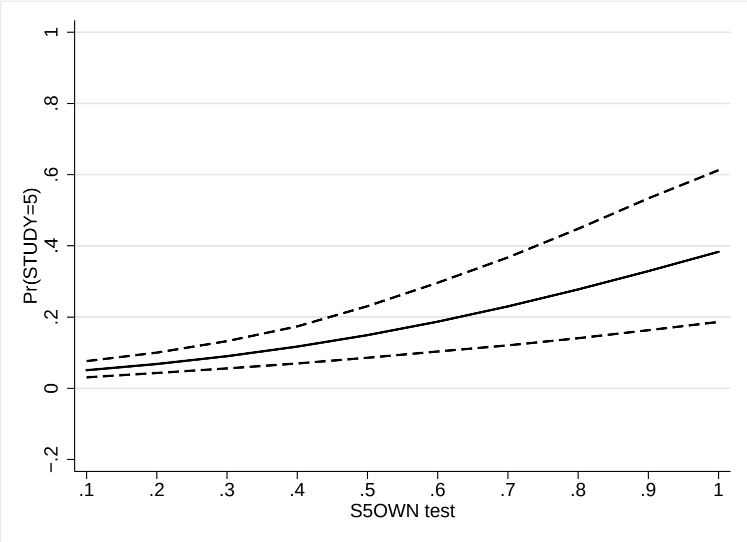


Figure A.53: Effect of S5OWN on Pr(STUDY=5), No School Dummies

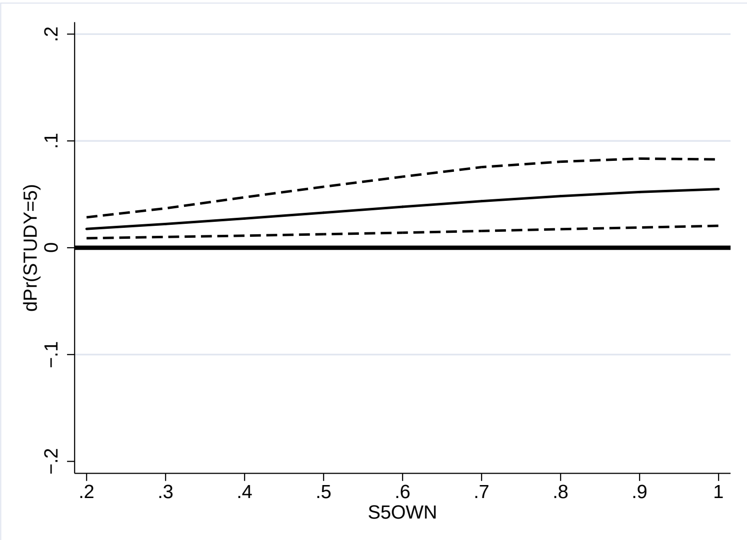


Figure A.54: Effect of S5OWN on dPr(STUDY=5), No School Dummies

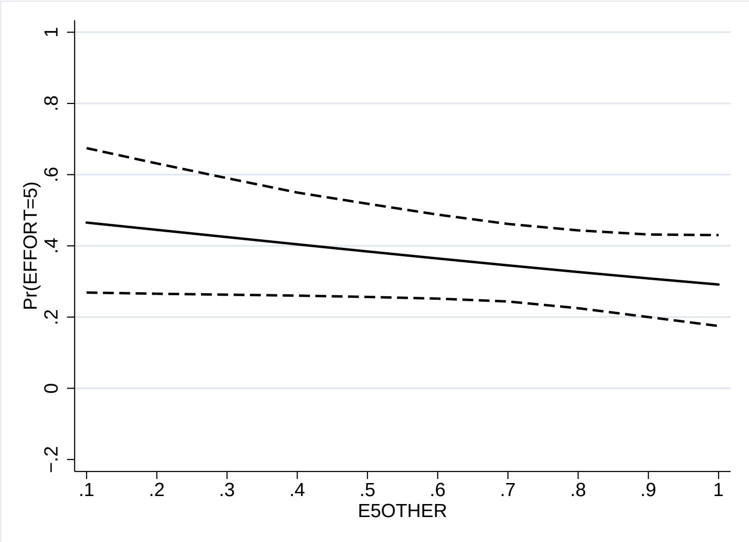


Figure A.55: Effect of E5OTHER on Pr(EFFORT=5), No School Dummies

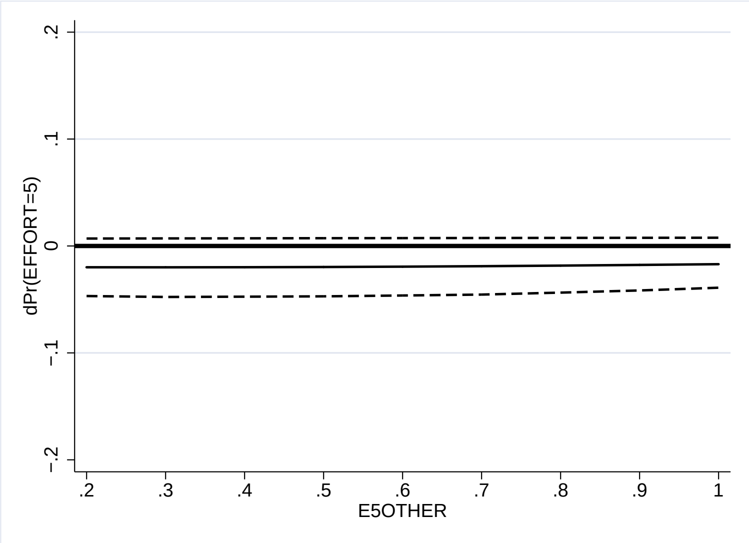


Figure A.56: Effect of E5OTHER on dPr(EFFORT=5), No School Dummies

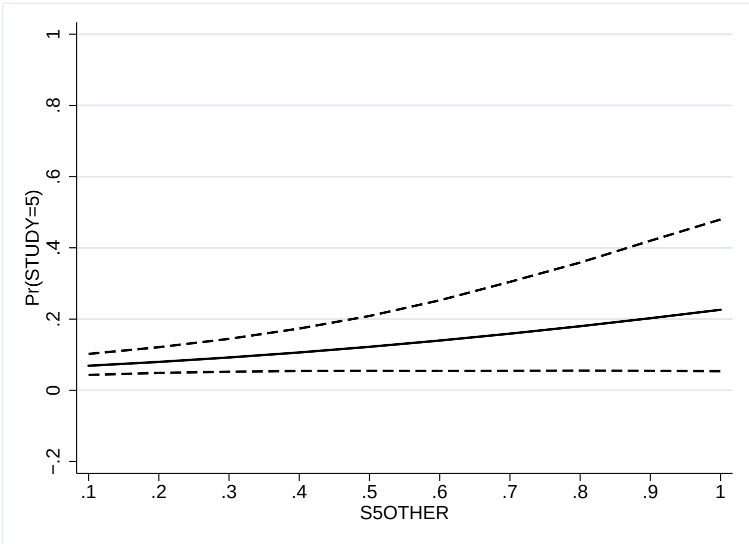


Figure A.57: Effect of S5OTHER on Pr(STUDY=5), No School Dummies

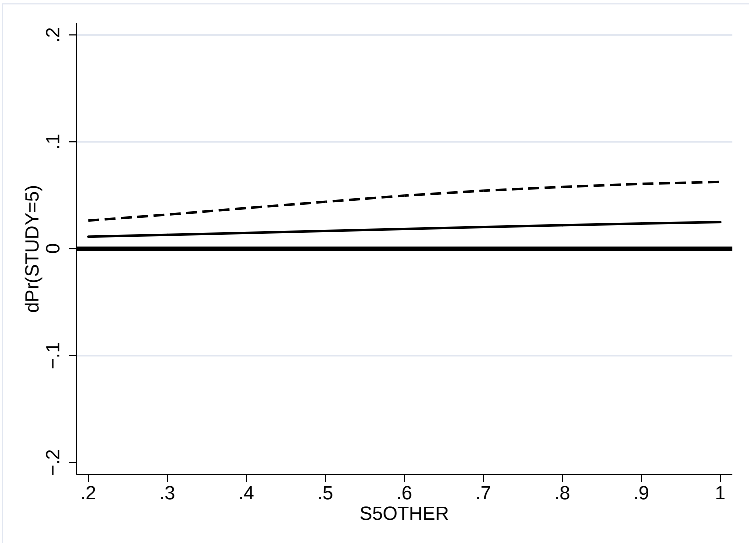
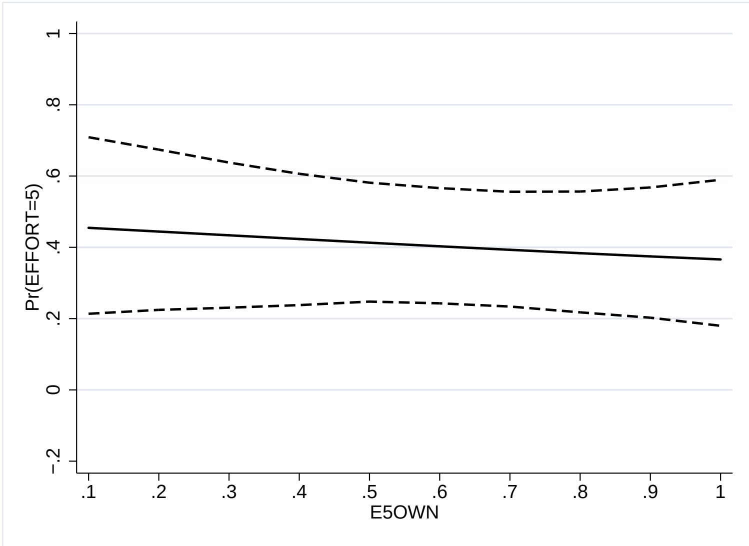
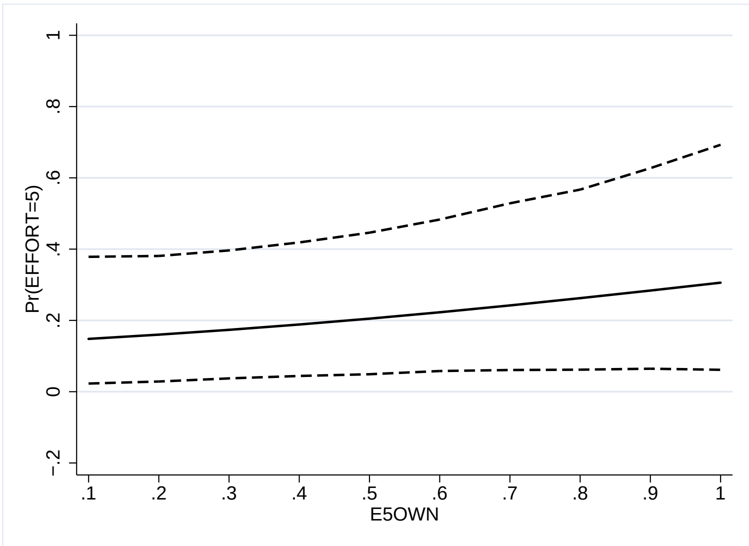


Figure A.58: Effect of S5OTHER on dPr(STUDY=5), No School Dummies

A.7.5 Separate MS and HS Estimation

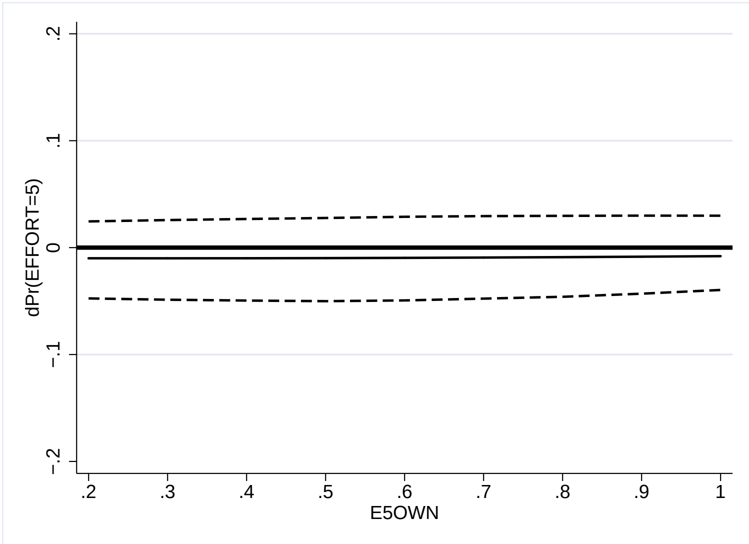


(a) MS

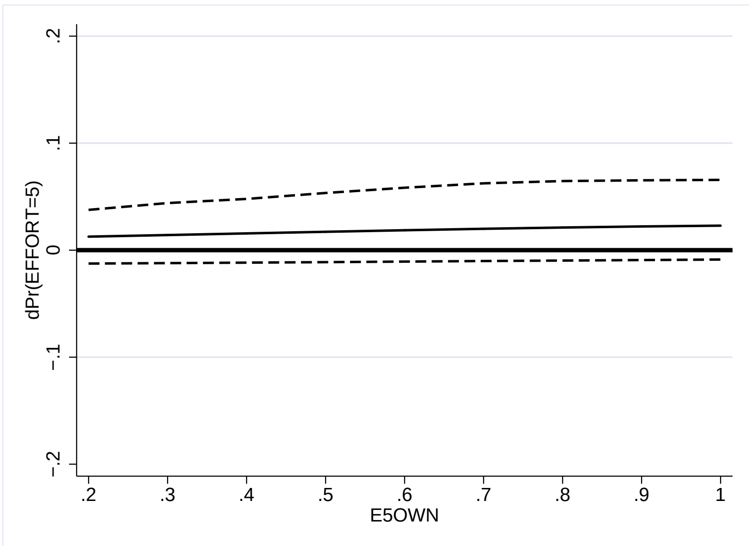


(b) HS

Figure A.59: Effect of E5OWN on Pr(EFFORT=5)

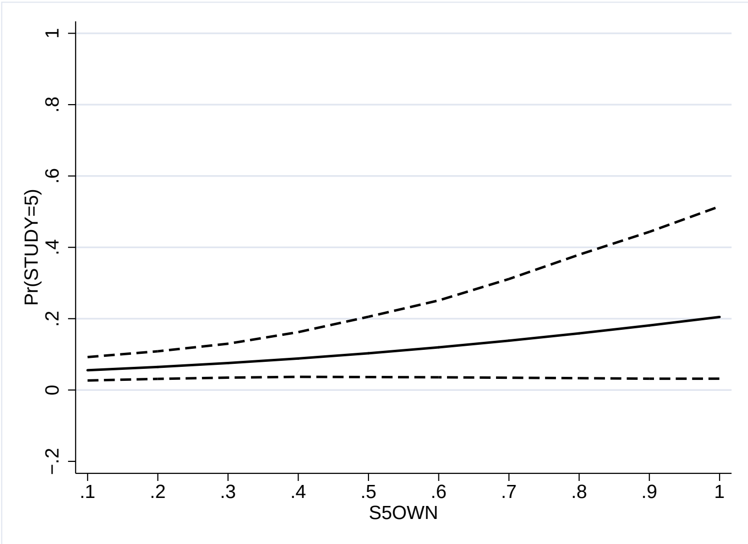


(a) MS

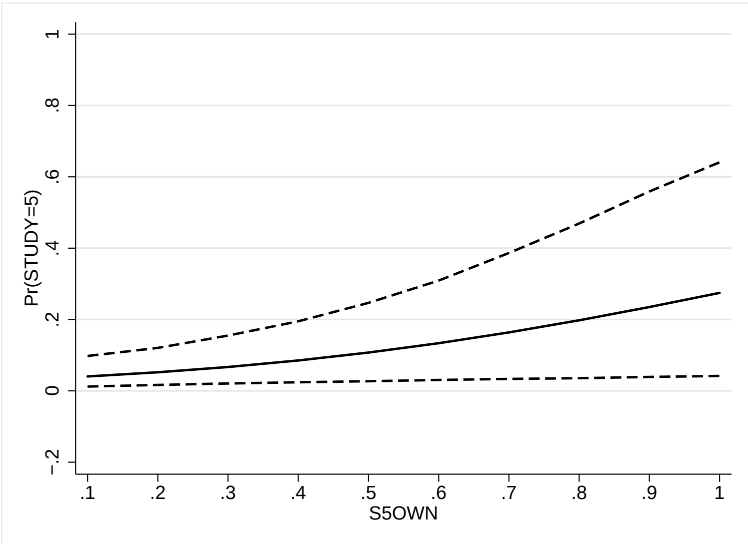


(b) HS

Figure A.60: Effect of E5OWN on dPr(EFFORT=5)

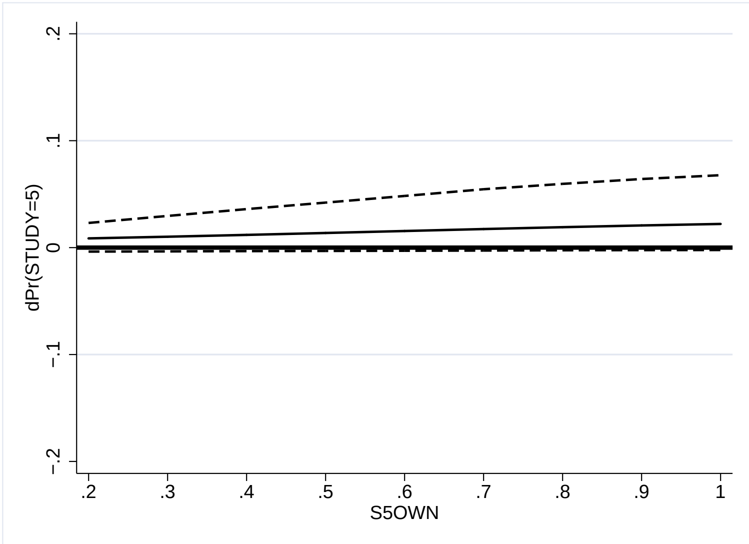


(a) MS

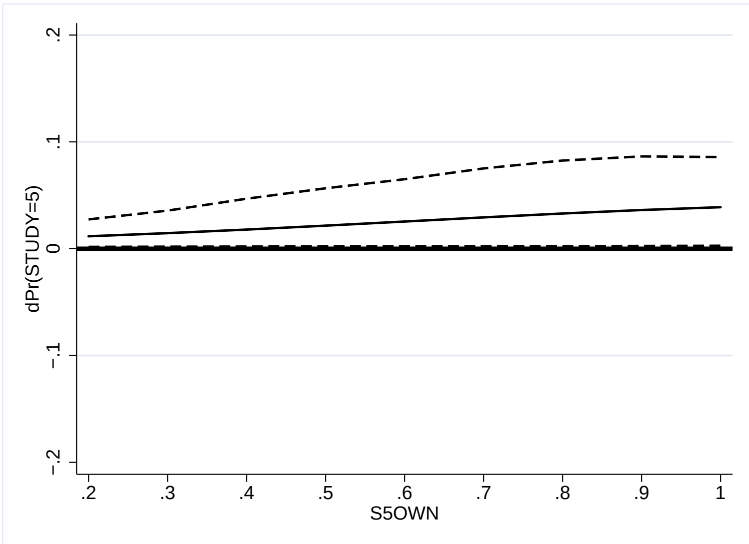


(b) HS

Figure A.61: Effect of S5OWN on Pr(STUDY=5)

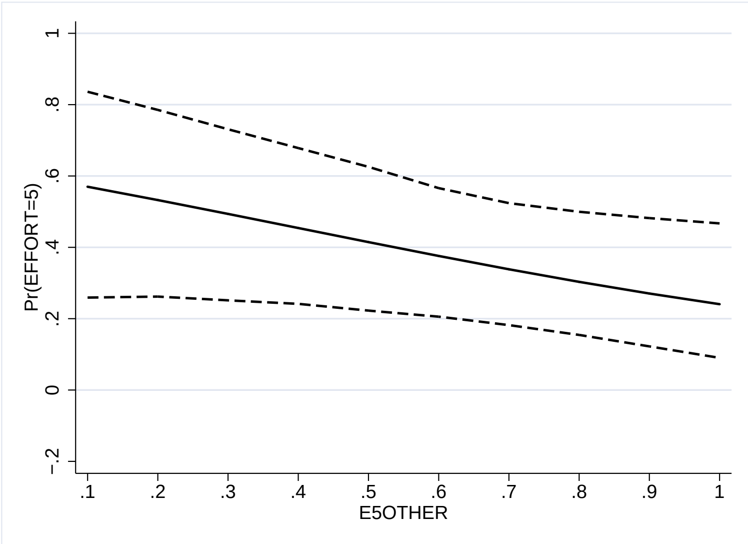


(a) MS

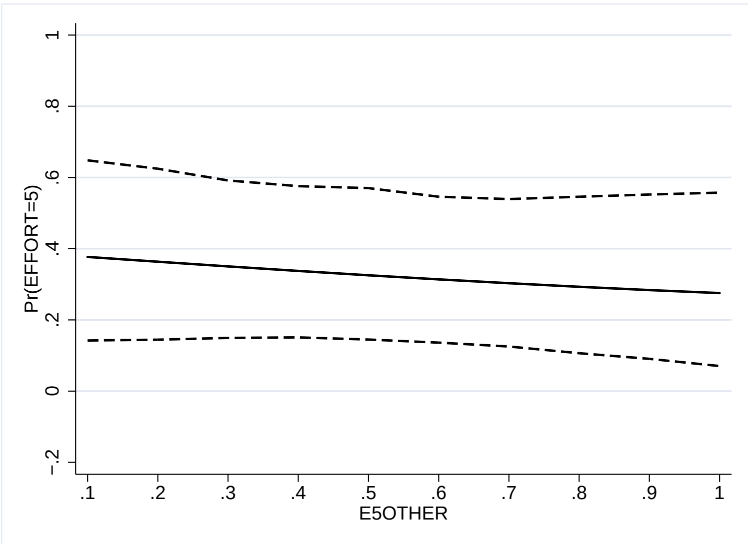


(b) HS

Figure A.62: Effect of S5OWN on dPr(STUDY=5)

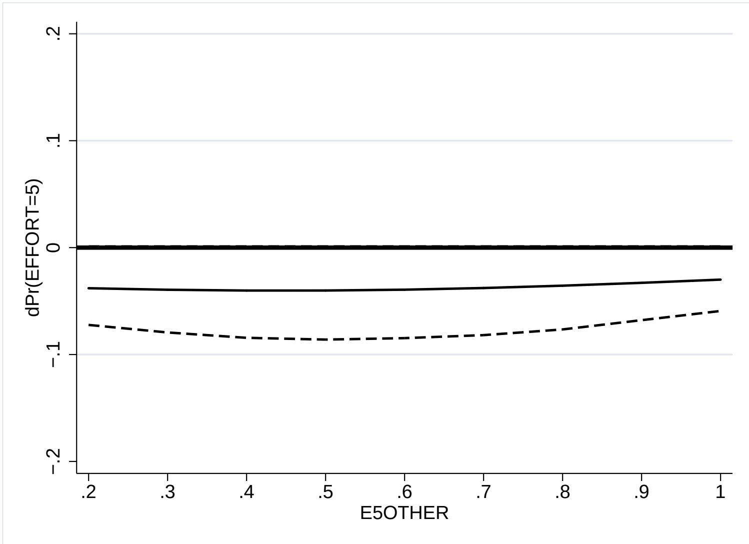


(a) MS

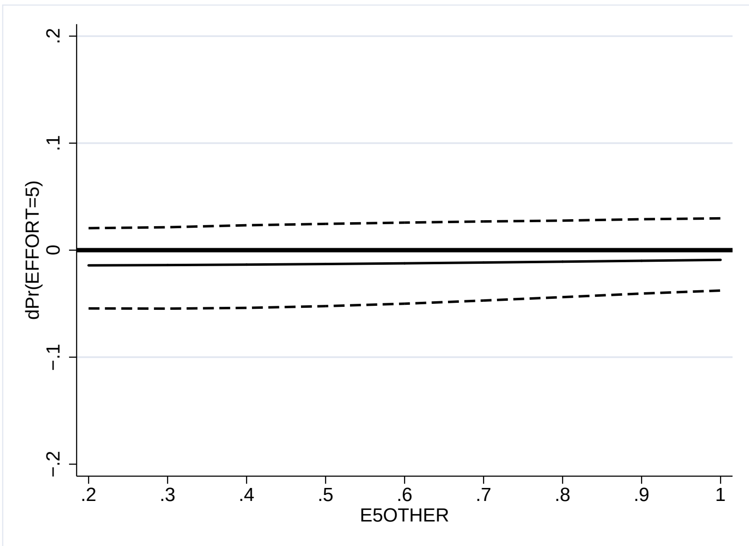


(b) HS

Figure A.63: Effect of E5OTHER on Pr(EFFORT=5)

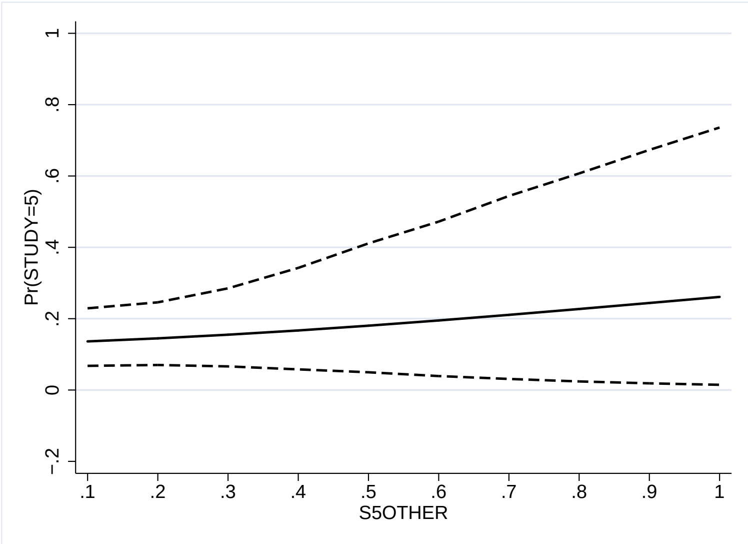


(a) MS

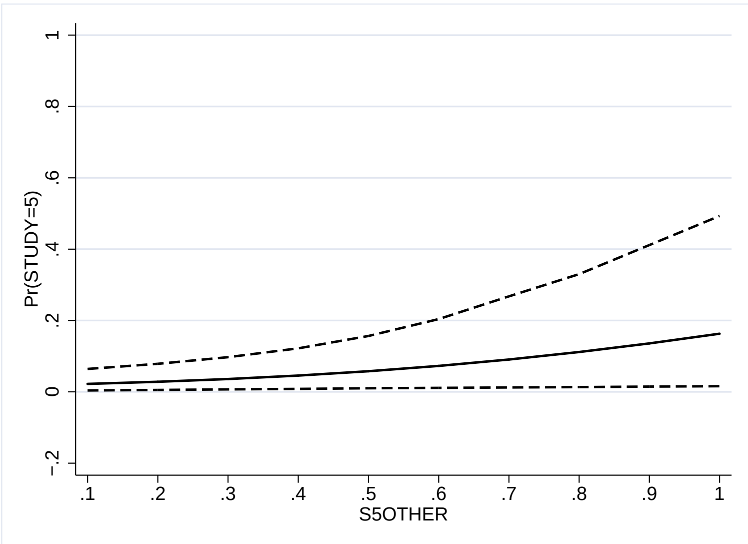


(b) HS

Figure A.64: Effect of E5OTHER on dPr(EFFORT=5)

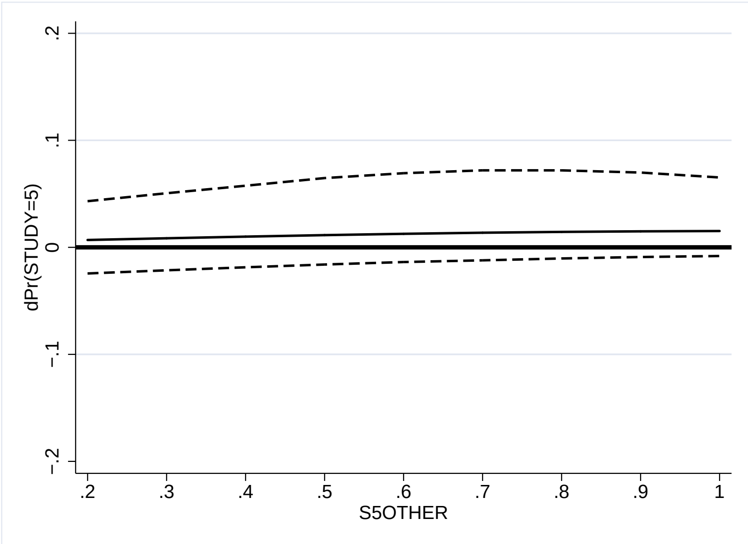


(a) MS

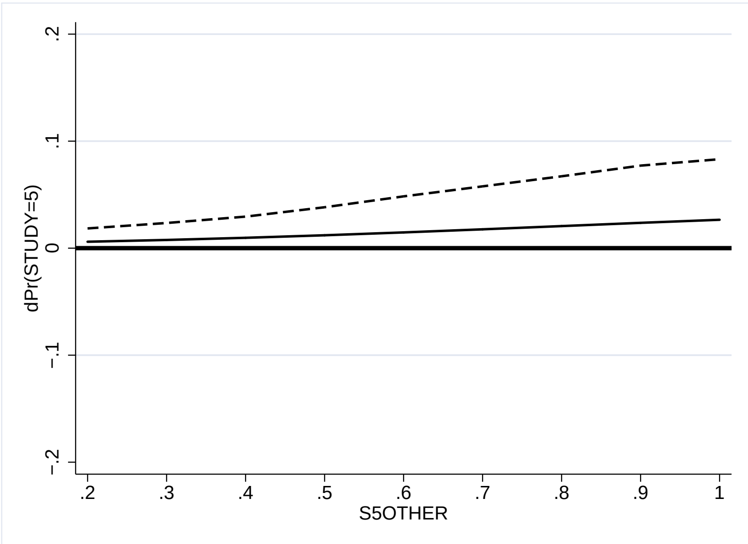


(b) HS

Figure A.65: Effect of S5OTHER on Pr(STUDY=5)



(a) MS



(b) HS

Figure A.66: Effect of S5OTHER on dPr(STUDY=5)

A.8 Alternative Specifications for Classroom Racial Composition Simulations

Figures A.67 and A.68 present the simulation results for alternative estimation specifications for changes in classroom racial composition.

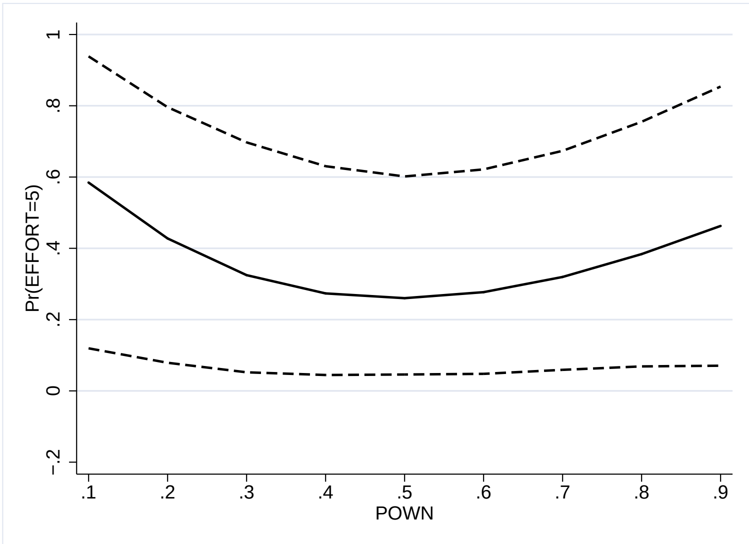


Figure A.67: Effect of POWN on Pr(EFFORT=5), Black Only

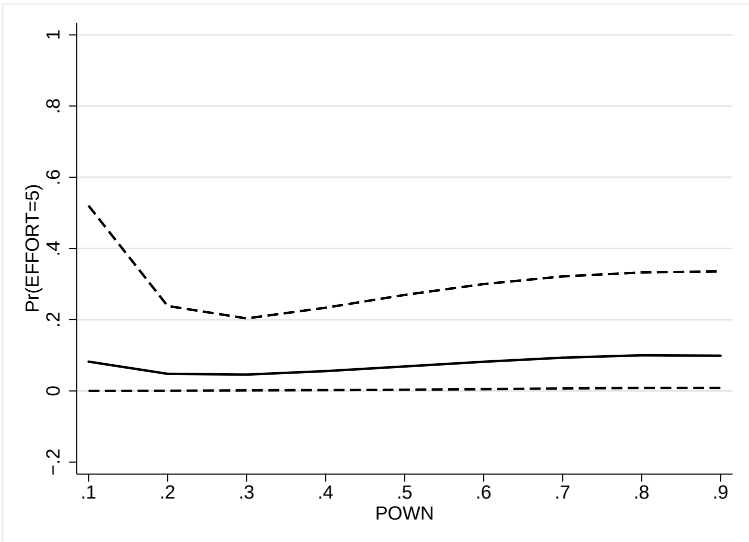


Figure A.68: Effect of POWN on Pr(EFFORT=5), White Only

A.9 Bivariate Ordered Probit

In this section, I present the estimation results for bivariate ordered probit estimation.[2] Bivariate ordered probit assumes that the distribution of the error terms of two separate ordered probit estimations is joint normal. It thereby accommodates correlation between the error terms of the EFFORT and STUDY equations and results in efficient estimation compared to separate univariate ordered probit estimation.

The error terms of the EFFORT and STUDY equations could be correlated for several reasons. A student who clashes with a specific teacher might decrease both EFFORT and STUDY because of the teacher. A student who is particularly motivated by a certain subject content would likely increase both EFFORT and STUDY in response. A reward for high achievement offered by parents could also cause a student to increase both EFFORT and STUDY.

To allow for correlation between the error terms, let $\rho > 0$. It follows that the distribution of the error terms ϵ_{1i} and ϵ_{2i} is

$$\begin{pmatrix} \epsilon_{1i} \\ \epsilon_{2i} \end{pmatrix} \sim \mathrm{N} \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right]. \tag{A.1}$$

[2]In the first application of bivariate ordered probit, Calhoun 1991 studies the relationship between desired family size and children actually born. Butler and Chatterjee 1995 analyze the joint ownership of dogs and cats. More recently, Scotti 2006 uses bivariate ordered probit to study the timing and magnitude of Federal Reserve and European Central Bank target interest rate changes. And Sajaia forthcoming analyzes the relationship between the individual's health and wealth status using survey data for Russia.

Let $j = 1, \ldots, 5$ and $k = 1, \ldots, 5$. The joint probability for EFFORT $= j$ and STUDY $= k$ is then

$$\text{Prob}\left(\text{EFFORT} = j, \text{STUDY} = k \mid \mathbf{x}_{1i}, \mathbf{x}_{2i}\right) =$$
$$\begin{pmatrix} \Phi_2\left(\mu_{1j} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1},\ \mu_{2,k} - \mathbf{x}_{2i}^{'}\boldsymbol{\beta_2},\ \rho\right) \\ -\Phi_2\left(\mu_{1j-1} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1},\ \mu_{2,k} - \mathbf{x}_{2i}^{'}\boldsymbol{\beta_2},\ \rho\right) \end{pmatrix} -$$
$$\begin{pmatrix} \Phi_2\left(\mu_{1j} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1},\ \mu_{2,k-1} - \mathbf{x}_{2i}^{'}\boldsymbol{\beta_2},\ \rho\right) \\ -\Phi_2\left(\mu_{1j-1} - \mathbf{x}_{1i}^{'}\boldsymbol{\beta_1},\ \mu_{2,k-1} - \mathbf{x}_{2i}^{'}\boldsymbol{\beta_2},\ \rho\right) \end{pmatrix}.$$

These probabilities then enter the log likelihood for a maximum likelihood estimator of the model's parameters β_1, β_2, and the threshold parameters.

Tables A.12-A.17 presents the estimation results for the BOP model.

Table A.12: BOP Estimation Results for EFFORT: Control Variables

	Coefficient	z-statistic
Female	0.462*	12.19
White	-0.104*	-2.64
Paid job, 1-5 hrs	0.040	0.85
Paid job, 6-10 hrs	-0.079	-1.34
Paid job, 11-19 hrs	-0.101	-1.88
Paid job, 20+ hrs	-0.185*	-3.19
Ability	-0.000	-0.22
Missing Ability	-0.076	-0.20
SES, 20-39 percentile	0.034	0.60
SES, 40-59 percentile	0.116*	2.02
SES, 60-79 percentile	0.117*	2.22
SES, 80+ percentile	0.075	1.44
Live with both parents	0.190*	5.13
High School	-0.465*	-2.94
Track: academic	0.002	0.05
Track: AP/IB	-0.118*	-2.03
EOWNVAR	0.138	0.94
EOTHERVAR	0.061	0.71
Percent black in school	0.001	0.37
% of school's teachers w/ tenure	0.012	1.62
% of school's teachers fully licensed	-0.010	-0.83
Magnet School	0.236*	3.28
N	4454	

* $p < 0.05$

Table A.13: BOP Estimation Results for EFFORT: School Dummies

	Coefficient	z-statistic
School 1	-0.262	-1.75
School 2	-0.198*	-2.35
School 4	0.056	0.46
School 6	-0.264	-1.44
School 7	0.182	1.38
School 8	0.077	0.79
School 9	-0.482*	-2.16
School 10	0.092	0.74
School 11	-0.004	-0.03
School 12	-0.091	-1.26
School 13	-0.217	-1.68
School 14	-0.439*	-3.18
School 15	-0.110	-1.03
School 16	-0.211	-1.69
School 18	0.045	0.23
School 19	-0.319*	-2.03
School 20	-0.140	-1.11
School 21	0.007	0.05
School 22	-0.477*	-2.45
School 23	0.283	1.67
School 32	0.031	0.26
School 33	-0.086	-0.75
School 34	-0.086	-0.62
School 35	-0.068	-0.65
School 36	0.319*	2.27
School 37	0.021	0.16
School 38	-0.132	-1.60
School 39	-0.150	-1.36
School 40	0.390	1.86
N	4454	

* $p < 0.05$

Table A.14: BOP Estimation Results for EFFORT: Interaction Terms

	Coefficient	z-statistic
POWN	-1.720*	-2.04
E2OWN	1.009	0.96
E2POWN	-4.305	-0.60
E2POWNSQ	11.140	0.74
E2POWNCU	-6.363	-0.66
E3OWN	0.106	0.18
E3POWN	4.575	1.27
E3POWNSQ	-10.736	-1.46
E3POWNCU	8.195	1.79
E4OWN	0.409	1.04
E4POWN	-1.203	-0.56
E4POWNSQ	5.902	1.39
E4POWNCU	-2.806	-1.06
E5OWN	0.412	0.54
E5POWN	3.320	0.64
E5POWNSQ	-3.164	-0.29
E5POWNCU	1.117	0.16
POTHER	-0.054	-0.22
E2OTHER	0.489	1.56
E2POTHER	-6.292	-1.80
E2POTHERSQ	19.575	1.84
E2POTHERCU	-15.413	-1.66
E3OTHER	-0.032	-0.26
E3POTHER	1.790	0.86
E3POTHERSQ	-7.631	-1.22
E3POTHERCU	7.177	1.45
E4OTHER	-0.046	-0.36
E4POTHER	0.839	0.65
E4POTHERSQ	-2.998	-0.81
E4POTHERCU	2.422	0.83
E5OTHER	-0.203	-1.35
E5POTHER	1.308	0.58
E5POTHERSQ	-3.679	-0.55
E5POTHERCU	2.650	0.48
N	4454	

* $p < 0.05$

Table A.15: BOP Estimation Results for STUDY: Control Variables

	Coefficient	z-statistic
Female	0.348*	10.53
White	-0.269*	-7.14
Paid job, 1-5 hrs	0.077	1.82
Paid job, 6-10 hrs	0.175*	2.94
Paid job, 11-19 hrs	-0.042	-0.67
Paid job, 20+ hrs	-0.214*	-3.24
Ability	-0.002*	-3.20
Missing Ability	-1.113*	-3.09
SES, 20-39 percentile	-0.052	-0.96
SES, 40-59 percentile	0.082	1.37
SES, 60-79 percentile	0.118*	2.11
SES, 80+ percentile	0.130*	2.27
High School	0.164	0.89
Track: academic	-0.012	-0.32
Track: AP/IB	0.007	0.12
Live with both parents	0.140*	3.58
SOWNVAR	-0.387*	-3.04
SOTHERVAR	-0.041	-0.53
% of school's teachers w/ tenure	0.008	1.46
% of school's teachers fully licensed	-0.017*	-2.65
Magnet School	0.142	1.32
N	4454	

* $p < 0.05$

Table A.16: BOP Estimation Results for STUDY: School Dummies

	Coefficient	z-statistic
School 1	-0.232*	-1.97
School 2	0.170	1.57
School 3	0.046	0.23
School 4	0.032	0.28
School 5	-0.019	-0.17
School 7	-0.070	-0.59
School 8	0.089	0.87
School 9	0.085	0.59
School 10	0.129	0.71
School 11	0.204	1.43
School 12	-0.048	-0.42
School 13	-0.049	-0.56
School 14	0.026	0.15
School 15	0.166	1.49
School 16	-0.084	-0.69
School 18	0.007	0.07
School 20	0.271*	2.49
School 21	0.076	0.96
School 22	0.037	0.25
School 23	-0.005	-0.04
School 31	-0.265	-1.53
School 32	-0.126	-0.70
School 33	-0.027	-0.17
School 34	0.073	0.47
School 35	-0.249	-1.60
School 36	0.021	0.14
School 37	0.188	1.70
School 38	0.163	1.41
School 39	-0.243	-1.61
School 40	0.048	0.31
N	4454	

* $p < 0.05$

Table A.17: BOP Estimation Results for STUDY: Interaction Terms

	Coefficient	z-statistic
POWN	-0.966*	-2.82
S2OWN	-0.135	-0.37
S2POWN	2.569	0.93
S2POWNSQ	-5.343	-0.88
S2POWNCU	4.129	1.05
S3OWN	0.298	0.78
S3POWN	-2.986	-0.89
S3POWNSQ	10.571	1.44
S3POWNCU	-6.883	-1.49
S4OWN	-1.136	-1.52
S4POWN	13.714*	2.32
S4POWNSQ	-25.609	-1.91
S4POWNCU	15.345	1.76
S5OWN	3.273*	2.54
S5POWN	-21.007*	-2.28
S5POWNSQ	48.805*	2.52
S5POWNCU	-30.770*	-2.54
POTHER	-0.226	-0.96
S2OTHER	-0.101	-0.73
S2POTHER	0.682	0.44
S2POTHERSQ	-0.301	-0.07
S2POTHERCU	-0.436	-0.13
S3OTHER	0.050	0.49
S3POTHER	0.570	0.32
S3POTHERSQ	-3.528	-0.62
S3POTHERCU	3.270	0.71
S4OTHER	-0.323	-1.70
S4POTHER	4.668	1.54
S4POTHERSQ	-12.011	-1.29
S4POTHERCU	8.121	1.05
S5OTHER	0.128	0.61
S5POTHER	1.317	0.35
S5POTHERSQ	-4.836	-0.40
S5POTHERCU	6.738	0.67
N	4454	

* $p < 0.05$

APPENDIX B

Stata Code for Simulations

This appendix presents the Stata code for the simulations run in Section 4.6. The code below is for the effect that the student's own racial peer group has on his EFFORT choice. The structure of the other simulations are exactly the same as for this one; only the starting values and variables being changed differ.

```
capture log close
log using allclarify2E1.smcl, replace text

// allclarify2E1: use clarify for marginal effects and s.e. estimates for EFFORT for d
> ifferent starting parameters for Specification 1
// Brent Edelman 03292010

version 8.2
clear
macro drop _all
set linesize 80
set more off
set memory 32m

set matsize 700


use "C:\DATA\CMS\Work\Datasets\CMS-all-07.dta", clear

local tag "allclarify2E1.do bme 03292010"

//-----ESTSIMP FOR EFFORT------------------
//-----THIS IS THE CODE FOR THE SIMULATIONS THAT GENERATE THE RELATIONSHIP BETWEEN THE
>  PROPORTION
//-----OF THE OWN ETHNIC GROUP THAT SELECTS EFFORT = 5 (E5OWN) AND THE PROBABILITY
//-----THAT THE STUDENT SELECTS EFFORT=5 (PR(EFFORT=5))

local indvar1e "magnet sd2 sd4 sd6 sd7 sd8 sd9 sd10 sd11 sd12 sd13 sd14 sd15 sd16 sd18
>  sd19 sd20 sd21 sd22 sd23 sd32 sd33 sd34 sd35 sd36 sd37 sd38 sd39 sd40 HSd pown EPGp
> erE2 EPGperE2p EPGperE2psq EPGperE2pcu EPGperE3 EPGperE3p EPGperE3psq EPGperE3pcu EP
> GperE4 EPGperE4p EPGperE4psq EPGperE4pcu EPGperE5 EPGperE5p EPGperE5psq EPGperE5pcu
> pother OEGperE2 OEGperE2p OEGperE2psq OEGperE2pcu OEGperE3 OEGperE3p OEGperE3psq OEG
> perE3pcu OEGperE4 OEGperE4p OEGperE4psq OEGperE4pcu OEGperE5 OEGperE5p OEGperE5psq O
> EGperE5pcu IQVEOWN IQVEOTHER gender white paidjob2 paidjob3 paidjob4 paidjob5 btss92
>  btssd ses2 ses3 ses4 ses5 livewithd track2 track3 pctblack tenure license"

estsimp oprobit effort ‘indvar1e’, level(90) cluster(classid) sims(1000)

// SIMQI AND PREDICT EFFORT=5

// WHITE FEMALE

generate plo = .
generate phi = .
generate plo2 = .
generate phi2 = .
generate Y0 = .
generate Eaxis = _n + 0 in 1/10
setx mean
setx pown .6 gender 1 white 1 paidjob2 1 track2 0 track3 0 livewithd 1 btss92 726 btss
> d 0 HSd 0
setx EPGperE4 0 EPGperE4p 0 EPGperE4psq 0 EPGperE4pcu 0
setx EPGperE2 .9 EPGperE2p .54 EPGperE2psq .324 EPGperE2pcu .1944
setx EPGperE3 0 EPGperE3p 0 EPGperE3psq 0 EPGperE3pcu 0
setx EPGperE5 0 EPGperE5p 0 EPGperE5psq 0 EPGperE5pcu 0
local a = 1
while ‘a’ <= 10 {
        setx EPGperE5 (‘a’/10) EPGperE5p ((‘a’/10)*pown) EPGperE5psq ((‘a’/10)*pown^2)
>  EPGperE5pcu ((‘a’/10)*pown^3) ///
        EPGperE2 (1-(‘a’/10))  EPGperE2p ((1-(‘a’/10))*pown)  EPGperE2psq ((1-(‘a’/10)
> )*pown^2)  EPGperE2pcu ((1-(‘a’/10))*pown^3)
        simqi, prval(5) genpr(pi)
        sum pi, meanonly
        replace Y0 = r(mean) if Eaxis == ‘a’
        _pctile pi, p(2.5,5,95,97.5)
        replace plo = r(r1) if Eaxis==‘a’
        replace phi = r(r4) if Eaxis==‘a’
        replace plo2 = r(r2) if Eaxis==‘a’
        replace phi2 = r(r3) if Eaxis==‘a’
        drop pi
        local a = ‘a’ + 1
}
```

```
//----THIS IS THE CODE THAT PRODUCES THE GRAPH OF THE RELATIONSHIP
//----BETWEEN THE E5OWN AND PR(EFFORT=5)

sort Eaxis
twoway (line plo Eaxis, clc(gs0) clw(medthick) clp(dash))  (line phi Eaxis, clc(gs0) c
> lw(medthick) clp(dash)) ///
(line Y0 Eaxis, clc(gs0) clw(medthick)),  ///
yscale(range(1)) ylabel(0(.2)1) xlabel(1 ".1" 2 ".2" 3 ".3" 4 ".4" 5 ".5" 6 ".6" 7 ".7
> " 8 ".8" 9 ".9" 10 "1") ylabel(-.2(.2)1) ///
legend(off) xtitle("E5OWN") ytitle("Pr(EFFORT=5)") graphregion(fcolor(gs16))
graph2tex, epsfile(Eown1)



//----THIS IS THE CODE THAT GENERATES THE FIRST DIFFERENCE
//----IN THE PROBABILITY THAT THE STUDENT SELECTS EFFORT=5
//----DUE TO THE CHANGE IN THE PROPORTION OF THE OWN ETHNIC
//----PEER GROUP THAT SELECTS EFFORT=5

// FD FOR EFFORT=5

setx mean
setx pown .6 gender 1 white 1 paidjob2 1 track2 0 track3 0 livewithd 1 btss92 726 btss
> d 0 HSd 0
setx EPGperE4 0 EPGperE4p 0 EPGperE4psq 0 EPGperE4pcu 0
setx EPGperE2 .9 EPGperE2p .54 EPGperE2psq .324 EPGperE2pcu .1944
setx EPGperE3 0 EPGperE3p 0 EPGperE3psq 0 EPGperE3pcu 0
setx EPGperE5 0 EPGperE5p 0 EPGperE5psq 0 EPGperE5pcu 0
local a = 1
while ‘a’ <= 9 {
        setx EPGperE5 (‘a’/10) EPGperE5p ((‘a’/10)*pown) EPGperE5psq ((‘a’/10)*pown^2)
>  EPGperE5pcu ((‘a’/10)*pown^3)
        simqi, prval(5) fd(prval(5) genpr(pi)) changex(EPGperE5 (‘a’/10) ((‘a’+1)/10)
> EPGperE5p ((‘a’/10)*pown) (((‘a’+1)/10)*pown) ///
        EPGperE5psq ((‘a’/10)*pown^2) (((‘a’+1)/10)*pown^2) EPGperE5pcu ((‘a’/10)*pown
> ^3) (((‘a’+1)/10)*pown^3) ///
        EPGperE2 (1-(‘a’/10)) (1-((‘a’+1)/10)) EPGperE2p ((1-(‘a’/10))*pown) ((1-((‘a’
> +1)/10))*pown)  ///
        EPGperE2psq ((1-(‘a’/10))*pown^2) ((1-((‘a’+1)/10))*pown^2) EPGperE2pcu ((1-(‘
> a’/10))*pown^3) ((1-((‘a’+1)/10))*pown^3))
        sum pi, meanonly
        replace Y0 = r(mean) if Eaxis==‘a’
        _pctile pi, p(2.5,97.5)
        replace plo = r(r1) if Eaxis==‘a’
        replace phi = r(r2) if Eaxis==‘a’
        drop pi
        local a = ‘a’ + 1
}


//----THIS IS THE CODE THAT PRODUCES THE GRAPH OF THE RELATIONSHIP
//----BETWEEN THE E5OWN AND dPR(EFFORT=5)

sort Eaxis
twoway (line plo Eaxis, clc(gs0) clw(medthick) clp(dash))  (line phi Eaxis, clc(gs0) c
> lw(medthick) clp(dash)) ///
(line Y0 Eaxis, clc(gs0) clw(medthick)),  ///
yscale(range(.2)) xlabel(1 ".2" 2 ".3" 3 ".4" 4 ".5" 5 ".6" 6 ".7" 7 ".8" 8 ".9" 9 "1"
> ) ylabel(-.2(.1).2) ///
legend(off) xtitle("E5OWN") ytitle("dPr(EFFORT=5)") graphregion(fcolor(gs16)) yline(0,
>  lpattern(solid) lwidth(thick) lcolor(blue))
graph2tex, epsfile(Eownfd1)



log close
exit
```